Real Analysis Problem Set 3, Part 1

03/12/2018

$\boxed{\text{Due 03/19, before class}}$

1. Read §1.5 "(三) Borel 集" (Page 40-44). Prove: each Borel set is Lebesgue measurable.

2. Read the proof of Thm 2.20 ("Steinhaus's Theorem"), (Page 85-86).

3. Let $L: \mathbb{R}^d \to \mathbb{R}^d$ be a linear transform.
   Prove: If $A \subset \mathbb{R}^d$ is Lebesgue measurable, then $L(A)$ is also Lebesgue measurable.
   [Hint: F_σ]

4. Let A be the subset of $[0,1]$ which consists of all numbers which do not have the digit 6 appearing in their decimal expansion. What is $m(A)$?

For those who want to challenge yourself. (NOT required to turn in)

Let $A_1, A_2 \in \mathbb{R}^d$ be measurable, and $m(A_1)>0$, $m(A_2)>0$.
Prove: $A_1 + A_2 = \{x+y \mid x \in A_1, y \in A_2\}$ contains a box.