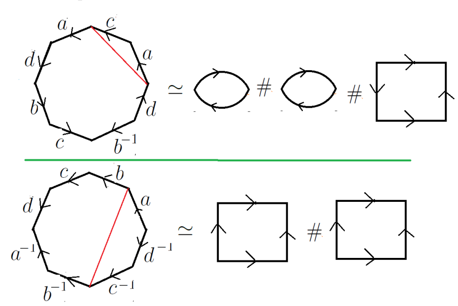
PROBLEM SET 14, PART 2: TOPOLOGY (H)
DUE: JUNE 8, 2022

(1) [Cut the Möbius band]
Prove your conclusion via polygonal presentation. (You may use scissor to find out the answers.)
  (a) Cut the Möbius band along the center circle, what do you get?
  (b) Cut the Möbius band along a circle that is close to the boundary circle, what do you get?
  (c) What if you cut the Möbius band along both circles mentioned above? Does the order of cutting matter?
  (d) What if you cut the Möbius band along k circles that are parallel to the center circle?

(2) [Cut and Paste Polygons]
Prove the following identities by doing "cutting and pasting" on the polygons.
[Hint: The first cutting is given. The first pasting is to eliminating a. You will need a second cutting and pasting.]



(3) [Triangulated surface]
Let S be a compact surface which is connected and without boundary.
  (a) Prove: If a finite simplicial complex K is a triangulation of S, then
    (i) Any 1-simplex in K is the intersection of exactly two 2-simplexes in K.
    [What if three triangles meeting at one edge? Use a theorem that we learned in this chapter.]
    (ii) For any 0-simplex v (i.e. vertex) in K, we can arrange the 2-simplexes containing v "cyclicly" as $\sigma_1, \sigma_2, \cdots, \sigma_k, \sigma_1$, so that $\sigma_i \cap \sigma_{i+1}$ is a 1-simplex (where we denote $\sigma_{k+1} = \sigma_1$).
    [What if these 2-simplexes can be arranged into more than two such "cycles"?]
  (b) (Not required) Conversely, suppose K is a simplicial complex consisting of finitely many 2-simplexes and their faces, so that the conditions (i) and (ii) are satisfied.

Show that $|K|$ is a surface.
[You need to show that any point has an Euclidean neighborhood.]

(4) (Not required) [Polygon presentation is a surface]
   (a) Complete the proof of Theorem 4.4.11 (the existence of polygonal presentation).
   (b) Prove: Any polygon presentation is a surface.
   [Again you need to show that any point has an Euclidean neighborhood. What if many vertices get glued into one point?]