

## ATTRACTOR SOLUTION IN COUPLED YANG–MILLS FIELD DARK ENERGY MODELS

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Received 16 September 2008

Revised 4 November 2008

Communicated by R. Gregory

We investigate the attractor solution in the coupled Yang–Mills field dark energy models with the general interaction term, and obtain the constraint equations for the interaction if the attractor solution exists. The research also shows that, if the attractor solution exists, the equation of state of dark energy must evolve from  $w_y > 0$  to  $w_y \leq -1$ , which is slightly suggested by the observation. At the same time, the total equation of state in the attractor solution is  $w_{\text{tot}} = -1$ , the universe is a de Sitter expansion, and the cosmic big rip is naturally avoided. These features are all independent of the interacting forms.

*Keywords:* Yang–Mills field; dark energy.

PACS Number(s): 98.80.-k, 98.80.Es, 04.30.-w, 04.62.+v

### 1. Introduction

The dark energy problem has been one of the most active fields in model cosmology since the discovery of accelerated expansion of the universe.<sup>1–13</sup> In observational cosmology, the equation of state (EOS) of dark energy  $w_{\text{DE}} \equiv p_{\text{DE}}/\rho_{\text{DE}}$  plays a central role;  $p_{\text{DE}}$  and  $\rho_{\text{DE}}$  are its pressure and energy density, respectively. To accelerate the expansion, the EOS of dark energy must satisfy  $w_{\text{DE}} < -1/3$ . The simplest candidate for dark energy is a tiny positive time-independent cosmological constant  $\Lambda$ , whose EOS is  $-1$ . However, it is difficult to understand why the cosmological constant is about 120 orders of magnitude smaller than its natural expectation, i.e. the Planck energy scale density. This is the so-called fine-tuning problem. Another puzzle of dark energy is the first cosmological coincidence problem.<sup>14–16</sup> Namely, *why has our universe begun the accelerated expansion recently? Why are we living in an epoch in which the dark energy density and the dust matter energy density are*

*comparable*? This problem becomes very serious especially for the cosmological constant as the dark energy candidate. The cosmological constant remains unchanged while the energy densities of dust matter and radiation decrease rapidly with the expansion of our universe. Thus, it is necessary to do some fine-tuning. In order to give a reasonable interpretation of the first cosmological coincidence problem, many dynamical dark energy models have been proposed as alternatives to the cosmological constant, such as quintessence,<sup>17–20</sup> phantom,<sup>21–25</sup> k-essence,<sup>26–29</sup> and quintom.<sup>30–39</sup>

Recently, by fitting the SNe Ia data, marginal evidence for  $w_{\text{DE}}(z) < -1$  at redshift  $z < 0.2$  has been found. In addition, many best fits of the present values of  $w_{\text{DE}}$  are less than  $-1$  in various data fittings with different parametrizations. The present observational data seem to slightly favor an evolving dark energy with  $w_{\text{DE}}$  crossing  $-1$  from above to below in the near past.<sup>40–43</sup> It has been found that the EOS of dark energy  $w_{\text{DE}}$  cannot cross the so-called phantom divide  $w_{\text{DE}} = -1$  for quintessence, phantom or k-essence alone.<sup>44</sup> A number of works have discussed the quintom models,<sup>30–39</sup> which are a combination of a quintessence and a phantom. Although many of these models provide the possibility that  $w_{\text{DE}}$  can cross  $-1$ , they do not answer another question: *Why has the crossing phantom divide occurred recently?* Since in many existing models whose EOS can cross the phantom divide,  $w_{\text{DE}}$  undulated around  $-1$  randomly, why are we living in an epoch  $w_{\text{DE}} < -1$ ? This is regarded as the second cosmological coincidence problem.<sup>45–47</sup>

As is well known, the most frequently used approach to alleviating the first cosmological coincidence problem is the tracker field dark energy scenario.<sup>48,49</sup> The dark energy can track the evolution of the background matter in the early stage, and only recently, the dark energy has negative pressure, and becomes dominant. Thus, the current condition of the dark energy is nearly independent of the initial condition. If the possible interaction between the dark energy and background matter<sup>50–58</sup> is considered, the whole system (including the background matter and dark energy) may eventually be attracted into the scaling attractor a balance achieved thanks to the interaction. In the scaling attractor, the effective densities of dark energy and background matter decrease in the same manner with the expansion of our universe, and the ratio of dark energy and background matter becomes a constant. So, it is not strange that we are living in an epoch when the densities of dark energy and matter are comparable. In this sense, the first cosmological coincidence problem is alleviated. On the other hand, if the scaling attractor also has the property that its EOS of dark energy is smaller than  $-1$ , the second cosmological coincidence problem, if it exists, is alleviated at the same time.<sup>45–47</sup> However, this is impossible in the interacting quintessence or phantom scenario.

Recently, a number of authors have discussed another class of models, which are based on the conjecture that a vector field can be the origin of the dark energy,<sup>59–65</sup> and have features different to those of a scalar field. In Refs. 66–72, it is suggested that the Yang–Mills (YM) field can be a kind of candidate for such a vector field. Compared with the scalar field, the YM field is the indispensable cornerstone of

particle physics and the gauge bosons have been observed. There is no room for adjusting the form of the effective YM Lagrangian, as it is predicted by quantum corrections according to field theory. In the previous works,<sup>69–73</sup> we have investigated the one-loop YM field case and found attractive features: the YM field dark energy models can naturally realize the EOS of  $w_y > -1$  and  $w_y < -1$ , and the current state of the YM dark energy is independent of the choice of the initial condition. The cosmic big rip is also avoided in the models.

In the recent works,<sup>74–76</sup> the two-loop and three-loop YM field dark energy are also considered. Although these cases are much more complicated than the one-loop case, they have not brought new features for the evolution of the universe. So, in this work, we shall focus only on the YM field with the one-loop case.

In this work, the cosmological evolution of the YM dark energy interacting with background perfect fluid is investigated. In fact, gauge fields play a very important role in, and are the indispensable cornerstone of, particle physics. All known fundamental interactions between particles are mediated through gauge bosons. Generally speaking, as a gauge field, the YM field under consideration may have interactions with other species of particles in the universe. However, unlike those well-known interactions in QED, QCD, and the electron-weak unification, here at the moment we do not yet have a model for the details of microscopic interactions between the YM field and other particles. In this work, instead of considering some specific assumed interactions between the YM field and matter and radiation, which have been adopted in Refs. 73–76, we shall consider the YM dark energy model with a general interacting term, and investigate the general feature of the attractor solution.

This paper is organized as follows. In Sec. 2, we give equations of the dynamical system of the interacting YM field dark energy models, and discuss the general features of the interacting models. In Sec. 3, we consider three special cases of the interaction terms and the holographic YM dark energy models, and investigate the constraints of these interaction terms. Finally, we present a brief conclusion and discussion in Sec. 4.

## 2. Dynamical System of Interacting Yang–Mills Dark Energy

The effective YM field cosmic model has been discussed in Refs. 66–72. The effective Lagrangian up to one-loop order is<sup>79–81</sup>

$$\mathcal{L}_{\text{eff}} = \frac{b}{2} F \ln \left| \frac{F}{e\kappa^2} \right|, \quad (1)$$

where  $b = 11N/24\pi^2$  for the generic gauge group  $SU(N)$  is the Callan–Symanzik coefficient.<sup>82,83</sup>  $F = -(1/2)F_{\mu\nu}^a F^{a\mu\nu}$  plays the role of the order parameter of the YM field.  $\kappa$  is the renormalization scale with the dimension of squared mass, the only model parameter. The attractive features of this effective YM Lagrangian include the gauge invariance, the Lorentz invariance, the correct trace anomaly, and the asymptotic freedom.<sup>79</sup> With the logarithmic dependence on the field strength,

$\mathcal{L}_{\text{eff}}$  has a form similar to the Coleman–Weinberg scalar effective potential<sup>84</sup> and the Parker–Raval effective gravity Lagrangian.<sup>85</sup> The effective YM field was first put into the expanding Robertson–Walker (R–W) space–time to study inflationary expansion<sup>66,67</sup> and dark energy.<sup>68</sup> We work in a spatially flat R–W space–time with a metric

$$ds^2 = a^2(\tau)(d\tau^2 - \delta_{ij}dx^i dx^j), \quad (2)$$

where  $\tau = \int (a_0/a)dt$  is the conformal time. For simplicity we study the SU(2) group and consider the electric case with  $B^2 \equiv 0$ . The energy density and pressure of the YM field are given by

$$\rho_y = \frac{E^2}{2}(\epsilon + b), \quad p_y = \frac{E^2}{2} \left( \frac{\epsilon}{3} - b \right), \quad (3)$$

where the dielectric constant is given by

$$\epsilon = b \ln \left| \frac{F}{\kappa^2} \right|, \quad (4)$$

and the EOS is

$$w_y = \frac{p_y}{\rho_y} = \frac{y - 3}{3y + 3}, \quad (5)$$

where  $y \equiv \epsilon/b = \ln |F/\kappa^2|$ . At the critical point with the order parameter  $F = \kappa^2$ , one has  $y = 0$  and  $w_y = -1$ , and the universe is in exact de Sitter expansion.<sup>66,67</sup> Around this critical point,  $F < \kappa^2$  gives  $y < 0$  and  $w_y < -1$ , and  $F > \kappa^2$  gives  $y > 0$  and  $w_y > -1$ . So, in the YM field model, the EOS of  $w_y > -1$  and  $w_y < -1$  can all be naturally realized. When  $y \gg 1$ , the YM field has a state of  $w_y = 1/3$ , becoming a radiation component. The effective YM equations are

$$\partial_\mu (a^4 \epsilon F^{a\mu\nu}) + f^{abc} A_\mu^b (a^4 \epsilon F^{c\mu\nu}) = 0, \quad (6)$$

the  $\nu = 0$  component of which is an identity, and the  $\nu = 1, 2, 3$  spatial components of which reduce to

$$\partial_\tau (a^2 \epsilon E) = 0. \quad (7)$$

In this work we will generalize the original YM dark energy model to include the interaction between the YM dark energy and dust matter. We assume that the YM dark energy and background matter interact through an interaction term  $Q$ , according to

$$\dot{\rho}_y + 3H(\rho_y + p_y) = -Q, \quad (8)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (9)$$

which preserves the total energy conservation equation  $\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0$ . It is worth noting that the equation of motion (7) should be changed when  $Q \neq 0$ . We introduce the dimensionless variables

$$x \equiv \frac{2\rho_m}{b\kappa^2}, \quad f \equiv \frac{2Q}{b\kappa^2 H}, \quad (10)$$

where  $f$  is the function of  $x$  and  $y$ . With the help of the definition of  $y$ , the evolution equations (8) and (9) can be rewritten as a dynamical system, i.e.

$$y' = -\frac{4y}{2+y} - \frac{f(x,y)}{(2+y)e^y}, \quad (11)$$

$$x' = -3x + f(x,y), \quad (12)$$

where a prime denotes the derivative with respect to the so-called e-folding time  $N \equiv \ln a$ . The fractional energy densities of dark energy and background matter are given by

$$\Omega_y = \frac{(1+y)e^y}{(1+y)e^y + x}, \quad \Omega_m = \frac{x}{(1+y)e^y + x}. \quad (13)$$

We can obtain the critical point  $(y_c, x_c)$  of the autonomous system by imposing the conditions  $y'_c = x'_c = 0$ . From Eqs. (11) and (12), we find that the critical state satisfies the simple relations

$$3x_c = f(x_c, y_c), \quad (14)$$

$$3x_c = -4y_c e^{y_c}, \quad (15)$$

and so we can get the critical state  $(y_c, x_c)$  by solving these two equations. In order to study the stability of the critical point, we substitute linear perturbations  $y \rightarrow y_c + \delta y$  and  $x \rightarrow x_c + \delta x$  about the critical point into the dynamical system equations (11) and (12) and linearize them, and obtain two independent evolutive equations, i.e.

$$\begin{pmatrix} \delta y' \\ \delta x' \end{pmatrix} \equiv M \begin{pmatrix} \delta y \\ \delta x \end{pmatrix} = \begin{pmatrix} G_y + R_y & R_x \\ f_y & f_x - 3 \end{pmatrix} \begin{pmatrix} \delta y \\ \delta x \end{pmatrix},$$

where

$$R_y \equiv \left. \frac{\partial R}{\partial y} \right|_{(y=y_c, x=x_c)}, \quad (16)$$

and the definitions of  $R_x$ ,  $f_y$ ,  $f_x$  and  $G_y$  are similar. The functions  $G$  and  $R$  are defined by

$$G = G(y) = \frac{4y}{2+y},$$

$$R = R(x, y) = -\frac{f(x, y)}{(2+y)e^y},$$

which are used for the simplification of the notation. The two eigenvalues of the coefficient matrix  $M$  determine the stability of the corresponding critical point. The critical point is an attractor solution, which is stable only if both of these two eigenvalues are negative (stable node), or real parts of these two eigenvalues are

negative and the determinant of the matrix  $M$  is negative (stable spiral), which requires that the critical point should satisfy the inequalities

$$G_y + R_y + f_x - 3 < 0, \quad (17)$$

$$[R_x f_y - (f_x - 3)(G_y + R_y)] [(G_y + R_y - f_x + 3)^2 - 4R_x f_y] < 0, \quad (18)$$

or that it should satisfy

$$G_y + R_y + f_x - 3 < 0, \quad (19)$$

$$(G_y + R_y - f_x + 3)^2 - 4R_x f_y = 0. \quad (20)$$

These generate a constraint of the interaction term  $Q$ , which will be shown in the following section.

Here we discuss some general features of the attractor solutions, regardless of the special form of the interaction term  $Q$ . From the expression (15), we find that  $x_c = -(4y_c/3)e^{y_c}$ . Substituting this into the formula (13), we obtain

$$\Omega_y = \frac{(y_c + 1)e^{y_c}}{(y_c + 1)e^{y_c} + x_c} = \frac{3 + 3y_c}{3 - y_c}. \quad (21)$$

Since  $0 \leq \Omega_y \leq 1$ , this formula follows a constraint of the critical point:

$$-1 \leq y_c \leq 0. \quad (22)$$

From the formulae (5) and (21), we obtain

$$\Omega_y w_y = -1. \quad (23)$$

This relation is kept for all attractor solutions, independent of the special form of the interaction. Since the value of  $\Omega_y$  is not larger than 1 in the attractor solution, we find that

$$w_y \leq -1, \quad (24)$$

and the EOS of the YM dark energy must not be larger than  $-1$ , phantom-like or  $\Lambda$ -like. Since in the early universe the value of the order parameter of the YM field  $F$  is much larger than that of  $\kappa^2$ , i.e.  $y \gg 1$ , the YM field is a kind of radiation component.<sup>70–72</sup> However, in the late attractor solution, the dark energy is phantom-like or  $\Lambda$ -like. So the phantom divide must be crossed in the former case, which is different from the interacting quintessence, phantom or k-essence models.

In order to investigate the final fate of the universe, we should investigate the total EOS in the universe, which is defined by

$$w_{\text{tot}} \equiv \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = \frac{p_y + p_m}{\rho_y + \rho_m} = \Omega_y w_y, \quad (25)$$

where  $p_m = 0$  is used. From the relation (23), we find that, in the attractor solution,

$$w_{\text{tot}} = -1. \quad (26)$$

This result is also independent of the special form of the interaction. So the universe is an exact de Sitter expansion, and the cosmic big rip is naturally avoided, although the YM field dark energy is phantom-like.

### 3. Several Interaction Models

In the previous section, we found that the critical point of the dynamical system of interacting YM dark energy models satisfies not only Eqs. (14) and (15), but also the constraint in (22). It is obvious that the expression (14) depends on the special form of the interacting term. If the critical point is an attractor, it also satisfies the constraint in (17) and (18), or in (19) and (20). These relations can give some constraints of the interaction term. In this section, we consider several cases with different interaction forms between the YM dark energy and background matter, which are taken as the most familiar interaction terms extensively considered in the literature.<sup>50–58</sup>

**Case a.**  $Q \propto H\rho_y$ , which is equivalent to the form  $f(x, y) = \alpha(y + 1)e^y$ , where  $\alpha$  is a dimensionless constant. From Eqs. (14) and (15), we obtain the critical point

$$y_c = -\frac{\alpha}{4 + \alpha}, \quad x_c = -\frac{4y_c}{3}e^{y_c}. \quad (27)$$

The constraint in (22) requires that

$$\alpha \geq 0, \quad (28)$$

and the attractor conditions in (17)–(20) require that

$$\alpha > -8. \quad (29)$$

So we obtain the constraint of the interaction form; if the attractor solution exists, the parameter  $\alpha$  satisfies

$$\alpha \geq 0, \quad (30)$$

and the EOS and the fractional energy density of the YM field in the attractor solution are

$$w_y = -\frac{1}{3}(\alpha + 3), \quad \Omega_y = \frac{3}{\alpha + 3}, \quad (31)$$

respectively. It is obvious that  $w_y \leq -1$ .

**Case b.**  $Q \propto H(\rho_y + \rho_m)$ , which is equivalent to the form  $f(x, y) = \beta[(y + 1)e^y + x]$ , where  $\beta$  is a dimensionless constant. From Eqs. (14) and (15), we obtain the critical point

$$y_c = \frac{3\beta}{\beta - 12}, \quad x_c = -\frac{4y_c}{3}e^{y_c}. \quad (32)$$

The constraint in (22) requires that

$$0 \leq \beta \leq 3, \quad (33)$$

and the attractor conditions in (17)–(20) require that

$$\beta < \frac{120}{31}. \quad (34)$$

So the parameter  $\beta$  satisfies

$$0 \leq \beta \leq 3, \quad (35)$$

if the critical state is an attractor solution. The EOS and the fractional energy density of the YM field in the attractor are

$$w_y = \frac{3}{\beta - 3}, \quad \Omega_y = \frac{3 - \beta}{3}, \quad (36)$$

respectively, and it follows that  $w_y \leq -1$ , and the YM field dark energy is phantom-like or  $\Lambda$ -like.

**Case c.**  $Q \propto H\rho_m$ , which is equivalent to the form  $f(x, y) = \gamma x$ , where  $\gamma$  is a dimensionless constant. From Eqs. (14) and (15), we easily find that they have no solution except that the value of  $\gamma$  is exactly zero, i.e. the case with no interaction.

**Case d.** Recently, a number of authors have discussed holographic dark energy, where the holographic principle has been put forward to explain the dark energy. According to the holographic principle, the number of degrees of freedom of a physical system scales with the area of its boundary. In this context, Cohen *et al.*<sup>86</sup> suggested that in quantum field theory a short distance cutoff is related to a long distance cutoff due to the limit set by formation of a black hole, which results in an upper bound on zero-point energy density. In line with this suggestion, Hsu and Li<sup>87,88</sup> argued that this energy density could be viewed as the holographic dark energy satisfying

$$\rho_{\text{DE}} = 3d^2 M_P^2 L^{-2}, \quad (37)$$

where  $d \geq 0$  is a numerical constant,  $M_P \equiv 1/\sqrt{8\pi G}$  is the reduced Planck mass and  $L$  is the size of the current universe. Li<sup>88</sup> proposed that the IR cutoff  $L$  should be taken as the size of the future event horizon:

$$L = R_{\text{eh}}(a) = a \int_t^\infty \frac{d\tilde{t}}{a(\tilde{t})} = a \int_a^\infty \frac{d\tilde{a}}{H\tilde{a}^2}. \quad (38)$$

In this paper, we consider the holographic YM field dark energy. From the relation (37), we obtain

$$\dot{\rho}_y = \dot{\rho}_{\text{DE}} = 6M_P^2 \Omega_y H^3 \left( \frac{\sqrt{\Omega_y}}{d} - 1 \right), \quad (39)$$

and it follows that the interaction form is

$$Q = -2\rho_y H \left( \frac{\sqrt{\Omega_y}}{d} - 1 \right) - 3H\rho_y(1 + w_y), \quad (40)$$

where the expression (8) has been used. This formula is equivalent to the form

$$f(x, y) = \left[ -\frac{4y}{y+1} - 2 \left( \frac{\sqrt{\frac{3+y}{3-y}}}{d} - 1 \right) \right] (y+1)e^y. \quad (41)$$



From Eqs. (14) and (15), we obtain the critical point

$$y_c = -\frac{3(d^2 - 1)}{3 + d^2}, \quad x_c = -\frac{4y_c}{3}e^{y_c}. \quad (42)$$

The attractor conditions in (17)–(20) require that

$$d < 0, \quad (43)$$

which conflicts with the previous assumption,  $d \geq 0$ . So we get the following conclusion: the holographic YM dark energy model has no attractor solution.

#### 4. Conclusion and Discussion

In summary, the cosmological evolution of the Yang–Mills field dark energy interacting with background matter is investigated in this paper. We find the features of the interacting YM dark energy models:

- (a) The interaction term between the YM dark energy model and the matter has a fairly tight constraint, if we require that the attractor solution of the model should exist.
- (b) If the attractor solution exists, the EOS of the YM field must evolve from  $w_y > 0$  to  $w_y < -1$  or  $w_y = -1$ .
- (c) The holographic YM dark energy model has no attractor solution, which is different from other holographic models.<sup>89–92</sup>
- (d) In the attractor solution, the total EOS is  $w_{\text{tot}} = -1$ , which is independent of the interacting forms. So the universe is in a de Sitter expansion, and the cosmic big rip does not exist in the models.

In the interacting YM dark energy models, we should notice the “fine-tuning” problem, which is reflected by the value of  $\kappa$ , the energy scale of the YM field dark energy models. In the interacting models, the total energy density in the universe is

$$\rho_{\text{tot}} = \frac{\rho_m}{\Omega_m} = \frac{b\kappa^2}{2}[(1 + y)e^y + x]. \quad (44)$$

In the attractor solution, we can obtain

$$\rho_{\text{tot}} = \frac{b\kappa^2}{2} \left(1 - \frac{1}{3}y_c\right) e^{y_c}, \quad (45)$$

where the expression (15) has been used. The value of  $\rho_{\text{tot}}$  should not be larger than that of the present total energy density in the universe,<sup>77,78</sup> i.e.

$$\rho_{\text{tot}} \leq 8.099h^2 \times 10^{-11} \text{ eV}^4, \quad (46)$$

which leads to

$$\kappa \leq 4.18h \times 10^{-5} \text{ eV}^2 \left(1 - \frac{1}{3}y_c\right)^{-1/2} e^{-y_c/2}. \quad (47)$$

For a fixed interacting model, where  $y_c$  can be obtained, one can exactly calculate the value of  $\kappa$ , which keeps the current energy density of YM dark energy being the

current observed value. From (47), we find that this energy scale  $\kappa$ , as well as the case with free YM field models, is very low compared to the typical energy scales in particle physics.

## Acknowledgment

This work is supported by the Chinese NSF under Grant Nos. 10703005 and 10775119.

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