PROBLEM SET 14, PART 1: TOPOLOGY (H)
DUE: JUNE 8, 2022

(1) [Maps on intervals]
  (a) Prove Lemma 4.3.5, Lemma 4.3.6 and Lemma 4.3.7.
  (b) Construct two coordinate charts on the "line with two doubled point" (see PSet8-2-3) that violates lemma 4.3.4.

(2) [Classification of 1-manifold with boundary]
  (a) Write down an analogue of Proposition 4.3.8 (and of Lemma 4.3.4 if you want) that can help you to prove the classification theorem of 1-manifold with boundary. [You don't need to prove your proposition.]
  (b) [NOT Required] Prove Theorem 4.3.3 (Classification of 1-manifold with boundary) using the proposition you wrote above.

(3) [Knot groups]
  (a) For any knot K, show that the abelianization of the knot group $\pi_1(\mathbb{R}^3 \setminus K)$ is $\mathbb{Z}$.
  (b) Write down the knot groups of the knots 4_1 and 7_1 (See figure 4.1).
  (c) [Not required] Show that the unknot, the knot 3_1 and the knot 4_1 are pairwise non-equivalent knots.

(4) [Knot in $\mathbb{R}^4$?] [Not required]
Let K be a polygonal knot in $\mathbb{R}^4$, that is, the image of an embedding of S^1 into $\mathbb{R}^4$ that consists of finitely many line segments.
  (a) Prove: There exists a direction $v \in S^3$ in $\mathbb{R}^4$ such that for any $x, y \in K$, $x - y$ is not parallel to v.
  (b) Use the projection $pr_v : \mathbb{R}^4 \to v^\perp$ to construct an ambient isotopy in $\mathbb{R}^4$ that converts the knot K to a polygonal knot in $v^\perp \simeq \mathbb{R}^3$.
  (c) Prove: Any knot in $\mathbb{R}^4$ is a trivial knot.