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Constraints on the extensions to the base ΛCDM model from BICEP2, Planck and WMAP†

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Recently Background Imaging of Cosmic Extragalactic Polarization (B2) discovered the relic gravitational waves at 7.0σ confidence level. However, the other cosmic microwave background (CMB) data, for example Planck data released in 2013 (P13), prefer a much smaller amplitude of the primordial gravitational waves spectrum if a power-law spectrum of adiabatic scalar perturbations is assumed in the six-parameter ΛCDM cosmology. In this paper, we explore whether the wCDM model and the running spectral index can relax the tension between B2 and other CMB data. Specifically we found that a positive running of running of spectral index is preferred at 1.7σ level from the combination of B2, P13 and WMAP Polarization data.

relic gravitational waves, spectral index, dark energy

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1 Introduction

In the early of 2013, Planck (P13) [1] released its data which precisely measured the temperature anisotropies of cosmic microwave background (CMB), and claimed that it strongly supports the standard spatially-flat six-parameter ΛCDM cosmology with a power-law spectrum of adiabatic scalar perturbations. Actually the relic gravitational waves could also make contributions to the temperature and polarization power spectra in the CMB [2–6]. Combining Wilkinson Microwave Anisotropy Probe (WMAP) 9-year data [7] with Baryon Acoustic Oscillation (BAO) [8], H_0 prior from Hubble Space Telescope (HST) [9] and other highL CMB data, including Atacama Cosmology Telescope (ACT) [10] and South Pole Telescope (SPT) [11], we obtained the constraint on the primordial gravitational waves before Planck as follows:

$$r_{0.002} < 0.12 \tag{1}$$

at 95% CL in ref. [12], where $r_{0.002}$ is the tensor-to-scalar ratio at the pivot scale $k_p = 0.002\ \mathrm{Mpc}^{-1}$ and a power-law spectrum of the primordial scalar perturbations is also assumed. A similar result was reported by Planck combining with WMAP polarization (WP) data and other highL CMB data, namely

$$r_{0.002} < 0.11 \tag{2}$$

at 95% CL in ref. [1]. In this paper, we shall fix the pivot scale as $k_p = 0.002\ \mathrm{Mpc}^{-1}$.

Considering that the primordial gravitational waves only make contributions to CMB power spectra at the very large scales, we fixed the background parameters as their best-fit values from Planck, and then run the CosmoMC to determine the amplitude of adiabatic scalar perturbations, spectral index and the tensor-to-scalar ratio by only using the low-multipole

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Planck TT [1] and WMAP TE (WP) data [7]. We found $r > 0$ at more than 68% confidence level with maximum likelihood at around $r \sim 0.2$ [13]. Our new result confirmed the previous one in ref. [14] where WMAP 7-year data were utilized. Recently Background Imaging of Cosmic Extragalactic Polarization (B2) [15] discovered the relic gravitational waves with the tensor-to-scalar ratio

$$r = 0.20^{+0.07}_{-0.05}, \quad (3)$$

and $r = 0$ is disfavored at 7.0σ. Using B2 only or the combination of B2, P13 and WP, the tilt n_t of relic gravitational waves spectrum is constrained to be around zero and $n_t = 2$ is ruled out at more than 5σ confidence level in refs. [16, 17] which strongly indicates that inflation [18–20] really happened in the early universe.

In this paper we hope to get a better understanding about the physics in our universe through a more careful investigation of the datasets. Comparing eq. (3) to eqs. (1) and (2), we see that there is a moderately strong tension between B2 and other CMB data in the base six-parameter ΛCDM+tensor cosmology. If all of these CMB datasets are trustable, it strongly implies that our Universe is much more complicated than what we expected before. In order to reconcile the tension on constraining the primordial gravitational waves between P13 and B2, we need to go beyond the ΛCDM+tensor model. There are several well-motivated extensions to the ΛCDM+tensor model which might relax such an inconsistency.

i) More complicated physics in the early universe can be involved. Here we consider that the spectrum of adiabatic scalar perturbations departures from a pure power-law form, and the running of spectral index ($\mathrm{d}n_s/\mathrm{d}\ln k$) and the running of running ($\mathrm{d}^2 n_s/\mathrm{d}\ln k^2$) are taken into account. Or the spatial curvature (Ω_k) of our universe deviates from exact flatness.

ii) We can consider more complicated physics about neutrino and relativistic components by relaxing the total mass of active neutrinos ($\sum m_\nu$), or the number of relativistic species (N_{eff}).

iii) The abundance of light elements, for example $Y_P \equiv 4n_{\mathrm{He}}/n_{\mathrm{b}}$ for helium-4, is taken as a free parameter.

iv) The dark energy is not a cosmological constant and its equation-of-state (EOS) parameter $w \equiv p_{\mathrm{de}}/\rho_{\mathrm{de}}$ is regarded as a free parameter.

An almost comprehensive investigation has been reported by Lewis in ref. [21] where the combination of B2 and P13 was considered. In this paper we will adopt B2, P13 and WP to explore two extensions: one is to relax the dark energy model from cosmological constant to one with a constant EOS parameter $w = p_{\mathrm{de}}/\rho_{\mathrm{de}}$; the other is to take into account the running of spectral index and the running of running. Here we fix the consistency relation to be $n_t = -r/8$.

2 Implications for cosmology from BICEP, Planck and WMAP

In this section we will use the CosmoMC [22] to work out the constraints on the cosmological parameters in different cosmological models from several different combinations of datasets respectively. Our results are summarized in Tables 1–3.

2.1 *w*CDM model

Herein we will extend the dark energy model from the cosmological constant to the dark energy with constant EOS parameter w. There are also several tensions between P13 and some local cosmological observations, including the H_0 prior from HST [9] and Supernova Legacy Survey (SNLS) samples [23]. For example, P13 prefers a larger matter density today compared to SNLS and a smaller Hubble constant compared to the H_0 prior from HST. However these two tensions can be significantly relaxed in the wCDM model [24] where the dark energy is preferred to be phantom-like, namely $w = -1.16 \pm 0.06$ from the combination of P13+WP+BAO+SNLS+HST.

Here we also wonder whether the dark energy EOS can help to relax the tension on the tensor-to-scalar ratio between P13 and B2. We constrain the cosmological parameters in the wCDM+r model by adopting the combinations of P13+WP and B2+P13+WP, respectively. See results in Table 1 and Figure 1.

We find that the constraint on r is given by

$$r_{0.002} < 0.16 \quad (4)$$

at 3σ confidence level from P13+WP. There is still a more than 3σ tension on r between P13+WP and B2 in the wCDM+r model. Therefore relaxing the dark energy model cannot reconcile the tension on r between P13 and B2. Because of such a tension, some exotic results appear. For example, the constraint on the dark energy EOS parameter becomes $w = -1.54^{+0.17}_{-0.32}$ in Table 1. A similar constraint on w from B2+P13 is $w = -1.55^{+0.18}_{-0.34}$ in ref. [25].

2.2 Running spectral index

Herein we extend the six-parameter base ΛCDM+r model to the ΛCDM+nrun+r and ΛCDM+nrun+nrunrun+r models, respectively, where nrun and nrunrun denote the running of spectral index ($\mathrm{d}n_s/\mathrm{d}\ln k$) and the running of running

Table 1 Constraints on the cosmological parameters in the wCDM+r model

wCDM+r	B2+P13+WP	
parameters	Best fit	68% limits
$\Omega_b h^2$	0.02222	$0.02210^{+0.00058}_{-0.00063}$
$\Omega_c h^2$	0.1161	$0.1172^{+0.0029}_{-0.0028}$
$100\theta_{\mathrm{MC}}$	1.04190	$1.04163^{+0.00067}_{-0.00068}$
τ	0.1001	$0.0888^{+0.0145}_{-0.0186}$
$\ln(10^{10}A_s)$	3.200	3.186 ± 0.034
n_s	0.9690	$0.9669^{+0.0140}_{-0.0163}$
$r_{0.002}$	0.16	$0.16^{+0.04}_{-0.05}$
w	−1.70	$-1.54^{+0.17}_{-0.32}$

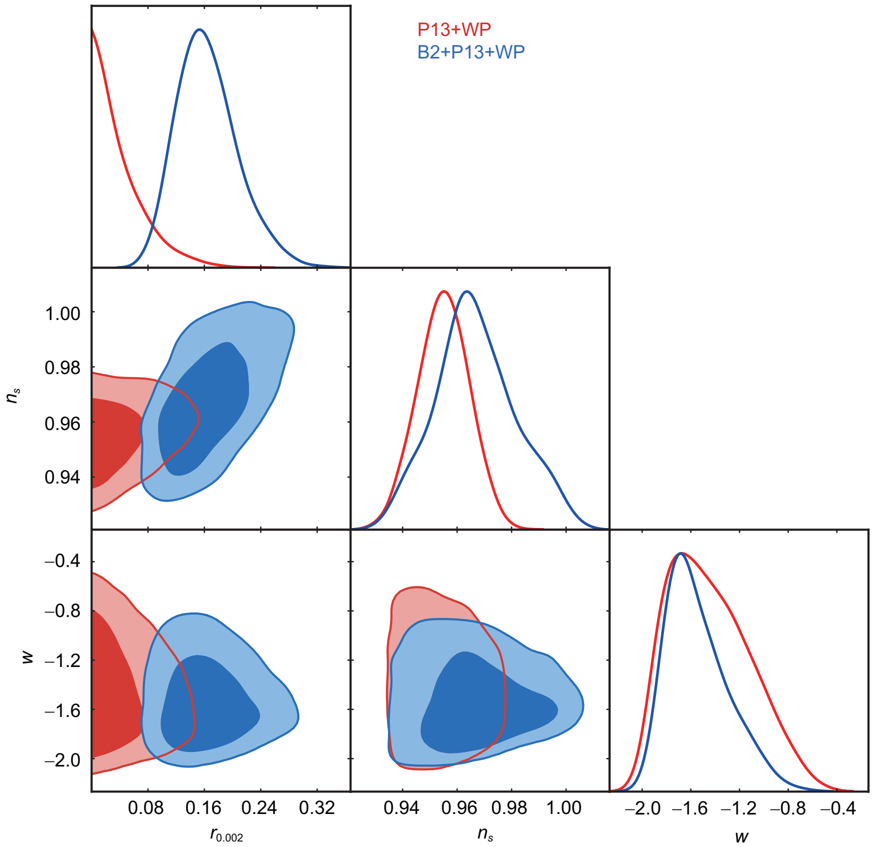

Figure 1 The constraint contours on r, n_s and w from the combinations of P13+WP and B2+P13+WP in the wCDM+r model.

($d^2n_s/d\ln k^2$). In this case the amplitude of scalar perturbation spectrum is parameterized by

$$P_s(k) = A_s\left(\frac{k}{k_p}\right)^{n_s-1+\frac{1}{2}\frac{dn_s}{d\ln k}\ln\frac{k}{k_p}+\frac{1}{6}\frac{d^2n_s}{d\ln k^2}\ln^2\frac{k}{k_p}}. \tag{5}$$

In ref. [12], the constraint on the tensor-to-scalar ratio from the combination of WMAP+ACT+SPT+BAO+HST is relaxed to be

$$r_{0.002} < 0.42 \tag{6}$$

at 95% CL if the running of spectral index is considered, and

$$r_{0.002} < 0.53 \tag{7}$$

at 95% CL if both the running and running of running are taken into account. In ref. [1], the constraint on the tensor-to-scalar ratio is relaxed to be

$$r_{0.002} < 0.26 \tag{8}$$

at 95% CL from the combination of P13+WP+ACT+SPT if the running of spectral index is considered. We see that the constraint on r can be significantly loosen to be consistent with B2 in the model with running spectral index.

First of all, we combine B2 with P13 and WP to constrain the cosmological parameters in the ΛCDM+nrun+r cosmology. Our results are given in Table 2 and Figure 2.

We see that a blue tilted scalar power spectrum at k_p = 0.002 Mpc^{-1} is preferred at 1.5σ level, and a negative running of spectral index is favored at around 2.7σ level. Combining with P13, WP and other highL CMB data, B2 implies $dn_s/d\ln k = -0.028 \pm 0.009$ [15]. In ref. [21], the combination of B2+P13 gives a constraint $dn_s/d\ln k = -0.028 \pm 0.020$. In ref. [25], $dn_s/d\ln k = -0.0281 \pm 0.0099$ from B2+P13+BAO+SN. See the analysis in refs. [26, 27] as well. Our results are consistent with all of these previous results.

Since a negative running of spectral index is preferred at high confidence level, we wonder whether the higher order terms in the parametrization of scalar perturbation spectrum are required. Here we further extend the previous model to

Table 2 Constraints on the cosmological parameters in the ΛCDM + nrun + tensor model

ΛCDM+nrun+r	B2+P13+WP	
parameters	Best fit	68% limits
$\Omega_b h^2$	0.02229	$0.02246^{+0.00030}_{-0.00032}$
$\Omega_c h^2$	0.1187	$0.1173^{+0.0022}_{-0.0021}$
$100\theta_{MC}$	1.04154	$1.04162^{+0.00061}_{-0.00062}$
τ	0.0991	$0.1011^{+0.0137}_{-0.0161}$
$\ln(10^{10}A_s)$	3.117	$3.098^{+0.045}_{-0.041}$
n_s	1.0336	$1.0447^{+0.0295}_{-0.0297}$
$dn_s/d\ln k$	−0.0228	-0.0253 ± 0.0093
$r_{0.002}$	0.18	$0.22^{+0.04}_{-0.07}$

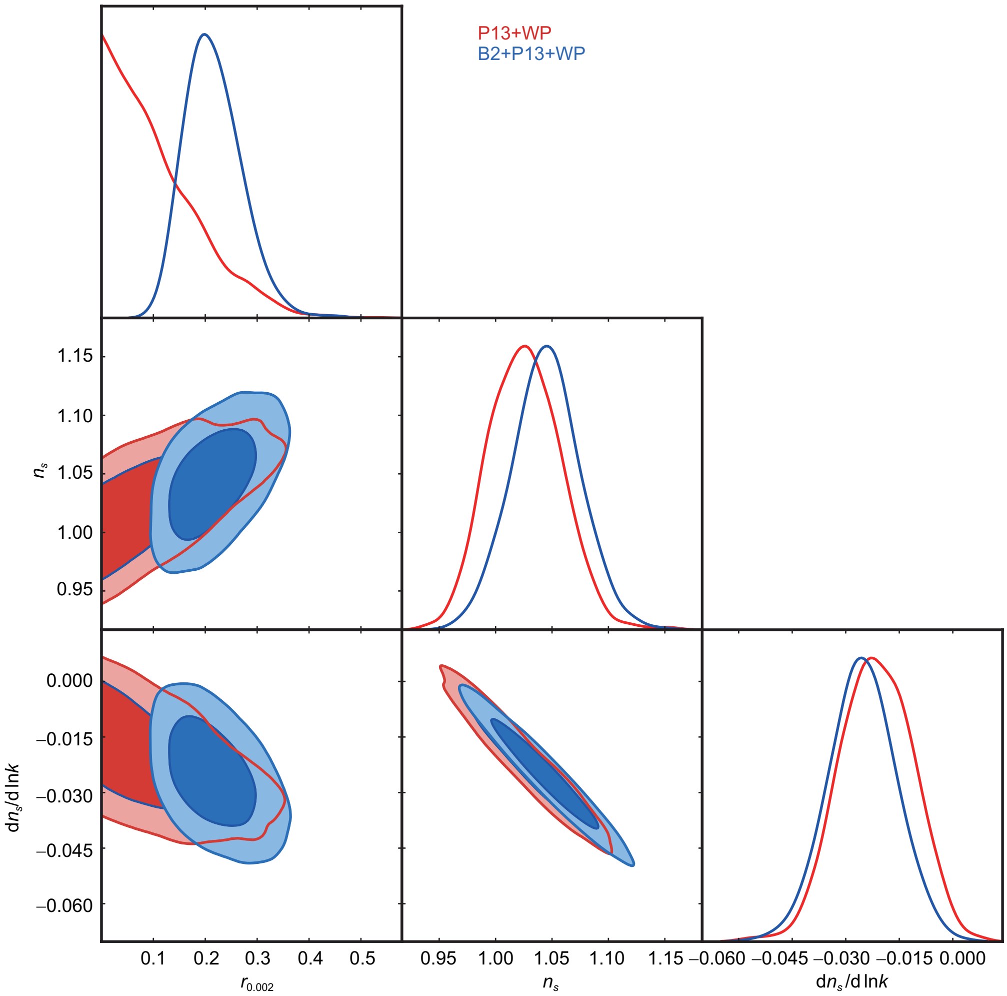

Figure 2 The constraint contours on r, n_s and d n_s/d ln$k$ from the combinations of P13+WP and B2+P13+WP in the ΛCDM+nrun+r model.

the ΛCDM+nrun+nrunrun+r model. The results show up in Table 3 and Figure 3.

Compared to the previous model with only the running of spectral index, $\Delta\chi^2 = 9852.70 - 9855.82 = -3.12$ which indicates that this further parameter extension is favored at more than 1σ level. From Table 3, we see that at the pivot scale $k_p = 0.002$ Mpc^{-1} the spectral index $n_s > 1$ is preferred at 2.2σ level, a negative running of spectral index is preferred at 2.2σ level and a positive running of running is preferred at 1.7σ level once the running of running is considered. Our results imply that higher order expansions might be considered in the future as well.

Table 3 Constraints on the cosmological parameters in the ΛCDM + nrun + nrunrun+tensor model

ΛCDM+nrun+nrunrun+r	B2+P13+WP	
parameters	Best fit	68% limits
$\Omega_b h^2$	0.02221	0.02217 ± 0.00035
$\Omega_c h^2$	0.1203	0.1184 ± 0.0023
$100\theta_{\rm MC}$	1.04145	$1.04142^{+0.00064}_{-0.00063}$
τ	0.0983	$0.1054^{+0.0142}_{-0.0168}$
$\ln(10^{10}A_s)$	3.047	$3.063^{+0.066}_{-0.050}$
n_s	1.1656	$1.1344^{+0.0612}_{-0.0608}$
$dn_s/d\ln k$	−0.139	$-0.108^{+0.049}_{-0.048}$
$d^2n_s/d\ln k^2$	0.045	$0.033^{+0.018}_{-0.019}$
$r_{0.002}$	0.22	$0.24^{+0.05}_{-0.07}$

3 Discussion

Discovery of relic gravitational waves opens a new window to explore cosmology. There are many possible sources for the relic gravitational waves, such as inflation [18–20] and cosmic string [28,29]. In this paper we extend the ΛCDM+r cosmology to wCDM+r model and ΛCDM+r model with running spectral index, and find that the tension between B2 and P13 can be reconciled if a running spectral index is taken into account, but relaxing dark energy model does not work.

Usually inflation model predicts $|n_s - 1| \lesssim \mathcal{O}(10^{-2})$, $|dn_s/d\ln k| \lesssim \mathcal{O}(10^{-3})$ and $|d^2n_s/d\ln k^2| \lesssim \mathcal{O}(10^{-4})$. Our results imply that the simple canonical single-field slow-roll inflation models are not compatible with the datasets and the physics in the early universe should be much more complicated than what we expect if all of B2, P13 and WP are

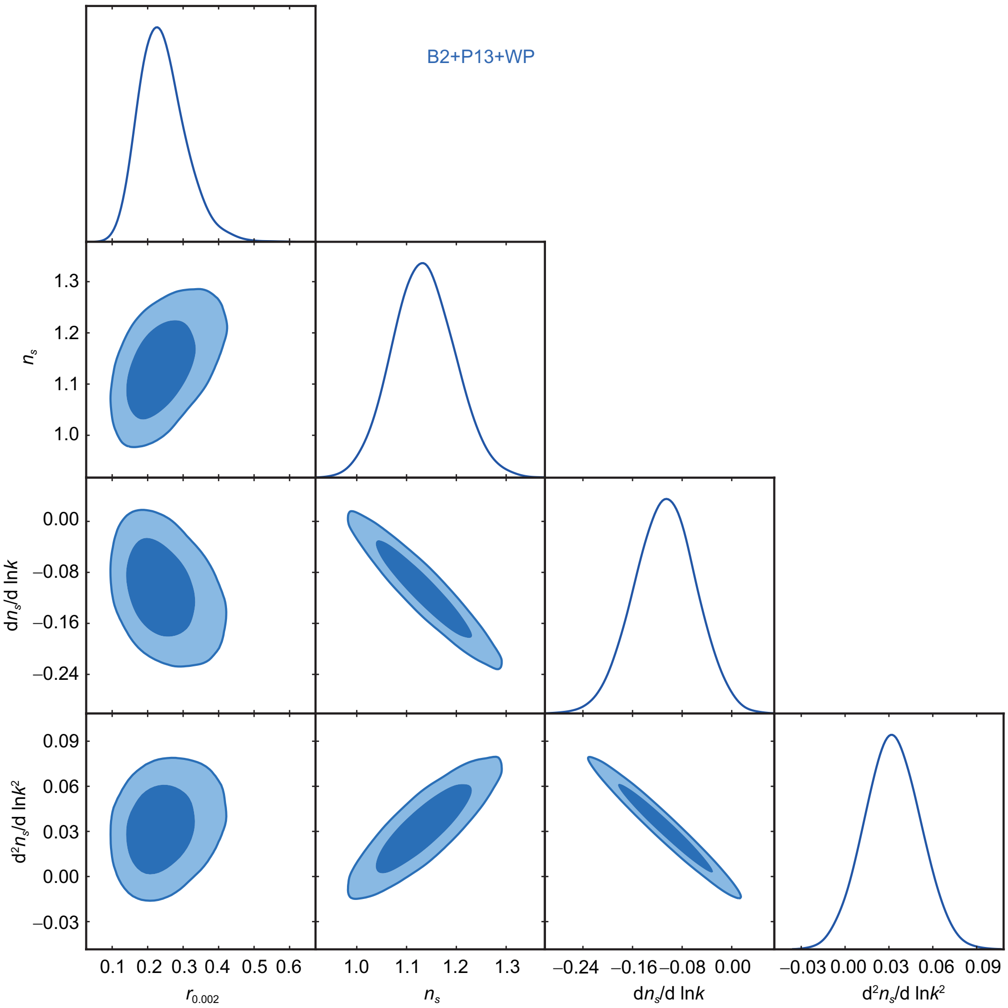

Figure 3 The constraint contours on r, n_s, $dn_s/d\ln k$ and $d^2n_s/d\ln k^2$ from the combination of B2+P13+WP in the ΛCDM+nrun+nrunrun+r model.

reliable. After B2 released its data, many researchers have investigated inflation models widely. See, for example, refs. [30–40]. However almost all of them only attempted to fit the value of tensor-to-scalar ratio and the spectral index. We believe that it is not sufficient because the combination of B2+P13+WP strongly implies a running spectral index. How to naturally achieve a significantly running spectral index is still an open question. As we known, the space-time noncommutative inflation [41–43] can generate a large negative running of spectral index. It can also be realized in the inflation with modulations [44–46] as well. Another possibility is that the Planck data is not reliable. In ref. [47] we combine B2 with WMAP 9-year data and find that the power-law spectrum of scalar perturbation is compatible with B2+WMAP, and the power-law inflation and inflation model with inverse power-law potential can fit the data nicely. In a word, we believe that the realistic inflation model is still unknown and further investigation is needed in the near future.

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