Waveform of gravitational waves in the general parity-violating gravities

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As an extension of our previous work [J. Qiao et al., arXiv:1909.03815.], in this article, we calculate the effects of parity violation on gravitational-wave (GW) waveforms during their propagation in the most general parity-violating gravities, including Chern-Simons modified gravity, ghost-free scalar-tensor gravity, the symmetric teleparallel equivalence of General Relativity theory, Hořava-Lifshitz gravity, and so on. For this purpose, we consider the GWs generated by the coalescence of compact binaries and concentrate on the imprints of the parity violation in the propagation of GWs. With a unified description of GW in the theories of parity-violating gravity, we study the effects of velocity and amplitude birefringence on the GW waveforms. Decomposing the GWs into the circular polarization modes, the two birefringence effects exactly correspond to the modifications in phase and amplitude of GW waveforms, respectively. We find that, for each circular polarization mode, the amplitude, phase, and velocity of GW can be modified by both the parity-violating terms and parity-conserving terms in gravity. Therefore, in order to test the parity symmetry in gravity, we should compare the difference between two circular polarization modes, rather than measure an individual mode. Combining two circular modes, we obtain the GW waveforms in the Fourier domain and obtain the deviations from those in General Relativity. The GW waveforms derived in this paper are also applicable to the theories of parity-conserving gravity, which have the modified dispersion relations (e.g., massive gravity, double special relativity theory, extra-dimensional theories, etc.) or/and have the modified friction terms (e.g., nonlocal gravity, gravitational theory with time-dependent Planck mass, etc.).

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I. INTRODUCTION

Gravitational waves (GWs) are always produced in the circumstances with extreme conditions (e.g., the strongest gravitational field, the densest celestial bodies, the earliest stage of the Universe, the highest energy scale physics, etc.) and have the weak interactions with other matters during the propagation [1–3]. Therefore, the GWs encode the cleanest information for these extreme conditions and provide the excellent opportunity to study the physics in these extreme circumstances. As an example of the applications, GWs can be used to test the theory of gravity [4,5], which has become an important topic in the era of GW astronomy [6–13]. Although Einstein's General Relativity (GR) has been considered to be the most successful theory

of gravity since it was proposed, it faces the difficulties in both theoretically (e.g., singularity, quantization, etc.) and observationally (e.g., dark matter, dark energy, etc.). Therefore, testing GR in various circumstances has been an important topic since its birth [14–19].

As is well known, symmetry permeates nature and is fundamental to all laws of physics. Thus, one important method for the gravity examination is to test the symmetries in gravity. In our series of works, we focus on the testing of parity symmetry in gravity. Parity symmetry implies that a directional flipping to the left and right does not change the laws of physics. It is well known that nature is parity violating. Since the first discovery of parity violation in weak interactions [20], the experimental tests have become more necessary in the other interactions, including gravity. Although the parity symmetry is maintained in GR, the gravitational terms with parity violation are always motivated by anomaly cancelation in particle physics and string

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theory [21,22], and various parity-violating (PV) theories of gravity have been proposed in the literature, including the Chern-Simons modified gravity [23–27], ghost-free scalar-tensor gravity [28–30], the symmetric teleparallel equivalence of GR theory [31], Hořava-Lifshitz gravity [32–38], and so on. As an extension of our previous work [39], in this article, we will calculate the waveform of GWs, produced by the coalescence of compact binaries, in the general framework of PV gravities.

The effect of parity violation on GW is the birefringence phenomenon during its propagation; i.e., the symmetry between the left-hand and right-hand circular polarization modes is broken [21,29,39,40]. For instance, the primordial GWs generated in the PV gravities are circularly polarized, which leaves the significant imprints in the temperature and polarization anisotropies of cosmic microwave background radiation [33,36,37,41-43]. However, it seems there is no hope of detecting these imprints in the near future [44], due to the small amplitude of primordial GWs. As one in our series of works, in this article, we will study the imprints of parity violation in gravity on the waveforms of GW, produced by the coalescence of compact binaries. Decomposing the GWs into the circular polarization modes, we find their propagation equations are decoupled, and the deviations from those in GR, caused by the PV terms, are quantified by two parametrized terms ν_A and μ_A , which exactly correspond to the effects of velocity and amplitude birefringence, respectively. As an application of these results, we focus on four specific theories of PV gravity (Chern-Simons gravity, ghost-free scalar-tensor gravity, the symmetric teleparallel equivalence of GR theory, and Hořava-Lifshitz gravity) and obtain the explicit expressions of these two terms for each theory. Converting the circular polarization modes to the general plus and cross modes, for the general theories of PV gravity, we obtain the GW waveforms in the Fourier domain. Because of the mixture of plus and cross modes during their propagation, we find both velocity and amplitude birefringence effects can contribute the modifications in GW phase and amplitude, and the deviations of GW waveforms from those in GR are also derived.

This paper is organized as follows. In Sec. II, in the effective field theory of gravity, we study the field equation of GW in the general theories of PV gravity. In Sec. III, we focus on the specific models of PV gravity and obtain the expression of corresponding quantities, which violate the parity symmetry. In Sec. IV, we discuss the amplitude and velocity birefringence effects of GWs. In Sec. V, we calculate the waveform of GWs produced by the coalescence of compact binary systems and particularly focus on the deviations from those in GR. The summary of this work is given in Sec. VI.

Throughout this paper, the metric convention is chosen as (-, +, +, +), and greek indices $(\mu, \nu, ...)$ run over 0, 1, 2, 3 and latin indices (i, j, k) run over 1, 2, 3. We set the units to $c = \hbar = 1$.

II. GRAVITATIONAL WAVES IN THE GENERAL PARITY-VIOLATING GRAVITIES

We consider the general theories of gravity with parity violation, the action of which can be written as

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [\mathcal{L}_{\rm GR} + \mathcal{L}_{\rm PV} + \mathcal{L}_{\rm other}], \qquad (2.1)$$

where \mathcal{L}_{GR} is the Einstein-Hilbert term *R* or its equivalent quantity in the non-Riemannian formulation of gravity [31]. \mathcal{L}_{PV} represents the PV Lagrangian density, which always consists of a number of terms in the specific models. \mathcal{L}_{other} is the Lagrangian density for the other matters, the scalar field, and the modification terms of gravity, which are not relevant to parity violation.

In the flat Friedmann-Robertson-Walker (FRW) universe, a GW is the tensor perturbation of the metric, i.e.,

$$ds^{2} = a^{2}(\tau)[-d\tau^{2} + (\delta_{ij} + h_{ij})dx^{i}dx^{j}], \qquad (2.2)$$

where $a(\tau)$ is the conformal scale factor, and we set its present value as $a_0 = 1$ in this paper. τ is the conformal time, which relates to the cosmic time by $dt = ad\tau$. x^i is the comoving coordinates. The quantity h_{ij} stands for the GW perturbation, which we take to be transverse and traceless gauge, $\delta^{ij}h_{ij} = 0$ and $\partial_i h^{ij} = 0$.

The equation of motion of GW is determined by the tensor quadratic action, which reads [45–47]

$$S^{(2)} = \frac{1}{16\pi G} \int dt d^3x a^3 [\mathcal{L}_{\rm GR}^{(2)} + \mathcal{L}_{\rm PV}^{(2)} + \mathcal{L}_{\rm other}^{(2)}], \quad (2.3)$$

where

$$\mathcal{L}_{\rm GR}^{(2)} = \frac{1}{4} [\dot{h}_{ij}^2 - a^{-2} (\partial_k h_{ij})^2]$$
(2.4)

is the standard Lagrangian obtained from the Einstein-Hilbert term *R*. In this article, a *dot* denotes the derivative with respect to the cosmic time *t*. In the viewpoint of effective fields [48], the operators \dot{h}_{ij}^2 and $(\partial_k h_{ij})^2$ are the only quadratic operators with two derivatives. Note that, in principle, this Lagrangian can be generalized by adding a time-dependent coefficient in each term, which might be caused by the Horndeski or Galileon scalar field, through the perturbation of extrinsic curvature of the spatial slices [49]. Similar terms also appear in the nonlocal gravity [50], Hořava-Lifsthitz gravity [36], and so on. Therefore, in this paper, we consider the extended version of the above Lagrangian, which takes the form

$$\mathcal{L}_{eGR}^{(2)} = \frac{1}{4} [b_1(t)\dot{h}_{ij}^2 - b_2(t)a^{-2}(\partial_k h_{ij})^2], \quad (2.5)$$

where b_1 and b_2 are the arbitrary functions of time. To avoid ghost instability, one has to require $b_1(t) > 0$. The propagation effects of GWs, due to these modifications, have been analyzed explicitly in the previous works [51,52].

The first possible corrections to the tensor mode mentioned above come from terms with three derivatives. There are only two possible terms [48],

$$\epsilon^{ijk}\dot{h}_{il}\partial_{j}\dot{h}_{kl}, \qquad \epsilon^{ijk}\partial^{2}h_{il}\partial_{j}h_{kl}, \qquad (2.6)$$

where ϵ^{ijk} is the antisymmetric symbol. So, the standard quadratic action is modified by the addition of [48]

$$\mathcal{L}_{\rm PV}^{(2)} = \frac{1}{4} \left[\frac{c_1}{aM_{\rm PV}} \epsilon^{ijk} \dot{h}_{il} \partial_j \dot{h}_{kl} + \frac{c_2}{a^3 M_{\rm PV}} \epsilon^{ijk} \partial^2 h_{il} \partial_j h_{kl} \right],$$
(2.7)

where a *dot* denotes the derivative with respect to the cosmic time t, c_1 and c_2 are dimensionless coefficients, and $M_{\rm PV}$ is the scale that suppresses these higher-dimension operators. Normally, $M_{\rm PV}$ can be constrained by the solar system experiments and various astrophysical observations. Since most of the experimental and astrophysical constraints are considered in the Chern-Simons (CS) modified gravity, here we consider constraints in this theory as examples to discuss the possible order of the magnitude of $M_{\rm PV}$. In the solar system experiment, the frame-dragging measurement with LAGEOS places the bound $M_{\rm PV}^{-1} \lesssim$ 2000 km [53]. For astrophysical test, the binary pulsar observations yield $M_{\rm PV}^{-1} \lesssim 0.4$ km [54]. The possible bounds on $M_{\rm PV}$ have also been explored with the detection of the GWs, and as pointed out in Refs. [27,40], the observation of the GWs from the compact binaries is expected to provide a stronger bound on $M_{\rm PV}$. In this paper, we assume the homogeneous and isotropic background of the Universe and take c_1 and c_2 to be functions of time.

The equation of motion for GWs can be derived by varying the quadratic action with respect to h_{ij} . We consider the GWs propagating in the vacuum, and the field equation for h_{ij} is given by

$$b_{1}h_{ij}'' + (2\mathcal{H}b_{1} + b_{1}')h_{ij}' - b_{2}\partial^{2}h_{ij} + \frac{\epsilon^{ilk}}{aM_{\rm PV}}\partial_{l}[c_{1}h_{jk}'' + (\mathcal{H}c_{1} + c_{1}')h_{jk}' - c_{2}\partial^{2}h_{jk}] = 0,$$
(2.8)

where a *prime* denotes the derivative with respect to the conformal time τ and $\mathcal{H} \equiv a'/a$. When $b_1 = 1 = b_2$, the above equation reduces to the same form as Eq. (3.9) in Ref. [39] in the ghost-free parity-violating gravities. This is easy to understand because the GR is only modified by the

parity-violating terms in the action of the ghost-free parityviolating gravities, and c_1 and c_2 represent their effects on the propagation equation of GWs [39]. However, in the general case, as one can see from (2.1), the action of the theory can also contain other matter terms or modifications of gravity, which are labeled by \mathcal{L}_{other} in (2.1) and are not relevant to parity violation. \mathcal{L}_{other} can also modify the propagation equation of GWs, and their effects are directly characterized by the parameters b_1 and b_2 in (2.8).

In the PV gravities, it is convenient to decompose the GWs into the circular polarization modes. To study the evolution of h_{ij} , we expand it over spatial Fourier harmonics,

$$h_{ij}(\tau, x^{i}) = \sum_{A=R,L} \int \frac{d^{3}k}{(2\pi)^{3}} h_{A}(\tau, k^{i}) e^{ik_{i}x^{i}} e^{A}_{ij}(k^{i}), \quad (2.9)$$

where e_{ij}^{A} denotes the circular polarization tensors and satisfies the relation $\epsilon^{ijk}n_je_{kl}^{A} = i\rho_A e_l^{iA}$, with $\rho_R = 1$ and $\rho_L = -1$. So, the equation of motion in Eq. (2.8) can be written as [29,39,40]

$$h_{\rm A}'' + \left(2 + \frac{b_1'/\mathcal{H} + \rho_A(k/aM_{\rm PV})(c_1 - c_1'/\mathcal{H})}{b_1 - \rho_A(k/aM_{\rm PV})c_1}\right)\mathcal{H}h_{\rm A}' + \left(1 + \frac{b_2 - b_1 + \rho_A(k/aM_{\rm PV})(c_1 - c_2)}{b_1 - \rho_A(k/aM_{\rm PV})c_1}\right)k^2h_{\rm A} = 0.$$
(2.10)

We expect the deviations from GR are small such that the terms with $\mathcal{O}(b'_1)$, $\mathcal{O}(c_1)$, $\mathcal{O}(c'_1)$, $\mathcal{O}(c_2)$, $\mathcal{O}(b_2 - b_1) \ll 1$. With this consideration, the above equation can be further simplified into the form

$$h_{\rm A}^{\prime\prime} + (2 + \bar{\nu} + \nu_{\rm A}) \mathcal{H} h_{\rm A}^{\prime} + (1 + \bar{\mu} + \mu_{\rm A}) k^2 h_{\rm A} = 0, \quad (2.11)$$

where A = R or L, standing for the right-hand or left-hand polarization mode, respectively, and

$$\begin{aligned} \mathcal{H}\bar{\nu} &= (\ln b_1)', \quad \bar{\mu} = b_2/b_1 - 1, \\ \mathcal{H}\nu_{\rm A} &= -[\rho_{\rm A}(k/aM_{\rm PV})(c_1/b_1)]', \\ \mu_{\rm A} &= \rho_{\rm A}(k/aM_{\rm PV})(c_1 - c_2)/b_1. \end{aligned} \tag{2.12}$$

The new effects arising from the generic PV gravities are fully characterized by four parameters: $\bar{\nu}$, $\bar{\mu}$, ν_A , and μ_A . While the parameters ν_A and μ_A label the effects of the parity violation, the parameters $\bar{\nu}$ and $\bar{\mu}$ arise from the other possible modifications which are not relevant to parity violation. In these four parameters, $\bar{\mu}$ and μ_A determine the speed of the gravitational waves, and $\bar{\nu}$ and ν_A provide an amplitude modulation to the gravitational waveform. Specific to parity violation, the parameter μ_A leads to different velocities of left-hand and right-hand circular polarizations of GWs; therefore, the arrival times of the two circular polarization modes could be different. For parameter ν_A , it is easy to see that it has the same value but opposite signs for the left-hand and right-hand GWs. As a result, the amplitude of left-hand circular polarization of gravitational waves will increase (or decrease) during the propagation, while the amplitude for the right-hand modes will decrease (or increase).

As an extension of the above theory, we can generalize the expressions of $\bar{\nu}$, $\bar{\mu}$, ν_A , and μ_A as the parametrized forms

$$\mathcal{H}\bar{\nu} = [\alpha_{\bar{\nu}}(\tau)(k/aM_{\rm PV})^{\beta_{\bar{\nu}}}]', \quad \bar{\mu} = \alpha_{\bar{\mu}}(\tau)(k/aM_{\rm PV})^{\beta_{\bar{\mu}}},$$
$$\mathcal{H}\nu_{\rm A} = [\rho_{\rm A}\alpha_{\nu}(\tau)(k/aM_{\rm PV})^{\beta_{\nu}}]', \quad \mu_{\rm A} = \rho_{\rm A}\alpha_{\mu}(\tau)(k/aM_{\rm PV})^{\beta_{\mu}},$$
$$(2.13)$$

where $\beta_{\bar{\nu}}$ and $\beta_{\bar{\mu}}$ are the arbitrary numbers and β_{ν} and β_{μ} are the arbitrary odd numbers. $\alpha_{\bar{\nu}}$, $\alpha_{\bar{\mu}}$, α_{ν} , and α_{μ} are the arbitrary functions of time. The formula in Eq. (2.11) with coefficients in Eq. (2.13) is the unifying description for the low-energy effective description of generic PV GWs. To our knowledge, all the known theories of PV gravity in the literature, even if including higher order derivative terms, can be casted into this form, which will be presented explicitly in Sec. III. Here, we should stress that, even for the theories of gravity with parity symmetry, the equation of motion of GWs is also described by this formula in (2.11). For instance, in the specific case with the only nonzero quantity $\bar{\mu}$ and $\beta_{\bar{\mu}} = -2$, this formula becomes the motion of equation of GW in the massive gravity [55].

III. SPECIFIC MODELS OF ALTERNATIVE GRAVITIES

A. Chern-Simons modified gravity

We first consider the CS modified gravity, which has been widely studied in the previous works [23–27]. CS modified gravity is an effective extension of GR that captures leading-order, gravitational PV term. The similar versions of this theory were suggested in the context of string theory [22] and three-dimensional topological massive gravity [56]. In this theory, the action in gravity is given by Eq. (2.1), and the PV term is

$$\mathcal{L}_{\rm PV} = \frac{1}{8} \vartheta(\phi) \varepsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\alpha\beta} R^{\alpha\beta}_{\mu\nu}, \qquad (3.1)$$

with $\varepsilon_{\rho\sigma\alpha\beta}$ being the Levi-Civita tensor defined in terms of the antisymmetric symbol $e^{\rho\sigma\alpha\beta}$ as $e^{\rho\sigma\alpha\beta} = e^{\rho\sigma\alpha\beta}/\sqrt{-g}$. The quantity ϑ is the so-called CS coupling field. If $\vartheta = \text{const.}$, CS modified gravity reduces to identically to GR, since the Pontryagin term in Eq. (3.1) can be expressed as the divergence of the CS topological current [21]. Therefore, in the dynamical CS gravity, ϑ is an arbitrary function of the scalar field ϕ , which is a function of spacetime. Similar to the previous works [24-27], in the FRW universe, we assume that the scalar field ϕ is a function of conformal time τ only to ensure that the background symmetry is preserved. The equation of motion of GW is determined by the tensor quadratic action, which in the CS modified gravity is given by Eq. (2.7) in which the coefficients are $c_1 = c_2 = \dot{\vartheta} M_{\rm PV}$. Therefore, the propagation equation of GW becomes that in (2.11) with

$$\mathcal{H}\bar{\nu} = 0, \quad \bar{\mu} = 0, \quad \mathcal{H}\nu_{\rm A} = -[\rho_A c_1 (k/aM_{\rm PV})]', \quad \mu_{\rm A} = 0.$$
(3.2)

The corresponding values of the parameters $(\alpha_{\bar{\nu}}, \beta_{\bar{\nu}}, \alpha_{\bar{\mu}}, \beta_{\bar{\mu}}, \alpha_{\nu}, \beta_{\nu}, \alpha_{\mu}, \beta_{\mu})$ defined in Eq. (2.13) are listed in Table I. Note that CS modified theory has higher-derivative field equations, which induces the dangerous Ostrogradsky ghosts [21]. For this reason, this theory can only be treated as a low-energy truncation of a fundamental theory.

B. Ghost-free scalar-tensor gravities

To avoid the Ostrogradsky ghost in the framework of scalar-tensor gravity with parity violation, in Ref. [28], the authors proposed an extension of CS modified gravity by considering the terms which involve the derivatives of the

TABLE I. The values of parameter set $(\alpha_{\bar{\nu}}, \beta_{\bar{\nu}}, \alpha_{\bar{u}}, \beta_{\bar{\mu}}, \alpha_{\nu}, \beta_{\nu}, \alpha_{\mu}, \beta_{\mu})$ defined in Eq. (2.13) in various modified gravities.

Theory of gravity	$lpha_{ar{ u}}$	$\beta_{ar{ u}}$	$lpha_{ar\mu}$	$eta_{ar\mu}$	$lpha_{ u}$	$\beta_{ u}$	$lpha_{\mu}$	eta_μ
Chern-Simons gravity	Zero		Zero		Nonzero	1	Zero	
Ghost-free scalar-tensor gravities	Zero		Zero		Nonzero	1	Nonzero	1
Symmetric teleparallel equivalence of GR theory	Zero		Zero		Nonzero	1	Nonzero	1
Hořava-Lifshitz gravity	Zero		Nonzero	0 or 2 or 4	Zero		Nonzero	1 or 3
Massive gravity	Zero		Nonzero	-2	Zero		Zero	
Double special relativity theory	Zero		Nonzero	-2 or 1	Zero		Zero	
Extra-dimensional theories	Zero		Nonzero	-2 or 2	Zero		Zero	
Noncommutative gravity	Zero		Nonzero	-2 or 0 or 2	Zero		Zero	
Nonlocal gravity	Nonzero	0	Zero		Zero		Zero	
Time-dependent Planck mass gravity	Nonzero	0	Zero		Zero		Zero	
f(T) gravity	Nonzero	0	Zero		Zero		Zero	

scalar field. If considering only the scalar field and its first derivatives in PV terms, the Lagrangian density \mathcal{L}_{PV} is given by

$$\mathcal{L}_{\rm PV} = \sum_{\rm A=1}^{4} a_{\rm A}(\phi, \phi^{\mu}\phi_{\mu})L_{\rm A}, \qquad (3.3)$$

where $\phi^{\mu} \equiv \nabla^{\mu} \phi$, and $a_{\rm A}$ are *a priori* arbitrary functions of ϕ and $\phi^{\mu} \phi_{\mu}$. The terms $L_{\rm A}$ are [28,30]

$$\begin{split} L_{1} &= \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R^{\rho}_{\mu\nu\lambda} \phi^{\sigma} \phi^{\lambda}, \quad L_{2} &= \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R^{\rho\sigma}_{\mu\lambda} \phi_{\nu} \phi^{\lambda}, \\ L_{3} &= \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R^{\sigma}_{\nu} \phi^{\rho} \phi_{\mu}, \quad L_{4} &= \varepsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\alpha\beta} R^{\alpha\beta}_{\mu\nu} \phi^{\lambda} \phi_{\lambda}. \end{split}$$

To avoid the Ostrogradsky modes in the unitary gauge (where the scalar field depends on time only), it is required that $4a_1 + 2a_2 + a_3 + 8a_4 = 0$. With this condition, the Lagrangian in Eq. (3.3) does not have any higher-order time derivative of the metric but has only higher-order space derivatives. In this theory, the tensor quadratic action is that in Eq. (2.7) with the coefficients c_1 and c_2 as [39]

$$c_{1}/M_{\rm PV} = -4\dot{a_{1}}\dot{\phi}^{2} - 8a_{1}\dot{\phi}\ddot{\phi} + 8a_{1}H\dot{\phi}^{2} - 2\dot{a_{2}}\dot{\phi}^{2}$$
$$- 4a_{2}\dot{\phi}\ddot{\phi} + \dot{a_{3}}\dot{\phi}^{2} + 2a_{3}\dot{\phi}\ddot{\phi} - 4a_{3}H\dot{\phi}^{2}$$
$$- 4\dot{a_{4}}\dot{\phi}^{2} - 8a_{4}\dot{\phi}\ddot{\phi} \qquad (3.4)$$

$$c_2/M_{\rm PV} = -2\dot{a_2}\dot{\phi}^2 - 4a_2\dot{\phi}\ddot{\phi} - \dot{a_3}\dot{\phi}^2 - 2a_3\dot{\phi}\ddot{\phi} - 4\dot{a_4}\dot{\phi}^2 - 8a_4\dot{\phi}\ddot{\phi}, \qquad (3.5)$$

where $H = \dot{a}/a$ is the Hubble parameter. Therefore, the propagation equation of GW becomes that in (2.11) with [39]

$$\mathcal{H}\bar{\nu} = 0, \quad \bar{\mu} = 0, \quad \mathcal{H}\nu_{\rm A} = -[\rho_A c_1 (k/aM_{\rm PV})]', \\ \mu_{\rm A} = \rho_{\rm A} (k/aM_{\rm PV}) (c_1 - c_2).$$
 (3.6)

One can also consider the terms which contain a second derivative of the scalar field. Focusing on only the terms that are linear in the Riemann tensor and linear/ quadratically in the second derivative of ϕ , the most general Lagrangian \mathcal{L}_{PV} with parity violation is given by [28,30]

$$\mathcal{L}_{\rm PV} = \sum_{\rm A=1}^{7} b_{\rm A}(\phi, \phi^{\lambda} \phi_{\lambda}) M_{\rm A}, \qquad (3.7)$$

$$\begin{split} M_{1} &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi^{\rho}\phi_{\mu}\phi_{\nu}^{\sigma}, \quad M_{2} &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi_{\mu}^{\rho}\phi_{\nu}^{\sigma}, \\ M_{3} &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi^{\sigma}\phi_{\mu}^{\rho}\phi_{\nu}^{\lambda}\phi_{\lambda}, \quad M_{4} &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi_{\nu}\phi_{\mu}^{\rho}\phi_{\lambda}^{\sigma}\phi^{\lambda}, \\ M_{5} &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\rho\sigma\lambda}\phi^{\rho}\phi_{\beta}\phi_{\mu}^{\sigma}\phi_{\nu}^{\lambda}, \quad M_{6} &= \varepsilon^{\mu\nu\alpha\beta}R_{\beta\gamma}\phi_{\alpha}\phi_{\mu}^{\gamma}\phi_{\nu}^{\lambda}\phi^{\lambda}, \\ M_{7} &= (\nabla^{2}\phi)L_{1}, \end{split}$$

with $\phi_{\nu}^{\sigma} \equiv \nabla^{\sigma} \nabla_{\nu} \phi$. Similarly, to avoid the Ostrogradsky modes in the unitary gauge, the following conditions should be imposed: $b_7 = 0$, $b_6 = 2(b_4 + b_5)$ and $b_2 = -A_*^2(b_3 - b_4)/2$, where $A_* \equiv \dot{\phi}(t)/N$ and N is the lapse function. After tedious calculations, we find that in this theory the tensor quadratic action is also given by Eq. (2.7), where $c_2 = 0$ and c_1 is [39]

$$c_{1}/M_{\rm PV} = -2b_{1}\dot{\phi}^{3} + 4b_{2}(2H\dot{\phi}^{2} - \dot{\phi}\,\ddot{\phi}) + 2b_{3}(\dot{\phi}^{3}\ddot{\phi} - H\dot{\phi}^{4}) + 2b_{4}(\dot{\phi}^{3}\ddot{\phi} - H\dot{\phi}^{4}) - 2b_{5}H\dot{\phi}^{4} + 2b_{7}\dot{\phi}^{3}\ddot{\phi}.$$
(3.8)

The propagation equation of GW is derived by varying the quadratic action with respect to h_{ij} , which is that in Eq. (2.11) with [39]

$$\mathcal{H}\bar{\nu} = 0, \quad \bar{\mu} = 0, \quad \mathcal{H}\nu_{\rm A} = -[\rho_A c_1 (k/aM_{\rm PV})]',$$

$$\mu_{\rm A} = \rho_{\rm A} (k/aM_{\rm PV}) c_1. \tag{3.9}$$

C. Symmetric teleparallel equivalence of GR theory

The parity-violating extension of the symmetric teleparallel equivalent of GR theory is a non-Riemannian formulation of gravity, which allows for a wider variety of consistent extensions than the metric formulation of gravity theory and has been studied explicitly in Ref. [31]. As is known, the Einstein-Hilbert action hides second derivatives, and its generalizations are severely restricted. However, combining the Riemannian formulation with the metric teleparallel [57] and the symmetric teleparallel [58] equivalents of GR provide a more flexible framework for generalizations, since their action principles can feature only first derivatives. For this reason, the analysis in the subsection is beyond the framework in Sec. II.

The nonmetricity tensor and its contractions are defined by $Q_{\alpha\mu\nu} \equiv \nabla_{\alpha}g_{\mu\nu}$, where the covariant derivative ∇ is with respect to a generic connection Γ , which is considered to be devoid of both curvature and torsion. This allows us to choose the coincident gauge [59] wherein we can simply write partial derivatives in place of the covariant operators above. The action of the nonmetricity equivalent of GR S_{OGR} is given in Eq. (2.15) in Ref. [31], which is related to the Einstein-Hilbert action by a total derivative. In this article, we investigate the PV gravity that is quadratic in nonmetricity in the symmetric teleparallel geometry and consider only the models that feature (at most) second time derivatives. Let us concentrate on the first class of Lagrangians, which are quadratic in nonmetricity, coupled quadratically to a scalar field and second-order in derivatives. The unique and nonvanishing Lagrangian for GW is given by [31]

$$\mathcal{L}_{\rm PV} = \sqrt{-g}\tilde{\alpha}\epsilon^{\mu\nu\theta\delta}\phi_{\theta}\phi^{\gamma}Q_{\mu\nu\alpha}Q_{\gamma\delta}{}^{\alpha},\qquad(3.10)$$

where $\tilde{\alpha}$ is an arbitrary function of field ϕ and its kinetic term. For the propagation of GW, we only need to consider the tensor quadratic action, which is

$$\mathcal{L}^{(2)} = \mathcal{L}^{(2)}_{\text{QGR}} + \mathcal{L}^{(2)}_{\text{PV}}, \qquad (3.11)$$

where

$$\mathcal{L}_{\text{QGR}}^{(2)} = \frac{1}{4} [\dot{h}_{ij}^2 - a^{-2} (\partial_k h_{ij})^2], \qquad \mathcal{L}_{\text{PV}}^{(2)} = \frac{H}{a} \tilde{\alpha} \epsilon^{ijk} h_k{}^l \partial_i h_{jl}.$$
(3.12)

From this action, the propagation equation of the GW is derived, that in Eq. (2.11) with [31]

$$\mathcal{H}\bar{\nu}=0, \ \bar{\mu}=0, \ \mathcal{H}\nu_{\mathrm{A}}=0, \ \mu_{\mathrm{A}}=4\tilde{\alpha}\rho_{\mathrm{A}}\mathcal{H}/k.$$
 (3.13)

In a similar fashion, we can discuss the PV Lagrangians in the symmetric teleparallel geometry with third-order derivatives. Let us focus on the action, which is quadratic in nonmetricity, and includes (at most) the second-order time derivatives. The most general Lagrangian density is given by [31]

$$\mathcal{L}^{(2)} = \mathcal{L}_{\text{QGR}}^{(2)} + \frac{\beta_1(t)}{a^3 M_{\text{PV}}} \mathcal{L}_{\text{PV1}}^{(2)} + \frac{\beta_2(t)}{a M_{\text{PV}}} \mathcal{L}_{\text{PV2}}^{(2)} + \frac{\beta_3(t)}{a M_{\text{PV}}} \mathcal{L}_{\text{PV3}}^{(2)},$$
(3.14)

where $\beta_i(t)$ is a dimensionless function of time, $M_{\rm PV}$ is an energy scale, and

$$\mathcal{L}_{PV1}^{(2)} \equiv \epsilon^{ijk} \partial^2 h_j{}^l \partial_i h_{kl}, \quad \mathcal{L}_{PV2}^{(2)} \equiv 2H \epsilon^{ijk} \dot{h}_j{}^l \partial_i h_{kl},$$
$$\mathcal{L}_{PV3}^{(2)} \equiv \epsilon^{ijk} \dot{h}_j{}^l \partial_i \dot{h}_{kl}. \tag{3.15}$$

Note that, in the symmetric teleparallel equivalence of GR theory, the extra term $\mathcal{L}_{PV2}^{(2)}$ can exist. Similarly, the propagation equation of GW is derived from the action, that in Eq. (2.11) with [31]

$$\mathcal{H}\bar{\nu} = 0, \quad \bar{\mu} = 0, \quad \mathcal{H}\nu_{\rm A} = [4\rho_A(k/aM_{\rm PV})\beta_3]', \mu_{\rm A} = 4\rho_A(k/aM_{\rm PV})(\beta_1 - \beta_3 + (a\mathcal{H}\beta_2)'/ak^2).$$
(3.16)

Note that, to avoid fine-tuning, β_1 , β_2 , and β_3 are expected to be of the same order. Thus, the last term in μ_A becomes ignorable.

D. Hořava-Lifshitz gravity

The Hořava-Lifshitz (HL) gravity is based on the perspective that the Lorentz symmetry appears only as an emergent symmetry at low energies but can be fundamentally absent at high energies [32,38]. This opens a completely new window to build a theory of quantum gravity without the Lorentz symmetry in the UV, using the

high-dimensional spatial derivative operators, while still keeping the time derivative operators to the second order, whereby the unitarity of the theory is reserved. Besides the original version of the theory by Hořava [32], there are several modifications, which are absent several inconsistent problems that appear in the original version. In this paper, we are going to focus on an extension of the HL gravity by abandoning the projectability condition but imposing an extra local U(1) symmetry that was proposed [35,60], in which the gravitational sector has the same degree of freedom as that in GR, i.e., only spin-2 massless gravitons exist.

By abandoning the Lorentz symmetry, the HL theory also provides a natural way to incorporate the parityviolation terms in the theory. For our current purpose, we consider the third- and/or fifth-order spatial derivative operators to the potential term \mathcal{L} of the total action in Refs. [35–37,60],

$$\mathcal{L}_{PV} = \frac{1}{M_{PV}} (\alpha_0 K_{ij} R_{ij} + \alpha_2 \varepsilon^{ijk} R_{il} \Delta_j R_k^l) + \frac{\alpha_1 \omega_3(\Gamma)}{M_{PV}} + \dots$$
(3.17)

Here, $M_{\rm PV}$ is the energy scale above which the high-order derivative operators become important. The coupling constants α_0 , α_1 , and α_2 are dimensionless and arbitrary, and $\omega_3(\Gamma)$ the three-dimensional gravitational CS term. "…" denotes the rest of the fifth-order operators given in Eq. (2.6) of Ref. [60]. Since they have no contributions to tensor perturbations, in this paper, we shall not write them out explicitly.

The general formulas of the linearized tensor perturbations were given in Ref. [37], so in the rest of this section, we give a very brief summary of the main results obtained there. Consider a flat FRW universe and assuming that matter fields have no contributions to tensor perturbations, the quadratic part of the total action can be cast in the form

$$S_{g}^{(2)} = \zeta^{2} \int d\eta d^{3}x \left\{ \frac{a^{2}}{4} (h_{ij}^{\prime})^{2} - \frac{1}{4} a^{2} (\partial_{k} h_{ij})^{2} - \frac{\hat{\gamma}_{3}}{4M_{PV}^{2}} (\partial^{2} h_{ij})^{2} - \frac{\hat{\gamma}_{5}}{4M_{PV}^{4} a^{2}} (\partial^{2} \partial_{k} h_{ij})^{2} - \frac{\alpha_{1} a \epsilon^{ijk}}{2M_{PV}} (\partial_{l} h_{i}^{m} \partial_{m} \partial_{j} h_{k}^{l} - \partial_{l} h_{im} \partial^{l} \partial_{j} h_{k}^{m}) - \frac{\alpha_{2} \epsilon^{ijk}}{4M_{PV}^{3} a} \partial^{2} h_{il} (\partial^{2} h_{k}^{l})_{,j} - \frac{3\alpha_{0} \mathcal{H}}{8M_{PV} a} (\partial_{k} h_{ij})^{2} \right\},$$
(3.18)

where $\hat{\gamma}_3 \equiv (2M_{\rm PV}/M_{\rm Pl})^2 \gamma_3$ and $\hat{\gamma}_5 \equiv (2M_{\rm PV}/M_{\rm Pl})^4 \gamma_5$, and γ_3 and γ_5 are the dimensionless coupling constants of the theory. To avoid fine-tuning, α_n and $\hat{\gamma}_n$ are expected to be of the same order. Then, the field equations for h_{ij} read

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \alpha^2 \partial^2 h_{ij} + \frac{\hat{\gamma}_3}{a^2 M_{\rm PV}^2} \partial^4 h_{ij} - \frac{\hat{\gamma}_5}{a^4 M_{\rm PV}^4} \partial^6 h_{ij} + e_i^{lk} \left(\frac{2\alpha_1}{aM_{\rm PV}} + \frac{\alpha_2}{a^3 M_{\rm PV}^3} \partial^2\right) (\partial^2 h_{jk})_{,l} = 0, \qquad (3.19)$$

where $\alpha^2 \equiv 1 + 3\alpha_0 \mathcal{H}/(2M_{PV}a)$. In the late Universe, $a \sim 1$, and $\mathcal{H} \ll M_{PV}$, so we find $\alpha^2 \to 1$. To study the evolution of h_{ij} , we expand it over spatial Fourier harmonics. For each circular polarization mode, the equation of motion of GW is given by

$$h''_{\rm A} + 2\mathcal{H}h'_{\rm A} + \omega_{\rm A}^2 h_{\rm A} = 0, \qquad (3.20)$$

with

$$\omega_{\rm A}^2(k,\eta) \equiv \alpha^2 k^2 [1 + \delta_1 \rho_{\rm A} (\alpha k/M_{\rm PV}a) + \delta_2 (\alpha k/M_{\rm PV}a)^2 - \delta_3 \rho_{\rm A} (\alpha k/M_{\rm PV}a)^3 + \delta_4 (\alpha k/M_{\rm PV}a)^4],$$
(3.21)

where $\delta_1 \equiv 2\alpha_1/\alpha^3$, $\delta_2 \equiv \hat{\gamma}_3/\alpha^4$, $\delta_3 \equiv \alpha_2/\alpha^5$, and $\delta_4 \equiv \hat{\gamma}_5/\alpha^6$. In comparison with the formula in Eq. (2.11), we find the coefficients in HL gravity as

$$\begin{aligned} \mathcal{H}\bar{\nu} &= 0, \\ \bar{\mu} &= \delta_2 (k/aM_{\rm PV})^2 + \delta_4 (k/aM_{\rm PV})^4 + 3\alpha_0 \mathcal{H}/(2M_{\rm PV}a), \\ \mathcal{H}\nu_{\rm A} &= 0, \\ \mu_{\rm A} &= \delta_1 \rho_{\rm A} (k/aM_{\rm PV}) - \delta_3 \rho_{\rm A} (k/aM_{\rm PV})^3, \end{aligned}$$
(3.22)

where we have considered the relation
$$\alpha^2 \rightarrow 1$$
. In the

expression of $\bar{\mu}$, the second term is always negligible, and the relative magnitude of the first and third terms depends on the values of *k* and $M_{\rm PV}$. In the theory, which includes both the third- and fifth-order operators [36,37], the first term in $\mu_{\rm A}$ is dominant. While for the theory, which includes only the fifth-order operator [33], only the second term in $\mu_{\rm A}$ exists.

E. Various theories of gravity without parity violation

It is important to mention that, although in this paper we focus on GWs in the parity-violating gravities, the equation of motion of GW in Eq. (2.11) can also include various cases in the parity-conserving gravities. In the most general scalar-tensor gravities, i.e., the Horndeski and beyond Horndeski gravities and degenerate higher-order scalar-tensor theories, the propagation equations of GW correspond to Eq. (2.11) with nonzero $\bar{\nu}$ and $\bar{\mu}$ [12,30,49,61].

Another case is the theories of gravity with modified dispersion relations; the propagation equation of GW is Eq. (2.11), where the parameters $\bar{\nu}$, ν_A , and μ_A are zero, but $\bar{\mu}$ is nonzero. A few examples of such modified dispersion relations include the following [55]:

- (i) *Massive gravity* has the dispersion relation $E^2 = p^2 + m_g^2$ with m_g the graviton's rest mass [55,56], which follows the equation of motion of GW in Eq. (2.11) with nonzero $\bar{\mu} = (m_g/k)^2$.
- (ii) The double special relativity theory has the relation $E^2 = p^2 + m_g^2 + \eta_{dsrt}E^3 + \cdots$ with η_{dsrt} a parameter of the order of Planck length [62], which follows Eq. (2.11) with nonzero $\bar{\mu} = (m_g/k)^2 + \eta_{dsrt}k + \cdots$.
- (iii) Extra-dimensional theories have the relation $E^2 = p^2 + m_g^2 \alpha_{\text{edt}} E^4$ with α_{edt} a constant related to the square of the Planck length [63], which follows Eq. (2.11) with nonzero $\bar{\mu} = (m_g/k)^2 \alpha_{\text{edt}}k^2$.
- (iv) Noncommutative gravity¹ has the relation $E^2g_1^2(E) = m_g^2 + p^2$ with $g_1 = (1 \sqrt{\alpha_{neg}\pi}/2)\exp(-\alpha_{ncg}E^2/E_p^2)$ with α_{ncg} a constant [64], which follows Eq. (2.11) with nonzero $\bar{\mu} = (1 + \sqrt{\alpha_{ncg}\pi}/2)(1 + (m_g/k)^2 + \alpha_{ncg}(k/E_p)^2 + (m_g/E_p)^2)$.

In some theories of parity-conserving gravity, the propagation of GW is described by Eq. (2.11) with nonzero $\bar{\nu}$, but the other parameters $\bar{\mu}$, μ_A , and ν_A are all zero. One example is the nonlocal gravity [50], in which the action of gravity is given by

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - \frac{1}{6} m^2 \frac{1}{\Box} R \right],$$

where *m* is a mass parameter that replaces the cosmological constant. The motion of equation of GW in this theory is given by Eq. (2.11) with $\mathcal{H}\bar{\nu} = -[3\bar{V}/(2-6\gamma\bar{V})]'$, where $\gamma = m^2/9H_0^2$ and \bar{V} is the background evolution of an auxiliary field in the theory [50,65]. In these theories, due to the modified friction term in the perturbation equation of cosmological GWs, the damping of GW amplitude with the expansion of the Universe is different from that in GR. As a result, the effective luminosity distance of GW is different from that of electromagnetic wave [50]. Similarly, in the theory of gravity with the time-dependent effective Planck mass, the motion of equation of the GW is also given by Eq. (2.11) with $\mathcal{H}\bar{\nu} = [\ln M_*^2]'$ with M_* the timedependent Planck mass [66]. Recently, the propagation of GW in the f(T) gravitational theory was also studied explicitly in Ref. [67], and the equation of motion was found to have the same form as $\mathcal{H}\bar{\nu} = |\ln f_T|'$, where T is the torsion scalar defined in the theory, f(T) is the arbitrary function of *T*, and $f_T \equiv df(T)/dT$.

¹In noncommutative gravity, the spacetime coordinates are promoted to operators, which satisfy the nontrivial commutation relations $[\hat{x}^{\mu}, \hat{x}_{\nu}] = i\Theta^{\mu\nu}$ with $\Theta^{\mu\nu}$ being a real constant antisymmetric tensor.

IV. AMPLITUDE AND VELOCITY BIREFRINGENCES

In this section, in the general theories of PV gravity, we study the velocity and amplitude birefringence effects during the propagation of GWs. As discussed in Sec. II, the equation of motion of GWs is Eq. (2.11), and the parameters are given by Eq. (2.13). For each circular polarization mode h_A , the velocity and amplitude birefringence effects induce the phase and amplitude corrections to the waveform of GWs. Similar to the previous work [39], to investigate these two effect separately, we decompose h_A as

$$h_{\rm A} = \bar{h}_{\rm A} e^{-i\theta(\tau)}, \qquad \bar{h}_{\rm A} = \mathcal{A}_{\rm A} e^{-i\Phi(\tau)}, \qquad (4.1)$$

where $\bar{h}_{\rm A}$ satisfies

$$\bar{h}_{\rm A}'' + 2\mathcal{H}\bar{h}_{\rm A}' + (1 + \bar{\mu} + \mu_{\rm A})k^2\bar{h}_{\rm A} = 0. \tag{4.2}$$

Here, \mathcal{A}_A and $\Phi(\tau)$ are the amplitude and phase of h_A , respectively. With this decomposition, $\theta(\tau)$ encodes the correction arising from $\bar{\nu}$ and ν_A , while the corrections due to $\bar{\mu}$ and μ_A are included in \bar{h}_A . In the ghost-free PV gravities, since $\bar{\nu} = 0 = \bar{\mu}$, the phase $\theta(\tau)$ only contains corrections from the parity violation, which is directly encoded by ν_A , while the amplitude correction from μ_A is included in \bar{h}_A , as one can see from Eq. (4.5) in Ref. [39]. In the generic case, the phase $\theta(\tau)$ and the amplitude \bar{h}_A can receive corrections arising from the modifications of gravity which are not relevant to the parity violation. As we mentioned, their effects are encoded by the parameters $\bar{\mu}$ and $\bar{\nu}$.

A. Phase modifications

We first concentrate on the corrections arising from the parameters μ and μ_A , which modify the dispersion relations of GWs. To proceed, we define $\bar{u}_A(\tau) = \frac{1}{2}a(\tau)M_{\rm Pl}\bar{h}_A(\tau)$, and then Eq. (4.2) can be written as

$$\frac{d^2\bar{u}_{\rm A}}{d\tau^2} + \left(\omega_{\rm A}^2 - \frac{a^{\prime\prime}}{a}\right)\bar{u}_{\rm A} = 0, \qquad (4.3)$$

where

$$\omega_{\rm A}^2 = k^2 (1 + \bar{\mu} + \mu_{\rm A})$$

= $k^2 (1 + \alpha_{\bar{\mu}}(\tau) (k/aM_{\rm PV})^{\beta_{\bar{\mu}}} + \rho_{\rm A} \alpha_{\mu}(\tau) (k/aM_{\rm PV})^{\beta_{\mu}})$
(4.4)

is the modified dispersion relation. With this relation, the speed of the graviton reads $v_A^2/c^2 \simeq 1 - \bar{\mu} - \mu_A$, which leads to

$$v_{\rm A}/c \simeq 1 - (1/2)\alpha_{\bar{\mu}}(\tau)(k/aM_{\rm PV})^{\beta_{\bar{\mu}}} - (1/2)\rho_{\rm A}\alpha_{\mu}(\tau)(k/aM_{\rm PV})^{\beta_{\mu}}.$$
 (4.5)

We find that the parameter $\bar{\mu}$ influences both circular modes in a same way, which has been tightly constrained by comparing the arrival times of GW170817 and GRB170817a [11]. However, for the parameter μ_A , since ρ_A have the opposite signs for left-hand and right-hand polarization modes, its effects on two modes are opposite, which induces the velocity birefringence effect of GWs. For this reason, in order to test the parity-violating effect by the velocity of GWs, we should measure the velocity difference of two circular polarization modes, instead of just measuring the speed of an individual mode [40].

Consider gravitons with the same ρ_A emitted at two different times t_e and t'_e , with wave numbers k and k', and received at corresponding arrival times (r_e is the same for both). Assuming $\Delta t_e \equiv t_e - t'_e \ll a/\dot{a}$, the difference of arrival times of these two waves is given by [39,40,55]

$$\Delta t_{0} = (1+z)\Delta t_{e} + \frac{\rho_{A}}{2} \left(\left(\frac{k}{M_{PV}} \right)^{\beta_{\mu}} - \left(\frac{k'}{M_{PV}} \right)^{\beta_{\mu}} \right)$$
$$\times \int_{t_{e}}^{t_{0}} \frac{\alpha_{\mu}}{a^{\beta_{\mu}+1}} dt + \frac{1}{2} \left(\left(\frac{k}{M_{PV}} \right)^{\beta_{\bar{\mu}}} - \left(\frac{k'}{M_{PV}} \right)^{\beta_{\bar{\mu}}} \right)$$
$$\times \int_{t_{e}}^{t_{0}} \frac{\alpha_{\bar{\mu}}}{a^{\beta_{\bar{\mu}}+1}} dt, \qquad (4.6)$$

where $z \equiv 1/a(t_e) - 1$ is the cosmological redshift. Let us focus on the GW signal generated by the nonspinning, quasicircular inspiral in the post-Newtonian approximation. Relative to the GW in GR, the terms $\bar{\mu}$ and μ_A modify the phase of GW $\Phi(\tau)$. The Fourier transform of \bar{h}_A can be obtained analytically in the stationary phase approximation, where we assume that the phase is changing much more rapidly than the amplitude, which is given by [2]

$$\tilde{\tilde{h}}_{\rm A}(f) = \frac{\mathcal{A}_{\rm A}(f)}{\sqrt{df/dt}} e^{i\Psi(f)},\tag{4.7}$$

where f is the GW frequency at the detector and Ψ is the phase of GWs. In Ref. [55], it was proved that the difference of arrival times in Eq. (4.6) induces the modification of GW phases Ψ as follows:

$$\Psi_{\rm A}(f) = \Psi_{\rm A}^{\rm GR}(f) + \rho_{\rm A}\delta\Psi_1(f) + \delta\Psi_2(f). \quad (4.8)$$

When $\beta_{\mu} \neq -1$,

$$\delta \Psi_1(f) = \frac{(2/M_{\rm PV})^{\beta_{\mu}}}{\beta_{\mu} + 1} \frac{u^{\beta_{\mu} + 1}}{\mathcal{M}^{\beta_{\mu} + 1}} \int_{t_e}^{t_0} \frac{\alpha_{\mu}}{a^{\beta_{\mu} + 1}} dt, \quad (4.9)$$

and when $\beta_{\mu} = -1$,

$$\delta \Psi_1(f) = \frac{M_{\rm PV}}{2} \ln u \int_{t_e}^{t_0} \alpha_\mu dt.$$
 (4.10)

Similarly, since $\beta_{\bar{\mu}}$ is an even number, we have

$$\delta \Psi_2(f) = \frac{(2/M_{\rm PV})^{\beta_{\bar{\mu}}}}{\beta_{\bar{\mu}} + 1} \frac{u^{\beta_{\bar{\mu}} + 1}}{\mathcal{M}^{\beta_{\bar{\mu}} + 1}} \int_{t_e}^{t_0} \frac{\alpha_{\bar{\mu}}}{a^{\beta_{\bar{\mu}} + 1}} dt.$$
(4.11)

Here, we have defined $u = \pi \mathcal{M} f$. The quantity $\mathcal{M} = (1+z)\mathcal{M}_c$ is the measured chirp mass, and $\mathcal{M}_c \equiv (m_1m_2)^{3/5}/(m_1+m_2)^{1/5}$ is the chirp mass of the binary system with component masses m_1 and m_2 . Note that the phase modification in Eqs. (4.9)–(4.11) is a simple propagation effect, which is independent of the generation effect of GWs. Although this formula is obtained in the stationary phase approximation [55], we expect this result is also applicable for the GWs in the more general cases in the presence of spin, precession, eccentricity of compact binaries, and/or for the GWs produced during the merger and ringdown of compact binaries. This extension has been adopted by LIGO and Virgo collaborations in Refs. [6,11].

B. Amplitude modifications

Now, let us turn to study the effects caused by $\bar{\nu}$ and ν_A . Plugging the second equation of the decomposition (4.1) into (4.2), one finds the equation for $\Phi(t)$,

$$i\Phi'' + \Phi'^2 + 2i\mathcal{H}\Phi' - (1 + \bar{\mu} + \mu_{\rm A})k^2 = 0. \tag{4.12}$$

Similarly, plugging the first equation of the decomposition (4.1) into (2.11), one obtains

$$i(\theta'' + \Phi'') + (\Phi' + \theta')^2 + i(2 + \bar{\nu} + \nu_A)\mathcal{H}(\theta' + \Phi') - (1 + \bar{\mu} + \mu_A)k^2 = 0.$$
(4.13)

Using the equation of motion (4.12) for Φ , the above equation reduces to

$$i\theta'' + 2\theta'\Phi' + \theta'^2 + i(2 + \bar{\nu} + \nu_A)\mathcal{H}\theta' + i(\bar{\nu} + \nu_A)\mathcal{H}\Phi' = 0.$$
(4.14)

The phase Φ is expected to be close to that in GR Φ_{GR} , and $\Phi'_{GR} \sim k$, where the wave number relates to the GW frequency by $k = 2\pi f/a_0$. Since the amplitude modification function θ is caused by the expansion of the Universe, we have $\theta' \sim \mathcal{H}$ and $\theta'' \sim \mathcal{H}^2$. Note that the conformal Hubble parameter $\mathcal{H} \sim 10^{-18}$ Hz, which is much smaller than the GW frequency (approximately 10^2 Hz) of compact binaries. Considering that $\theta'' \ll \Phi'\theta' \sim k\theta'$, $k \gg \mathcal{H}$, and keeping only the leading-order terms, the above equation can be simplified into the form $2\theta' + i\mathcal{H}(\bar{\nu} + \nu_A) = 0$, which has the solution

$$\theta = -\frac{i}{2} \int_{\tau_e}^{\tau_0} \mathcal{H}(\bar{\nu} + \nu_{\rm A}) d\tau.$$
(4.15)

We observe that the contributions of $\bar{\nu}$ and ν_A in the phase are purely imaginary. This indicates that these parameters lead to modification of the amplitude of the GWs during the propagation. As a result, relative to the corresponding mode in GR, the amplitude of left-hand circular polarization of GWs will increase (or decrease) during the propagation, while the amplitude for the right-hand mode will decrease (or increase).

More specifically, for the parameters $\bar{\nu}$ and ν_A given in Eq. (2.13), one can write the waveform of GWs with parity-violation effects in the form

$$h_{\rm A} = h_{\rm A}^{\rm GR} (1 + \rho_{\rm A} \delta h_1 + \delta h_2) e^{-i\delta \Phi_{\rm A}}, \qquad (4.16)$$

where

$$\delta h_1 = -\frac{1}{2} \left[a_\nu \left(\frac{k}{aM_{\rm PV}} \right)^{\beta_\nu} \right] \Big|_{a_e}^{a_0},$$

$$\delta h_2 = -\frac{1}{2} \left[a_{\bar{\nu}} \left(\frac{k}{aM_{\rm PV}} \right)^{\beta_{\bar{\nu}}} \right] \Big|_{a_e}^{a_0}.$$
 (4.17)

Using the notations u and \mathcal{M} , one can rewrite δh_1 and δh_2 in the forms

$$\delta h_{1} = -\frac{1}{2} \left(\frac{2u}{M_{\rm PV} \mathcal{M}} \right)^{\beta_{\nu}} [a_{\nu}(\tau_{0}) - a_{\nu}(\tau_{e})(1+z)^{\beta_{\nu}}],$$

$$\delta h_{2} = -\frac{1}{2} \left(\frac{2u}{M_{\rm PV} \mathcal{M}} \right)^{\beta_{\bar{\nu}}} [a_{\bar{\nu}}(\tau_{0}) - a_{\bar{\nu}}(\tau_{e})(1+z)^{\beta_{\bar{\nu}}}]. \quad (4.18)$$

C. GW waveforms of the circular polarization modes

For each circular polarization mode, we summarize the modification of GW waveforms in this subsection. Similar to the above discussion, we consider the GWs produced during the inspiraling stage of the compact binaries. In this scheme, in the stationary phase approximation, the Fourier transform of h_A can be calculated directly, which is given by (similar to the parametrized post-Einsteinian formulas in Ref. [68])

$$\tilde{h}_{\rm A}(f) = \tilde{h}_{\rm A}^{\rm GR}(f)(1+\rho_{\rm A}\delta h_1+\delta h_2)e^{i(\rho_{\rm A}\delta\Psi_1+\delta\Psi_2)},\qquad(4.19)$$

where $\tilde{h}_{A}^{GR}(f)$ is the corresponding waveform in GR, and the explicit formula can be found in the previous works [1,2]. Here, we focus on the correction terms on both the amplitude and phase of GWs. The former ones are caused by the parameters $\bar{\nu}$ and ν_{A} in Eq. (2.11), while the latter ones are caused by the parameters $\bar{\mu}$ and μ_{A} . Since in the theories of PV gravity the GW propagation equations of left-hand and right-hand polarization modes are decoupled, the parameters ν_{L} and μ_{L} (ν_{R} and μ_{R}) can only affect the amplitude and phase of the left-hand (right-hand) polarization mode. Because of the opposite signs of ν_{L} and ν_{R} (as well as μ_{L} and μ_{R}), they induce the birefringence effects of GWs. However, the parameters $\bar{\nu}$ and $\bar{\mu}$ influence the amplitudes and phases of two modes in the same manner. Therefore, as expected, they are not relevant to parity violation in gravity and cannot cause the GW birefringence effects.

V. MODIFICATIONS TO THE GW WAVEFORM

To make contact with observations, it is convenient to analyze the GWs in the Fourier domain, and the responses of detectors for the GW signals $\tilde{h}(f)$ can be written in terms of waveforms of \tilde{h}_+ and \tilde{h}_{\times} as

$$\tilde{h}(f) = [F_+ \tilde{h}_+(f) + F_\times \tilde{h}_\times(f)] e^{-2\pi i f \Delta t}, \qquad (5.1)$$

where F_+ and F_{\times} are the beam pattern functions of GW detectors, depending on the source location and polarization angle [69]. Δt is the arrival time difference between the detector and the geocenter. In GR, the waveforms of the two polarizations $\tilde{h}_+(f)$ and $\tilde{h}_{\times}(f)$ are given, respectively, by [1,2,70]

$$\tilde{h}_{+}^{\text{GR}} = (1 + \chi^2) \mathcal{A} e^{i\Psi}, \qquad \tilde{h}_{\times}^{\text{GR}} = 2\chi \mathcal{A} e^{i(\Psi + \pi/2)}, \quad (5.2)$$

where \mathcal{A} and Ψ denote the amplitude and phase of the waveforms $h_{+\times}^{\text{GR}}$ and $\chi = \cos i$ with *i* being the inclination angle of the binary system. In GR, the explicit forms of \mathcal{A} and Ψ have been calculated in the high-order PN approximation (see, for instance, Ref. [1] and references therein). Now, we would like to derive how the parity violation in the modified gravities can affect both the amplitude and the phase of the above waveforms. The circular polarization modes \tilde{h}_R and \tilde{h}_L relate to the modes \tilde{h}_+ and \tilde{h}_{\times} via

$$\tilde{h}_{+} = \frac{\tilde{h}_{\rm L} + \tilde{h}_{\rm R}}{\sqrt{2}}, \qquad \tilde{h}_{\times} = \frac{\tilde{h}_{\rm L} - \tilde{h}_{\rm R}}{\sqrt{2}i}. \tag{5.3}$$

Similar to the previous work [27,39], throughout this paper, we ignore the parity-violating generation effect, which is caused by a modified energy loss, inspiral rate, and chirping rate of the binaries. Since the generation effect occurs on a radiation-reaction timescale, which is much smaller than the GW time of flight, its impact on the evolution of the GW phase is negligible [26]. In addition, the extra polarization modes of GW can also been produced in the modified gravities. For instance, in the scalar-tensor gravities, the breathing mode and/or longitude mode can be generated [71,72] by the coalescence of compact binaries. However, the amplitudes of these extra modes are always much smaller than the tensorial modes h_+ and h_{\times} , or $h_{\rm R}$ and $h_{\rm L}$, if considering the model constraints from the observations in solar systems, or binary pulsars, and in cosmology [71,73]. For these reasons, in this paper, we do not consider this effect. Thus, the circular polarization modes h_A are given in (4.19), and the waveforms for the plus and cross modes become

$$\tilde{h}_{+} = \tilde{h}_{+}^{\text{GR}} (1 + \delta h_2 + i\delta \Psi_2) - \tilde{h}_{\times}^{\text{GR}} (i\delta h_1 - \delta \Psi_1) \quad (5.4)$$

$$\tilde{h}_{\times} = \tilde{h}_{\times}^{\text{GR}} (1 + \delta h_2 + i \delta \Psi_2) + \tilde{h}_{+}^{\text{GR}} (i \delta h_1 - \delta \Psi_1).$$
 (5.5)

Therefore, the Fourier waveform $\tilde{h}(f)$ becomes

$$\tilde{h}(f) = \mathcal{A}\delta\mathcal{A}e^{i(\Psi + \delta\Psi)},\tag{5.6}$$

where

$$\begin{split} \delta \mathcal{A} &= \sqrt{(1+\chi^2)^2 F_+^2 + 4\chi^2 F_\times^2} \\ &\times \left[1 + \delta h_2 + \frac{2\chi(1+\chi^2)(F_+^2 + F_\times^2)}{(1+\chi^2)^2 F_+^2 + 4\chi^2 F_\times^2} \delta h_1 \right. \\ &- \frac{(1-\chi^2)^2 F_+ F_\times}{(1+\chi^2)^2 F_+^2 + 4\chi^2 F_\times^2} \delta \Psi_1 \right], \\ \delta \Psi &= \tan^{-1} \left[\frac{2\chi F_\times}{(1+\chi^2) F_+} \right] + \delta \Psi_2 \\ &+ \frac{(1-\chi^2)^2 F_+ F_\times}{(1+\chi^2)^2 F_+^2 + 4\chi^2 F_\times^2} \delta h_1 \\ &+ \frac{2\chi(1+\chi^2)(F_+^2 + F_\times^2)}{(1+\chi^2)^2 F_+^2 + 4\chi^2 F_\times^2} \delta \Psi_1, \end{split}$$
(5.7)

where δh_1 and δh_2 are given by Eq. (4.18), while $\delta \Psi_1$ and $\delta \Psi_2$ are given by Eqs. (4.9)–(4.11).

From these expressions, we find that relative to the waveforms in GR the modifications of GWs are quantified by the terms δh_i and $\delta \Psi_i$ (i = 1, 2). In the specific case with $\delta h_i = \delta \Psi_i = 0$, the formula in (5.6) returns to that in GR. We observe that when $\delta h_1 = \delta \Psi_1 = 0$ it returns to the theory without parity violation. The correction term δh_2 , caused by the parameter $\bar{\nu}$ in Eq. (2.11), influences only on the amplitude of $\tilde{h}(f)$. In particular, if δh_2 is frequency independent, i.e., $\beta_{\bar{\nu}} = 0$, the effect of GW amplitude modification is equivalent to modifying the effective luminosity distance of GW sources [50]. On the other hand, the correction term $\delta \Psi_2$, caused by the parameter $\bar{\mu}$ in Eq. (2.11), influences only the phase of $\tilde{h}(f)$. These are consistent with the results in the previous works [51,52,55], in which the specific theories in this case are discussed.

The modifications of GW waveforms, due to the PV terms in the theories of gravity, are represented by the terms δh_1 and $\delta \Psi_1$. As mentioned in Sec. IV, δh_1 is the result of the amplitude birefringence effect between left-hand and right-hand polarization modes, which is caused by the parameter ν_A in Eq. (2.11), and $\delta \Psi_1$ is result of the amplitude birefringence effect between two circular polarization modes, caused by the parameter ν_A in Eq. (2.11). Since in the parity-violating gravities the evolution of polarization modes is inevitable. For this reason, we find that both terms δh_1 and $\delta \Psi_1$ appear in the phase and amplitude modifications of $\tilde{h}(f)$. As the extension of our previous work [39], we find that when $\beta_{\nu} = \beta_{\mu} = 1$ the

above results automatically return to those in Ref. [39]. In particular, in the CS gravity, only $\delta \Psi_1$ is nonzero, and the formulas in Eq. (5.7) return to the corresponding quantities in Ref. [27]. However, in general, both δh_1 and $\delta \Psi_1$ are nonzero in the theories of PV gravity. In the leading order, the modification $\delta \mathcal{A}$ (or $\delta \Psi$) linearly depends on them, and it is important to estimate their relative magnitudes. Let us assume the GW is emitted at the redshift $z \sim O(1)$ and approximately treat α_{ν} and α_{μ} as constants during the propagation of GW. In addition, we assume $\beta_{\nu} = \beta_{\mu}$, which is retained in most cases. Therefore, we find the ratio of two correction terms is $\delta \Psi_1 / \delta h_1 \sim t_0 f$, where f is the GW frequency and $t_0 = 13.8$ billion years is the cosmic age [74]. As is known, $f \sim 100$ Hz for the ground-based GW detectors, and $f \sim 0.01$ Hz for the space-borne detectors. For both cases, we find $\delta \Psi_1$ is more than 10 orders of magnitude larger than δh_1 . So, we arrive at the conclusion: in the PV gravities with both velocity and amplitude birefringence effects, both the amplitude and phase corrections of the GW waveform $\tilde{h}(f)$ mainly come from the contribution of velocity birefringence rather than that of amplitude birefringence.

VI. CONCLUSIONS

With the discovery of GW sources by the aLIGO and aVirgo collaborations, the testing of gravity in the strong gravitational fields becomes possible. Therefore, the studies on GWs in the alternative theories of gravity are important. In the series of our works, we focus on how to test the parity symmetry in gravity. In the theories of PV gravity, the symmetry between left-hand and right-hand circular polarization modes of GWs is broken. So, the effects of birefringence between these two modes occur during their propagation in the Universe. In this article, we investigate the waveforms of GWs in the general PV gravity, and the existing models in the literature, including Chern-Simons modified gravity, ghost-free scalar-tensor gravity, the symmetric teleparallel equivalence of GR theory, and Hořava-Lifshitz gravity, are all the specific cases of this general theory. We find that, in general, both amplitude and velocity birefringence effects exist in these gravities, which exactly correspond to the amplitude and phase modifications of waveforms for each mode. For an individual circular polarization mode, the amplitude and/or phase of GW waveform as well as the velocity of GW are modified by two parameters: one describes the parity violation in gravity, and the other describes the modification of GR but keeps the parity symmetry. For this reason, in order to model-independently test the parity symmetry of gravity, we should investigate the difference between two circular polarizations, rather than focus on an individual mode. Combining these two modes, we obtain the GW waveforms produced by the compact binary coalescence and derive the correction terms relative to that in GR. We find that if the parity symmetry is not broken, the correction term in the amplitude (phase) of $\tilde{h}(f)$ only comes from δh_2 ($\delta \Psi_2$), which is the amplitude (phase) modification in each circular polarization mode. However, if the parity symmetry is violated in the theory, in addition to these terms, the corrections in the amplitude (phase) of $\hat{h}(f)$ also comes from both δh_1 and $\delta \Psi_1$ terms, where δh_1 encodes the effect of amplitude birefringence and $\delta \Psi_1$ represents the effect of velocity birefringence. The mixture of them in the waveform of $\tilde{h}(f)$ is caused by the fact that in the PV gravities the general plus and cross polarization modes are not independent in their propagations. Comparing the contributions of δh_1 and $\delta \Psi_1$ in the modification of GW waveforms, we find the latter one is completely dominant, unless the velocity birefringence does not exist in the theory. All these conclusions are consistent with those derived from the previous work [39]. We should mention that, considering the current and potential observations of ground-based and space-borne GW detectors, the explicit waveforms of GWs derived in this article can be used as the template to constrain the these gravities with parity violation. The comprehensive analysis on this topic will be carried on in a separate paper of this series of works.

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