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Yang–Mills condensate dark energy coupled with matter and radiation

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Abstract

The coincidence problem is studied for the dark energy model of effective Yang–Mills condensate (YMC) in a flat expanding universe during the matter-dominated stage. The YMC energy $\rho_y(t)$ is taken to represent the dark energy, which is coupled either with the matter $\rho_m(t)$, or with both the matter and the radiation components $\rho_r(t)$. The effective YM Lagrangian is completely determined by the quantum field theory up to 1-loop order with an energy scale $\sim 10^{-3}$ eV as a model parameter, and for each coupling, there is an extra model parameter. We have studied extensively the coupling models: the YMC decaying into the matter and the radiation; or vice versa the matter and radiation decaying into the YMC. It is found that, starting from the equality of radiation-matter $\rho_{mi} = \rho_{ri}$, for a wide range of initial conditions of $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, the models have a scaling solution during the early stages, and the YMC levels off and becomes dominant at late time, and the present state with $\Omega_y \simeq 0.7$, $\Omega_m \simeq 0.3$ and $\Omega_r \simeq 10^{-5}$ is always achieved. If the YMC decays into a component, then this component also levels off later and approaches a constant value asymptotically, and the equation of state (EoS) of the YMC $w_y = \rho_y/p_y$ crosses over -1 and takes the value $w_y \simeq -1.1$ at $z = 0$. If the matter and radiation decay into the YMC, then $\rho_m(t) \propto a(t)^{-3}$ and $\rho_r(t) \propto a(t)^{-4}$ approximately for all the time, and w_y approaches -1 but does not cross over -1. We have also demonstrated that, at $t \to \infty$, the coupled dynamics for $(\rho_y(t), \rho_m(t), \rho_r(t))$ is a stable attractor. Therefore, under generic circumstances, the existence of the scaling solution during the early stages and the subsequential exit from the scaling regime around $z \simeq (0.3–0.5)$ are inevitable. Thus the coincidence problem can be naturally solved in the YMC dark energy models.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The observations on the cosmic microwave background radiation (CMB) [1] suggest a flat universe. This observation, together with that from the Type Ia Supernova (SNIa) [2], implies that the universe consists of some mysterious dark energy ($\Omega_\Lambda \sim 0.73$), dark matter ($\Omega_d \sim 0.23$), ordinary baryon matter ($\Omega_b \sim 0.04$) and a radiation component $\Omega_r \sim 10^{-5}$. This is also supported by the large scale structure of the universe [3]. The dark energy as the dominant cosmic energy component drives the current accelerating expansion of the universe. The simplest model for the dark energy is the cosmological constant Λ, which corresponds to a homogeneous and time-independent energy density $\rho_\Lambda \sim 5.8h_0^2 \times 10^{-11}$ eV^4 with an EoS of $w_\Lambda = p_\Lambda/\rho_\Lambda = -1$. Although this model can account for the observations so far, it has a coincidence problem. The observations show that the present value of the energy density for the matter component $\rho_m = \rho_d + \rho_b$ is about one third of ρ_Λ, but it varies with time as $\rho_m(t) \propto a(t)^{-3}$. So, for example, at an earlier time of radiation-matter equality with redshift $z \simeq 3454$ [4], ρ_Λ should be a very fine tuned value $\simeq 6.3 \times 10^{-11}\rho_m(t)$. Otherwise, a slightly variant initial value of ρ_Λ would lead to a value of the ratio ρ_Λ/ρ_m drastically different from the observed one. This is called the coincidence problem.

One class of models aiming at solving the coincidence problem is based upon the dynamics of some scalar field ϕ, such as quintessence [5], K-essence [6], tachyon [7], phantom [8] and quintom [9], etc. These models can give rise to certain desired features of evolutional dynamics, such as scaling solutions [10] and tracking behaviour. As a common point, these scalar models need to make use of some special forms of the potential $V(\phi)$ with certain chosen parameters. For example, in the quintessence model, one needs to choose $V(\phi) \propto (M/\phi)^\alpha$ with α and M being a positive number, or $\propto e^{-\phi/m_{pl}}$ [11]. In the phantom model one may take $V(\phi) = V_0[\cosh(\alpha\phi/m_{pl})]^{-1}$ [12]. Moreover, phantom models typically introduce a negative kinetic energy term $-\dot{\phi}^2/2$. Some of these models are expected to be the low energy effective field theory, coming from some fundamental field theory; others are simply introduced by hand. The approach based on the Born–Infeld quantum condensate [13] is interesting, which seeks an alternative candidate for the dark energy other than the scalar fields. But there the important issue of evolution has not been addressed in detail.

As far as cosmological observations are concerned, a cosmological model has to give the current status: $\Omega_\Lambda \sim 0.7$ and $\Omega_m \sim 0.3$. Besides, it is safer for a model not to contradict the conventional scenario in the Big Bang model from the energy scale $\sim$1 MeV down to the present. Therefore, one idea to solve the coincidence problem is that during the early stages of the expansion the dark energy density need not be a constant, but varies with time. Even its EoS w need not be close to -1 at early stages. Only at some rather recent moment has it become dominant and acquired an EoS $w \sim -1$. In order to allow the conventional cosmological processes, such as the nucleosynthesis and the recombination, etc, to have been occurred in the past, the dark energy component should be subdominant to the matter component at early stages.

In the approach of the scalar field models, certain coupling has been introduced between the scalar field dark energy and the matter [14–16]. With some particular choice of the model parameters, there can exist a scaling solution of dynamics, in which the dark energy density is proportional to that of matter during the early stages of evolution. However, to achieve the tracking solution, some scalar models need to have a very large coupling, so that the universe would enter the acceleration stage soon after the matter era. This would result in a picture of structure formation, totally different from that required by the observations. To remedy this defect some models [17, 18] introduce certain particular forms of couplings, but still the entrance to the accelerating stage is a little earlier than what observations suggested. On the other hand, it has been pointed out that the k-essence models always have, at some stage, the difficulty of superluminal propagation, leading to violation of causality, and the EoS and the sound speed of k-essence could be greater than >1 [19]. Besides the unconventional negative kinetic energy, after the EoS w cross -1, a class of scalar models may suffer from severe quantum instabilities and the Big Rip singularity, i.e. either the energy density ρ, or the pressure p, may grow to infinity within a finite time. More recently, an overall estimate of scalar models has been given on the issue of coincidence problem. By examining the most general form of scalar Lagrangian with a generic coupling between scalar dark energy and dark matter, it has been shown that a vast class of scalar models, including all the models in the current literature, have the difficulty of implementing both a scaling solution without singularity and a sequence of expansion epochs required by standard cosmology, such as the radiation-dominated, matter-dominated and dark energy-dominated epochs [20]. Therefore, the coincidence problem still remains after over a decade of extensive studies [21].

The introduction of the quantum effective YMC into cosmology [22] has been motivated by the fact that the $SU(3)$ YMC has given a phenomenological description of the vacuum within hadrons confining quarks, and yet at the same time all the important properties of a proper quantum field are kept, such as the Lorentz invariance, the gauge symmetry and the correct trace anomaly [23]. Quarks inside a hadron would experience the existence of the Bag constant, B, which is equivalent to an energy density $\rho = B$ and a pressure $p = -B$. So quarks would feel an energy–momentum tensor of the vacuum as $T_{\mu\nu} = B\,\mathrm{diag}(1, -1, -1, -1)$. This non-trivial vacuum has been formed mainly by the contributions from the quantum effective YMC, and from the possible interactions with quarks. Our thinking has been that, like the vacuum of QCD inside a hadron, what if the vacuum of the universe as a whole is also filled with some kind of YMC. Gauge fields play a very important role in, and are the indispensable cornerstone to, particle physics. All known fundamental interactions between particles are mediated through gauge bosons. Generally speaking, as a gauge field, the YMC under consideration may have interactions with other species of particles in the universe. In our previous studies of [27, 28], the possible interactions of the YMC with other cosmic components have not been examined. However, unlike those well known interactions in QED, QCD and the electro-weak unification, at the moment we do not yet have a model for the details of the microscopic interactions between the YMC and other particles. Therefore, in this paper on the dark energy model, we will adopt a simple description of the possible interactions between the YMC and other cosmic particles. That is, we introduce coupling terms in the continuity equations of cosmic energy densities, such as the YMC, the matter and the radiation, study the cosmic evolution of the universe from the matter-dominated era up to the present. As we will show, in our model the current status of the universe turns out to be a natural result of evolutional dynamics driven by the effective YMC as the dark energy, plus the matter and the radiation that are coupled to the former. What is important is that this has been achieved with a choice of the initial value of the fractional energy density of the YMC ranging from 10^{-10} to 10^{-2}. As a novel feature in contrast to the non-coupling models, the coupling YMC dark energy models can give rise to an EoS of dark energy w_y crossing -1, say $w_y \sim -1.1$ at present. The coincidence problem can be solved.

In section 2, as a basis for the setup, an introduction is given to the effective Yang–Mills condensate theory, and the dynamic equations for the three components in the Robertson–Walker spacetime are derived. Section 3 is about the simplest case of non-coupling. Section 4 studies the dynamic cosmic evolution with the effective YMC decaying into the matter component. There are two dynamic equations for the YMC and the matter, respectively. Section 5 studies the model in which the matter decays into the YMC. The matter component has different dynamic evolutions, especially at late stages for these two coupling models.

Nevertheless, in both these two models, the YMC has a scaling solution and a tracking behaviour as a natural outcome of the dynamic evolution. Section 6 extends to the general case with the YMC coupling to both the matter and the radiation. Now one has one more coupled dynamic equation and an extra coupling for the radiation component. Two cases are studied, the YMC decaying into the matter and radiation, and the matter and radiation decaying into the YMC. The dynamics are examined parallel to sections 4 and 5. In each of these four models, various functional forms of coupling have been explored. The major part of the study is in sections 4, 5 and 6, which contain the calculations and the results. Section 7 gives an analysis on asymptotic behaviour of the dynamic system at $t \to \infty$. It is found that there exists a unique attractor in the asymptotic region, which is stable against perturbations. Section 8 contains a summary and discussions.

Throughout this paper we will work with unit, in which $c = \hbar = k_B = 1$.

2. YM condensate as dark energy

In the effective YMC dark energy model, the effective YM field Lagrangian is given by [22, 23]:

$$L_{\text{eff}} = \frac{1}{2} b F \left(\ln \left| \frac{F}{\kappa^2} \right| - 1 \right) \tag{1}$$

where κ is the renormalization scale of dimension of squared mass, $F \equiv -\frac{1}{2} F^a_{\mu\nu} F^{a\mu\nu} = E^2 - B^2$ plays the role of the order parameter of the YMC. In this paper, for simplicity, we only discuss the pure 'electric' case, $F = E^2$. For a general case with both 'electric' and 'magnetic' fields being present, the extended model of YM dark energy also works, and the behaviour of evolution of dark energy is similar to the pure 'electric' case. Specifically, in the expanding universe, a given 'magnetic' component of YM field decreases quite rapidly, and the YM field becomes the 'electric' type. The Callan–Symanzik coefficient $b = (11N - 2N_f)/24\pi^2$ for $SU(N)$ with N_f being the number of quark flavours. For the gauge group $SU(2)$ considered in this paper, one has $b = 2 \times 11/24\pi^2$ when the fermion's contribution is neglected, and $b = 2 \times 5/24\pi^2$ when the number of quark flavours is taken to be $N_f = 6$. For the case of $SU(3)$ the effective Lagrangian in equation (1) leads to a phenomenological description of the asymptotic freedom for the quarks inside hadrons [23]. It should be noted that the $SU(2)$ YM field is introduced here as a model for the cosmic dark energy, it may not be directly identified as the QCD gluon fields, nor the weak-electromagnetic unification gauge fields, such as Z^0 and $W^\pm$. As will be seen later, the YMC has an energy scale characterized by the parameter $\kappa^{1/2} \sim 10^{-3}$ eV, much smaller than that of QCD and of the weak-electromagnetic unification. An explanation can be given for the form in equation (1) as an effective Lagrangian up to 1-loop quantum correction [23–25]. A classical $SU(N)$ YM field Lagrangian is

$$L = \frac{1}{2g_0^2} F,$$

where g_0 is the bare coupling constant. As is known, when the 1-loop quantum corrections are included, the bare coupling g_0 will be replaced by a running one g as the following [26]

$$g_0^2 \to g^2 = \frac{4 \times 12\pi^2}{11N \ln\left(\frac{k^2}{k_0^2}\right)} = \frac{2}{b \ln\left(\frac{k^2}{k_0^2}\right)},$$

where k is the momentum transfer and k_0 is the energy scale. To build up an effective theory [23–25], one may just replace the momentum transfer k^2 by the field strength F in the following manner:

$$\ln\left(\frac{k^2}{k_0^2}\right) \to 2\ln\left|\frac{F}{\kappa^2 e}\right| = 2\left(\ln\left|\frac{F}{\kappa^2}\right| - 1\right),$$

yielding equation (1). We would like to point out that the renormalization scale κ is the only parameter of this effective YM model, and its value should be determined by comparing the observations. In contrast to the scalar-field dark energy models, the YMC Lagrangian is completely fixed by quantum corrections up to order of 1-loops, and there is no room for adjusting its functional form. This is an attractive feature of the effective YMC dark energy model.

From equation (1) we can derive the energy density and the pressure of the condensate [22, 27] in the flat R-W spacetime:

$$\rho_y = \tfrac{1}{2}\epsilon E^2 + \tfrac{1}{2}bE^2, \tag{2}$$

$$p_y = \tfrac{1}{6}\epsilon E^2 - \tfrac{1}{2}bE^2, \tag{3}$$

where ϵ is called the dielectric constant of the YMC, given by [23]

$$\epsilon = 2\frac{\partial L_{\rm eff}}{\partial F} = b\ln\left|\frac{F}{\kappa^2}\right|. \tag{4}$$

The EoS of YMC is given by

$$w_y = \frac{p_y}{\rho_y} = \frac{y-3}{3y+3}, \tag{5}$$

where

$$y \equiv \frac{\epsilon}{b} = \ln\left|\frac{E^2}{\kappa^2}\right| \tag{6}$$

is a dimensionless quantity, in terms of which the energy density and pressure of the YMC will be given by

$$\rho_y = \tfrac{1}{2}b\kappa^2(y+1)\,{\rm e}^y, \tag{7}$$

$$p_y = \tfrac{1}{2}b\kappa^2\left(\tfrac{1}{3}y - 1\right){\rm e}^y. \tag{8}$$

One sees that, to ensure that the energy density be positive in any physically viable model, the allowance for the quantity y should be $y > -1$, i.e. $F > \kappa^2/e \simeq 0.368\kappa^2$. Before setting up a cosmological model, the EoS w_y itself as a function of F is interesting. From equations (2) and (3) one sees that the YMC exhibits an EoS of radiation with $p_y = \frac{1}{3}\rho_y$ and $w = 1/3$ for a large dielectric $\epsilon \gg b$ (i.e. $F \gg \kappa^2$). On the other hand, for $\epsilon = 0$ (i.e. $F = \kappa^2$), which is called the critical point, the YMC has an EoS of the cosmological constant with $p_y = -\rho_y$ and $w = -1$. The latter case occurs when the YMC energy density takes on the value of the critical energy density $\rho_y = \frac{1}{2}b\kappa^2$ [22]. It is this interesting property of the EoS of YMC, going from $w = 1/3$ at higher energies ($F \gg \kappa^2$) to $w = -1$ at low energies ($F = \kappa^2$), that makes it possible for the scaling solution [10] for the dark energy component to exist in our model. More interestingly, this transition is smooth since w is a smooth function of y in the range $(-1, \infty)$. Now we ask the question: can w_y cross over -1? By looking at equation (5) for w_y, we see that w_y only depends on the value of the condensate strength F. In principle, $w_y < -1$ can be achieved as soon as $F < \kappa^2$. Moreover, in regard to the behaviour of w_y as a function of F, this crossing is also smooth. However, as shall be shown explicitly later, when the YMC is put into a cosmological model as the dark energy component, together with the radiation and matter components, to drive the expansion of the universe, the value of F cannot be arbitrary, it comes out as a function of time t and has to be determined by the dynamic evolution. Specifically, when the YMC does not decay into the matter and radiation, w_y can only approach -1 asymptotically, but will not cross over -1. On the other hand, when the YMC decays into the matter and/or radiation, w_y does cross over -1, and, depending on the strength of the coupling, w_y will settle down to an asymptotic value ~ -1.17. As a merit, in this lower region of $w_y < -1$, all the physical quantities ρ_y, p_y and w_y behave smoothly, there are no finite-time singularities that are suffered by a class of scalar models.

Now we put the YMC into the cosmic setting, which is assumed to be a spatially flat ($k = 0$) Robertson–Walker spacetime

$$ds^2 = dt^2 - a^2(t)\delta_{ij}\, dx^i\, dx^j. \tag{9}$$

As it stands, the present universe is filled with three kinds of major energy components, the dark energy, the matter, including both baryons and dark matter, and the radiation. In our model, the dark energy component is represented by the YMC, and the matter component is simply described by a non-relativistic dust with negligible pressure, and the radiation component consists of CMB and possibly other particles, such as neutrinos, if they are massless. Since the universe is assumed to be flat, the sum of the fraction densities is $\Omega = \Omega_y + \Omega_m + \Omega_r = 1$, where the fractional energy densities are $\Omega_y = \rho_y/\rho$, $\Omega_m = \rho_m/\rho$ and $\Omega_r = \rho_r/\rho$. The overall expansion of the universe is determined by the Friedmann equations

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}(\rho_y + \rho_m + \rho_r), \tag{10}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho_y + 3p_y + \rho_m + \rho_r + 3p_r), \tag{11}$$

in which all these three components of energy contribute to the source on the right-hand side of the equations. The dynamical evolutions of the three components are determined by their equations of motion, which can be written as equations of conservation of energy [22, 27]:

$$\dot\rho_y + 3\frac{\dot a}{a}(\rho_y + p_y) = -Q_m - Q_r, \tag{12}$$

$$\dot\rho_m + 3\frac{\dot a}{a}\rho_m = Q_m, \tag{13}$$

$$\dot\rho_r + 3\frac{\dot a}{a}(\rho_r + p_r) = Q_r, \tag{14}$$

where Q_m represents the energy exchange between the YMC and the matter, and Q_r between the YMC and the radiation, respectively. In the natural unit, both quantities have the dimension of $[energy]^5$. The couplings Q_m and Q_r are phenomenological, and their specific forms will be addressed later. The sum of equations (12), (13) and (14) guarantees that the total energy is still conserved. As is known, equation (11) is not independent and can be derived from equations (10), (12), (13) and (14). It is noted that once the couplings Q_m and Q_r are introduced as above, they will bring two new parameters in our model. When $Q_m > 0$, the YMC transfers energy into the matter, and this could be implemented, for instance, by the processes with the YMC decaying into pairs of matter particles. On the other hand, when $Q_m < 0$, the matter transfers energy into the YM condensate. Similarly, when $Q_r > 0$, the

YMC transfers energy into the radiation. Therefore, in the most general case of coupling, there will be three model parameters: Q_m, Q_r and κ.

In the following computations, it is simpler to employ the following functions rescaled by the critical energy density $\frac{1}{2}b\kappa^2$ of YMC,

$$x \equiv \frac{\rho_m}{\frac{1}{2}b\kappa^2}, \tag{15}$$

$$r \equiv \frac{\rho_r}{\frac{1}{2}b\kappa^2}, \tag{16}$$

$$q_m \equiv \frac{Q_m}{\frac{1}{2}b\kappa^2}, \tag{17}$$

$$q_r \equiv \frac{Q_r}{\frac{1}{2}b\kappa^2}. \tag{18}$$

Here the dimensionless functions x and r are simply the rescaled energy density of the matter and radiation, respectively, and the rescaled exchange rates q_m and q_r have unit of $[time]^{-1}$. Then, in terms of x, y and r, the dynamical evolutions given in equations (10), (12)–(14) can be recast into:

$$\frac{\mathrm{d}y}{\mathrm{d}N} + \frac{4y}{2+y} = -\frac{q_m + q_r}{Hh(2+y)\,\mathrm{e}^y}, \tag{19}$$

$$\frac{\mathrm{d}x}{\mathrm{d}N} + 3x = \frac{q_m}{Hh}, \tag{20}$$

$$\frac{\mathrm{d}r}{\mathrm{d}N} + 4r = \frac{q_r}{Hh}, \tag{21}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2h^2, \tag{22}$$

where the variable $N = \ln a(t)$, the function $h = \sqrt{(1+y)\,\mathrm{e}^y + x + r}$ and the constant $H = \sqrt{4\pi G b\kappa^2/3}$. Note that H is not exactly the present Hubble constant H_0. From equation (22) one can see that it is the quantity Hh that determines the actual expansion rate of the universe, and the present value of Hh is identified as the Hubble constant H_0. However, as our calculations will show later, the value of $h \sim 1.07$ at present, so approximately $H \simeq H_0$. It should be emphasized that H is not an independent parameter, it is fixed by the model parameter κ. Once the YMC is put into the cosmological context, one can estimate the order of magnitude of κ as follows. The critical density $\rho_c = 8.099h_0^2 \times 10^{-11}$ eV^4 with the Hubble parameter $h_0 \simeq 0.72$, and the current value of the dark energy density should be $\rho_y = \Omega_y\rho_c \simeq 0.7\rho_c$. As calculations will show, the present value of the factor $(1+y)\,\mathrm{e}^y \simeq 0.8$ in equation (7), and $\rho_y \simeq 0.8 \times \frac{1}{2}b\kappa^2$. So one has

$$\kappa^{1/2} \simeq 5 \times 10^{-3}h_0\,\mathrm{eV}. \tag{23}$$

This energy scale is much smaller than those typical energy scales occurring in the standard model of particle physics, such as $\sim 10^2$ MeV for QCD, and $\sim 10^2$ GeV for the weak-electromagnetic unification. Therefore, for the lack of an explanation of the origin of this energy scale κ within the standard model, we may have to regard the YM condensate as a new physics beyond the standard model of particle physics. In this sense, like other dark energy models, the fine-tuning problem, i.e. why κ has just this small value, also exists in our model.

The set of equations (19)–(22) holds for a generic stage of cosmic expansion driven by the combination of the radiation, the matter and the dark energy. In this paper we focus on the matter-dominated era and the subsequent accelerating era. In particular, we like to see how the cosmic expansion evolves and transits from the matter-dominated to the accelerating era. In the remaining of the paper, we always take the initial condition to be at the time t_i of the equality of radiation-matter (with a redshift $z = 3454^{+385}_{-392}$ [4])

$$\Omega_{mi} = \Omega_{ri}, \tag{24}$$

with the subscript i denoting the initial value. Of course, once the initial value Ω_{yi} for the YMC is given, one has immediately

$$\Omega_{mi} = \Omega_{ri} = \tfrac{1}{2} - \tfrac{1}{2}\Omega_{yi}. \tag{25}$$

In order to keep the main features of the conventional scenario of cosmic expansion, it is also assumed that initially the matter and the radiation are dominant, $\Omega_{mi} = \Omega_{ri} \simeq 1/2$, and the YMC is subdominant

$$\Omega_{yi} \ll 1/2. \tag{26}$$

In [28] we considered the constraints on the YMC energy density at an earlier stage. There the initial condition was taken at a redshift $z \simeq 10^{10}$, corresponding to an energy ~1 MeV, during the radiation stage, when the Big Bang nucleosynthesis processes took place. The upper bound has been found to be $\Omega_y \leqslant 0.26\Omega_r$ at $z \simeq 10^{10}$. Afterwards up to $z \sim 3000$, both ρ_r and ρ_y evolved approximately in a similar way $\propto a(t)^{-3}$. Thus, at the equality of radiation-matter with $z \simeq 3500$, this consideration will give an upper bound $\Omega_{yi} \sim 0.1$. Within this restriction, the initial value Ω_{yi} may still be allowed to vary in a very broad range. In the following we take a safe value

$$\Omega_{yi} \leqslant 10^{-2} \tag{27}$$

as the upper bound both for illustration purposes. This choice has been made in concordance with the thinking that the dark energy component has been existing in the universe from the equality of radiation-matter, but its initial relative contribution is bounded by a few per cent of the total, so that during most of the history of the universe the cosmic evolution will be the same as in the standard Big Bang model. Only quite recently has the dark energy component become dominant and modified the cosmic evolution considerably.

We like to mention that the initial condition in equation (27) has been taken so as not to spoil the conventional Big Bang nucleosynthesis processes that occurred at $T \sim 1$ Mev, i.e. $z \sim 10^{10}$. As is known, an extra amount of energy around this period will speed up the expansion, enhancing the effective species of neutrinos, thus might endanger the BBN. As has been analysed in [28], as long as the ratio (dark energy)/(radiation) <0.26 around the period of $T \sim 1$ Mev, the BBN remains valid. In our present context, for computational simplicity, the initial condition of dark energy is taken to be at the equality of radiation-matter $z \sim 3454$, and its upper limit is given in equation (27). Moreover, as will be explicitly shown later, tracing back to $z \sim 10^{10}$, the YM component has the similar behaviour to the radiation with $\rho_y \propto \rho_r \propto a^{-4}$ during the radiation-dominated stage, thus their ratio remains almost the same. Hence, BBM is to be spoiled by the presence of the YM dark energy.

3. Non-coupling case

The simplest case is the non-coupling with $Q_m = 0$ and $Q_r = 0$ in equations (12), (13) and (14). Then there is only one model parameter κ that has already been fixed in equation (23).

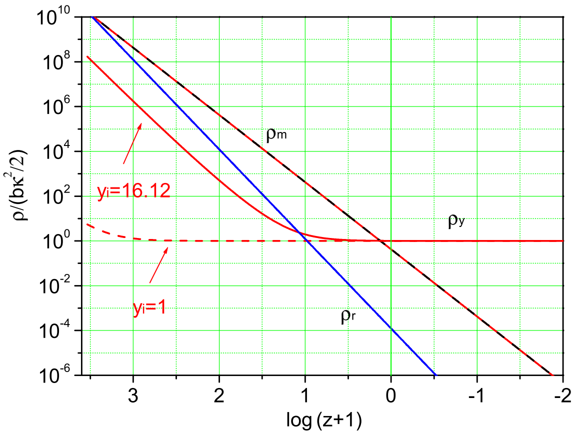

Figure 1. Non-coupling case: the evolution of energy densities. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.35$.

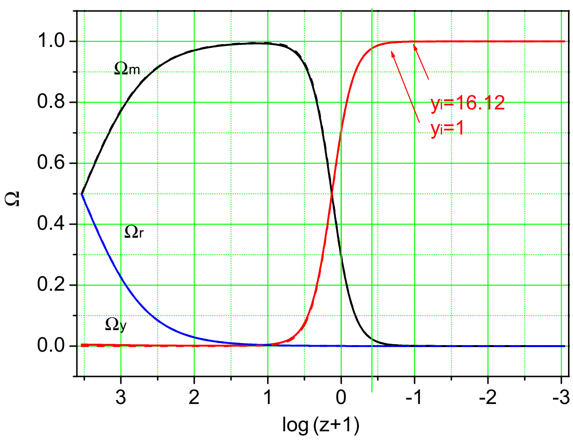

Figure 2. Non-coupling case: the evolution of fraction of energy densities. The two curves of Ω_y for $y_i = 1$ and $y_i = 16.12$ are very close to each other, and nearly overlap in this figure.

Each component evolves independently in the expanding RW spacetime. To solve the dynamic equations, we take the initial YMC at the time t_i to be in the broad range

$$\Omega_{yi} = (10^{-10}, 10^{-2}), \tag{28}$$

consistent with restriction (27), that is, the initial value of ρ_{yi} ranges over eight orders of magnitude. In terms of y, x and r, the initial condition above can be written as

$$y_i = (1, 16.12), \tag{29}$$

$$x_i = r_i = 1.7 \times 10^{10}. \tag{30}$$

Equations (19), (20) and (21) with $q_m = q_r = 0$ are solved easily for $\rho_y(t)$, $\rho_m(t)$ and $\rho_r(t)$, which are shown as a function of redshift z in figure 2. Both $\rho_m(t) \propto a^{-3}(t)$ and $\rho_r(t) \propto a^{-4}(t)$ decrease monotonically at their fixed slope, respectively, and do not level off as $t \to \infty$. More

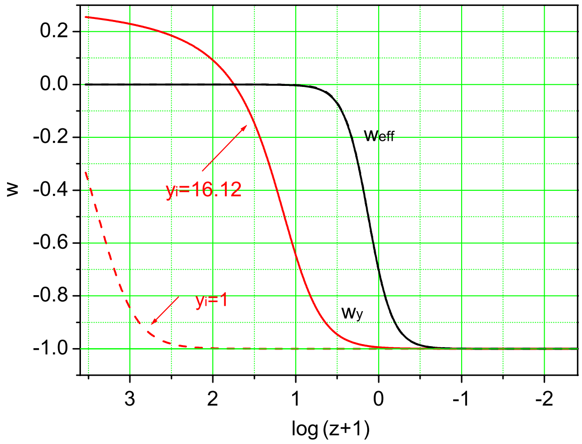

Figure 3. Non-coupling case: the evolution of EoS. w_y approaches -1 as $t \to \infty$, but does not cross over -1.

interesting is the evolution of YMC energy density. At the early stage $\rho_y(t)$ is subdominant to $\rho_m(t)$ and $\rho_r(t)$, and decreases at a slope between those of $\rho_r(t)$ and $\rho_m(t)$, tracking the matter. Later, $\rho_y(t)$ gradually levels off at and approaches a constant. At $z \sim 0.35$, $\rho_y(t)$ starts to dominate over $\rho_m(t)$ and the accelerating expansion takes over. This exit from the subdominant region (scaling) to the dominant region is naturally realized. It is important to note that, as long as the initial value is in the broad range of equation (28), $\rho_y(t)$ always has the same asymptotic value as $t \to \infty$. By the way, the first-order differential equations (20) and (21) for $x(t)$ and $r(t)$ have no fixed points since $\mathrm{d}x/\mathrm{d}N \neq 0$ and $\mathrm{d}r/\mathrm{d}N \neq 0$ during the course of evolution, but the differential equation (19) for $y(t)$ has a fixed point $y_f = 0$ as a solution of $\mathrm{d}y/\mathrm{d}N = 0$ as $t \to \infty$. Figure 3 gives the corresponding evolution of the fractional energy densities. Starting with the initial value $\Omega_{ri} \simeq 1/2$, the radiation component Ω_r has a simple evolution of monotonic decrease. In contrast, the matter component Ω_m, starting with $\simeq 1/2$, increases quickly and approaches ~ 1 around a redshift $(1+z) \sim 174$. At $(1+z) \sim 2.7$, Ω_m drops down and is dominated by Ω_y at $z \sim 0.35$. The YMC component Ω_y starts with the very small initial value and increases slowly and monotonically. Around $(1+z) \sim 2.7$, Ω_y has a quick increase, and around $z \sim 0.35$ it dominates over Ω_m. Observe in figure 3 that the two curves of Ω_y for the two different initial values 10^{-10} and 10^{-2}, respectively, are almost overlapped into one curve. This pattern of degeneracy for Ω_y demonstrates vividly the fact that the cosmic evolution and the current status are insensitive to the initial condition of the YMC. As the result of evolution, at present ($z = 0$), one has $\Omega_y \sim 0.7$, $\Omega_m \sim 0.3$ and $\Omega_r \sim 10^{-5}$. Figure 1 shows the evolution of w_y and that of the effective EoS w_{eff} defined by

$$w_{\text{eff}} = \frac{p_m + p_y}{\rho_m + \rho_y}. \tag{31}$$

Both approach -1 as $t \to \infty$, but, they cannot across -1. So there is no super-accelerating stage in the non-coupling model. Looking back at equations (5) and (6), it is clear that in the non-coupling case the YM field strength will always stay above the critical value: $F \geqslant \kappa^2$. The asymptotic region at $t \to \infty$ corresponds to $F = \kappa^2$. This is a state with the dielectric constant $\epsilon = 0$. Thus, for the non-coupling case, no matter what kind of initial condition is given, the YMC always settles down to the state of $\epsilon = 0$ as a result of dynamic evolution. As for the current status $\Omega_y \sim 0.7$ and $\Omega_m \sim 0.3$, it has been achieved for the whole range of initial values in equation (29) at the fixed model parameter κ in equation (23). Therefore, the coincidence problem is solved in this model, but the fine-tuning problem still exists, i.e., we do not have an answer to the question why κ should have such a value as in equation (23). The non-coupling case with $Q_m = 0$ and $Q_r = 0$ has also been studied in [27, 28] with the initial condition being taken at a redshift $z \sim 10^{10}$ at the radiation-dominated stage. There the evolution behaviour found for the matter-dominated era is similar to what is obtained here.

4. YMC decaying into matter

Consider the case that the YMC couples to the matter component only. In terms of the cosmic energy densities today, the radiation fraction is roughly $\Omega_r \sim 10^{-5}$, a very small contribution relatively, much lower than the other two components. Therefore, in this section we temporarily neglect its coupling with the YMC by setting $Q_r = 0$. There is only one free parameter Q_m since κ has been fixed. Then equations (19), (20) and (21) reduce to

$$\frac{\mathrm{d}y}{\mathrm{d}N} + \frac{4y}{2+y} = -\frac{q_m}{Hh(2+y)\,\mathrm{e}^y}, \tag{32}$$

$$\frac{\mathrm{d}x}{\mathrm{d}N} + 3x = \frac{q_m}{Hh}, \tag{33}$$

$$\frac{\mathrm{d}r}{\mathrm{d}N} + 4r = 0. \tag{34}$$

The dynamic evolution of the radiation is independent of the other two, and $\rho_r(t) \propto a^{-4}(t)$, as is seen in equation (34). To proceed further, one needs to know the coupling q_m to solve equations (32) and (33). As mentioned earlier, in general there are two kinds of models depending on whether $Q_m > 0$ or $Q_m < 0$. In this section, we examine the model $Q_m > 0$ with the YMC decaying constantly into the matter. We call this model 1. It can be generically expressed as

$$Q_m = \Gamma\rho_y, \tag{35}$$

where $\Gamma > 0$ is of dimension $[time]^{-1}$ and measures the decay rate of the YMC energy density as well as the production rate of the matter energy density. Here Γ is simply taken as a model parameter describing phenomenologically the interactions between the YMC and the matter. In the following we discuss several cases for the parameter Γ. Substituting equation (35) into equations (32) and (33) yields

$$\frac{\mathrm{d}x}{\mathrm{d}N} = \frac{\Gamma}{H}\frac{(1+y)\,\mathrm{e}^y}{h} - 3x, \tag{36}$$

$$\frac{\mathrm{d}y}{\mathrm{d}N} = -\frac{\Gamma}{H}\frac{1+y}{(2+y)h} - \frac{4y}{2+y}. \tag{37}$$

1. Consider the simple case of a constant rate with

$$\Gamma/H = 0.5. \tag{38}$$

We have taken this magnitude of the rate Γ, so that the present status of the universe will be $\Omega_y \simeq 0.7$ and $\Omega_m \simeq 0.3$ as the outcome from our computation. In fact, this status can be also achieved by taking a whole family of values for Γ between 0 and $0.5H$, and we have checked this property by carrying out computations for other cases $\Gamma = 0.01H$ and $0.1H$. Now at the time t_i with $z \simeq 3454$ the initial YMC energy density is taken to be in the range

$$\Omega_{yi} = (10^{-10}, 3\times 10^{-3}), \tag{39}$$

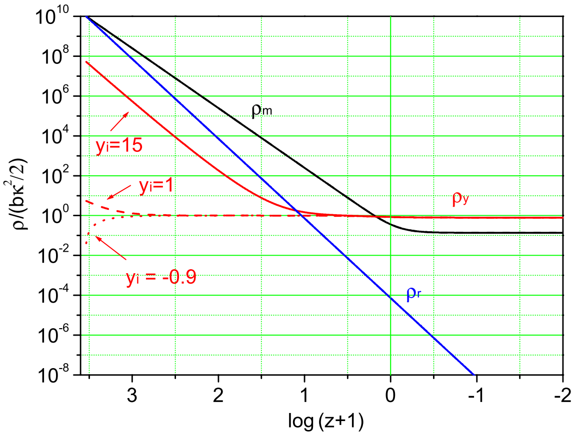

Figure 4. Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: the evolution of energy densities with the YMC decaying into the matter. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, i.e., $y_i = (1, 15)$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.48$. Due to the coupling, $\rho_m(t)$ also levels off at late time.

similar to equation (28), which corresponds to

$$y_i = (1, 15). \tag{40}$$

The initial values for the matter and the radiation components are given by

$$x_i = r_i = 1.0 \times 10^{10}, \tag{41}$$

which are a little bit smaller than that in equation (30). This is because in the case here the matter is being generated out of the decaying YMC during the course of evolution. Consequently, smaller initial values x_i and r_i are needed to arrive at the current status. Given the different initial values of Ω_{yi} in equation (39), the corresponding initial values of $\Omega_{mi} = \Omega_{ri} = (1 - \Omega_{yi})/2$ also vary by a small amount. But for the different values of y_i in equation (40) we have taken the same set of values in equation (41) since the initial total energy density $\rho_{yi} + \rho_{mi} + \rho_{ri}$ itself at $z \simeq 3454$ has some errors. The results are given as functions of the redshift z in figures 4–6 . The evolution of $\rho_y(t)$ is also similar to the non-coupling case. During the early stage $\rho_y(t)$ is lower than, and keeps track of $\rho_m(t)$. Later, $\rho_y(t)$ levels off and approaches a constant, and around $z \sim 0.48$, it starts to dominate over $\rho_m(t)$, and the accelerating stage begins. In fact in figure 4 for three different y_i there are three curves of the matter $\rho_m(t)$, respectively. However, these three curves are too close to each other so that they are overlapped. The similar is for $\rho_r(t)$. Note that, in the presence of coupling Q_m, the evolution of $\rho_m(t)$ is different from the non-coupling case in that at the late stage around $z \sim 0$ it also levels off just like $\rho_y(t)$. In this sense, the evolution of the matter component is sort of bound to the YMC. In fact, as $t \to \infty$, the set of equations (36) and (37) has the asymptotic behaviour

$$\frac{\mathrm{d}x}{\mathrm{d}N} \to 0, \qquad \frac{\mathrm{d}y}{\mathrm{d}N} \to 0, \tag{42}$$

and both $\rho_y(t)$ and $\rho_m(t)$ have asymptotic values. This will be addressed later in section 6.

As a novel feature of the coupling model, in contrast to the non-coupling case, now the EoS of the YMC as a function of time t, $w_y(t)$ crosses over -1 around $z \sim 2$, takes a value $w_y \simeq -1.1$ at present $z = 0$, and approaches $w_y \simeq -1.17$ asymptotically, as shown in figure 6. We have also checked that the value of EoS depends directly on the value Γ, and a

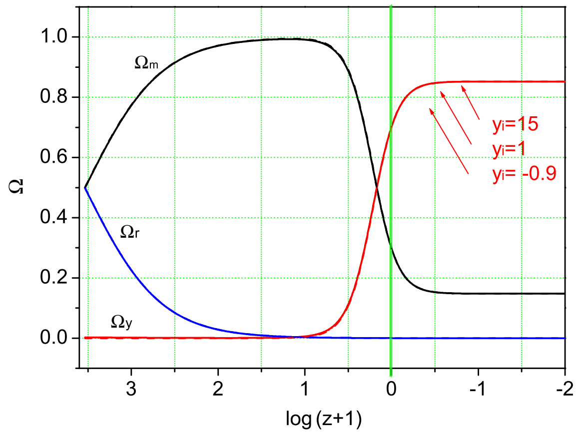

Figure 5. Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: the evolution of fractional energy densities in the same model as in figure 4.

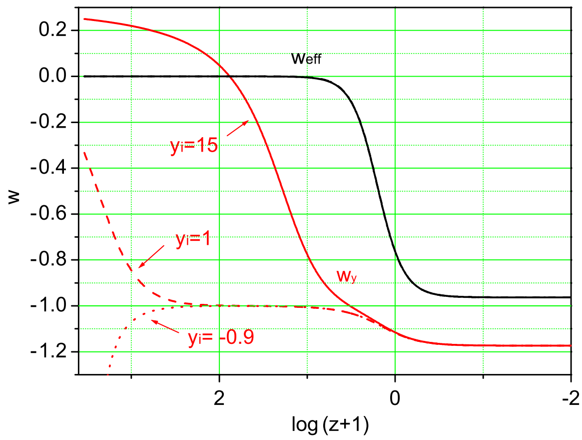

Figure 6. Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: the evolution of EoS in the same model as in figures 4 and 5. Due to the coupling, w_y crosses over -1, takes on value ~ -1.1 at $z = 0$ and approaches -1.17 asymptotically. This is in contrast to the non-coupling case. The trend of $w(z)$ here agrees with the reconstructed one from the supernova data given by [31].

smaller value of $\Gamma < 0.5H$ will yield a greater $w > -1.17$. If w turns out to be some other value by the future observations, then we can adjust Γ accordingly. The occurrence of crossing over -1 in this model can be understood, since the coupling makes the YMC lose energy into the matter; consequently, the YM field strength F will drop down below the critical value κ^2, leading to $\epsilon = by < 0$ and $w_y < -1$ as in equation (5). This can also be arrived at by looking at the asymptotic region determined by the equation $\frac{\mathrm{d}y}{\mathrm{d}N} = 0$, which by equation (37) is just

$$\frac{\Gamma}{H}\frac{1+y}{h} + 4y = 0. \tag{43}$$

Recall that for the non-coupling $\Gamma = 0$ the asymptotic value is $y_f = 0$, yielding $w_y = -1$ by equation (5). Once $\Gamma > 0$, equation (43) yields an asymptotic value $y_f < 0$ as the solution, hence $w_y < -1$. Thus, when transferring energy to the matter, the YMC will eventually settle down in the state of $w_y < -1$, which is equivalent to a negative dielectric $\epsilon < 0$. Recently, there have been some observational indications that the current value of EoS of dark energy w is less than -1; for instance, $w = -1.023 \pm 0.090(\text{stat}) \pm 0.054(\text{sys})$ from the 71 high redshift supernovae discovered during the first year SNLS [29], and $w = -1.21^{+0.15}_{-0.12}$ from the blind analysis of 21 high redshift supernovae by CMAGIC technique [30]. Reference [31] uses two supernovae datasets, Gold+HST and SNLS, to reconstruct the dark energy properties, yielding an estimate of the EoS going from $w \sim -0.5$ at $z = 1.6$ to $w \sim -1.5$ at $z = 0$ at 2σ CL. Reference [32] uses the neutrino mass result from the Heidelberg–Moscow double beta decay experiment, combines it with the WMAP 3-year data and constrains the EoS to $-1.67 < w < -1.05$ at 95% c.l., ruling out a cosmological constant at more than 95% c.l. Reference [33] combines gamma-ray bursts, SNe Ia Gold sample, WMAP, SDSS, and 2dFGRS data, and obtains the EoS $w = -1.09 + 0.89\frac{z}{1+z}$. Analysing the ESSENCE supernova survey, reference [34] gives the EoS $w = -1.05^{+0.13}_{-0.12}$ (stat 1σ) $\pm$ 0.13(sys), and, when combining it with the SNLS, gives a joint constraint $w = -1.07^{+0.09}_{-0.09}$ (stat 1σ)$\pm$0.13(sys); the data are still fully consistent with a cosmological constant. Of course, these are still to be observationally examined with a higher confidence level in future. However, the crossing-over -1 would be difficult for scalar models, except for quintom models at a price of introducing two scalar fields and an artificially designed potential [35, 36]. As we just have demonstrated, in the YMC dark energy model with coupling $Q_m > 0$, this crossing is realized naturally. In general, the asymptotic value of w_y at $t \to \infty$ and the current value w_y at $z = 0$ are determined by the asymptotic value of y through equation (5), and the latter is obtained from the combination of equations (37) and (42), and thus depends on the ratio Γ/H. Thus the asymptotic value of w_y is determined by the ratio Γ/H of the two parameters of our model. For instance, the observed EoS of dark energy $w = -1.023$ from SNLS [29] can be obtained, in our model, by taking a slightly smaller decay rate $\Gamma/H = 0.13$ and slightly higher initial densities $x_i = r_i = 1.5 \times 10^{10}$, and the value $w = -1.21$ from the analysis by CMAGIC [30] can be obtained by taking a slightly larger decay rate $\Gamma/H = 0.81$ and a slightly lower density $x_i = r_i = 0.7 \times 10^{10}$, respectively. Figure 6 also shows that the effective w_{eff} cannot cross -1 yet, and its asymptotic value is ~ -0.96. Interestingly, this model predicts that in the upcoming future the dark energy density ρ_y will remain a constant slightly larger than today, and the matter energy density ρ_m will be a constant slightly lower than today. Eventually the universe will settle down to a steady state with $\Omega_y \sim 0.85$, $\Omega_m \sim 0.15$ and $\Omega_r \sim 0$.

Therefore, this model yields a picture of the evolutional cosmos, the early part of which can account for the past history of the expanding universe, i.e., that of the standard Big-Bang model, and the late part of which, i.e. the future of the universe, is similar to the steady-state model [37, 38]. As a plus for the YMC model, there are no Big Rip singularities in finite time, since all the quantities ρ_y, p_y and w_y are smooth function of the time t.

Although in the above we have only presented the results for the initial values within the range $\Omega_{yi} > 10^{-10}$ given in equation (28), as a matter of fact, we have worked out for even lower values $\Omega_{yi} < 10^{-10}$. For instance, we have calculated the case of $\Omega_{yi} \simeq 2 \times 10^{-12}$, i.e., $y_i = -0.9$. The initial energy density ρ_{yi} is even lower than the current values $\rho_y \simeq 0.7\rho_c$. The details are given by the curves denoted by $y_i = -0.9$ in figures 4–6. The evolution is such that $\rho_y(t)$ starts initially from the given very low value, increases (instead of decreasing) very quickly with time t and approaches its corresponding asymptotic value $\simeq 0.7\rho_c$, as is seen in figure 4. The evolutions for $\rho_m(t)$ and $\rho_r(t)$ are similar to the cases in equation (39). Thus, except for the initial increase in $\rho_y(t)$, the current status of the universe is the same as those in the range of equation (39).

2. Now consider a case of the YMC decaying into fermion pairs. As is known in QED, a constant electric field is unstable against decay into pair of particles [39]. Analogously, a constant 'electric' $SU(3)$ Yang–Mills field configuration of QCD is unstable against decay

into quark pairs and gluon pairs [40–42]. Calculations have shown that for a gauge group $SU(N)$, due to the decay into fermion pairs and gauge boson pairs, the average local colour electric field decreases at a rate

$$\frac{\partial E}{\partial t} = -cE^{3/2}, \tag{44}$$

where c is a dimensionless constant [22, 41, 42]. Note that this rate is derived from the energy–momentum conversation of pair creation. From the point of view of particle physics, the $SU(2)$ YM field in our case should be allowed to have some fundamental interactions with other microscopic particles. At the moment we do not intend to proceed further to build up the detail of its interaction. Instead, we simply assume that, similar to the $SU(3)$ gauge field in QCD, the YM condensate, here a constant in space, also has the property of being unstable against decay into other particles, and the decay rate of the field strength has the same form as equation (44). Consequently, this in turn will give rise to the decay rate of the YMC energy density

$$\frac{\partial \rho_y}{\partial t} = -\Gamma \rho_y. \tag{45}$$

Making use of the chain rule relation $\frac{\partial \rho_y}{\partial t} = \frac{\partial \rho_y}{\partial E}\frac{\partial E}{\partial t}$ and equation (44), one obtains the following expression for the decay rate [22],

$$\Gamma = 2c\kappa^{\frac{1}{2}}\frac{2+y}{1+y}\,\mathrm{e}^{\frac{y}{4}}, \tag{46}$$

depending on the coefficient constant c, which is treated as the model parameter in place of Γ. We choose the value of $c = 0.125H/\kappa^{1/2}$, so that

$$\frac{\Gamma}{H} = 0.25\frac{2+y}{1+y}\,\mathrm{e}^{\frac{y}{4}}. \tag{47}$$

As our computations show, the value of variable y at the present stage is very small $y \sim 0$, so equation (47) yields $\Gamma \sim 0.5H$, quite close to that in equation (38). The initial condition is taken to be the same as equations (40) and (41). Figures 7, 8 and 9 show the results, which are very similar to that in the previous case of the constant rate. From equation (13) it is seen that, in the late-time asymptotic region when $\dot{\rho}_m \simeq 0$, the matter generation rate is estimated as $Q_m \simeq 3H\rho_m \simeq 10^{-46} g\ \mathrm{cm}^{-3}\ \mathrm{s}^{-1}$, a very small rate, equivalent to generation of ~0.3 protons in a cubic kilometer per year. This value is approximately equal to that in the steady-state model [37, 38]. Thus in our model the particle pairs are continuously generated, at a very low rate, out of the vacuum filled with the YMC. As is known, in order to continuously generate the cosmic matter, the steady-state model has to introduce some C-field with negative energy [38], which is problematic in a physical theory. Here in our model, the effective quantum YMC plays the role of a matter generator; there is no negative energy to occur in the proper range. The YMC has a positive energy and a negative pressure.

From equations (38) and (47) it seems that the overall behaviour of the dynamic evolution is not sensitive to the particular form of the coupling Γ, as long as its magnitude is $\Gamma \sim 0.5H$. This has been confirmed in our examinations. For example, we have also investigated the case with the decay rate of the form

$$\Gamma/H = 0.5\,\mathrm{e}^{-y}, \tag{48}$$

and the results are very similar to that in the previous case. Therefore, in model 1 of the YM–matter coupling with the coupling $Q_m = \Gamma\rho_y$, as long as Γ is constant or depends on ρ_y, the overall features of the dynamic evolution are similar. We have also studied the cases with

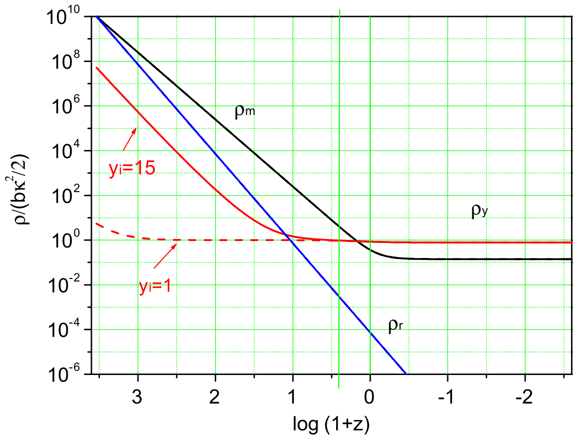

Figure 7. Model 1 with $Q_m > 0$ for $\Gamma/H = 0.25\,\mathrm{e}^{\frac{y}{4}}\,\frac{2+y}{1+y}$: the evolution of energy densities with the YMC decaying into the matter. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.48$. Due to the coupling, $\rho_m(t)$ also levels off at late time. This is quite similar to the case of a constant rate $\Gamma/H = 0.5$ in figure 4.

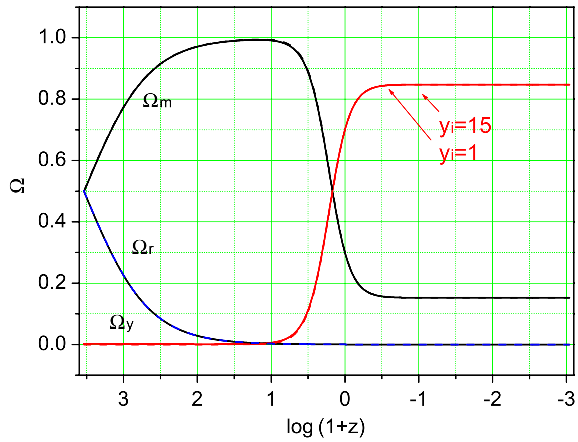

Figure 8. Model 1 with $Q_m > 0$ for $\Gamma/H = 0.25\,\mathrm{e}^{\frac{y}{4}}\,\frac{2+y}{1+y}$: the evolution of fractional energy densities in the same model as in figure 7.

Γ depending on the matter ρ_m. It is found that, if the decay rate Γ depends on the matter ρ_m, for instance

$$\Gamma/H = bx, \tag{49}$$

where b is some constant and x is defined in equation (15), then for $b \leqslant 10^{-5}$ the evolution will be similar to the non-coupling case. When the constant $b \geqslant 10^{-3}$, the decay rate is too fast, and at the start $\rho_y(t)$ drops down quickly, later it increases to its asymptotic value from below. To keep the paper short, we do not demonstrate these detailed graphs of the cases of equations (48) and (49).

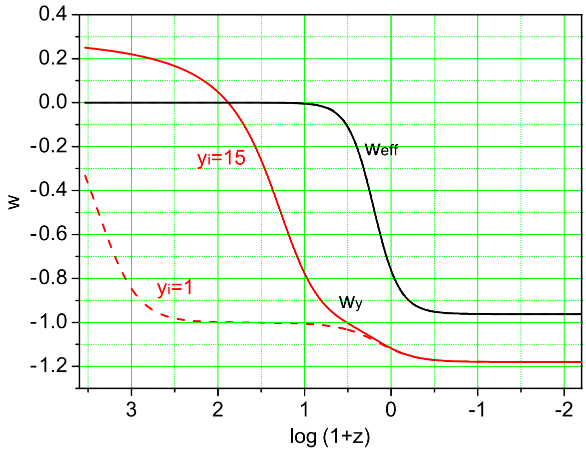

Figure 9. Model 1 with $Q_m > 0$ for $\Gamma/H = 0.25\,\mathrm{e}^{\frac{y}{4}}\,\frac{2+y}{1+y}$: the evolution of EoS in the same model as in figures 7 and 8. Again, due to the coupling, w_y crosses over -1, and takes on value ~ -1.1 at $z = 0$.

5. Matter decaying into YMC

In this section we study the situation in which the matter decays constantly into the YMC with $Q_m < 0$, just opposite to model 1. We call this model 2. It can be generically expressed as

$$Q_m = -\Gamma \rho_m, \tag{50}$$

i.e. the matter transports energy into the YM condensate. Then equations (32) and (33) reduce to

$$\frac{\mathrm{d}x}{\mathrm{d}N} + 3x = -\frac{\Gamma}{H}\frac{x}{h}, \tag{51}$$

$$\frac{\mathrm{d}y}{\mathrm{d}N} + \frac{4y}{2+y} = \frac{\Gamma}{H}\frac{x}{(2+y)\,\mathrm{e}^{y} h}. \tag{52}$$

1. Consider the simple case of a constant decay rate

$$\Gamma/H = 0.02. \tag{53}$$

Again, the value 0.02 has been taken here, so that the resulting energy densities from our computation will be $\Omega_y \simeq 0.7$ and $\Omega_m \simeq 0.3$ at present. So in model 2 the decay rate of the matter into the YM condensate needs to be almost two orders of magnitude smaller than the expansion rate. The initial values of x_i are taken to be

$$x_i = r_i \simeq 1.8 \times 10^{10}. \tag{54}$$

Since the matter is decaying and constantly being converted into the YMC component, a slightly larger initial value of the matter has been taken than that in equation (41). The initial value of the YMC is taken to be $y_i = (1, 15)$, the same as equation (40). The results are given in figures 10–12. Now w_y does not cross over -1. It is interesting to find out that the evolution of the YM condensate behaves differently for two different ranges of y_i. For the higher range

$$y_i = (5, 15), \tag{55}$$

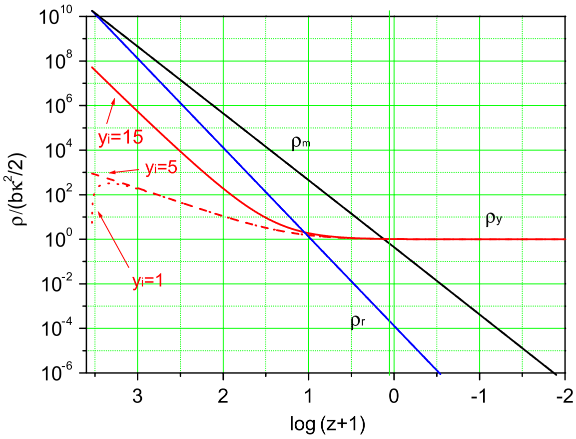

Figure 10. Model 2 with $Q_m < 0$ for the constant coupling $\Gamma/H = 0.02$: the evolution of energy densities with the matter decaying into the YMC.

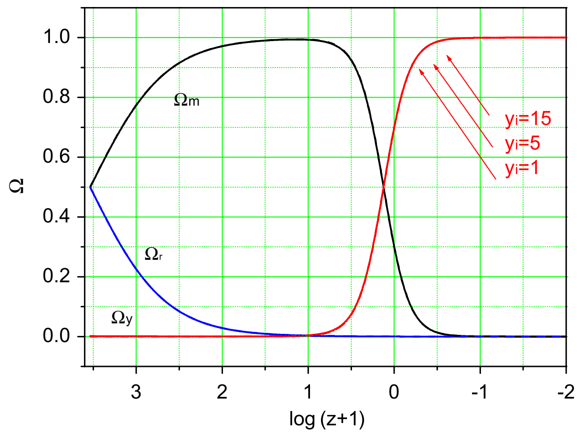

Figure 11. Model 2 with $Q_m < 0$ for the constant coupling $\Gamma/H = 0.02$: the evolution of fractional energy densities in the same model as in figure 10.

corresponding to $\Omega_{yi} = (5 \times 10^{-8}, 3 \times 10^{-3})$, all the quantities have similar evolution to the non-coupling case in figures 1–3. For the lower range

$$y_i = (1, 5), \tag{56}$$

however, the YMC has an instantly sudden increase during the initial stage, and quickly catches up the evolution pattern of the $y_i = 5$ case. This is in contrast to the smooth behaviour on the higher range. Thus, in order to have a rather smooth evolution within model 2, the initial value y_i should be given by the higher range in equation (55). Moreover, we have also found that the overall behaviour of the dynamic evolution is not sensitive to the particular form of the coupling Q_m. For instance, we have checked a case of

$$\Gamma/H = 0.02\,\mathrm{e}^{-x}, \tag{57}$$

and the resulting evolutions are similar to the case of equation (53). Thus, the coincidence problem can also be solved in model 2.

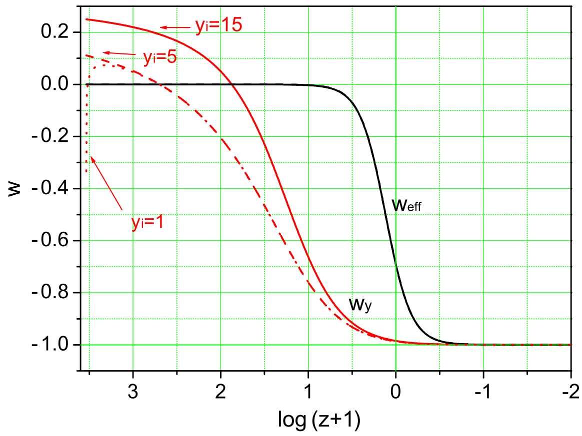

Figure 12. Model 2 with $Q_m < 0$ for the constant coupling $\Gamma/H = 0.02$: the evolution of EoS in the same model as in figures 10 and 11. Since the YMC gets energy from the matter, w_y approaches -1, but does not cross over -1.

So far in model 1 and model 2, in regard to the coupling between the YMC and the matter, we have not explicitly distinguished the baryons and the dark matter, and have assumed, for simplicity, the same coupling Q_m for both the baryons and the dark matter. We can roughly estimate the current value of the cross section corresponding to the collisions involving the baryons as in equations (38), (47), (48), (49) with $b \leqslant 10^{-5}$, (53) and (57). For instance, take the baryon decay rate $\Gamma \sim bxH$ as in equation (49). Then, by definition, the rate is $\Gamma \sim vn\sigma$, where v is the baryon velocity, the baryon number density is $n = \rho_b/m_b \sim 0.04\rho_c/m_b$, and σ is the crossing section for the collisions between the baryons and the YM gauge bosons for this type of interaction. Then we can get an estimate

$$\sigma \sim 25bm_b H/v\rho_c. \tag{58}$$

Taking $b \sim 10^{-5}$, $v \sim 10^3$ km s^{-1}, the baryon mass $m_b \sim 0.94$ GeV, the current Hubble constant for H, one has $\sigma \sim 6 \times 10^{-26}$ cm^2. This is an order lower than the Thomson's cross section $\sigma_T \simeq 6.7 \times 10^{-25}$ cm^2 for QED. Similarly, letting $\Gamma \sim 0.5H$ as in equations (38) and (47) for the YMC decaying into baryons, we would get $\sigma \sim 20 \times 10^{-25}$ cm^2 analogously, slightly greater than σ_T. Therefore, given this magnitude for the cross section σ in both cases, we would, in principle, be able to observe these kinds of interactions occurring, either with the baryon decaying into the YM boson pairs or the baryon pairs jumping out of the vacuum. However, as said earlier, the rate Γ for these kinds of events is too low, giving $Q_m \sim 0.3$ proton generated in one cubic kilometer per year. For the galaxy of a volume $\sim 10^3$(kpc)^3, this rate is roughly equivalent to an amount of mass $\sim 10^{-6} M_\odot$ generated per year, a small production rate. The chance of directly detecting the event may be small. Even if future experiments rule out or restrict the coupling with the baryons, one has to drop it or reduce its magnitude as a model parameter. Nevertheless, the coupling with the dark matter probably still remains. This is because the dark matter is usually assumed not to have interactions with ordinary particles, such as baryons, photons, etc. So it is difficult to directly detect productions of dark particle pairs and decays of dark particles.

6. Coupling with both matter and radiation

It is quite natural to allow the YMC to couple with both the matter and the radiation simultaneously. Now we study model 3, that the YMC decays into the matter and the

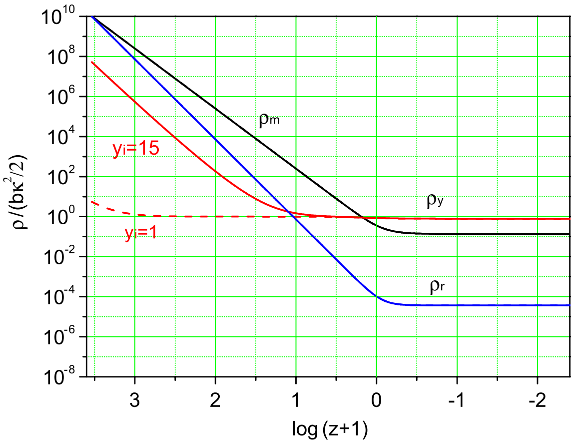

Figure 13. Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.000\,18$: the evolution of energy densities with the YMC decaying into both the matter and the radiation. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.48$. Note that, due to coupling, both $\rho_m(t)$ and $\rho_r(t)$ level off like $\rho_y(t)$.

radiation as well:

$$Q_m = \Gamma\rho_y > 0, \qquad Q_r = \Gamma'\rho_y > 0. \tag{59}$$

Then equations (19)–(21) reduce to

$$\frac{\mathrm{d}y}{\mathrm{d}N} = -\frac{\Gamma + \Gamma'}{H}\frac{1+y}{(2+y)h} - \frac{4y}{2+y}, \tag{60}$$

$$\frac{\mathrm{d}x}{\mathrm{d}N} = \frac{\Gamma}{H}\frac{(1+y)\,\mathrm{e}^{y}}{h} - 3x, \tag{61}$$

$$\frac{\mathrm{d}r}{\mathrm{d}N} = \frac{\Gamma'}{H}\frac{(1+y)\,\mathrm{e}^{y}}{h} - 4r. \tag{62}$$

Consider the case of the constant decay rates

$$\Gamma/H = 0.5, \qquad \Gamma'/H = 1.8 \times 10^{-4}. \tag{63}$$

Note that Γ' is lower than Γ by three orders of magnitude. These values of coupling are taken so that the current values are $\Omega_y \simeq 0.7$, $\Omega_m \simeq 0.3$, $\Omega_\gamma \simeq 8.6 \times 10^{-5}$ (including massless neutrinos). The initial condition is the same as in equations (40) and (41). The results are given in figures 13–15. As before, the particular form of the couplings is not important for the overall behaviour of evolution. For instance, we have also examined the case

$$\Gamma/H = 0.25\frac{2+y}{1+y}\,\mathrm{e}^{\frac{y}{4}}, \qquad \Gamma'/H = 0.9 \times 10^{-4}\frac{2+y}{1+y}\,\mathrm{e}^{\frac{y}{4}}, \tag{64}$$

based on an analogous consideration to equation (47). The evolution is similar to the case of equation (63). In these two cases of model 3, due to the couplings with the YMC, both the energy densities, ρ_m and ρ_r, level off around $z \sim 0$, and w_y crosses over -1 around $z \simeq 2.5$. Thus all the three components of cosmic energy will remain constant in future, and the state of the universe will remain almost the same as it is today. This is similar to the steady-state universe [37, 38]. Hence, according to model 3, the past history of the universe is consistent

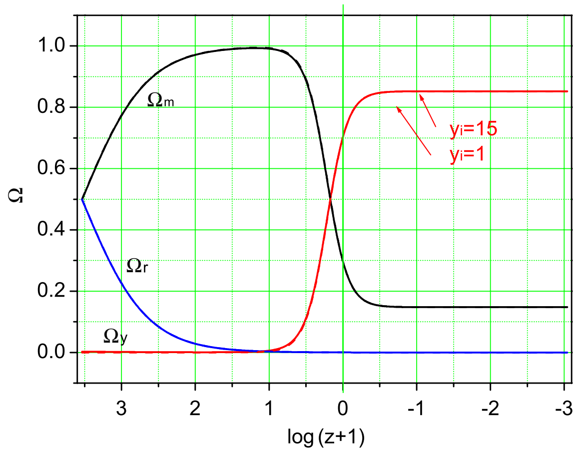

Figure 14. The evolution of a fraction of energy densities in the same model as in figure 13.

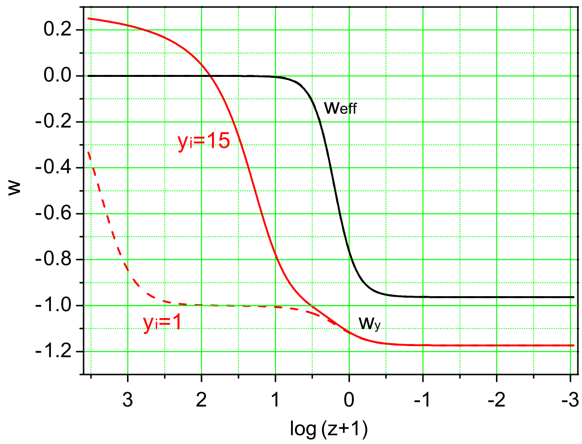

Figure 15. Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.000\,18$: the evolution of EoS in the same model as in figures 13 and 14. Note that w_y also crosses over -1 and takes on a value ~ -1.1 at $z = 0$.

with the conventional standard Big Bang model, and from now on, the cosmic evolution tends to that of a steady state, that is, the universe will remain similar to what it is today, but with $\Omega_y \sim 0.85$, $\Omega_m \sim 0.15$ and $\Omega_r \sim 10^{-5}$.

We have also studied the case with both the matter and radiation decaying into the YMC. We call this model 4. The couplings are such that

$$Q_m = -\Gamma\rho_m < 0, \qquad Q_r = -\Gamma'\rho_r < 0. \tag{65}$$

We take

$$\Gamma/H = 0.02, \qquad \Gamma'/H = 1.8 \times 10^{-4}. \tag{66}$$

The initial condition for the YMC is the same as equations (55) and (56) and the initial densities for the matter and radiation are the same as in equation (54). The resulting evolution is qualitatively similar to those in model 2 with $Q_m < 0$. Approximately,

$\rho_m(t) \propto a(t)^{-3}$, $\rho_r(t) \propto a(t)^{-4}$, and $\rho_y(t)$ also has a scaling solution and exits the scaling regime, just as in the non-coupling case. The EoS of the YMC w_y approaches -1 from above, but does not cross over -1. To keep the paper short and concise, we will not repeat these details and no longer give the corresponding graphs here.

7. Asymptotic behaviour and stable attractor

It is interesting to investigate the asymptotic behaviour of the dynamical evolution. First we study model 1 with the YMC coupling to the matter component only. Now since the evolution of the radiation component is independent of the YMC and the matter, and the value of r at late time is much less than the other variables, it can be neglected in the analysis of the fixed point. To find the fixed points, one sets $\mathrm{d}x/\mathrm{d}N = \mathrm{d}y/\mathrm{d}N = 0$ in equations (32) and (33), and obtains the relations at the fixed point:

$$x_f = -\frac{4}{3} y_f \, \mathrm{e}^{y_f}, \tag{67}$$

$$\frac{\Gamma}{H}\left(1 + \frac{1}{y_f}\right) = -4\sqrt{1 - \frac{y_f}{3}} \, \mathrm{e}^{y_f/2}, \tag{68}$$

where the sub-index f refers to the respective values at the fixed point. From these two equations one can write the asymptotic value of the fractional matter density

$$\Omega_{mf} = \frac{4y_f}{y_f - 3}. \tag{69}$$

Equations (67) and (68) depend on the value of the ratio Γ/H, so does the solution (x_f, y_f). Here we consider the case that Γ/H is constant. Because Ω_{mf} must be larger than 0 and smaller than 1, so y_f must be in the range from -1 to 0 at the fixed point. Thus, as long as $\Gamma/H \in (0, \infty)$ there will exist fixed points. To be specific, consider $\Gamma/H = 0.5$. In the region of physics there is only one fixed point:

$$(x_f, y_f) = (0.136\,66, -0.114\,99). \tag{70}$$

This is, in terms of the respective densities,

$$(\rho_{mf}, \rho_{yf}) = \tfrac{1}{2} b\kappa^2 (0.136\,66, 0.788\,88). \tag{71}$$

The stability of this fixed point can be analysed in the conventional way as follows. Because the two equations for x and y are nonlinear, a local analysis can be given by linearizing the two evolution equations (36) and (37). By a standard procedure, expanding $x = x_f + \varepsilon$ and $y = y_f + \eta$, where ε and η are small perturbations around the fixed point, and keeping up to the first order of small perturbations, equations (36) and (37) reduce to

$$\frac{\mathrm{d}}{\mathrm{d}N}\begin{pmatrix}\varepsilon \\ \eta\end{pmatrix} = M \begin{pmatrix}\varepsilon \\ \eta\end{pmatrix}, \tag{72}$$

where M is a 2×2 matrix depending on the values of x_f, y_f and Γ/H, whose elements are

$$M_{11} = -\left[\frac{\Gamma}{H}\frac{(1 + y_f)\,\mathrm{e}^{y_f}}{2h_f^3} + 3\right],$$

$$M_{12} = \frac{\Gamma}{H}\frac{\mathrm{e}^{y_f}}{h_f}\left\{(1 + y_f)\left[1 - \frac{(2 + y_f)\,\mathrm{e}^{y_f}}{2h_f^2}\right] + 1\right\},$$

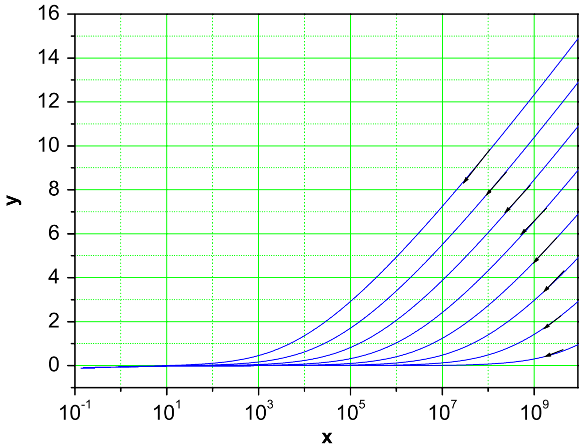

Figure 16. Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: the trajectories in the phase plane. Each trajectory starts with a different initial condition. All of them approach the fixed point $(x_f, y_f) = (0.136\,66, -0.114\,99)$.

$$M_{21} = \frac{\Gamma}{H} \frac{1+y_f}{2(2+y_f)h_f^3},$$

$$M_{22} = \frac{\Gamma}{H}\left[\frac{(1+y_f)\,\mathrm{e}^{y_f}}{2h_f^3} - \frac{1}{(2+y_f)^2 h_f}\right] - \frac{8}{(2+y_f)^2},$$

where $h_f = \sqrt{(1+y_f)\,\mathrm{e}^{y_f} + x_f + r_f}$. The general solution for the linear perturbations is of the form

$$\varepsilon = C_1\,\mathrm{e}^{\mu_1 N} + C_2\,\mathrm{e}^{\mu_2 N} \tag{73}$$

$$\eta = C_3\,\mathrm{e}^{\mu_1 N} + C_4\,\mathrm{e}^{\mu_2 N} \tag{74}$$

where μ_1 and μ_2 are eigenvalues of M. If they are both negative, the fixed point (x_f, y_f) is stable, and the solution is called an attractor. For the case of $\Gamma/H = 0.5$ one finds the matrix

$$M = \begin{pmatrix} -3.221\,46 & 0.501\,12 \\ 0.131\,80 & -2.176\,25 \end{pmatrix},$$

and its two eigenvalues $\mu_1 = -3.281\,23$ and $\mu_2 = -2.116\,48$, respectively, both negative. Thus the fixed point of this model is stable, and is an attractor. For illustration, we plot in figure 16 the phase graph of trajectories, each trajectory starts with a different initial condition, and ends up at the fixed point of equation (70). As a matter of fact, one can check that the asymptotic behaviour of other cases of model 1 are also stable fixed points.

The analysis of the asymptotic behaviour can be done analogously for model 3 with the couplings to the matter and the radiation. Setting $\mathrm{d}x/\mathrm{d}N = \mathrm{d}y/\mathrm{d}N = \mathrm{d}r/\mathrm{d}N = 0$ in equations (60), (61) and (62) one has the following three relations at the fixed point:

$$4y_f = -\frac{\Gamma + \Gamma'}{H}\frac{1+y_f}{h_f}, \tag{75}$$

$$3x_f = \frac{\Gamma}{H}\frac{(1+y_f)\,\mathrm{e}^{y_f}}{h_f}, \tag{76}$$

$$4r_f = \frac{\Gamma'}{H} \frac{(1+y_f)\,\mathrm{e}^{y_f}}{h_f}. \tag{77}$$

The fixed point (x_f, y_f, r_f) as the solution of this set of equations depends on the ratios of rates Γ/H and Γ'/H as well. Consider the constant Γ and Γ'. The local analysis of the stability of the fixed point can be carried out similarly. By setting $x = x_f + \varepsilon$, $y = y_f + \eta$, and $r = r_f + \gamma$, where ε, η, and γ are small perturbations around the fixed point, one has the equations

$$\frac{\mathrm{d}}{\mathrm{d}N} \begin{pmatrix} \varepsilon \\ \eta \\ \gamma \end{pmatrix} = M' \begin{pmatrix} \varepsilon \\ \eta \\ \gamma \end{pmatrix}, \tag{78}$$

where M' is a 3×3 matrix depending on the values of x_f, y_f, γ_f, Γ/H and Γ'/H, whose the elements are:

$$M'_{11} = -\left[\frac{\Gamma}{H} \frac{(1+y_f)\,\mathrm{e}^{y_f}}{2h_f^3} + 3 \right],$$

$$M'_{12} = \frac{\Gamma}{H} \frac{\mathrm{e}^{y_f}}{h_f} \left\{ (1+y_f) \left[1 - \frac{(2+y_f)\,\mathrm{e}^{y_f}}{2h_f^2} \right] + 1 \right\},$$

$$M'_{13} = -\frac{\Gamma}{H} \frac{(1+y_f)\,\mathrm{e}^{y_f}}{2h_f^3},$$

$$M'_{21} = \frac{\Gamma + \Gamma'}{H} \frac{1+y_f}{2(2+y_f)h_f^3},$$

$$M'_{22} = \frac{\Gamma + \Gamma'}{H} \left[\frac{(1+y_f)\,\mathrm{e}^{y_f}}{2h_f^3} - \frac{1}{(2+y_f)^2 h_f} \right] - \frac{8}{(2+y_f)^2},$$

$$M'_{23} = \frac{\Gamma + \Gamma'}{H} \frac{1+y_f}{2(2+y_f)h_f^3},$$

$$M'_{31} = -\frac{\Gamma'}{H} \frac{(1+y_f)\,\mathrm{e}^{y_f}}{2h_f^3},$$

$$M'_{32} = -\frac{\Gamma'}{H} \frac{(2+y_f)\,\mathrm{e}^{y_f}}{h_f} \left[\frac{(1+y_f)\,\mathrm{e}^{y_f}}{2h_f^2} + 1 \right]$$

$$M'_{33} = -\left[\frac{\Gamma'}{H} \frac{(1+y_f)\,\mathrm{e}^{y_f}}{2h_f^3} + 4 \right].$$

Consider the specific case $\Gamma/H = 0.5$ and $\Gamma'/H = 1.8 \times 10^{-4}$. Substituting these into equations (75), (76) and (77) yields the unique fixed point given by

$$(x_f, y_f, r_f) = (0.136\,66, -0.115\,03, 4 \times 10^{-5}), \tag{79}$$

and the matrix

$$M' = \begin{pmatrix} -3.221\,49 & 0.501\,10 & -0.221\,49 \\ 0.131\,87 & -2.176\,31 & 0.131\,87 \\ -0.000\,08 & 0.000\,45 & -4.000\,08 \end{pmatrix}.$$

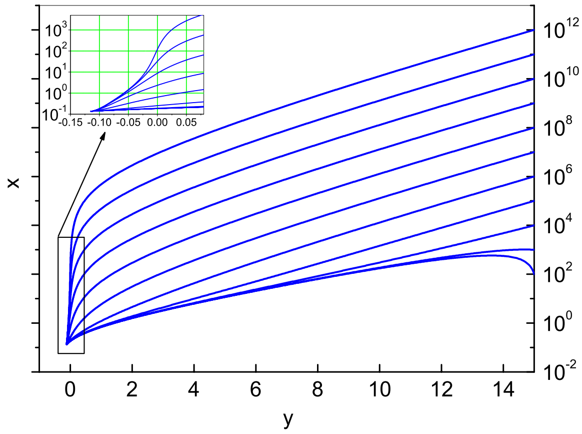

Figure 17. Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.000\,18$: the trajectories in the phase plane (x, y). Each trajectory starts with a different initial condition. All of them approach the fixed point $(x_f, y_f, r_f) = (0.136\,66, -0.115\,03, 4 \times 10^{-5})$. The parameters are the same as in figure 13.

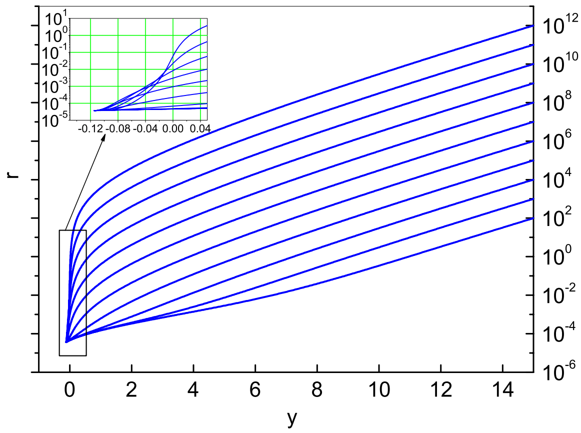

Figure 18. Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.000\,18$: the trajectories in the phase plane (r, y). The parameters are the same as in figure 13.

The three eigenvalues of the matrix M' are found to be $-4.000\,15$, $-3.281\,24$ and $-2.116\,49$, each being negative. Therefore, this attractor of model 3 is also stable. Notice that there are three quantities (x, y, r) in model 3, so we need the two phase graphs of trajectories. They are plotted in figure 17 for (x, y) and figure 18 for (r, y), separately. Each trajectory starts with a different initial condition, and ends up at the fixed point of equation (79). One can check the dynamics of other cases in model 3 also have a stable attractor.

As for model 2 and model 4, only $\rho_y(t)$ has an asymptotic constant value and has a stable attractor.

8. Conclusion and discussion

Our motivation of this study is to investigate the coincidence problem for the cosmic dark energy in a spatially flat universe. We have presented a detailed and comprehensive analysis of the model of the effective YMC dark energy interacting with the matter and radiation. This work has been an extended development of our previous work on the non-coupling YMC dark energy model. Through the Friedmann equation and the dynamic equations for each cosmic component, once the couplings between these components are specified, the overall cosmic evolution is fully determined by the initial conditions of these three components. We have studied the evolution for the matter-dominated era starting from the equality of radiation matter at $z \sim 3454$.

The major results of this work are the following.

Given the initial dominant matter and radiation $\Omega_{mi} = \Omega_{yi} \simeq 1/2$ and the subdominant YMC energy density $\Omega_{yi} \leqslant 10^{-2}$, no matter what kind of coupling between the YMC and the matter, or between the YMC and the radiation, the evolution is such that the YMC is subdominant to, and keeps track of, the matter, until later, at a redshift $z \sim 0.48$ for a coupling $Q_m > 0$, or $z \sim 0.35$ for a coupling $Q_m < 0$, the YMC becomes dominant over the matter. The era is followed by a subsequent accelerating era driven by the dominant YM dark energy. As the evolution outcome, the universe arrives at the present state with $\Omega_y \sim 0.7$, $\Omega_r \sim 0.3$ and $\Omega_r \sim 10^{-5}$. It is very important to note that this has been achieved for a variety of coupling forms Q_m and Q_r, and, nevertheless, under a very broad range of initial conditions $\Omega_{yi} \simeq (10^{-10}, 3 \times 10^{-3})$.

If the YMC decays only into the matter, as a result of the coupling in model 1, to achieve the present state of the universe, the decay rate needs to be of the same order of magnitude of the expansion rate of the universe, i.e., $\Gamma \sim 0.5H$. Moreover, for this coupled system, as $t \to \infty$, both ρ_y and ρ_m asymptotically approach constants, respectively. That is, for the system there is a unique attractor. Furthermore, as our analysis has shown, this attractor is stable. As an interesting behaviour, the EoS for the YMC w_y always crosses over -1 around $z \simeq 2.5$, and the present value is $w_y \sim -1.1$. This crossing -1 seems to have preliminary indications by the recent observations on SN 1a.

When the YMC decays into both the matter and the radiation as in model 3, there are two parameters $\Gamma \sim 0.5H$ and $\Gamma' \sim 1.8 \times 10^{-4}H$, representing the respective decay rate. The evolutional behaviour is almost the same as model 1, and w_y crosses over -1. Moreover, the radiation energy density also asymptotically approaches a constant, as $t \to \infty$, like the YMC and the matter components. Most of the conclusions are the same as for model 1.

On the other hand, if the matter decays into the YMC as in model 2 with a rate $\Gamma \sim 0.02H$, or if both the matter and radiation decay into the YMC as in model 4 with rates $\Gamma \sim 0.02H$ and $\Gamma' \sim 1.8 \times 10^{-4}H$, respectively, then only ρ_y asymptotically approaches a constant. w_y approaches -1, but does not cross over -1. The evolution is nearly similar to the non-coupling case.

Therefore, for all four types of models that we have studied, the coincidence problem can be naturally solved by introducing the effective YMC as the dark energy at the fixed parameter κ given in equation (38). The present state of the universe is a natural result of the dynamic evolution. The past history of the evolving universe is that of the standard Big Bang model, and the future of the universe depends on the details of the coupling. If there is no coupling, or if the matter decays, or both matter and radiation decay, into the YMC, as in model 2 and in model 4, the matter and the radiation will keep on decreasing as $\rho_m(t) \propto a(t)^{-3}$ and $\rho_r(t) \propto a(t)^{-4}$. If the YMC decays into the matter only as in model 1, then $\rho_m(t)$ will asymptotically remain constant, like $\rho_y(t)$ does, but $\rho_r(t) \propto a(t)^{-4}$. If the YMC decays into both the matter and radiation as in model 3, then all the components $\rho_y(t)$, $\rho_m(t)$, and $\rho_r(t)$ will asymptotically remain as constant. In model 1 and model 3 the future of the universe is a steady state, quite similar to that of the Steady State model, thus, in a sense, these two models bridge between the Big Bang model and the Steady State model.

The distinguished characteristics of the YMC dark energy model are the following.

The YM field is known to be indispensable to particle physics, the effective YMC employed in our work comes from quantum corrections up to 1-loop . Therefore, there is no room to adjust the form of the effective Lagrangian. This is in contrast to scalar field models, which have to design the form of potential and sometimes even the form of kinetic energy.

The solution of the coincidence problem has relied on the parameter κ in all our models. Viewed from the standard model of particle physics, the energy scale by κ is much smaller than the other known microscopic energy scales. And this stands as the fine-tuning problem for any current cosmological model so far, and for our model as well. However, if the YM field in our model is regarded as a fundamental gauge field with κ being the energy scale for this new physics, then the fine-tuning problem is traced up to the new physics. When the couplings are included, there are two more parameters Γ and Γ'. But the present state of the universe requires that Γ be roughly the same order of magnitude of the expansion rate $\Gamma \sim 0.5H$, and Γ' be roughly three orders lower. Since $H \sim \sqrt{G\kappa^2} = \kappa/m_{pl}$ with m_{pl} being the Planck mass, the couplings $\Gamma \sim \kappa/m_{pl}$ and $\Gamma' \sim 10^{-3}\kappa/m_{pl}$ are also associated with the scale κ.

On the dynamic evolution, in comparison with scalar models, our models have the following features. All our models, for a broad range of the initial conditions and for a variety of the coupling forms, automatically have the scaling property, i.e., $\rho_y(t)$ is initially subdominant to, and keeps track of, the matter. The accelerating stage has begun only quite recently around a redshift $z \sim (0.35, 0.48)$. This will allow the Big Bang cosmology to remain without drastic modifications. Besides, all our models have only one stable fixed point, uniquely determined by the ratio Γ/H, and have nothing to do with the initial conditions Ω_{yi}. Moreover, all the quantities in our model, especially $\rho_y(t)$ and $p_y(t)$ are continuous functions of t. So there are no Big Rip singularities in our models. Interestingly, as a function of t, w_y behaves quite smoothly during the evolution, going from $\sim 1/3$, approaching -1. And in the case of the YMC decaying, w_y crosses over -1 at $z \sim 2$, acquires the present value ~ -1.1 and settles down to an asymptotic value ~ -1.17.

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