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Testing inflationary consistency relations by the potential CMB observations

Wen Zhao[1,2] **and Qing-Guo Huang**[3]

[1] International Center for Astrophysics, Korea Astronomy and Space Science Institute, Daejeon, 305-348, Korea
[2] Niels Bohr Institute, Copenhagen University, Blegdamsvej 17, Copenhagen DK-2100, Denmark
[3] Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

E-mail: wzhao7@mail.ustc.edu.cn and huangqg@itp.ac.cn

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Abstract
Testing the so-called consistency relations plays an important role in distinguishing the different classes of inflation models. In this paper, we investigate the possible testing for various single-field inflation models based on the potential future observations of the cosmic microwave background (CMB) radiation, including the planned CMBPol mission and the ideal CMB experiment where only the reduced cosmic weak lensing contamination for the B-mode polarization is considered. We find that for the canonical single-field inflation, the phantom inflation and the potential-driven G-inflation, the consistency relations are quite hard to be tested: the testing is possible only if $r > 0.14$ for the CMBPol mission, and $r > 0.06$ for the ideal experiment. However, the situation could become much more optimistic for the general Lorentz-invariant single-field inflation model with large non-local non-Gaussian signal. We find that testing the latter class of inflation is possible if $r \gtrsim 10^{-2}$ or even smaller for both CMBPol and ideal CMB experiments.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Understanding the expansion history of the Universe is a fundamental task of modern cosmology. The standard hot big-bang cosmological model is the most successful model to explain various observations [1]. However, in this scenario, one has to face the flatness, horizon and monopole puzzles. In order to solve these problems, various inflation-like scenarios for the expansion history of the Universe at the very early time have been proposed [2]. The necessity of this stage can also be understood in the following way: it seems logical to suggest

that our Universe came into being as a configuration with a Planckian size and a Planckian energy density, and with a total energy, including gravity, equal to zero (see [3] and references therein). The newly created classical configuration cannot reach the average energy density and size of the presently observed Universe, unless the configuration experienced a primordial kick, i.e. an inflation-like stage [3].

Nowadays, there are various inflation models in the market [2]. The problem is how to distinguish these quite different models from observations. It is well known that the strong variable gravitational field of the inflationary Universe inevitably generates primordial density perturbations (i.e. scalar perturbations) and relic gravitational waves (i.e. tensor perturbations) [4, 5]. The former provides the seed of the large-scale structure formation, and the latter faithfully encodes the information of the whole expansion history of the Universe [6]. Searching for the evidence of these perturbations provides a way to study the physics in the inflationary stage, and opens an observational window to explore the physics around the very high energy scale.

The current observations on the cosmic microwave background (CMB) radiation and the large-scale structure support that the primordial density perturbations have the nearly scale-invariant power spectrum which are predicted by the inflation models. However, it is not sufficient to distinguish various inflation models, i.e. almost all the models can explain the present observations, so long as the proper model parameters are chosen. At present, we do not even have an obscure picture of this stage: Was the inflationary stage promoted by a single effective scalar field, multiple scalar fields or some effective fields with non-canonical kinetic terms?

Recently, a number of authors have discussed how to test the inflation models by the current and potential future CMB observations (see, for instance, [7–13]). In these works, the authors mainly focused on testing the canonical single-field slow-roll inflation models, by determining the scalar spectrum index n_s, its running α_s and the tensor-to-scalar ratio r. Different from these works, in this paper, we investigate the possibilities of confirming or ruling out the different single-field inflationary scenarios (including canonical single-field models, phantom models, general Lorentz-invariant models and so on) by the potential future CMB observations.

One of the most powerful tools to distinguish these different scenarios is to test the so-called consistency relations, which are independent of the inflaton potential and are quite different in different scenarios. The experimental determination of the parameters specifying the relic gravitational waves plays a crucial role in approaching this aim. The CMB has proved to be a valuable tool in this respect. Relic gravitational waves leave an observable imprint in the temperature and polarization anisotropies on the CMB [14], which provides the unique way to detect the relic gravitational waves with the largest wavelength.

Although the recent effort, including the WMAP satellite [15, 16], QUaD [17], BICEP [18] and QUIET [19], has not found the definite evidence of relic gravitational waves, their purpose remains one of the key tasks for the current, upcoming and future CMB observations on the ground [17–26], on balloons [27–29] and in the space [11, 30–34].

The accurate measurement of the parameters specifying the relic gravitational waves, e.g. the spectral index n_t, depends on the full-sky observations of the CMB B-mode polarization field with the sensitive experiments [35–38]. The proposed CMBPol project [11], which is taken as a next-generation mission of the Planck satellite [31], provides an excellent opportunity to realize this aim. In this paper, we will carefully discuss the possibility of testing the consistency relations in various single-field inflation models by the CMBPol mission and an ideal CMB experiment where only the reduced cosmic weak lensing contamination for the B-mode polarization is considered. We find that although it is a very hard task which has been

claimed by many other authors, it is still possible to test these consistency relations by the CMBPol mission and the ideal CMB experiment, so long as the amplitude of relic gravitational waves is not too small. If so, these observations will certainly provide a great chance for us to investigate the physics in the early Universe and figure out a natural inflationary scenario.

The outline of the paper is as follows. In section 2, we introduce how to measure the parameters: the tensor-to-scalar ratio r and the spectral index n_t, by the potential future CMB observations, including the planned CMBPol mission and the ideal CMB experiment. Based on these results, in section 3, we carefully discuss the possibility of testing the inflationary consistency relations in the canonical single-field slow-roll inflation, the general Lorentz-invariant single-field inflation, the phantom inflation and the potential-driven G-inflation. Section 4 is a simple conclusion, which summarizes the main results in this paper.

2. Detecting relic gravitational waves in the CMB

The main contribution to the observed temperature and polarization anisotropies of the CMB comes from two types of the cosmological perturbations, density perturbations and relic gravitational waves. These perturbations are generally characterized by their primordial power spectra. These power spectra are usually assumed to be power-law, which is a generic prediction of a wide range of scenarios of the early Universe, for example the inflation models[4]. Thus, the power spectra of the perturbation fields take the form

$$P_s(k) = A_s(k_0)(k/k_0)^{n_s-1}, \qquad P_t(k) = A_t(k_0)(k/k_0)^{n_t}, \tag{1}$$

for density perturbations and relic gravitational waves, respectively. In the above expression, k_0 is an arbitrarily chosen pivot wavenumber, n_s is the primordial power spectral index for density perturbations and n_t is the spectral index for gravitational waves. $A_s(k_0)$ and $A_t(k_0)$ are the normalization coefficients determining the absolute values of the primordial power spectra at the pivot wavenumber k_0.

We can also define the tensor-to-scalar ratio as follows:

$$r(k_0) \equiv \frac{A_t(k_0)}{A_s(k_0)}, \tag{2}$$

which describes the relative contribution of density perturbations and gravitational waves. The amplitude of gravitational waves $A_t(k_0) = r(k_0)A_s(k_0)$ provides us with direct information on the Hubble parameter in the very early Universe [39]. More specifically, this amplitude is directly related to the value of the Hubble parameter H at a time when perturbation mode with wavenumber k_0 crossed the horizon [8]:

$$A_t^{1/2}(k_0) = \left.\frac{\sqrt{2}}{M_{\rm pl}}\frac{H}{\pi}\right|_{k_0/a=H}, \tag{3}$$

where $M_{\rm pl} = 1/\sqrt{8\pi G}$ is the reduced Planck mass. If we adopt $A_s = 2.430 \times 10^{-9}$ from the 7 year WMAP observations [15], the Hubble parameter is $H \simeq 2.67r^{1/2} \times 10^{14}$ GeV which only depends on the value of r. In the standard single-field slow-roll inflation models, the Hubble parameter directly relates to the energy scale of inflation $V^{1/4}$. Relation (3) follows that $V^{1/4} \simeq 3.35r^{1/4} \times 10^{16}$ GeV, which has been emphasized by a number of authors.

Density perturbations and gravitational waves produce temperature and polarization anisotropies in the CMB, which are characterized by four angular power spectra C_ℓ^T, C_ℓ^C, C_ℓ^E and C_ℓ^B as functions of the multipole number ℓ. Here, C_ℓ^T is the power spectrum of the

[4] In general there might be deviations from a power-law, which can be parameterized in terms of the running of the spectral index (see for example [39, 40]), but we will not consider this possibility in this paper.

temperature anisotropies, C_ℓ^E and C_ℓ^B are the power spectra of the so-called E-mode and B-mode of polarization (note that, density perturbations do not generate B-mode of polarization [14]) and C_ℓ^C is the power spectrum of the temperature-polarization cross correlation.

In general, the power spectra C_ℓ^Y (where $Y = T, E, B$ or C) can be presented in the following form:

$$C_\ell^Y = C_\ell^Y(\mathrm{dp}) + C_\ell^Y(\mathrm{gw}), \tag{4}$$

where $C_\ell^Y(\mathrm{dp})$ is the power spectrum due to the density perturbations and $C_\ell^Y(\mathrm{gw})$ is the spectrum due to gravitational waves.

Since we are primarily interested in the parameters of the gravitational-wave field, in the following discussion, we work with a fixed cosmological background model. More specifically, we work in the framework of the ΛCDM model, and keep the background cosmological parameters fixed at the values determined by a typical model [15] $h = 0.705$, $\Omega_b h^2 = 0.02255$, $\Omega_m h^2 = 0.1126$, $\Omega_k = 0$, $\tau_{\mathrm{reion}} = 0.088$, $A_s = 2.430 \times 10^{-9}$. Furthermore, the spectral indices of density perturbations and gravitational waves are adopted as follows for simplicity:

$$n_s = 1, n_t = 0. \tag{5}$$

Note that, although the constraint of n_s is quite tight based on the WMAP observations, its value strongly depends on the assumption of the primordial power spectrum, i.e. running or no running (see [15] for details). In this section, in order to simplify the calculation and without loss of generality, we assume the scale-invariant power spectra for both density perturbations and gravitational waves in the fiducial model.

The CMB power spectra C_ℓ^Y are theoretical constructions determined by ensemble averages over all possible realizations of the underlying random process. However, in real CMB observations, we only have access to a single sky, and hence to a single realization. In order to obtain information on the power spectra from a single realization, it is necessary to construct estimators of power spectra. In order to differentiate the estimators from the actual power spectra, we use the notation D_ℓ^Y to denote the estimators while retaining the notation C_ℓ^Y to denote the power spectrum. The probability distribution functions for the estimators predict the expectation values of the estimators

$$\langle D_\ell^Y \rangle = C_\ell^Y, \tag{6}$$

and the standard deviations

$$\begin{aligned} \left(\sigma_{D_\ell^X}\right)^2 &= \frac{2\left(C_\ell^X + N_\ell^X\right)^2}{(2\ell+1) f_{\mathrm{sky}}}, \qquad (X = T, E, B) \\ \left(\sigma_{D_\ell^C}\right)^2 &= \frac{\left(C_\ell^T + N_\ell^T\right)\left(C_\ell^E + N_\ell^E\right) + \left(C_\ell^C + N_\ell^C\right)^2}{(2\ell+1) f_{\mathrm{sky}}}, \end{aligned} \tag{7}$$

where f_{sky} is the sky-cut factor and N_ℓ^Y are the noise power spectra, which are all determined by the specific experiments.

In order to estimate the parameters r and n_t characterizing the gravitational-wave background, we use an analysis based on the likelihood function [41]. In previous work [35], we have analytically discussed how to constrain the parameters of the relic gravitational waves, r and n_t, by the CMB observations. We found that in general, the constraints on r and n_t correlate with each other. However, if we consider the tensor-to-scalar ratio at the best-pivot wavenumber k_t, the constraints on r and n_t become independent of each other, and the uncertainties Δr and Δn_t have the minimum values. We have derived the analytical formulae to calculate the following quantities: the best-pivot wavenumber k_t, and the uncertainties of the parameters Δr and Δn_t, which provides a simple and quick method to investigate the

detection abilities of the future CMB observations. In [35], we have also found that these analytical results are well consistent with the simulation results by using the Markov-chain Monte Carlo method. We briefly introduce these results in this section.

It is convenient to define the quantities as follows:

$$a_\ell^Y \equiv \frac{C_\ell^Y(\mathrm{gw})}{\sigma_{D_\ell^Y}}, \qquad b_\ell \equiv \ln\left(\frac{\ell}{\ell_t}\right), \qquad d_\ell^Y \equiv \frac{D_\ell^Y - C_\ell^Y(\mathrm{dp})}{\sigma_{D_\ell^Y}}, \tag{8}$$

where $\sigma_{D_\ell^Y}$ is the standard deviation of the estimator D_ℓ^Y, which can be calculated by equation (7). We should note that the quantity d_ℓ^Y is dependent on the random date D_ℓ^Y. By considering the relations in (6) and (4), we can obtain that $\langle d_\ell^Y \rangle = a_\ell^Y$, which shows that d_ℓ^Y is an unbiased estimator of a_ℓ^Y. ℓ_t is the so-called best-pivot multipole, which is determined by solving the following equation [35]:

$$\sum_\ell \sum_Y a_\ell^{Y2} b_\ell = 0. \tag{9}$$

So, the value of ℓ_t depends on the cosmological model, the amplitude of gravitational waves and noise power spectra by the quantity a_ℓ^Y. The best-pivot wavenumber k_t relates to ℓ_t by the approximation relation [35]

$$k_t \simeq \ell_t \times 10^{-4}\ \mathrm{Mpc}^{-1}. \tag{10}$$

Note that, in our previous work [35], the best-pivot wavenumber k_t and multipole ℓ_t have been denoted as k_t^* and ℓ_t^*, respectively. Once the value of ℓ_t is obtained, the uncertainties Δr and Δn_t can be calculated by the following simple formulae:

$$\Delta r = r \Bigg/ \sqrt{\sum_\ell \sum_Y a_\ell^{Y2}}, \qquad \Delta n_t = 1 \Bigg/ \sqrt{\sum_\ell \sum_Y \left(a_\ell^Y b_\ell\right)^2}. \tag{11}$$

As usual, we can define the signal-to-noise ratio $S/N \equiv r/\Delta r$. Using (11), we obtain

$$S/N = \sqrt{\sum_\ell \sum_Y a_\ell^{Y2}}. \tag{12}$$

Here, we mention that in equation (11) and throughout the paper below, the quantity r denotes the tensor-to-scalar ratio at the best-pivot wavenumber, i.e. $r \equiv r(k_t)$, which has been written as r^* in the previous work [35].

For a given gravitational-wave background, the values of S/N and Δn_t mainly depend on two experimental quantities: the total noise level of the experiment and the surveyed sky area. The lower noise and larger sky survey follow a larger S/N and a smaller Δn_t. In the previous works [16, 35–38], we have carefully investigated the detection abilities of various future CMB experiments. We found that in the optimistic case, the launched Planck satellite is expected to find the signal of gravitational waves if $r > 0.03$, consistent with [42]. The ground-based experiments, such as QUIET and POLARBEAR, are expected to have a detection if $r > 0.01$. However, both of them cannot well determine the spectral index n_t, due to the large noise level for the Planck satellite and the small sky-cut factor for the ground-based experiments. Even if we combine the Planck and ground-based POLARBEAR experiments, we only obtain $\Delta n_t = 0.1$ for the case with the tensor-to-scalar ratio $r = 0.1$ [36], which is not accurate enough to distinguish different inflation models.

It has been noted that the well detection of gravitational waves needs a full-sky observation by the high-sensitivity detectors. The proposed CMBPol mission provides an excellent opportunity in this respect [11]. The CMBPol mission is expected to have a full-sky survey for the CMB temperature and polarization fields, and the instrumental noises are close to, or even lower than, the cosmic weak lensing contamination for the B-mode polarization. So,

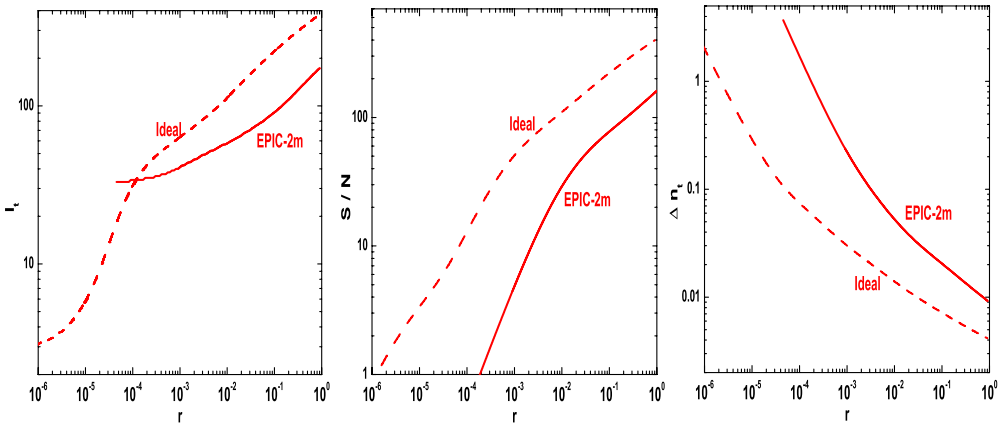

Figure 1. The figures show the values of the best-pivot multipole ℓ_t (left panel), signal-to-noise ratio S/N (middle panel) and the uncertainty of the spectral index Δn_t (right panel) as functions of the tensor-to-scalar ratio r.

in this paper, we discuss the detection of relic gravitational waves and the distinguishing of the inflation models by the potential CMBPol observations. For the CMBPol mission, we consider a sky-cut factor $f_{\rm sky} = 0.8$, proposed in the CMBPol white book [11]. The total noises of CMBPol observation mainly include three parts: the instrumental noises, the foreground contaminations (including the synchrotron and dust emissions) and cosmic weak lensing contamination for the B-mode polarization. For the instrumental noises, there are several proposals [11, 43]. In this paper, we focus on the middle-cost EPIC-2m proposal. The detailed calculation of the total noise power spectra of EPIC-2m is given in our recent work [38], where the analytical formulae are given to calculate the noise power spectrum. Throughout this paper, we have used the CAMB package [44] to calculate the CMB power spectra, and the contaminations due to the cosmic weak lensing. We note that, the total noise power spectra depend on the assumed parameters $\sigma^{\rm fore}$ and $\sigma^{\rm lens}$, which describe the residual fractions of foreground emissions and lensed B-mode polarization considered as the effective noises. In this paper, we focus on the optimistic case with the assumed parameters $(\sigma^{\rm fore}, \sigma^{\rm lens})$ being (0.01, 0.5). Note that, in our discussion, we have not considered the leakage from the E-mode into the B-mode polarization due to the partial sky analysis. We assume that this E–B mixture can be properly avoided (or deeply reduced) by constructing the pure E-mode and B-mode polarization fields [45].

Taking into account the total noises, and using the formulae in equations (9), (11) and (12), we calculate the quantities ℓ_t, S/N and Δn_t as functions of the input tensor-to-scalar ratio r. The results are shown in figure 1 (red solid lines), which are consistent with the previous works [11, 37, 38]. Figure 1 shows that as expected, the model with a larger r follows a larger S/N, and a smaller Δn_t. It is interesting to find that when $r > 0.001$, EPIC-2m can detect the signal of gravitational waves at more than 5σ level. For the $r = 0.01$ case, we have $S/N = 29$, and for $r = 0.1$, we have $S/N = 78$. So we conclude that EPIC-2m can well detect the signal of gravitational waves so long as the tensor-to-scalar ratio is larger than 0.001. At the same time, the determination of the spectral index n_t is also quite accurate. If $r = 0.001$, one has $\Delta n_t = 0.2$, and if $r = 0.01$, one has $\Delta n_t = 0.05$. Especially, for $r = 0.1$, the uncertainty reduces to $\Delta n_t = 0.02$. So, it would be a quite powerful tool to study the physics in the

early Universe, especially for distinguishing different inflation models, which will be shown in section 3.

It was noted that the detection abilities of the future CMB experiments for the relic gravitational waves are limited by the cosmic variance and cosmic weak lensing effect (see [46] and references therein). Especially, when the instrumental noise power spectra of the future experiment become smaller than $\sim 10^{-6}\mu\mathrm{K}^2$, the weak lensing contamination for the B-mode polarization could be dominant among the total noises, and forms a detection limit for the CMB experiments [35, 47]. A number of works have discussed methods to subtract the lensed B-mode signal (see [48, 49]). In [49], the authors claimed that a reduction in lensing power by a faction of 40, i.e. a residual faction $\sigma^{\mathrm{lens}} = 0.025$, is possible using an approximate iterative maximum-likelihood method. For this reason, as an idealized scenario, we also consider the case with reduced cosmic lensing noise with σ^{lens}. In this ideal case, we assume an exactly full sky survey with $f_{\mathrm{sky}} = 1$. We also assume that there are no instrumental noises or foreground emissions.

Using similar steps, we calculate the quantities ℓ_t, S/N and Δn_t as functions of the input tensor-to-scalar ratio r in this ideal case. The results are also shown in figure 1 with dashed lines, consistent with [35]. We find that the gravitational waves with $r > 3.7 \times 10^{-6}$ can be detected at more than 2σ level. This can be treated as the detection limit of the CMB experiments. This lower limit corresponds to the Hubble parameter $H \simeq 3.1 \times 10^{11}$ GeV, and the energy scale of inflation $V^{1/4} \simeq 1.5 \times 10^{15}$ GeV. From figure 1, we also find that when $r > 2 \times 10^{-5}$, the signal-to-noise ratio becomes quite large, i.e. $S/N > 5$. In this idealized situation, the uncertainty of the spectral index n_t also becomes very small. We can constrain n_t to the level $\Delta n_t = 0.014$ if $r = 0.01$, and $\Delta n_t = 0.007$ for $r = 0.1$. Distinguishing different inflation models in this ideal case will also be presented in section 3.

3. Testing the inflationary consistency relations

Nowadays, the inflationary scenario has been widely accepted by almost all of the cosmologists. Inflation can naturally explain the well-observed primordial density perturbations with a nearly scale-invariant power spectrum. However, we still do not know which inflation model describes our real Universe, since almost all the models are compatible with the current observations if the proper model parameters are adopted. So, distinguishing different inflation models, especially different classes of models, is the key task for the future research.

The so-called consistency relation gives a clear difference for different classes of inflation. For example, for the canonical single-field slow-roll inflation models, the consistency relation $r = -8n_t$ is held, which provides a unique way to confirm or rule out this class of models. In this section, we carefully investigate the possibility of testing the consistency relations for the canonical single-field slow-roll inflation, the general Lorentz-invariant single-field inflation, the phantom inflation and the potential-driven G-inflation, by the potential future observations, such as the planned CMBPol mission and the ideal CMB experiment.

3.1. Canonical single-field slow-roll inflation model

First of all, we focus on the simplest version of inflation, i.e. the canonical single-field slow-roll inflation model. In this scenario, the dynamics of the Universe is governed by a scalar field (the inflaton) ϕ with a canonical kinetic term. The inflaton slowly rolled down its flat potential in the inflationary stage. Inflation ended when the slow-roll conditions were broken down, and the inflaton decayed into the relativistic particles and re-heated the Universe. See some recent discussions in [50].

The Lagrangian for the canonical single-field inflation model is given by [2]

$$\mathcal{L}(\phi) = \tfrac{1}{2}(\partial\phi)^2 - V(\phi), \tag{13}$$

where $V(\phi)$ is the potential of the inflaton field ϕ. Different $V(\phi)$ describes the different inflation models. In the inflationary stage, the potential energy of the inflaton dominates over its kinetic energy, and $V(\phi)$ should be quite flat. Thus, we can define the slow-roll parameter

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2, \tag{14}$$

where the *dot* denotes d/dt, and the *prime* denotes $d/d\phi$. This parameter ϵ should be much smaller than the one during inflation.

The tensor-to-scalar ratio r and the tensor spectral index n_t in this scenario are related to the slow-roll parameter ϵ by [2]

$$r = 16\epsilon, \qquad n_t = -2\epsilon. \tag{15}$$

The above equations lead to the so-called consistency relation for the canonical single-field slow-roll inflation [2] (for a detailed critical discussion of this consistency relation see the last paper in [4]):

$$n_t = -r/8. \tag{16}$$

This consistency relation is independent of the form of the potential and valid for all the single-field slow-roll inflation models with canonical kinetic terms[5]. So, testing this relation provides a model-independent criteria to confirm or rule out the canonical single-field slow-roll inflation models.

From (16), we find that testing this relation depends on the measurement of the gravitational-wave's parameters r and n_t, since the absolute value of n_t is expected to be one order smaller than that of r (see equation (16)), and the measurement of n_t is much more difficult than that of r [35–37]. How well we can measure the spectral index n_t plays a crucial role in testing the consistency relation (16).

Now, let us discuss how well this consistency relation can be tested by the potential future CMB observations, which has been partly discussed in the previous works [13, 37, 38, 52]. Here, we will revisit this problem based on the discussion in section 2, where a best-pivot wavenumber and the best determinations of the parameters r and n_t are considered.

The uncertainty Δn_t as a function of the input r is replotted in figure 2 (red lines). We investigate if the future experiments might distinguish the tilted gravitation waves from the scale-invariant one (i.e. $n_t = 0$). To attain the goal of this testing, in figure 2 we compare the value of $|n_t| = r/8$ with that of Δn_t [6]. If $\Delta n_t < |n_t|$, then the constraint on n_t is tight enough to allow the consistency relation to be tested. We find that $\Delta n_t < |n_t|$ is satisfied only if $r > 0.14$ for the EPIC-2m experiment, which is quite close to the current upper limit of r obtained from the 7 year WMAP observations [15]. So we conclude that there is only a small space left for testing the consistency relation for CMBPol. This is the reason why many people claimed that the consistency relation is difficult to test [11]. However, from figure 2, we find that this situation can be slightly alleviated for the ideal observation. In this case, $\Delta n_t < |n_t|$

[5] It was pointed out that this consistency relation could be violated if considering the trans-Planckian physics in the early Universe [51]. So in this sense, the testing of this consistency relation also provides the change to study the trans-Planckian physics.

[6] In principle, for any given n_t, we should compare its value with that of Δn_t, derived from the model with this n_t as input. However, we note that in the inflation models considered in this paper, the value of n_t is always very close to zero, which follows that the difference between these Δn_t's and that in section 2 is very small. Throughout this paper, we ignore this little difference.

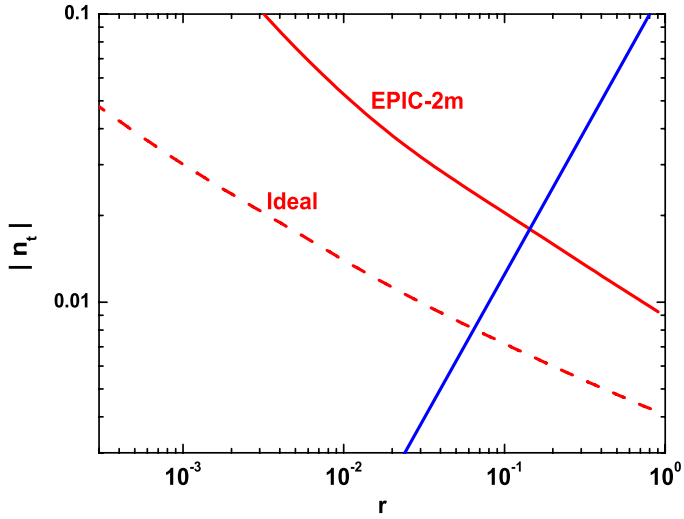

Figure 2. For the EPIC-2m mission and the ideal CMB experiment, the value of Δn_t compares with that of $|n_t| = r/8$ (grey (blue) solid line). Note that the red lines are identical to those in figure 1 (right panel).

is satisfied so long as $r > 0.06$. Thus, it is possible to test the consistency relation (16) only for some inflation models with fairly large r.

Let us consider two specific models. First, we discuss the *chaotic inflation*. The prototype for chaotic inflation involves a single polynomial term $V(\phi) = \Lambda_p(\phi/\mu)^p$ with $p > 0$ [53]. Here, the scale $\mu < M_{\rm pl}$ is relevant for the higher dimensional terms in this effective potential. The chaotic inflation models of this form make the following prediction [11]:

$$r = 8\left(\frac{p}{p+2}\right)(1-n_s). \tag{17}$$

Assuming $n_s = 0.968$ [15] and $p = 2$, we have $r = 0.128$. So in this model, the consistency relation could be tested by the ideal experiment. Another typical model we consider is the *hill-top model* with quadratic term, which has the potential form $V(\phi) = V_0\left[1-(\phi/\mu)^p\right]$ with $p \geqslant 2$ and $\phi < \mu$. This potential is considered as an approximation to a generic symmetry-breaking potential [2, 11]. If $p = 2$, the value of r can be expressed as [11]

$$r = 8(1-n_s)\exp[-1-N_e(1-n_s)], \tag{18}$$

where N_e is the number of e-folds, taking to be in the range $N_e \in [40, 70]$ based on the current observations of the CMB [15]. For $n_s = 0.968$, we find that $r \in [0.010, 0.026]$. So, if this model describes our real Universe, the consistency relation cannot be tested even in the ideal case.

3.2. General Lorentz-invariant single-field inflation model

In this subsection, we will consider the general Lorentz-invariant single-field inflation model, in particular the model with non-canonical kinetic terms, such as K-inflation [54], Dirac–Born–Infeld (DBI) inflation [55], the power-law kinetic inflation [56, 57] and so on. These models may be motivated by the high-dimensional superstring theory or brane theory [11].

The Lagrangian for a general Lorentz-invariant single-field inflation takes the form

$$\mathcal{L}(\phi) = P(X, \phi), \tag{19}$$

where $X \equiv (\partial\phi)^2/2$. The canonical single-field inflation is included in (19). The function $P(X, \phi)$ corresponds to the pressure of the scalar fluid, while the energy density is $E = 2XP_{,X} - P$, where $P_{,X} \equiv \partial P/\partial X$. An important parameter in this model is the speed of sound, which is defined as

$$c_s^2 \equiv \frac{P_{,X}}{E_{,X}} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}. \tag{20}$$

For the canonical single-field inflation we have $c_s^2 = 1$. However, in the general K-inflation model, the value of c_s^2 can be larger or smaller than the speed of light [54].

The parameters related to the gravitational waves can be quantified in terms of the sound speed c_s and the slow-roll parameter $\epsilon \equiv -\dot{H}/H^2$ [54], i.e.

$$r = 16\epsilon c_s, \qquad n_t = -2\epsilon. \tag{21}$$

Thus, the consistency relation becomes

$$n_t = -\frac{r}{8c_s}. \tag{22}$$

When $c_s = 1$, equation (16) is naturally recovered. However, when $c_s \gg 1$ or $c_s \ll 1$, the difference between (22) and (16) becomes obvious, and makes it possible to distinguish these two classes of models.

Due to the dependence of c_s in the consistency relation (22), it is necessary to consider another observation to constrain the sound speed in the models. In addition to directly constraining c_s, there are some other possibilities of constraining the sound speed by the observations. For the models with $c_s \neq 1$, higher derivative terms in the Lagrangian are included and usually a large non-local form bispectrum is predicted. However, the full bispectrum is controlled by two parameters [58]: c_s^2 and $\lambda/\Sigma = \left(X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}\right)\big/(XP_{,X} + 2X^2P_{,XX})$. Therefore, we need two observables to fix c_s^2. Fortunately, two independent templates have been well defined for measuring the non-local form bispectrum: equilateral and orthogonal forms whose sizes are respectively measured by two non-Gaussian parameters $f_{\rm NL}^{\rm equil}$ and $f_{\rm NL}^{\rm orth}$ [15, 59–61]. In some specific cases, such as the DBI inflations [55] and power-law kinetic inflations [56, 57], c_s^2 and λ/Σ can be determined by the unique parameter $f_{\rm NL}^{\rm equil}$. However, for the general Lorentz-invariant single-field inflations, in [62] one of us (QGH) found that the speed of sound c_s can be fixed once both $f_{\rm NL}^{\rm equil}$ and $f_{\rm NL}^{\rm orth}$ are detected, i.e.

$$\frac{1}{c_s^2} - 1 = -1.260 f_{\rm NL}^{\rm equil} - 23.19 f_{\rm NL}^{\rm orth}. \tag{23}$$

The constraints on the non-Gaussian parameters by the current and potential future CMB observations have been discussed by a number of authors [15, 59, 63–65]. Different from the constraint on the relic gravitational waves, the detection of a non-Gaussian signal mainly depends on the observations of CMB TT, TE and EE power spectra. So, we expect that there is no degeneration between constraints of the non-Gaussian parameters and those of the gravitational-wave's parameters. The current 7 year WMAP data indicate a constraint on the non-Gaussian parameters as follows [15]:

$$-214 < f_{\rm NL}^{\rm equil} < 266, \qquad \text{and} \qquad -410 < f_{\rm NL}^{\rm orth} < 6 (95\%\text{C.L.}). \tag{24}$$

These constraints have been obtained using the temperature signal only. The upcoming Planck satellite will improve this to the level $\Delta f_{\rm NL}^{\rm equil} \simeq \Delta f_{\rm NL}^{\rm orth} \simeq 25$, mainly by the observations of the E-mode polarization signal. In addition, a satellite mission such as CMBPol, dedicated to polarization and cosmic variance limited up to $\ell \sim 2000$, would be able to further improve on Planck by a factor of order 1.6, reaching $\Delta f_{\rm NL}^{\rm equil} \simeq \Delta f_{\rm NL}^{\rm orth} \simeq 14$ [11, 66]. This can also be treated as the detection ability of the ideal CMB experiment, due to the cosmic variance limit.

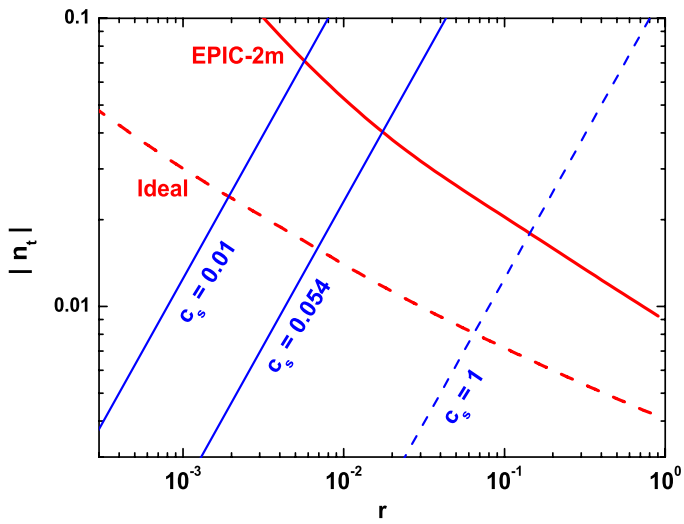

Figure 3. For the EPIC-2m mission and the ideal CMB experiment, the value of Δn_t compares with that of $|n_t| = r/(8c_s)$ (grey (blue) lines). Note that, the red lines are identical to those in the figure 1 (right panel), while the dashed blue line is identical to the blue line in figure 2.

In the limit of $f_{\rm NL}^{\rm equil} = -214$ and $f_{\rm NL}^{\rm orth} = -410$, signal of non-Gaussianity will be quite well observed by the future observations, such as the Planck satellite [31]. In this limit, we have $c_s = 0.01$ and the consistency relation becomes $n_t = -12.5r$. In figure 3, we compare the values of $|n_t| = 12.5r$ (blue solid line) with Δn_t. We find that $\Delta n_t < |n_t|$ is satisfied if $r > 0.006$ for EPIC-2m experiment, and $r > 0.002$ for the ideal experiment. Comparing with the conclusion in section 3.1, the quite promising results are expected for the study of inflation.

However, we have to mention that this is a too optimistic case. Actually, the values of $|f_{\rm NL}|$ could be much smaller. With the decrease of $|f_{\rm NL}|$, the value of c_s increases, and the testing of the consistency relation becomes more and more difficult. A non-Gaussian signal exceeding $|f_{\rm NL}| \geqslant 14$ will be detectable from the future CMB observations [11]. So as another limit case, let us assume $f_{\rm NL}^{\rm equil} = f_{\rm NL}^{\rm orth} = -14$, which follows that $c_s = 0.054$ and the consistency relation $n_t = -2.31r$. From figure 3, we find that $\Delta n_t < |n_t|$ is satisfied if $r > 0.017$ for EPIC-2m experiment, and $r > 0.007$ for the ideal experiment. We mention that when $\left|f_{\rm NL}^{\rm equil}\right| < 14$ or $\left|f_{\rm NL}^{\rm orth}\right| < 14$, the detection of the non-Gaussian signal becomes impossible, and testing of the consistency relation by this method becomes impossible as well. In figure 4, we plot the value of $r_{\rm min}$ (where $|n_t| = \Delta n_t$ is satisfied) for different values of c_s by the blue lines. In this figure, we use the dashed grey lines to label two limit cases considered above. It is important to mention that these results are only correct for the non-canonical Lorentz-invariant single-field inflation models. As a subclass of the general single-field inflation model, the canonical inflation model has a definite prediction $c_s = 1$. So the sound speed is not a free parameter for this subclass of models, and we do not need to constrain c_s for the testing of the consistency relation, which has been clearly discussed in section 3.1

Recall that an important goal of inflation programs is to test whether inflation really arises from the canonical single-field slow-roll models. To attain the goal of this testing, we should compare Δn_t with the quantity $\delta n_t \equiv |n_t - r/8|$. If $\Delta n_t < \delta n_t$, the constraint of n_t is tight enough to allow this test. Similar to above, for any given c_s, we define $r'_{\rm min}$ (where $\delta n_t = \Delta n_t$ is satisfied). This is the minimal r, when the test is allowed. In figure 4, we also plot the value of $r'_{\rm min}$ for different values of c_s by the black lines. As expected, the black lines are quite close to the corresponding blue ones.

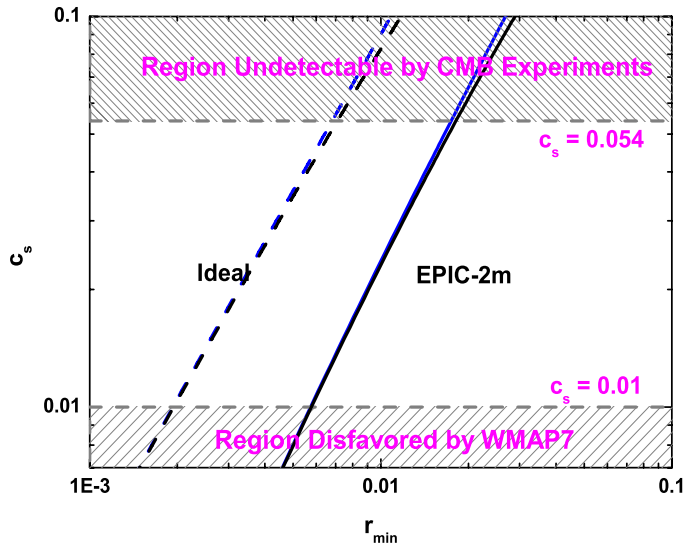

Figure 4. The values of $r_{\min}$ (grey (blue) lines) and $r'_{\min}$ (black lines) as functions of the speed parameter c_s for both the EPIC-2m mission and the ideal CMB experiment.

3.3. Phantom inflation model

In a spatially flat Friedmann–Lemaître–Robertson–Walker Universe, the null energy condition corresponds to the inequality $\dot{H} < 0$ and hence $n_t < 0$. In order to obtain blue tilted gravitational waves, one needs to break the null energy condition, for example phantom inflation [67] in which the Lagrangian is given by

$$\mathcal{L}(\phi) = -\tfrac{1}{2}(\partial\phi)^2 - V(\phi). \tag{25}$$

The tensor-to-scalar ratio r and the tensor spectral index n_t in this scenario are related to the slow-roll parameter ϵ by

$$r = 16\epsilon, \qquad n_t = 2\epsilon, \tag{26}$$

which leads to the consistency relation for the phantom inflation [67]:

$$n_t = r/8. \tag{27}$$

This consistency relation is the same as that in (16), except the sign of n_t. Therefore, the analysis on phantom inflation is exactly the same as that in the canonical single-field inflation discussed in section 3.1. So, we can easily obtain the conclusion: $\Delta n_t < |n_t|$ is satisfied only if $r > 0.14$ for the EPIC-2m experiment, $r > 0.06$ for the ideal CMB experiment. However, distinguishing phantom inflation from the canonical single-field slow-roll inflation is a little easier. In order to attain this goal, we compare Δn_t with $\delta n_t = r/4$. We find that $\Delta n_t < \delta n_t$ is satisfied if $r > 0.09$ for EPIC-2m experiment, and $r > 0.06$ for the ideal experiment.

3.4. Potential-driven G-inflation model

Recently, an inflation model dubbed as 'G-inflation' was proposed in [68–70]. In this class of models, inflation is driven by a scalar field with Galileon-like kinetic term. In [68], the authors found that the model can generate a scale-invariant power spectrum of density perturbations,

and significantly large amplitude of gravitational waves. The general Lagrangian of G-inflation is of the form

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \tag{28}$$

where K and G are general functions of ϕ and $X \equiv (\partial\phi)^2/2$. If $G = 0$, this model returns to the general Lorentz-invariant single-field inflation discussed in section 3.2. In [70], the potential-driven G-inflations were discussed. In this subclass of models, the Lagrangian in (28) has the following form:

$$K(\phi, X) = X - V(\phi), \qquad G(\phi, X) = g(\phi)X. \tag{29}$$

In the inflationary stage, the energy density is dominated by the potential $V(\phi)$ under the slow-roll condition. In these models, the model-independent consistency relation between the tensor-to-scalar ratio and the tensor spectral index is satisfied [70]:

$$n_t = -\frac{9r}{32\sqrt{6}}, \tag{30}$$

which is the smoking-gun evidence for the potential-driven G-inflation.

First, we investigate whether it is possible to test the consistency relation in (30) by discriminating the tilted gravitational waves from the scale-invariant one. Very similar to the case with canonical single-field slow-roll inflation, we find that $\Delta n_t < |n_t|$ is satisfied if $r > 0.14$ for the EPIC-2m mission, and $r > 0.06$ for the ideal experiment.

Another test for the consistency relation is discriminating this G-inflation from the canonical single-field inflation. To attain the goal, we compare Δn_t with $\delta n_t \equiv |n_t - r/8|$. Since the two consistency relations (16) and (30) are very close to each other, which induces a very small $\delta n_t = 0.01r$, we find that it is impossible to obtain the condition $\Delta n_t < \delta n_t$ for the EPIC-2m mission so long as $r < 1$. Even if we consider the ideal CMB observations, $\Delta n_t < \delta n_t$ is satisfied only if $r > 0.47$, which is conflicted with the current constraint on r [15]. So we conclude that by the CMB observations, it is impossible to discriminate the potential-driven G-inflation from the canonical single-field inflation by testing the consistency relations.

4. Conclusions

The inflationary scenario has been accepted by most cosmologists, which naturally solves the flatness, horizon and monopole puzzles in the standard hot big-bang cosmological model, and predicts the nearly scale-invariant power spectra of the primordial density perturbations and gravitational waves. Although the recent experimental efforts, including the CMB and the large-scale structure, have led to a robust detection of the primordial density perturbations, and indirectly supported the existence of the early inflationary stage, how to distinguish different inflation models still remains an outstanding experimental challenge. It depends on how accurately we can measure the primordial perturbations, in particular the primordial gravitational waves.

In this paper, based on the potential future CMB observations by the planned CMBPol mission and an ideal CMB experiment, we investigated the possible tests for the consistency relations in several classes of single-field inflation models: the canonical single-field slow-roll inflation, the general Lorentz-invariant single-field inflation, the phantom inflation and the potential-driven G-inflation. For the canonical single-field inflation, phantom inflation and the potential-driven G-inflation, we found that the consistency relations are quite hard to be tested, due to the smallness of the spectral index n_t. For example, the testing is possible only if $r > 0.14$ for the CMBPol mission and $r > 0.06$ for the ideal experiment.

Dramatically the situation becomes quite promising for the general Lorentz-invariant single-field inflation with large non-local non-Gaussianity, because the value of $|n_t|$ could be quite large compared to that in the canonical single-field inflation for a given r in these cases. For the general Lorentz-invariant single-field inflation with the non-Gaussian parameters $f_{\rm NL}^{\rm equil} = f_{\rm NL}^{\rm orth} = -20$, the testing is possible if $r > 0.015$ for the CMBPol mission and $r > 0.006$ for the ideal experiment.

At the end of this paper, it is worthy to point out that the similar analysis can be applied to observationally test some other inflationary scenarios [71, 72]. At the same time, in addition to the relations related to r and n_t, the other inflationary consistency relations (see [62, 73, 74] for instance) can also be used to distinguish different inflation models. We leave it as a future work.

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