Polarized primordial gravitational waves in the ghost-free parity-violating gravity

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The tests of parity symmetry in the gravitational interaction is an attractive issue in gravitational-wave astronomy. In the general theories of gravity with parity violation, one of the fundamental results is that primordial gravitational waves (PGWs) produced during slow-roll inflation is circularly polarized. In this article, we investigate the polarization of PGWs in the recently proposed ghost-free parity-violating gravity, which generalizes Chern-Simons gravity by including higher derivatives of the coupling scalar field. For this purpose, we first construct the approximate analytical solution to the mode function of the PGWs during slow-roll inflation by using the uniform asymptotic approximation. With the approximate solution, we explicitly calculate the power spectrum and the corresponding circular polarization of the PGWs analytically, and find that the contributions of the higher derivatives of the coupling scalar field to the circular polarization are of the same order of magnitude as that of Chern-Simons gravity. The degree of circular polarization of PGWs is suppressed by the energy scale of parity violation in gravity, which is unlikely to be detected using only the two-point statistics of future cosmic microwave background data.

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I. INTRODUCTION

Precise measurements of the cosmic microwave background (CMB), in particular by the WMAP and *Planck* missions, provide invaluable information about the physics of the very early Universe. Their angular power spectra directly reflect statistical properties of primordial density fluctuations and primordial gravitational waves (PGWs) [1–4], which are in good agreement with the theoretical predictions of the slow-roll inflation model with a single scalar field [5–10]. For standard slow-roll inflation in the framework of general relativity (GR), the PGWs have two polarization modes which share exactly the same statistical properties and their primordial power spectra take the same form. Such PGWs produce the TT, EE, BB, and TE spectra of CMB, but the TB and EB spectra vanish because of the parity symmetry of GR [11–15]. Since nonzero TB and EB spectra of CMB implies parity violation in the gravitational sector, the precise measurement of TB and EB spectra could be important evidence of parity violation of the gravitational interaction [16–20].[1]

In fact, the gravitational terms with parity violation are ubiquitous in numerous candidates of quantum gravity, such as string theory, loop quantum gravity, and Hořava-Lifshitz gravity. One well-studied example is the gravitational Chern-Simons term, which arises from string theory [16–20,25–28] and loop quantum gravity [29–33]. In Hořava-Lifshitz gravity, the parity-violating third and fifth spatial derivative terms are allowed in the gravitational action of the theory [34–36]. In the literature, parity violation can also arise from graviton self-couplings [37,38], gaugeflation and chromonatural inflation [39–42], Holst gravity [43], and in models that connect leptogenesis to PGWs [27,44]. In all of these examples, a fundamental

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[1]Note that another effect that produces the TB and EB power spectra of CMB is the so-called cosmological birefringence effect, which can be caused by the possible coupling between the electromagnetic field and the scalar field through the Chern-Simons term [17,21–24].

effect of parity violation is the circular polarization of PGWs, i.e., the left-hand and right-hand polarization modes of GWs propagate with different behaviors. As we mentioned above, such asymmetry between two chiral modes of PGWs can induce a significant parity-violating signature in the CMB polarization (E/B) power spectra, which has motivated a lot of works in this direction (see Refs. [20,34,45–54] and references therein for examples).

Recently, based on Chern-Simons modified gravity, a ghost-free parity-violating theory of gravity was explored in Ref. [55] by including higher derivatives of the coupling scalar field. The observational implications of this theory, as well as its extensions of the gravitational waves generated by the compact binaries, have been explored in a series papers [56–59]. In comparison with Chern-Simons gravity, one of the distinguishing features of higher derivatives of the coupling scalar field is that they lead to the velocity birefringence phenomenon, i.e., the velocities of left-hand and right-hand circular polarizations of GWs are different in ghost-free parity-violating gravities. Thus, it is expected that such a velocity birefringence effect could induce some distinguishable signatures in the power spectra of PGWs. With these motivations, in this paper we study circularly polarized PGWs in this theory of gravity with parity violation, and the possibility of detecting the chirality of PGWs in future potential CMB observations.

The rest of the paper is organized as follows. In Sec. II we give a very brief review of ghost-free parity-violating gravities, and in Sec. III we consider a flat Friedmann-Robertson-Walker (FRW) universe and derive the equation of motion for PGWs. In Sec. IV we first construct the approximate analytical solution of PGWs by using the uniform asymptotic approximation, and then explicitly calculate the power spectrum and polarization of PGWs during slow-roll inflation. The effects of parity violation in the CMB spectra and their detectability are also briefly discussed. We end in Sec. V, in which we summarize our main conclusions and provide some outlooks.

II. PARITY-VIOLATING GRAVITIES

In this section we present a brief introduction to ghost-free parity-violating gravity proposed in Ref. [55]. The action of parity-violating gravity has the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R + \mathcal{L}_{\rm PV} + \mathcal{L}_\phi), \tag{2.1}$$

where R is the Ricci scalar, $\mathcal{L}_{\rm PV}$ is a parity-violating Lagrangian, and $\mathcal{L}_\phi$ is the Lagrangian for the scalar field, which is coupled nonminimally to gravity. As a simplest example, we consider the action of the scalar field as

$$\mathcal{L}_\phi = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi). \tag{2.2}$$

Here $V(\phi)$ denotes the potential of the scalar field. The parity-violating Lagrangian of the theory can be written in the form

$$\mathcal{L}_{\rm PV} = \mathcal{L}_{\rm CS} + \mathcal{L}_{\rm PV1} + \mathcal{L}_{\rm PV2}, \tag{2.3}$$

where the Chern-Simons term $\mathcal{L}_{\rm CS}$ is given by

$$\mathcal{L}_{\rm CS} = \frac{1}{8}\vartheta(\phi)\varepsilon^{\mu\nu\rho\sigma}R_{\rho\sigma\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}, \tag{2.4}$$

where the Levi-Civita tensor $\varepsilon_{\rho\sigma\alpha\beta}$ is defined in terms of the antisymmetric symbol $\epsilon^{\rho\sigma\alpha\beta}$ as $\varepsilon^{\rho\sigma\alpha\beta} = \epsilon^{\rho\sigma\alpha\beta}/\sqrt{-g}$. $\mathcal{L}_{\rm PV1}$ contains the first derivative of the scalar field and is given by

$$\begin{aligned}
\mathcal{L}_{\rm PV1} &= \sum_{\mathcal{A}=1}^{4} a_{\mathcal{A}}(\phi, \phi^\mu\phi_\mu)L_{\mathcal{A}},\\
L_1 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}R_{\mu\nu}{}^{\rho}{}_{\lambda}\phi^\sigma\phi^\lambda,\\
L_2 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}R_{\mu\lambda}{}^{\rho\sigma}\phi_\nu\phi^\lambda,\\
L_3 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}R^{\sigma}{}_{\nu}\phi^\rho\phi_\mu,\\
L_4 &= \varepsilon^{\mu\nu\rho\sigma}R_{\rho\sigma\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}\phi^\lambda\phi_\lambda,
\end{aligned} \tag{2.5}$$

with $\phi^\mu \equiv \nabla^\mu\phi$. It has been shown that in order to avoid Ostrogradsky modes, it is required that $4a_1 + 2a_2 + a_3 + 8a_4 = 0$. The term $\mathcal{L}_{\rm PV2}$, which contains the second derivatives of the scalar field, is described by

$$\begin{aligned}
\mathcal{L}_{\rm PV2} &= \sum_{\mathcal{A}=1}^{7} b_{\mathcal{A}}(\phi, \phi^\lambda\phi_\lambda)M_{\mathcal{A}},\\
M_1 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi^\rho\phi_\mu\phi^\sigma_\nu,\\
M_2 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi^\rho_\mu\phi^\sigma_\nu,\\
M_3 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi^\sigma\phi^\rho_\mu\phi^\lambda_\nu\phi_\lambda,\\
M_4 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma}\phi_\nu\phi^\rho_\mu\phi^\sigma_\lambda\phi^\lambda,\\
M_5 &= \varepsilon^{\mu\nu\alpha\beta}R_{\alpha\rho\sigma\lambda}\phi^\rho\phi_\beta\phi^\sigma_\mu\phi^\lambda_\nu,\\
M_6 &= \varepsilon^{\mu\nu\alpha\beta}R_{\beta\gamma}\phi_\alpha\phi^\gamma_\mu\phi^\lambda_\nu\phi^\lambda,\\
M_7 &= (\nabla^2\phi)L_1,
\end{aligned} \tag{2.6}$$

with $\phi^\sigma_\nu \equiv \nabla^\sigma\nabla_\nu\phi$. Similarly, in order to avoid Ostrogradsky modes in the unitary gauge, the following conditions should be imposed: $b_7 = 0$, $b_6 = 2(b_4 + b_5)$ and $b_2 = -A_*^2(b_3 - b_4)/2$, where $A_* \equiv \dot{\phi}(t)/N$ and N is the lapse function of the spacetime.

In the current paper we focus only on the terms coupled with the first and second derivatives of the scalar field. More general forms of the Lagrangian, which contain higher-order derivatives of the scalar field, can be found in Ref. [56].

III. GRAVITATIONAL WAVES IN PARITY-VIOLATING GRAVITIES

In a flat FRW universe, the background metric is given by

$$ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j), \tag{3.1}$$

where $a(\tau)$ denotes the scale factor of the universe and τ represents the conformal time, which is related to the cosmic time t via $dt = a(\tau)d\tau$. In the parity-violating gravities considered in this paper, we observe that all of the parity-violating terms in the action (2.1) have no effect on the background evolution. We further assume that the universe is dominated by the scalar field ϕ which plays the role of the inflaton field during slow-roll inflation. In this case, the Friedmann equation, which governs the background evolution, takes exactly the same form as that in GR, i.e.,

$$H^2 = \frac{8\pi G}{3}\rho, \tag{3.2}$$

where H denotes the Hubble parameter during the inflationary stage, and the energy density of the scalar field is $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$. The evolution of the scalar field ϕ is also the same as that in GR,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \tag{3.3}$$

It is worth noting that in the standard slow-roll inflation, the scalar field is assumed to satisfy the slow-roll conditions,

$$|\ddot{\phi}| \ll |3H\dot{\phi}|, \qquad |\dot{\phi}^2| \ll V(\phi). \tag{3.4}$$

With the above slow-roll conditions, it is convenient to define the following Hubble slow-roll parameters:

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \qquad \epsilon_2 = \frac{d\ln\epsilon_1}{d\ln a}, \qquad \epsilon_3 = \frac{d\ln\epsilon_2}{d\ln a}. \tag{3.5}$$

Primordial gravitational waves are the tensor perturbations of the homogeneous and isotropic background, and we now turn to study their propagation. With the tensor perturbations, the spatial metric is written as

$$g_{ij} = a^2(\tau)(\delta_{ij} + h_{ij}(\tau, x^i)), \tag{3.6}$$

where h_{ij} represents the transverse and traceless metric perturbations, i.e.,

$$\partial^i h_{ij} = 0 = h^i_i. \tag{3.7}$$

In order to derive the equation of motion for the tensor perturbations, we substitute the metric perturbation into the action (2.1) and expand it to the second order in h_{ij}. After tedious calculations, we find

$$S^{(2)} = \frac{1}{16\pi G}\int d\tau d^3x a^4(\tau)[\mathcal{L}^{(2)}_{\rm GR} + \mathcal{L}^{(2)}_{\rm PV}], \tag{3.8}$$

where

$$\mathcal{L}^{(2)}_{\rm GR} = \frac{1}{4a^2}[(h'_{ij})^2 - (\partial_k h_{ij})^2], \tag{3.9}$$

$$\mathcal{L}^{(2)}_{\rm PV} = \frac{1}{4a^2}\left[\frac{c_1(\tau)}{aM_{\rm PV}}\epsilon^{ijk}h'_{il}\partial_j h'_{kl} + \frac{c_2(\tau)}{aM_{\rm PV}}\epsilon^{ijk}\partial^2 h_{il}\partial_j h_{kl}\right], \tag{3.10}$$

where $M_{\rm PV}$ denotes the characteristic energy scale of the parity violation in the theory.

In the above expression, c_1 and c_2 are dimensionless coefficients normalized by the characteristic energy scale $M_{\rm PV}$ and are given by [57]

$$\begin{aligned}\frac{c_1(\tau)}{M_{\rm PV}} = {}& \dot{\vartheta} - 4\dot{a}_1\dot{\phi}^2 - 8a_1\dot{\phi}\ddot{\phi} + 8a_1H\dot{\phi}^2 - 2\dot{a}_2\dot{\phi}^2 - 4a_2\dot{\phi}\ddot{\phi} \\ & + \dot{a}_3\dot{\phi}^2 + 2a_3\dot{\phi}\ddot{\phi} - 4a_3H\dot{\phi}^2 - 4\dot{a}_4\dot{\phi}^2 - 8a_4\dot{\phi}\ddot{\phi} \\ & - 2b_1\dot{\phi}^3 + 4b_2(2H\dot{\phi}^2 - \dot{\phi}\ddot{\phi}) \\ & + 2b_3(\dot{\phi}^3\ddot{\phi} - H\dot{\phi}^4) + 2b_4(\dot{\phi}^3\ddot{\phi} - H\dot{\phi}^4) \\ & - 2b_5H\dot{\phi}^4 + 2b_7\dot{\phi}^3\ddot{\phi},\end{aligned} \tag{3.11}$$

$$\begin{aligned}\frac{c_2(\tau)}{M_{\rm PV}} = {}& \dot{\vartheta} - 2\dot{a}_2\dot{\phi}^2 - 4a_2\dot{\phi}\ddot{\phi} - \dot{a}_3\dot{\phi}^2 - 2a_3\dot{\phi}\ddot{\phi} \\ & - 4\dot{a}_4\dot{\phi}^2 - 8a_4\dot{\phi}\ddot{\phi}.\end{aligned} \tag{3.12}$$

Then, by varying the action with respect to h_{ij} we obtain the field equation for h_{ij} [57],

$$\begin{aligned}& h''_{ij} + 2\mathcal{H}h'_{ij} - \partial^2 h_{ij} \\ & + \frac{\epsilon^{ilk}}{aM_{\rm PV}}\partial_l[c_1 h''_{jk} + (\mathcal{H}c_1 + c'_1)h'_{jk} - c_2\partial^2 h_{jk}] = 0.\end{aligned} \tag{3.13}$$

IV. POLARIZATION OF PGWs

A. Equation of motion for GWs

In parity-violating gravities the propagation equations for the two circular polarization modes of GWs are decoupled. To study the evolution of h_{ij} we expand it over spatial Fourier harmonics,

$$h_{ij}(\tau, x^i) = \sum_{A={\rm R,L}}\int\frac{d^3k}{(2\pi)^3}\tilde{h}_A(\tau, k^i)e^{ik_i x^i}e^A_{ij}(k^i), \tag{4.1}$$

where e^A_{ij} denote the circular polarization tensors and satisfy the relation

$$\epsilon_{ilm}k^l e^A_{ij} = ik\rho_A e^A_{mj}, \tag{4.2}$$

with $\rho_R = 1$ and $\rho_L = -1$. Thus, the field equation in Eq. (3.13) can be cast in the form [57]

$$\tilde{h}''_A + (2+\nu_A)\mathcal{H}\tilde{h}'_A + (1+\mu_A)k^2\tilde{h}_A = 0, \tag{4.3}$$

where a prime denotes a derivative with respect to the conformal time τ. The deviations from that in GR are quantified by the quantities ν_A and μ_A, which are given by

$$\nu_A = \frac{\rho_A k(c_1\mathcal{H} - c'_1)/(a\mathcal{H}M_{\rm PV})}{1-\rho_A k c_1/(aM_{\rm PV})}, \tag{4.4}$$

$$\mu_A = \frac{\rho_A k(c_1 - c_2)/(aM_{\rm PV})}{1-\rho_A k c_1/(aM_{\rm PV})}. \tag{4.5}$$

The quantity ν_A describes the modification of the friction term, and μ_A describes the modification of the dispersion relation of GWs. In parity-violating gravities, the former induces the amplitude birefringence effect of GWs, while the latter induces the velocity birefringence of GWs. In the specific case with $c_1/M_{\rm PV} = c_2/M_{\rm PV} = \dot{\vartheta}$, this equation reduces to that in Chern-Simons gravity, in which we have $\mu_A = 0$, i.e., only the amplitudes of GWs are modified during the propagation through the term ν_A. However, in the general case of ghost-free parity-violating gravity, both ν_A and μ_A are nonzero. In particular, $\mu_A \neq 0$ represents a distinguishable feature of the higher derivatives of the coupling scalar field in this theory, which leads to the velocity birefringence phenomenon, i.e., the velocities of left-hand and right-hand modes of GWs are different in ghost-free parity-violating gravity.

As usual, we define the variable $u_A \equiv zh_A$ and rewrite Eq. (4.3) as

$$u''_A + [(1+\mu_A)k^2 - z''/z]u_A = 0, \tag{4.6}$$

where $z = a(1 - c_1 k\rho_A/(aM_{\rm PV}))^{1/2}$. In this paper we consider PGWs during the inflationary stage, and assume that the background evolution during inflation is slowly varying. In addition, we expect the deviations from GR arising from parity violation to be small. With these considerations, we can expand the effective time-dependent mass term z''/z in Eq. (4.6) in terms of the slow-roll parameters and corrections from the parity violation as

$$\begin{aligned}\frac{z''}{z} &= \frac{a''}{a} + \frac{1}{2}\frac{(\frac{a''}{a}c_1 - c''_1)k\rho_A/aM_{\rm PV}}{1 - c_1 k\rho_A/aM_{\rm PV}} \\ &\quad + \frac{1}{4}\left[\frac{(c_1\mathcal{H} - c'_1)k\rho_A/aM_{\rm PV}}{1 - c_1 k\rho_A/aM_{\rm PV}}\right]^2 \\ &\simeq \frac{v_t^2 - \frac{1}{4}}{\tau^2} - \rho_A\frac{k}{\tau}c_1\epsilon_*,\end{aligned} \tag{4.7}$$

where

$$v_t \simeq \frac{3}{2} + 3\epsilon_1 + 4\epsilon_1^2 + 4\epsilon_1\epsilon_2 + \mathcal{O}(\epsilon^3), \tag{4.8}$$

and $\epsilon_* \equiv H/M_{\rm PV}$ denotes the ratio between the energy scale of inflation and the characteristic energy scale of the parity violation, which determines the magnitude of the corrections to GR. From the expressions for z''/z in Eq. (4.7) and μ_A in Eq. (4.4), one observes that there is a divergence if $k_{\rm phys}/M_{\rm PV} \sim 1$ (with $k_{\rm phys} \equiv k/a$) and $c_1 \sim \mathcal{O}(1)$ during slow-roll inflation. As pointed out in Ref. [50], the amplitude of the modes at all scales blows up at the time corresponding to this divergence. At this point, the linear theory of cosmological perturbations that we used is no longer valid. One way to avoid this problem is to assume that all of the physical wave numbers $k_{\rm phys} < M_{\rm PV}$ at the beginning of inflation. On the other hand, we also require that all of the relevant perturbation modes are well inside the Hubble horizon, i.e., $k_{\rm phys} > H$ at the beginning of inflation, such that the quantum tensor perturbations originate from a Bunch-Davies vacuum state. With the above two assumptions, it is obvious that $\epsilon_* \ll 1$ during slow-roll inflation.

Similarly, the parameter μ_A, which modifies the dispersion relation of the tensor modes, can be expressed in the form

$$\mu_A = \frac{\rho_A k(c_1 - c_2)/(aM_{\rm PV})}{1-\rho_A k c_1/(aM_{\rm PV})} \simeq -\rho_A k\tau(c_1 - c_2)\epsilon_*. \tag{4.9}$$

It is worth noting that, in order to obtain the above expansions, we have used the relation

$$a = -\frac{1}{\tau H}(1 + \epsilon_1 + \epsilon_1^2 + \epsilon_1\epsilon_2) + \mathcal{O}(\epsilon^3). \tag{4.10}$$

With the expressions for z''/z and μ_A, one observes that the equation of motion in Eq. (4.6) can be cast in the form

$$\begin{aligned}&u''_A + \left\{[1 - \rho_A k\tau(c_1 - c_2)\epsilon_*]k^2 - \frac{v_t^2 - \frac{1}{4}}{\tau^2} + \rho_A\frac{k}{\tau}c_1\epsilon_*\right\}u_A \\ &\quad = 0.\end{aligned} \tag{4.11}$$

When $c_1 = c_2$, this equation reduces to the same form as that in Chern-Simons gravity, which admits an exact solution in terms of confluent hypergeometric functions [46]. However, when $c_1 \neq c_2$, this equation does not have exact solutions. In order to obtain its solution, we have to consider some approximations. In general, the most widely considered approach is the WKB approximation, if the WKB condition is satisfied during the whole process. However, in some cases, the WKB condition may be violated or not be fulfilled completely (see Ref. [60]). Recently, we have developed a mathematical approximation (the uniform asymptotic approximation) to better treat equations with turning points and poles, an approximation that has been verified to be powerful and robust in calculating primordial spectra for various inflation models [60,60–76] and applications in studying the reheating process [77] and quantum mechanics [78]. In the following subsections, we apply this approximation to construct the approximate solution of Eq. (4.11) and calculate the primordial tensor power spectrum in general ghost-free parity-violating gravity.

B. Uniform asymptotic approximation

In this subsection, we will apply the uniform asymptotic approximation method to construct approximate asymptotic solutions. To proceed, let us first rewrite the equation of motion (4.11) in the following standard form [60,76]:

$$\frac{d^2 u_A(y)}{dy^2} = [g(y) + q(y)]u_A(y), \tag{4.12}$$

where $y \equiv -k\tau$ is a dimensionless variable and

$$g(y) + q(y) \equiv \frac{v_t^2 - \frac{1}{4}}{y^2} + \frac{\rho_A c_1 \epsilon_*}{y} - \rho_A y(c_1 - c_2)\epsilon_* - 1. \tag{4.13}$$

In general, $g(y)$ and $q(y)$ have two poles (singularities): one is at $y = 0^+$ and the other is at $y = +\infty$. Now, in order to construct the approximate solution in the uniform asymptotic approximation, one has to choose [60,75]

$$q(y) = -\frac{1}{4y^2} \tag{4.14}$$

to ensure the convergence of the errors of the approximate solutions around the second-order pole at $y = 0^+$. With this choice, the function $g(y)$ is given by

$$g(y) = \frac{v_t^2}{y^2} - 1 - \rho_A y(c_1 - c_2)\epsilon_* + \frac{\rho_A c_1 \epsilon_*}{y}. \tag{4.15}$$

Except for the two poles at $y = 0^+$ and $y = +\infty$, $g(y)$ may also have a single zero in the range $y \in (0, +\infty)$, which is called a single turning point of $g(y)$. By solving the equation $g(y) = 0$, we obtain the turning point

$$y_0^A = -\frac{1}{3\rho_A(c_1 - c_2)\epsilon_*}[1 - 2^{1/3}(1 + 3\rho_A^2(c_1 - c_2)c_1\epsilon_*^2)/Y - 2^{-1/3}Y], \tag{4.16}$$

where

$$Y = (Y_1 + \sqrt{-4(1 + 3\rho_A^2(c_1 - c_2)c_1\epsilon_*^2)^3 + Y_1^2})^{1/3},$$
$$Y_1 = -2 + 27v_T^2(\rho_A c_1 - \rho_A c_2)^2\epsilon_*^2 - 9\rho_A^2(c_1 - c_2)c_1\epsilon_*^2.$$

In the uniform asymptotic approximation, the approximate solution depends on the type of turning point. Thus, in the following discussion, we will discuss the solution around this turning point in detail.

For the single turning point y_0, the approximate solution of the equation of motion around this turning point can be expressed in terms of Airy-type functions as [60,75]

$$u_A = \alpha_0 \left(\frac{\xi}{g(y)}\right)^{1/4} \mathrm{Ai}(\xi) + \beta_0 \left(\frac{\xi}{g(y)}\right)^{1/4} \mathrm{Bi}(\xi), \tag{4.17}$$

where $\mathrm{Ai}(\xi)$ and $\mathrm{Bi}(\xi)$ are the Airy functions, α_0 and β_0 are two integration constants, and ξ is a function of y given by [60,75]

$$\xi(y) = \begin{cases} \left(-\frac{3}{2}\int_{y_0}^{y}\sqrt{g(y')}dy'\right)^{2/3}, & y \leq y_0, \\ -\left(\frac{3}{2}\int_{y_0}^{y}\sqrt{-g(y')}dy'\right)^{2/3}, & y \geq y_0. \end{cases} \tag{4.18}$$

With this solution, we need to determine the coefficients α_0 and β_0 by matching it with the initial condition in the limit $y \to +\infty$. For this purpose, we assume that the universe was initially in an adiabatic vacuum [60,75],

$$\lim_{y\to+\infty} u_k(y) = \frac{1}{\sqrt{2\omega_k}} e^{-i\int \omega_k d\eta} = \sqrt{\frac{1}{2k}}\left(\frac{1}{-g}\right)^{1/4} \exp\left(-i\int_{y_i}^{y}\sqrt{-g}dy\right). \tag{4.19}$$

When $y \to +\infty$, we note that $\xi(y)$ is very large and negative. In this limit, the asymptotic forms of the Airy functions read

$$\mathrm{Ai}(-x) = \frac{1}{\pi^{1/2}x^{1/4}}\cos\left(\frac{2}{3}x^{3/2} - \frac{\pi}{4}\right), \tag{4.20}$$

$$\mathrm{Bi}(-x) = -\frac{1}{\pi^{1/2}x^{1/4}}\sin\left(\frac{2}{3}x^{3/2} - \frac{\pi}{4}\right). \tag{4.21}$$

Combining the initial condition and the approximate analytical solution, we obtain

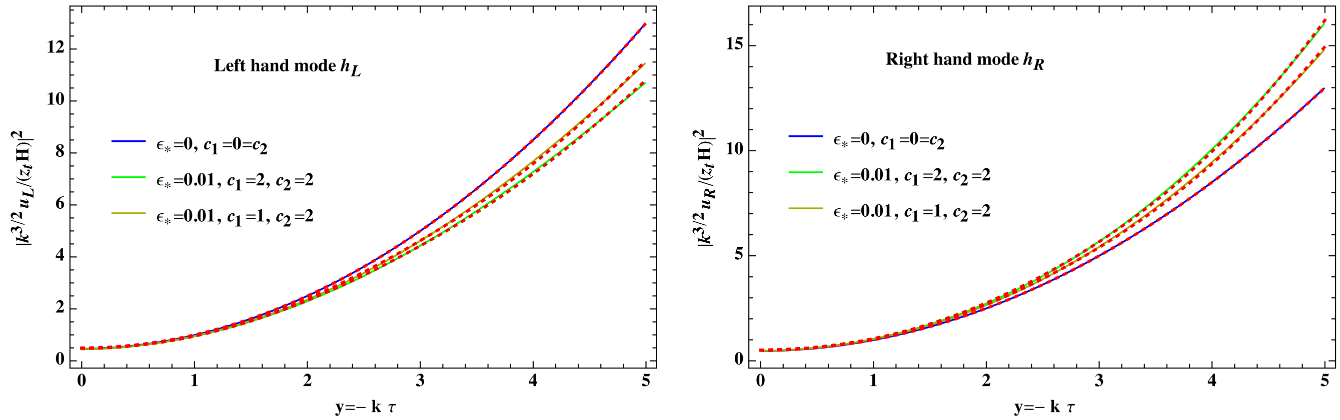

FIG. 1. Uniform asymptotic approximate solutions of mode functions $|k^{3/2}u_A/(z_t H)|^2$ (solid curves) and the corresponding numerical solutions (dotted curves). The left and right panels show the solutions of the left-hand and right-hand modes, respectively. In each panel, the solid blue, green, and darker yellow curves correspond to the solutions for general relativity, Chern-Simons theory, and ghost-free parity-violating gravities, respectively. The numerical solution associated with each analytical solution is shown by the red dotted curves.

$$\alpha_0 = \sqrt{\frac{\pi}{2k}} e^{i\frac{\pi}{4}}, \qquad \beta_0 = i\sqrt{\frac{\pi}{2k}} e^{i\frac{\pi}{4}}. \tag{4.22}$$

Having obtained the approximate solutions of the mode functions $u_{\rm R,L}(y)$ as given by Eq. (4.17) with the coefficients α_0 and β_0 given above, let us compare them with the numerical solutions. The results are presented in Fig. 1, in which we present the uniform asymptotic approximate solutions (solid blue, green, darker yellow curves) and numerical solutions (red dotted curves) of mode functions $|k^{3/2}u_A/(z_t H)|^2$ in general relativity, Chern-Simons theory, and ghost-free parity-violating gravities, respectively. The left and right panels show the solutions of the left-hand and right-hand modes, respectively. From this figure, one can clearly see that our analytical solutions are extremely close to the numerical ones; at times they are indistinguishable. For the values of c_1, c_2, and ϵ_* used in this figure, we see that the right-hand and left-hand modes tend to be enhanced and suppressed by the parity violation, respectively. This feature is also consistent with the analytical results of the power spectra calculated later in the next subsection.

C. Power spectra of PGWs

Once the approximate solutions of the PGWs have been derived in the manner described above, the relevant power spectra $\mathcal{P}_{\rm T}^{L,R}$ can be computed in the limit $y \to 0$ via

$$\mathcal{P}_{\rm T}^{\rm L} = \frac{2k^3}{\pi^2}\left|\frac{u_k^{\rm L}(y)}{z}\right|^2, \qquad \mathcal{P}_{\rm T}^{\rm R} = \frac{2k^3}{\pi^2}\left|\frac{u_k^{\rm R}(y)}{z}\right|^2. \tag{4.23}$$

When $y \to 0^+$, the argument of the Airy functions $\xi(y)$ becomes very large and positive, allowing the use of the following asymptotic forms:

$$\mathrm{Ai}(x) = \frac{1}{2\pi^{1/2}x^{1/4}} \exp\left(-\frac{2}{3}x^{3/2}\right), \tag{4.24}$$

$$\mathrm{Bi}(x) = -\frac{1}{\pi^{1/2}x^{1/4}} \exp\left(\frac{2}{3}x^{3/2}\right). \tag{4.25}$$

From the Airy functions (4.24) we observe that, in this limit, only the growing mode of $u_k^A(y)$ is relevant, so we have

$$\begin{aligned} u_k^A(y) &\approx \beta_0 \left(\frac{1}{\pi^2 g(y)}\right)^{1/4} \exp\left(\int_y^{y_0} dy\sqrt{g(y)}\right) \\ &= i\frac{1}{\sqrt{2k}}\left(\frac{1}{g(y)}\right)^{1/4} \exp\left(\int_y^{y_0} dy\sqrt{g(y)}\right). \end{aligned} \tag{4.26}$$

The power spectra of PGWs are then given by

$$\begin{aligned} \mathcal{P}_{\rm T}^A &= \frac{k^2}{\pi^2}\frac{1}{z^2}\frac{y}{v_t} \exp\left(2\int_y^{y_0} dy\sqrt{g(y)}\right) \\ &\simeq 18\frac{H^2}{\pi^2 e^3} e^{\frac{\pi\rho_A\epsilon_*}{16}(9c_2-c_1)} \\ &\simeq 18\frac{H^2}{\pi^2 e^3}\left[1 + \frac{\pi\rho_A}{16}\mathcal{M}\epsilon_* + \frac{\pi^2}{2\times 16^2}\mathcal{M}^2\epsilon_*^2 + \mathcal{O}(\epsilon_*)^3\right], \end{aligned} \tag{4.27}$$

where we define the dimensionless parameter $\mathcal{M} \equiv 9c_2 - c_1$ and

$$\begin{aligned}\frac{9c_2 - c_1}{M_{\rm PV}} &= 8\dot{\vartheta} + 4\dot{a}_1\dot{\phi}^2 + 8a_1\dot{\phi}\ddot{\phi} - 8a_1H\dot{\phi}^2 - 16\dot{a}_2\dot{\phi}^2 \\ &\quad - 32a_2\dot{\phi}\ddot{\phi} - 10\dot{a}_3\dot{\phi}^2 - 20a_3\dot{\phi}\ddot{\phi} + 4a_3H\dot{\phi}^2 \\ &\quad - 32\dot{a}_4\dot{\phi}^2 - 64a_4\dot{\phi}\ddot{\phi} + 2b_1\dot{\phi}^3 \\ &\quad - 4b_2(2H\dot{\phi}^2 - \dot{\phi}\ddot{\phi}) - 2b_3(\dot{\phi}^3\ddot{\phi} - H\dot{\phi}^4) \\ &\quad - 2b_4(\dot{\phi}^3\ddot{\phi} - H\dot{\phi}^4) + 2b_5H\dot{\phi}^4 - 2b_7\dot{\phi}^3\ddot{\phi}.\end{aligned} \tag{4.28}$$

Obviously, the power spectra can be modified due to the presence of parity-violating terms in the action (2.1). As expected, one can check that when $\mathcal{M}\epsilon_* = 0$ the standard GR result is recovered. Therefore, we can rewrite the power spectra in Eq. (4.27) as follows:

$$\mathcal{P}_{\rm T}^{A} = \frac{\mathcal{P}_{\rm T}^{\rm GR}}{2}\left[1 + \frac{\pi\rho_A}{16}\mathcal{M}\epsilon_* + \frac{\pi^2\rho_A^2}{2\times 16^2}\mathcal{M}^2\epsilon_*^2 + \mathcal{O}(\epsilon_*)^2\right], \tag{4.29}$$

where

$$\mathcal{P}_{\rm T}^{\rm GR} = \frac{2k^3}{\pi^2}\left(\left|\frac{u_k^{\rm L}(y)}{z}\right|^2 + \left|\frac{u_k^{\rm R}(y)}{z}\right|^2\right) \tag{4.30}$$

denotes the standard nearly scale-invariant power-law spectrum calculated using the uniform asymptotic approximation in the framework of GR [60]. For the two circular polarization modes, i.e., $A = {\rm R}$ and $A = {\rm L}$, the spectra $\mathcal{P}_{\rm T}^{\rm GR}$ have exactly the same form. The quantity $\mathcal{M}$ depends on the coefficients ϑ, $a_{\mathcal{A}}$, and $b_{\mathcal{A}}$, as well as the evolution of the scalar field. It is interesting to observe that for positive value of $\mathcal{M}$, the parity violation tends to enhance (suppress) the power spectra of the right- (left-)hand modes. During slow-roll inflation the scalar field is slow rolling, which satisfies the slow-roll conditions (3.4). With this condition, the quantities c_1 and c_2 are assumed to be slowly varying during the expansion of the universe and can be approximately treated as constants during slow-roll inflation. We observe that the expression for $9c_2 - c_1$ contains terms with ϑ, $a_{\mathcal{A}}$, $b_{\mathcal{A}}$ and their derivatives with respect to ϕ. Considering the scalar field ϕ with the slow-roll condition (3.4), the leading contribution to $9c_2 - c_1$ reads

$$\frac{9c_2 - c_1}{M_{\rm PV}} \simeq 8\dot{\vartheta} - 8\left(a_1 - \frac{a_3}{2} + b_2\right)H\dot{\phi}^2. \tag{4.31}$$

Therefore, the leading contribution to the power spectra of PGWs depends only on the coefficients $\dot{\vartheta}$, a_1, a_3, and b_2.

D. Circular polarization and detectability

Now we are in a position to calculate the degree of the circular polarization of PGWs, which is defined by the differences of the amplitudes between the two circular polarization states of PGWs as

$$\begin{aligned}\Pi &\equiv \frac{\mathcal{P}_{\rm T}^{\rm R} - \mathcal{P}_{\rm T}^{\rm L}}{\mathcal{P}_{\rm T}^{\rm R} + \mathcal{P}_{\rm T}^{\rm L}} \simeq \frac{\pi}{16}(9c_2 - c_1)\epsilon_* + \mathcal{O}(\epsilon_*^3) \\ &\simeq \frac{\pi}{2}\dot{\vartheta}M_{\rm PV}\epsilon_* - \frac{\pi}{2}\left(a_1 - \frac{a_3}{2} + b_2\right)H\dot{\phi}^2M_{\rm PV}\epsilon_* + \mathcal{O}(\epsilon_*^3).\end{aligned} \tag{4.32}$$

As expected, when $a_1 = a_3 = b_2 = 0$ the above expression of the circular polarization Π exactly reduces to that in Chern-Simons gravity [79,80]. Obviously, under the conditions (3.4), we observe that the degree of the circular polarization Π is very small due to the suppressing parameter ϵ_*.

As we mentioned in the Introduction, the parity-violating effect in PGWs—which is measured by the observable Π—can produce the TB and EB spectra in the CMB data. This provides the opportunity to directly detect the chiral asymmetry of gravity in observational data, which has been discussed in the literature (see Refs. [17,18,20] for examples). However, as pointed out in Ref. [34], the detectability of the circular polarization of PGWs is sensitive to the values of the tensor-to-scalar ratio r and Π. According to the combination of *Planck* 2018 data and the BICEP2/Keck Array BK15 data [4], r has been tightly constrained as $r \lesssim 0.065$. For this case, in order to detect any signal of parity violation in forthcoming CMB experiments, Π must be larger than $\mathcal{O}(0.5)$ as discussed, even in an ideal case with the cosmic variance limit. On the other hand, since the condition $\epsilon_* \ll 1$ is imposed for the considerations made in constructing the theory, the order of magnitude of Π is roughly $\lesssim\mathcal{O}(0.5)$. For these reasons, we conclude that it would be difficult to detect or efficiently constrain parity violation effects using only the two-point statistics of future cosmic microwave background data.

V. CONCLUSIONS AND OUTLOOK

In this paper, we studied the circular polarization of PGWs in the ghost-free parity-violating theory of gravity, which generalizes Chern-Simons gravity by including the first and second derivatives of the coupling scalar field. Applying the uniform asymptotic approximation to the equation of motion for the PGWs, we constructed the approximate analytical solution to the PGWs during slow-roll inflation. Using this approximate solution, we explicitly calculated both the power spectra for the two polarization modes and the corresponding degree of circular polarization of PGWs. It was shown that in the presence of parity violation the power spectra of PGWs are slightly modified and the degree of circular polarization becomes nonzero. However, the circular polarization generated in the ghost-free parity-violating theory of gravity is quite small, which is suppressed by the energy scale of parity violation of the theory, and it would be difficult to detect using only the power spectra of future CMB data.

It should be noted that in all of the above discussions the effect of parity violation on the non-Gaussianity of PGWs was not considered. Although there is very little hope of detecting parity-violation signatures in the two-point correlation of CMB data, a calculation in Chern-Simons gravity shows that parity-violation signatures in the bispectrum could be large enough to be detectable in future CMB data [46] (Note that a similar analysis for Hořava-Lifshitz gravity with parity violation was also carried out in Ref. [36]). According to the analysis in Ref. [46], the tensor-tensor-scalar bispectra for each polarization state can be peaked in the squeezed limit by setting the level of parity violation during inflation. Therefore, it would be interesting to further explore whether the ghost-free parity-violating theory of gravity could lead to any parity-violation signatures in the non-Gaussianity of PGWs. We leave this topic to future work.

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