Real Analysis Problem Set 13, Part 1

05/28/2018

$\boxed{\text{Due 06/04, before class}}$

1. Let μ be a signed measure on $(X, \mathcal{F})$. Prove:
   (1). If $A_n \in \mathcal{F}$, and $A_1 \subset A_2 \subset \cdots$, then $\mu(\bigcup_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n)$
   (2) If ---------- $A_1 \supset A_2 \supset \cdots$, and $|\mu(A_1)| < +\infty$, then $\mu(\bigcap_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n)$.

2. (1) Let μ, ν be signed measures. Prove: $\mu \perp \nu \Leftrightarrow |\mu| \perp \nu$.
   (2) ---------- : $\mu \ll \nu \Leftrightarrow |\mu| \ll \nu$.
   (3) Let $\nu_1, \nu_2, \cdots$ be a sequence of (positive) measures. Prove: $\nu_j \perp \mu, \forall j \Rightarrow \sum_{j=1}^{\infty} \nu_j \perp \mu$.
   (4) ---------- $\nu_j \ll \mu, \forall j \Rightarrow \sum_{j=1}^{\infty} \nu_j \ll \mu$.

3. Let ν, μ, ρ be σ-finite positive measures on $(X, \mathcal{F})$.
   Suppose $\nu \ll \mu \ll \rho$.
   (1) Let $f \in L^1(X, \nu)$. Prove: $f \frac{d\nu}{d\mu} \in L^1(X, \mu)$, and $\int_X f \, d\nu = \int_X f \frac{d\nu}{d\mu} \, d\mu$.
   (2) Prove: $\nu \ll \rho$, and $\frac{d\nu}{d\rho} = \frac{d\nu}{d\mu} \cdot \frac{d\mu}{d\rho}$ ρ.a.e. holds.

4. Prove the proposition at the bottom of page 4.

$\boxed{\text{Not required to turn in}}$

- How to define complex measure?
- Which theorem(s)/definition(s) in today's lecture extends to complex measures?