

量子混沌与分形不确定性原理

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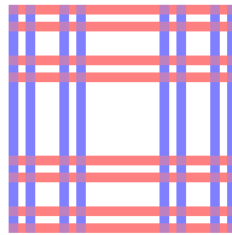
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课程概述

在这个短课程中, 我们简要讨论量子混沌领域中的一些最新进展。特别地, 我们引入一个新的工具称为分形不确定性原理 (Fractal Uncertainty Principle, 简记为 FUP)。课程计划如下:

- (1) 引论: 量子混沌中的一些问题;
- (2) 简单模型: 开放量子映射与离散 FUP;
- (3) 预备知识: 双曲曲面的几何与动力系统性质;
- (4) 双曲曲面上的半经典微局部分析;
- (5) 紧致双曲曲面上的控制问题;
- (6) 分形不确定性原理和未解问题。

Dyatlov-Zahl 在 2016 年的文章最早提出了 FUP 这一工具来研究凸余紧双曲曲面的谱隙; Bourgain-Dyatlov 在一维情形给出了完整的证明, 并证明了谱隙的存在性; Dyatlov-Jin 以及 Dyatlov-Jin-Nonnenmacher 将其应用到紧致双曲曲面的半经典测度的研究中。我们希望能够在这个短课程中讲述 Bourgain-Dyatlov 以及 Dyatlov-Jin 对于双曲曲面上 Laplace 特征函数的半经典测度的全支集性质的证明的主要思想。下面是关于 FUP 的一些参考文献。



参考文献

1. 概述性文章

- Semyon Dyatlov, *Control of eigenfunctions on hyperbolic surfaces: an application of fractal uncertainty principle*, Journées équations aux dérivées partielles, 2017.
- Semyon Dyatlov, *An introduction to fractal uncertainty principle*, J. Math. Phys. **60** (2019), 081505.
- Steve Zelditch, *Mathematics of Quantum Chaos in 2019*, Not. Am. Math. Soc. **66**(2019).
- Long Jin, *Quantum chaos and fractal uncertainty principle*, to appear in Proceedings of ICCM 2018.

2. 量子混沌的一些新进展

- Semyon Dyatlov and Joshua Zahl, *Spectral gaps, additive energy, and a fractal uncertainty principle*, Geom. Funct. Anal. **26** (2016), 1011–1094.
- Semyon Dyatlov and Long Jin, *Resonances for open quantum maps and a fractal uncertainty principle*, Comm. Math. Phys. **354** (2017), 269–316.
- Semyon Dyatlov and Long Jin, *Semiclassical measures on hyperbolic surfaces have full support*, Acta Math. **220** (2018), 297–339.
- Semyon Dyatlov, *Improved fractal Weyl bounds for hyperbolic manifolds*, with an appendix with David Borthwick and Tobias Weich, J. Eur. Math. Soc. **21** (2019), 1595–1639.
- Semyon Dyatlov and Maciej Zworski, *Fractal uncertainty for transfer operators*, Int. Math. Res. Not. (2020), 781–812.
- Semyon Dyatlov, Long Jin and Stéphane Nonnenmacher, *Control of eigenfunctions on surfaces of variable curvature*, preprint, arXiv:1906.08923.

3. 分形不确定性原理

- Jean Bourgain and Semyon Dyatlov, *Fourier dimension and spectral gaps for hyperbolic surfaces*, Geom. Funct. Anal. **27** (2017), 744–771.

- Jean Bourgain and Semyon Dyatlov, *Spectral gaps without the pressure condition*, Ann. of Math. (2) **187** (2018), 825–867.
- Semyon Dyatlov and Long Jin, *Dolgopyat’s method and the fractal uncertainty principle*, Anal. PDE, **11** (2018), 1457–1485.
- Long Jin and Ruixiang Zhang, *Fractal uncertainty principle with explicit exponent*, Math. Ann. **376** (2020), 1031–1057.
- Rui Han and Wilhelm Schlag, *A higher dimensional Bourgain-Dyatlov fractal uncertainty principle*, Anal. PDE, **13** (2020), 813–863.
- Laura Cladek and Terence Tao, *Additive energy of regular measures in one and higher dimensions, and the fractal uncertainty principle*, preprint, arXiv:2012.02747

4. 在偏微分方程中的应用

- Long Jin, *Control for Schrödinger equation on hyperbolic surfaces*, Math. Res. Lett. **25** (2018), 1865–1877.
- Jian Wang, *Strichartz estimates for convex co-compact hyperbolic surfaces*, Proc. Amer. Math. Soc. **147** (2019), 873–883.
- Long Jin, *Damped wave equations on compact hyperbolic surfaces*, Comm. Math. Phys. **373** No. 3 (2020), 771–794.
- Jefferey Galkowski and Steve Zelditch, *Lower bounds for Cauchy data on curves in a negatively curved surface*, preprint, arXiv:2002.09456.