Class. Quantum Grav. 23 (2006) 3405-3417

The state equation of Yang–Mills field dark energy models

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Received 5 January 2006 Published 24 April 2006 Online at stacks.iop.org/CQG/23/3405

Abstract

In this paper, we study the possibility of building Yang–Mills (YM) field dark energy models with equation of state (EoS) crossing -1, and find that it cannot be realized by the single YM field models, no matter what kind of Lagrangian or initial condition. But the states of $-1 < \omega < 0$ and $\omega < -1$ all can be naturally got in this kind of model. The former is like a quintessence field, and the latter is like a phantom field. This states that one can build a model with two YM fields, one with the initial state of $-1 < \omega < 0$, and the other with $\omega < -1$. We give an example model of this kind, and find that its EoS is larger than -1 in the past and less than -1 at the present time. We also find that this change must be from $\omega > -1$ to < -1, and it will go to the critical state of $\omega = -1$ with the expansion of the universe, which character is the same as the single YM field models, and the big rip is naturally avoided.

PACS numbers: 98.80.-k, 98.80.Es, 04.30.-w, 04.62.+v

1. Introduction

Recent observations on the type Ia supernova (SNIa) [1], cosmic microwave background radiation (CMB) [2] and large scale structure (LSS) [3] all suggest that the universe mainly consists of dark energy (73%), dark matter (23%) and baryon matter (4%). How to understand the physics of the dark energy is an important mission in the modern cosmology, which has the EoS of $\omega < -1/3$, and leads to the recent accelerating expansion of the universe. Several scenarios have been put forward as a possible explanation of this. A positive cosmological constant is the simplest candidate, but it needs extreme fine tuning to account for the observed

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accelerating expansion of the universe. This fact has led to models where the dark energy component varies with time, such as quintessence models [4], which assume that the dark energy is made of a single (light) scalar field. Despite some pleasing features, these models are not entirely satisfactory, since in order to achieve $\Omega_{de} \sim \Omega_m$ (where Ω_{de} and Ω_m are the dark energy and matter energy densities at present, respectively) some fine tuning is also required. Many other possibilities have been considered for the origin of this dark energy component such as a scalar field with a non-standard kinetic term and k-essence models [5]. It is also possible to construct models which have the EoS of $\omega = p/\rho < -1$, the so-called phantom [6]. Some other models such as the generalized Chaplygin gas (GCG) models [7] and the vector field models [8] have also been studied by a lot of authors. Although these models achieve some success, some problems also exist. To understand the nature of the dark energy, it is essential to detect the value and evolution of its EoS. The observation data show that the cosmological constant is a good candidate [9], which has the effective equation $p = -\rho$, i.e. $\omega \equiv -1$. However, there is evidence to show that the dark energy might evolve from $\omega > -1$ in the past to $\omega < -1$ today, and cross the critical state of $\omega = -1$ in the intermediate redshift [10]. If such a result holds with accumulation of observational data, this would be a great challenge to the current models of dark energy. It is obvious that the cosmological constant as a candidate will be excluded, and the dark energy must be dynamical. But the normal models, such as the quintessence fields, can only give the state of $-1 < \omega < 0$. Although the k-essence models and the phantom models can get the state of $\omega < -1$, the behaviour of ω crossing -1 cannot be realized, and all these will lead to theoretical problems in the field theory. To answer this crossing phenomenon of ω , many people have advised some more complex models, such as the quintom models [11, 12], which are made of a quintessence field and a phantom field. A model with a higher derivative term has been suggested in [13], which also can get from $\omega > -1$ to $\omega < -1$, but it also will lead to theoretical difficulty in the field theory.

We have advised that the YM field [14, 15] can be used to describe the dark energy. There are two major reasons that prompt us to study this system. First, in the normal scalar models the connection of field to particle physics models has not been clear so far. The second reason is that the weak energy condition cannot be violated by the field. The YM field we have advised has the desired interesting feature: the YM field is the indispensable cornerstone to any particle physics model with interactions mediated by gauge bosons, so it can be incorporated into a sensible unified theory of particle physics. Besides, the EoS of matter for the effective YM condensate is different from that of ordinary matter as well as the scalar fields, and the states of $-1 < \omega < 0$ and $\omega < -1$ can also be naturally realized. But is it possible to build a YM field model with EoS crossing -1? In this paper, we focus on this topic. First we consider the YM field with a general Lagrangian, and find that the state of $\omega \sim -1$ is easily realized, as long as it satisfies some constraint. From the kinetic equation of the YM field, we find that $\omega + 1 \propto a^{-2}$ with the expansion of the universe. But no matter what kind of Lagrangian and initial condition we choose, this model cannot get the behaviour of ω crossing -1. But it can be easily got in the models with two YM fields, one with the initial condition of $\omega > -1$, which is like a quintessence field, and the other with $\omega < -1$ like a phantom field.

This paper is organized as follows. In section 2 we discuss the general YM field model, and study the evolution of its EoS by solving its kinetic equation. But we find that this kind of model cannot get the state of ω crossing -1. Then we study the two YM fields model in section 3, and solve the evolution of ω with scale factor for an example model. We find that ω crossing -1 can be easily realized in this model, which is very like the quintom models. Finally, we have a conclusion and discussion in section 4.

2. Single YM field model

In [15], we have discussed the EoS of the YM field dark energy models, which has the effective Lagrangian [16, 17]

$$\mathcal{L}_{\rm eff} = F/2g^2,\tag{1}$$

where $F = -\frac{1}{2}F^a_{\mu\nu}F^{a\mu\nu}$ plays the role of the order parameter of the YM condensate, and g is the running coupling constant which, up to one-loop order, is given by

$$\frac{1}{g^2} = b \ln \left| \frac{F}{\kappa^2} - 1 \right|. \tag{2}$$

Thus the effective Lagrangian is

$$\mathcal{L}_{\rm eff} = \frac{b}{2} F \ln \left| \frac{F}{e\kappa^2} \right|,\tag{3}$$

where $e \simeq 2.72$, $b = 11N/24\pi^2$ for the generic gauge group SU(N) is the Callan–Symanzik coefficient [18], κ is the renormalization scale with the dimension of squared mass, the only model parameter. The attractive features of this effective YM action model include the gauge invariance, the Lorentz invariance, the correct trace anomaly and the asymptotic freedom [16]. With the logarithmic dependence on the field strength, \mathcal{L}_{eff} has a form similar to the Coleman–Weinberg scalar effective potential [19], and the Parker–Raval effective gravity Lagrangian [20].

It is straightforward to extend this model to the expanding Robertson–Walker (R–W) spacetime. For simplicity we will work in a spatially flat R–W spacetime with a metric

$$\mathrm{d}s^2 = a^2(\tau) \,(\mathrm{d}\tau^2 - \gamma_{ij} \,\mathrm{d}x^i \,\mathrm{d}x^j),\tag{4}$$

where we have set the speed of light $c \equiv 1$, $\gamma_{ij} = \delta_j^i$ denoting the background space is flat and $\tau = \int (a_0/a) dt$ is the conformal time. Consider the dominant YM condensate minimally coupled to the general relativity with the effective action,

$$S = \int \sqrt{-\tilde{g}} \left[-\frac{R}{16\pi G} + \mathcal{L}_{\text{eff}} \right] \mathrm{d}^4 x, \tag{5}$$

where \tilde{g} is the determinant of the metric $g_{\mu\nu}$. By variation of *S* with respect to the metric $g^{\mu\nu}$, one obtains the Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, where the energy-momentum tensor is given by

$$T_{\mu\nu} = \sum_{a} \frac{g_{\mu\nu}}{4g^2} F^a_{\sigma\delta} F^{a\sigma\delta} + \epsilon F^a_{\mu\sigma} F^{a\sigma}_{\nu}.$$
 (6)

The dielectric constant is defined by $\epsilon = 2\partial \mathcal{L}_{eff} / \partial F$, and in this one-loop order it is given by

$$\epsilon = b \ln \left| \frac{F}{\kappa^2} \right|. \tag{7}$$

This energy-momentum tensor is the sum of the several different energy-momentum tensors of the vectors, $T_{\mu\nu} = \sum_{a} {}^{(a)}T_{\mu\nu}$, neither of which is of prefect-fluid form, which can make the YM field anisotropic. This is one of the most important characters of the vector field dark energy models [8]. If it is true and this anisotropic YM field is dominant in the universe, this will make the universe anisotropic, one would expect an anisotropic expansion of the universe, in conflict with the significant isotropy of the CMB [21]. But on the other hand there also appear to be hints of statistical anisotropy in the CMB perturbations [22]. But here we only consider the other case. To keep the total energy-momentum tensor $T_{\mu\nu}$ homogeneous and isotropic, here we assume that the gauge fields are functions only of time t, and $A_{\mu} = \frac{i}{2}\sigma_a A^a_{\mu}(t)$ (here σ_a are Pauli matrices) are given by $A_0 = 0$ and $A^a_i = \delta^a_i A(t)$. Define the YM field tensor as usual:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu, \tag{8}$$

where f^{abc} is the structure constant of the gauge group and $f^{abc} = \epsilon^{abc}$ for the SU(2) case. This tensor can be written in the form with the electric and magnetic fields as

$$F_{\nu}^{a\mu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}.$$
(9)

It can easily be found that $E_1^2 = E_2^2 = E_3^2$, and $B_1^2 = B_2^2 = B_3^2$. Thus *F* has a simple form with $F = E^2 - B^2$, where $E^2 = \sum_{i=1}^3 E_i^2$ and $B^2 = \sum_{i=1}^3 B_i^2$. In this case, each component of the energy-momentum tensor is

$${}^{(a)}T^{0}_{\mu} = \frac{1}{6g^{2}} \left(B^{2} - E^{2}\right)\delta^{0}_{\mu} + \frac{\epsilon}{3}E^{2}\delta^{0}_{\mu}, \tag{10}$$

$${}^{(a)}T^{i}_{j} = \frac{1}{6g^{2}} \left(B^{2} - E^{2}\right)\delta^{i}_{j} + \frac{\epsilon}{3}E^{2}\delta^{i}_{j}\delta^{a}_{j} - \frac{\epsilon}{3}B^{2}\delta^{i}_{j}\left(1 - \delta^{a}_{j}\right).$$
(11)

Although this tensor is not isotropic, its value along the j = a direction is different from the one along the directions perpendicular to it. Nevertheless, the total energy–momentum tensor $T_{\mu\nu} = \sum_{a=1}^{3} {}^{(a)}T_{\mu\nu}$ has isotropic stresses, and the corresponding energy density and pressure are given by (here we only consider the condition of $B^2 \equiv 0$) [15]

$$\rho = \frac{E^2}{2} (\epsilon + b), \qquad p = \frac{E^2}{2} \left(\frac{\epsilon}{3} - b\right), \tag{12}$$

and its EoS is

$$\omega = \frac{\epsilon - 3b}{3\epsilon + 3b}.\tag{13}$$

It is easily found that at the critical point of $\epsilon = 0$, which follows that $\omega = -1$, the universe is exactly a de Sitter expansion. Near this point, if $\epsilon < 0$, we have $\omega < -1$, and $\epsilon > 0$ follows $\omega > -1$. So in these models, the EoS of $0 < \omega < -1$ and $\omega < -1$ all can be naturally realized.

To study the evolution of this EoS, we should solve the YM field equations, which is equivalent to solving the Einstein equation [15]. By variation of S with respect to A^a_{μ} , one obtains the effective YM equations

$$\partial_{\mu}(a^{4}\epsilon F^{a\mu\nu}) + f^{abc}A^{b}_{\mu}(a^{4}\epsilon F^{c\mu\nu}) = 0.$$
(14)

As we have assumed the YM condensate is homogeneous and isotropic, from the definition of $F^a_{\mu\nu}$, it is easily found that the $\nu = 0$ component of YM equations is an identity and the i = 1, 2, 3 spatial components are

$$\partial_{\tau}(a^2 \epsilon E) = 0. \tag{15}$$

If $\epsilon = 0$, this equation is also an identity. When $\epsilon \neq 0$, this equation follows that [15]

$$\beta e^{\beta/2} \propto a^{-2}, \tag{16}$$

where we have defined $\beta \equiv \epsilon/b$, and used the expression of ϵ in equation (7). In this equation, the proportion factor can be fixed by the initial condition. This is the main equation, which determines the evolution of β , and β directly relates to the EoS of the YM field. Combining

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Figure 1. The evolution of the state equation ω with the scale factor *a*. Model 1 denotes the single YM field model with the present EoS $\omega_0 = -1.2$, model 2 denotes the model with $\omega_0 = -0.8$, and model 3 denotes the two YM fields model, where we have set $\kappa_1^2 = 1.5\kappa_2^2$, and $\beta_1 = -0.4$, $\beta_2 = 0.2$ at present with $a_0 = 1$.

equations (13) and (16), one can obtain the evolution of EoS in the YM field dark energy universe. In figure 1, we plot the evolution of ω in the YM field dark energy models with the present value $\omega_0 = -1.2$ and $\omega_0 = -0.8$, and find that the former one is very like the evolution of the phantom field, and the latter is like a quintessence field. They all have the same attractor solution with $\omega = -1$. So in these models, the big rip is naturally avoided. This is the most attractive feature of the YM field models.

In equation (16), the undetermined factor can be fixed by the present value of EoS ω_0 , which must be determined by observations on SNIa, CMB or LSS. In this paper, we will only show that the observation of the CMB power spectrum is an effective way to determine it. The dark energy can influence the CMB temperature anisotropy power spectrum (especially at the large scale) by the integral Sachs–Wolfe (ISW) effect [23]. Consider the flat R–W metric with the scalar perturbation in the conformal Newtonian gauge,

$$ds^{2} = a^{2}(\tau)[(1+2\phi) d\tau^{2} - (1-2\psi)\gamma_{ij} dx^{i} dx^{j}].$$
(17)

The gauge-invariant metric perturbation ψ is the Newtonian potential and ϕ is the perturbation to the intrinsic spatial curvature. Always the background matters in the universe are perfect fluids without anisotropic stress, which follows that $\phi = \psi$. So there is only one perturbation function ϕ in the metric of (17), and its evolution is determined by [24]

$$\phi'' + 3\mathcal{H}\left(1 + \frac{p'}{\rho'}\right)\phi' - \frac{p'}{\rho'}\nabla^2\phi + \left[\left(1 + 3\frac{p'}{\rho'}\right)\mathcal{H}^2 + 2\mathcal{H}'\right]\phi = 4\pi Ga^2\left(\delta p - \frac{p'}{\rho'}\delta\rho\right), \quad (18)$$

where $\mathcal{H} \equiv a'/a$, and 'prime' denotes $d/d\tau$. The pressure $p = \sum_i p_i$, and energy density $\rho = \sum_i \rho_i$, which should include the contribution of baryon, photon, neutron, cold dark matter and the dark energy. Especially at late time of the universe, the effect of the dark energy is very important. We recall that the ISW effect stems from the time variation of the metric perturbations,

$$C_l^{\rm ISW} \propto \int \frac{\mathrm{d}k}{k} \left[\int_0^{\chi_{\rm LSS}} \mathrm{d}\chi \left(\phi' + \psi'\right) j_l(k\chi) \right]^2, \tag{19}$$



Figure 2. The CMB anisotropy power spectrum. Model 1 denotes the single YM field model with the present EoS $\omega_0 = -1.2$, model 2 denotes the model with $\omega_0 = -0.8$ and model 3 denotes the two YM fields model, where we have set $\kappa_1^2 = 1.5\kappa_2^2$, and $\beta_1 = -0.4$, $\beta_2 = 0.2$ at present with $a_0 = 1$. In this figure, the dots denote observation results from the WMAP [2] satellite. Here we have used the CMBFAST program [25].

where χ_{LSS} is the conformal distance to the last scattering surface and j_l is the *l*th spherical Bessel function. The ISW effect occurs because photons can gain energy as they travel through time-varying gravitational wells. One always solves the CMB power spectrum in the numerical methods [25, 26]. In figure 2, we plot the CMB power spectrum at large scale with these two kinds of YM dark energy models, where we have chosen the cosmological parameters as the Hubble parameter h = 0.72, the energy density of baryon $\Omega_b h^2 = 0.024$ and dark matter $\Omega_{\rm dm}h^2 = 0.14$, the reionization optical depth $\tau = 0.17$, the spectrum index and amplitude of the primordial perturbation spectrum being $n_s = 0.99$ without running and A = 0.9. Here we have not considered the perturbation of the dark energy. From this figure, one can find that the values of the CMB power spectra are very sensitively dependent on ω_0 . Comparing with the Λ CDM model (which is equivalent to the YM model with $\omega_0 = -1$), the model with $\omega_0 = -0.8 > -1.0$, which is like the quintessence field model, the CMB spectra have smaller values, especially at a scale of l < 10 the difference is very obvious; but the model with $\omega_0 = -1.2 < -1.0$, which is like the phantom field model, the CMB spectra have larger values. As the evolution of EoS is only determined by ω_0 , the value of it can be determined by fitting the CMB observation. It is obvious that the recent observations on the CMB power spectra at large scale from the WMAP satellite have large error. Further results will depend on the observation of the following WMAP and Planck satellites.

Now let us return to the evolution of ω . From figure 1, one finds that ω crossing -1 cannot be realized in these models with a single YM field, no matter what values of ω_0 we have chosen. To study it more clearly, assume that the YM field has an initial state of $|\omega + 1| \ll 1$, which follows that $\beta \ll 1$, equation (16) becomes

$$\beta \propto a^{-2}$$
. (20)

The value of β will go to zero with the expansion of the universe. This means that *E* will go to a critical state of $E^2 = \kappa^2$. And the EoS is

$$\omega + 1 \simeq \frac{4\beta}{3} \propto a^{-2}.$$
(21)

This result has two important characters: (i) with the expansion of the universe, ω will go to the critical point of $\omega = -1$. This is the most important character of this dark energy model, which is very like the behaviour of the vacuum energy with $\omega \equiv -1$; (ii) the values of $\omega > -1$ and $\omega < -1$ all can be realized, but it cannot cross -1 from one area to another. This character is the same as the scalar field such as the quintessence field, the k-essence and the phantom field models.

It is interesting to ask if these characters are correct just for the YM model with the Lagrangian as formula (3) and whether or not one can build a model whose EoS can cross -1. So let us consider the YM field model with a general effective Lagrangian as

$$\mathcal{L}_{\rm eff} = G(F)F/2,\tag{22}$$

where G(F) is the running coupling constant, which is a general function of *F*. If we choose $G(F) = b \ln \left| \frac{F}{e\kappa^2} \right|$, this effective Lagrangian returns to the form in equation (3). The dielectric constant can also be defined by $\epsilon = 2\partial \mathcal{L}_{\text{eff}} / \partial F$, which is

$$\epsilon = G + FG_F. \tag{23}$$

Here G_F represents dG/dF. We also discuss the homogeneous and isotropic YM field with electric field (B = 0); then the energy density and the pressure of the YM field are

$$\rho = E^2 \left(\epsilon - \frac{G}{2} \right),\tag{24}$$

$$p = -E^2 \left(\frac{\epsilon}{3} - \frac{G}{2}\right). \tag{25}$$

The energy density $\rho > 0$ follows a constraint $G > -2FG_F$. The EoS of this YM field is

$$\omega = -\frac{3-2\gamma}{3-6\gamma},\tag{26}$$

where we have defined that $\gamma \equiv \epsilon/G$. When the condition of $\gamma = 0$ can be got at some state with $E^2 \neq 0$ and $G(F) \neq 0$, the state of $\omega = -1$ is naturally realized. This condition can be easily satisfied. In the discussion below we only consider these kinds of YM fields. For example, in the model with the Lagrangian (3), $\gamma = 0$ is got at the state $E^2 = \kappa^2$. Near this state, $\gamma > 0$ leads to $\omega < -1$, and $\gamma < 0$ leads to $\omega > -1$. But if the YM field has a trivial Lagrangian with G = constant, it follows that $\gamma \equiv 1$, and $\omega \equiv 1/3$. This is exactly the EoS of the relativistic matter, and it cannot generate the state of $\omega < 0$.

To study the evolution of EoS, we also consider the YM equation, which can be got by variation of S with respect to A_{μ}^{a} :

$$\partial_{\mu}(a^{4}\epsilon F^{a\mu\nu}) + f^{abc}A^{b}_{\mu}(a^{4}\epsilon F^{c\mu\nu}) = 0; \qquad (27)$$

from the definition of $F^a_{\mu\nu}$, it is found that these equations become a simple relation

$$\partial_{\tau}(a^2 \epsilon E) = 0, \tag{28}$$

where *E* is defined by $E^2 = \sum_{i=1}^{3} E_i^2$. If $\epsilon = 0$, this equation is an identity, and from (26), we know $\omega = -1$, which cannot be differentiated from the cosmological constant. When $\epsilon \neq 0$, this equation can be integrated to give

$$a^2 \epsilon E = \text{constant.}$$
 (29)

We want to study whether or not the EoS of this YM field can cross $\omega = -1$, here we assume that its initial state is $\omega \sim -1$. Under this condition, from the expression of p and ρ , it follows that $\epsilon \sim 0$, E and G(F) are nearly kept constant, which for the universe is nearly de Sitter

expansion and $\rho \sim -G(F)E^2/2$ is nearly a constant in this universe. So the YM equation suggests that

$$\epsilon \propto a^{-2}$$
. (30)

From the EoS of (26), one knows that

$$\omega + 1 \propto a^{-2}.\tag{31}$$

This is the EoS evolution equation of the general YM field dark energy models. It is exactly the same as the special case of equation (21). So it also keeps the characters of the special case with the Lagrangian (3): ω will run to the critical point $\omega = -1$ with the expansion of the universe. But it cannot cross this critical point. These are the general characters of these kinds of YM field dark energy models. To show this more clearly, we discuss two example models.

First we consider the YM field with the running coupling constant

$$G(F) = B\left(F^n - F_c^n\right),\tag{32}$$

where *B* and F_c are quantities with positive value, and *n* is a positive number. The constraint of $\rho > 0$ follows that

$$F > \frac{F_c}{\sqrt[n]{1+2n}}.$$
(33)

The dielectric constant can be easily got as

F

$$= G + FG_F = B(n+1)F^n - BF_c^n,$$
(34)

and

$$\gamma = (n+1) + \frac{nF_c^n}{F^n - F_c^n}.$$
(35)

It is obvious that when $F = F_c / \sqrt[n]{n+1}$, $\gamma = 0$ is satisfied, which leads to $\omega = -1$. Near this critical state, $E \sim \sqrt[2n]{F_c^n}/(n+1)$. So the YM equation of (29) becomes

$$\frac{An}{n+1}\sqrt[2n]{\frac{F_c^{3n}}{n+1}}\gamma \propto a^{-2},$$
(36)

which follows that $\gamma \propto a^{-2}$. From the expression of ω in equation (26), one can easily get

$$\omega + 1 \simeq -\frac{4\gamma}{3} \propto a^{-2}.$$
(37)

This is exactly the same as the evolution behaviour shown in formula (31).

In another example, we consider the YM field with the coupling constant of

$$G(F) = 1 - \exp\left(1 - \frac{F}{F_c}\right),\tag{38}$$

where the constant quantity $F_c \neq 0$. When $F \gg F_c$, this Lagrangian becomes the trivial case with G(F) = 1, but when F is near F_c , the nonlinear effect is obvious. Then

$$\epsilon = 1 + \left(\frac{F}{F_c} - 1\right) \exp\left(1 - \frac{F}{F_c}\right),$$

so the critical state of $F = 0.433 F_c$ leads to $\gamma = 0$ and $\omega = -1$. By similar discussion, from the YM field (29), one can also get $\gamma \propto a^{-2}$ near this critical state, which generates $\omega + 1 \propto a^{-2}$.

3. Two YM fields model

In the former section, we have discussed that the dark energy models with single YM field cannot form a state of ω crossing -1, no matter what kind of Lagrangian or initial condition. But we should note another character: the YM field has the EoS of $\omega \propto -1 + a^{-2}$, when its initial value is near the critical state of $\omega = -1$. So if the YM field has an initial state of $\omega > -1$, it will keep this state with the evolution of the universe, which is like the quintessence models. But if its initial state is $\omega < -1$, it will also keep it, which is like the phantom models. This states that we can build a model with two different free YM fields, one having an initial state of $\omega > -1$ and the other being $\omega < -1$. In this kind of model, the behaviour of ω crossing -1 is easily got. This idea is like the quintom models [11], where the authors built the model with a quintessence field and a phantom field.

In the discussion below, we will build a toy example of this kind of model. Assume that the dark energy is made of two YM fields with the effective Lagrangian as equation (3)

$$\mathcal{L}_{i} = \frac{b}{2} F_{i} \ln \left| \frac{F_{i}}{e \kappa_{i}^{2}} \right|, \qquad (i = 1, 2),$$
(39)

where $F_i = E_i^2$ (*i* = 1, 2), and $\kappa_1 \neq \kappa_2$. Their dielectric constants are

$$\epsilon_i \equiv \frac{2\partial \mathcal{L}_i}{\partial F_i} = b \ln \left| \frac{F_i}{\kappa_i^2} \right|. \tag{40}$$

From the YM field kinetic equations, we can also get the relations

$$a^2 \epsilon_i E_i = C_i, \tag{41}$$

where C_i (i = 1, 2) are the integral constants, which are determined by the initial state of the YM fields. If the YM field is a phantom-like field with $\omega_i < -1$, then $\epsilon_i < 0$ and $C_i < 0$. At the same time, a quintessence-like YM field follows that $C_i > 0$. Here we choose the YM field of \mathcal{L}_1 as the phantom-like field with $C_1 < 0$, and \mathcal{L}_2 as the quintessence-like field with $C_2 > 0$. The energy density and pressure are

$$\rho_i = \frac{E_i^2}{2} (\epsilon_i + b), \qquad p_i = \frac{E_i^2}{2} \left(\frac{\epsilon_i}{3} - b\right), \tag{42}$$

so the total EoS is

$$\omega \equiv \frac{p_1 + p_2}{\rho_1 + \rho_2} = \frac{E_1^2 \left(\frac{\beta_1}{3} - 1\right) + E_2^2 \left(\frac{\beta_2}{3} - 1\right)}{E_1^2 (\beta_1 + 1) + E_2^2 (\beta_2 + 1)},\tag{43}$$

where we have also defined that $\beta_i \equiv \epsilon_i/b$. Using the relation of β_i and E_i , we can simplify the equation of state as

$$\omega + 1 = \frac{4}{3} \frac{e^{\beta_1} \beta_1 \alpha + e^{\beta_2} \beta_2}{e^{\beta_1} (\beta_1 + 1) \alpha + e^{\beta_2} (\beta_2 + 1)},$$
(44)

where $\alpha \equiv \kappa_1^2 / \kappa_2^2$. We need this dark energy to have the initial state of $\omega > -1$, which requires that the field of ρ_2 is dominant at the initial time. This is easily obtained as long as at this time $E_1^2(\beta_1 + 1) < E_2^2(\beta_2 + 1)$ is satisfied. In the final state, we need to get $\omega < -1$, which means that ρ_1 is dominant, and the behaviour of crossing -1 is realized in the intermediate time. But how to get this? From the previous discussion, we know in the universe with only one kind of YM field (i = 1 or 2), the YM equation follows that $\epsilon_i \propto a^{-2}$. And it will go to the critical state $\epsilon_i = 0$ with the expansion of the universe. At this state, $E_i^2 = \kappa_i^2$, and $\rho_i = bE_i^2/2 = b\kappa_i^2/2$ stays constant. So in this two YM fields model, if we choose the condition $\kappa_1^2 > \kappa_2^2(\alpha > 1)$, this may follow that the final energy density $\rho_1 > \rho_2$, and ρ_1 is dominant. With this intent,



Figure 3. The evolution of β_1 and β_2 with the scale factor *a*, where we have set $\kappa_1^2 = 1.5\kappa_2^2$, and $\beta_1 = -0.4$, $\beta_2 = 0.2$ at present with $a_0 = 1$.

we build this model with the condition as below: choosing $\alpha = 1.5$, which can ensure the final state, the first kind of YM field (i = 1) is the dominant matter. For the present state, corresponding to the scale factor $a_0 = 1$, we choose $\beta_1 = -0.4 < 0$ and $\beta_2 = 0.2 > 0$, which keeps the first field always having a state of $\omega_1 < -1$ (like the phantom) and the second field with $\omega_2 > -1$ (like the quintessence). This choice of β_i leads to the present EoS

$$\omega = -1 + \frac{4}{3} \frac{e^{\beta_1} \beta_1 \alpha + e^{\beta_2} \beta_2}{e^{\beta_1} (\beta_1 + 1) \alpha + e^{\beta_2} (\beta_2 + 1)} = -1.10 < -1,$$
(45)

which is like the phantom field. Since ρ_1 increases, and ρ_2 decreases with the expansion of the universe, there must exist a time before which ρ_2 is dominant, and this leads to the total EoS $\omega > -1$ at that time.

Combining equations (41) and (44), we can solve the evolution of EoS ω with the scale time in a numerical calculation, where the relation of C_1 and C_2 is easily obtained

$$\frac{C_1}{C_2} = \frac{\beta_1 \, \mathrm{e}^{\beta_1/2}}{\beta_2 \, \mathrm{e}^{\beta_2/2}} = -1.48.$$

For each kind of YM field, its EoS is

$$\omega_i = \frac{\beta_i - 3}{3\beta_i + 3} \qquad (i = 1, 2). \tag{46}$$

The condition $\beta_i > 0$ ($\beta_i < 0$) will generate $\omega_i > -1$ ($\omega_i < -1$). The evolution of these is shown in figure 3. This is the exact result we expect; β_i runs to the critical point of $\beta_i = 0$ with the expansion of the universe, which makes ω_i run to $\omega_i = -1$, no matter what kind of initial values. This is the same as the single YM field model. With $\beta_i \rightarrow 0$, the strength of the field E_i^2 will also go to its critical point $E_i^2 = \kappa_i^2$. This is shown in figure 4. As we have chosen the condition $\alpha \equiv \kappa_1^2 / \kappa_2^2 > 1$, it must lead to $E_1^2 > E_2^2$ at some time, the first kind of YM field becomes dominant, and the total EoS $\omega < -1$ is realized. This can be shown in figure 1. In figure 2, we also plot the CMB power spectrum in the universe with this kind of YM field dark energy, and find that it is difficult to distinguish from the Λ CDM universe, which for the effect of the dark energy on the CMB power spectrum is an integral effect from CMB



Figure 4. The evolution of E_1 and E_2 with the scale factor *a*, where we have set $\kappa_1^2 = 1.5\kappa_2^2$, and $\beta_1 = -0.4$, $\beta_2 = 0.2$ at present with $a_0 = 1$.

decoupling time to now. But the evolution detail of ω is not obvious. This is the disadvantage of this way to detect dark energy.

Now, let us conclude this dark energy model, which is made of two YM fields. One has the EoS of $\omega_1 < -1$ and the other has $\omega_2 > -1$. At the initial time, we choose their condition to make $\rho_1 < \rho_2$, and the second kind of YM field is dominant, which makes the total EoS $\omega > -1$ at this time. This is like the quintessence model. For $\omega_1 < -1$ field for all time, from the Friedmann equations, one knows that its energy density will enhance with the expansion of the universe. And finally it will run to its critical point $\rho_1 = b\kappa_1^2/2$. And at the same time, ρ_2 will decrease to its critical state $\rho_2 = b\kappa_2^2/2$. For we have chosen $\kappa_1^2/\kappa_2^2 > 1$, which must make $\rho_1 = \rho_2$ at some time, and after this, ρ_1 is dominant, the total EoS $\omega < -1$. So the equation of state ω crossing -1 is realized. It is simply found that this kind of crossing must be from $\omega > -1$ to < -1, which is exactly the same as the observations. But the contrary condition, from $\omega < -1$ to > -1, cannot be realized in this kind of model.

4. Conclusion and discussion

In summary, in this paper we have studied the possibility of ω crossing -1 in the YM field dark energy models, and found that the single YM field models cannot realize this no matter what their effective Lagrangian, although this kind of model can naturally give a state of $\omega > -1$ or $\omega < -1$, which depends on their initial state. Near the critical state of $\omega = -1$, the evolution of their EoS with the expansion of the universe is the same, $\omega + 1 \propto a^{-2}$, which means that the universe will be a nearly de Sitter expansion. This is the most attractive character of this kind of model, and this makes it very like the cosmological constant. So the big rip is naturally avoided in this model. But this evolution behaviour also shows that the single field models cannot realize ω crossing -1. This is same as the single scalar field models.

But in these models, $\omega > -1$ and $\omega < -1$ can both be easily got. The former behaviour is like a quintessence field, and the latter is like a phantom field. So one can build a model with two YM fields, one field with $\omega < -1$ and the other with $\omega > -1$. This idea is very like the quintom models. Then we give an example model and find that in this model, the

property of crossing the cosmological constant boundary can be naturally realized, and we also found that this crossing must be from $\omega > -1$ to < -1, which is exactly the observation result. In this model, the state will also go to the critical state of $\omega = -1$ with the expansion of the universe, as the single YM field models. This is the main character of the YM field dark energy models, which means the big rip is avoided. The present models we discuss in this paper are in the almost standard framework of physics, e.g. in general relativity in four dimension. There does not exist a phantom or higher derivative term in the model, which will lead to theory problems in the field theory. Instead, the YM field as in (3) is introduced, which includes the gauge invariance, the Lorentz invariance, the correct trace anomaly and the asymptotic freedom. These are the advantages of this kind of dark energy model. But these models also have some disadvantages: first, what is the origin of the YM field and why is its renormalization scale κ^2 as low as the present density of the dark energy? In the two YM fields model, we must choose $\alpha > 1$ to realize the ω crossing -1, which is a mild fine-tuning problem. All these make this kind of model unnatural. These are the universal problems which exist in most dark energy models. If considering the possible interaction between the YM field and other matter, especially dark matter, which may have some new character [27]. This topic has been deeply discussed in the scalar field dark energy models [28], but has not been considered in this paper.

Acknowledgments

YZ's research work has been supported by the Chinese NSF (10173008) and by NKBRSF (G19990754).

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