



New method to constrain the relativistic free-streaming gas in the Universe

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ABSTRACT

We discuss a method to constrain the fraction density f of the relativistic gas in the radiation-dominant stage, by their impacts on a relic gravitational waves and the cosmic microwave background (CMB) B -polarization power spectrum. We find that the uncertainty of f strongly depends on the noise power spectra of the CMB experiments and the amplitude of the gravitational waves. Taking into account of the CMBPol instrumental noises, an uncertainty $\Delta f = 0.046$ is obtained for the model with tensor-to-scalar ratio $r = 0.1$. For an ideal experiment with only the reduced cosmic lensing as the contamination of B -polarization, $\Delta f = 0.008$ is obtained for the model with $r = 0.1$. So the precise observation of the CMB B -polarization provides a great opportunity to study the relativistic components in the early Universe.

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1. Introduction

Understanding the cosmic components in the Universe is one of the main tasks for cosmology. The current observations from cosmic microwave background, large scale structure, Type Ia supernova, etc., have already indicated $\sim 72\%$ dark energy, $\sim 23\%$ dark matter, $\sim 5\%$ baryons and $\sim 0.005\%$ photons as the main components in the present Universe [1–3].

With the upcoming of the more precise observations, it becomes possible and necessary to determine other components. In this Letter, we shall focus on the determination of the relativistic components in the Universe. In addition to the photons and the gravitational wave background [4], these components also include the massless (or tiny massive) neutrinos, the possible scalar field, the Yang–Mills field dark energy in the scaling stage [5–7], and some unknown massless (or tiny massive) particles, such as the sterile neutrinos [8]. As known, a large relativistic component in the Universe during the big bang nucleosynthesis (BBN) stage can enhance the expansion rate of the Universe, leading to a change the primordial abundances of the light elements. Thereby, one can constrain the total energy density of the relativistic components during the BBN stage [9], but unable to distinguish each component, as the expansion rate is determined by the total of all the relativistic components.

If a relativistic component behaves as a free-streaming gas of massless particles at the photon decoupling, they will also affect the growth of density perturbations, in addition to the change of the expansion rate. So by the observation of CMB spectra, especially the temperature anisotropy spectrum and the matter perturbation, one can constrain the fraction density f of the relativistic free-streaming gas among all the relativistic components [10–12]. However, there are various degeneracies between f and other cosmological parameters, which need to be broken for the method to work.

The stochastic gravitational waves backgrounds, generated in the very early Universe due to the superadiabatic application of zero point quantum fluctuations of the gravitational field [4], provide a much cleaner way to study the evolution of the Universe. The effect of the neutrino free-streaming gas on the spectrum of the relic gravitational waves (RGWs) has been examined in the previous works [16–19]. In particular, it has been found that the neutrino free-streaming gas causes a reduction of the spectral amplitude by 20% in the range $(10^{-16} - 10^{-10})$ Hz, and leaves the other portion of the spectrum almost unchanged [19].

This reduced RGWs leave observable imprints on the CMB temperature and polarization anisotropies power spectra [13,14]. Especially, the B -polarization power spectrum, only generated by RGWs, is reduced by (20–35%) when $\ell > 200$. In Ref. [15] it is pointed out that the similar effect can also be generated by other relativistic free-streaming gas. In this Letter, we introduce a new method to constrain the fraction energy density f of the relativistic free-streaming gas by the future CMB B -polarization observations. It will be shown that the value of Δf , the uncertainty of f

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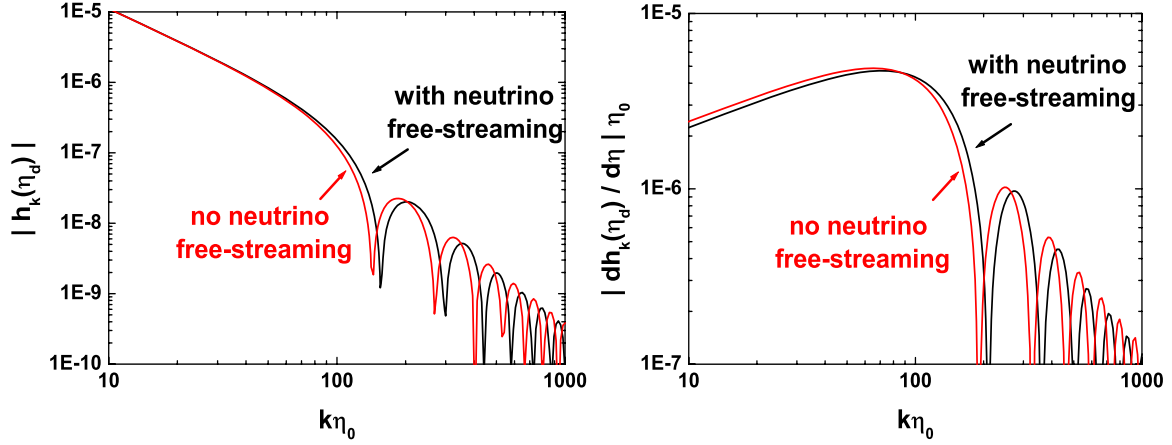


Fig. 1. The RGWs $|h_k(\eta_d)|$ and $|\dot{h}_k(\eta_d)|$ at the decoupling, where we have adopted the parameters of the primordial power spectra $r = 0.1$ and $n_t = 0$.

in the radiation-dominant stage, strongly depends on the value of tensor-to-scalar ratio r , and is limited by the noise power spectra of the CMB experiments. For the model with $r = 0.1$, CMBPol experiment can give $\Delta f = 0.046$. If considering the ideal case, where only the reduced cosmic lensing effect on the B -polarization is included, then one has $\Delta f = 0.008$.

2. Effects of free-streaming gas on RGWs and CMB polarizations

Incorporating the perturbations to the spatially flat Friedmann-Lemaître-Robertson-Walker space-time, the metric is

$$ds^2 = a^2(\eta)[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \quad (1)$$

where the perturbations of space-time h_{ij} is a 3×3 symmetric matrix. The gravitational wave field is the tensorial portion of h_{ij} , which is transverse-traceless $h_{ij,j} = 0$, $h^i_i = 0$. Since the relic gravitational waves are very weak, $|h_{ij}| \ll 1$, one needs to just study the linearized field equation:

$$\partial_\nu(\sqrt{-g}\partial^\nu h_{ij}) = -16\pi G\pi_{ij}. \quad (2)$$

The relativistic free-streaming gas gives rise to an anisotropic portion π_{ij} , which is also transverse and traceless. By the Fourier decomposition of h_{ij} and π_{ij} , for each mode \mathbf{k} and each polarization, Eq. (2) can be put into the form (see for instance [16])

$$\ddot{h}_k + 2\frac{\dot{a}}{a}\dot{h}_k + k^2 h_k = 16\pi G a^2 \pi_k, \quad (3)$$

where the overdot denotes a conformal time derivative $d/d\eta$. This equation can be modified to the following integro-differential equation [16]

$$\ddot{h}_k + 2\frac{\dot{a}}{a}\dot{h}_k + k^2 h_k = -24f\left(\frac{\dot{a}}{a}\right)^2 \int_{\eta_{rd}}^{\eta} \dot{h}_k(\eta') K(k(\eta - \eta')) d\eta', \quad (4)$$

where the kernel function in Eq. (4) is

$$K(x) \equiv -\frac{\sin x}{x^3} - \frac{3 \cos x}{x^4} + \frac{3 \sin x}{x^5},$$

$f \equiv \frac{\rho_f}{\rho_0}$ is the fractional density of the relativistic free-streaming gas in the radiation-dominant stage, and η_{rd} is the decoupling time of the relativistic free-streaming gas. One has $f = 0.41$ for the decoupled neutrino background with the number of species $N_\nu = 3$ as the relativistic free-streaming gas. However, if the other relativistic free-streaming gases also exist in the early Universe, the

value of f should be larger than 0.41. On the other hand, if the neutrinos do not free-stream, due to some possible couplings [20], then the value of f should be smaller than 0.41. So the determination of f provides a chance to study the relativistic components in the early Universe.

In the analytic approach, Eq. (4) is approximately reduced to the following form [14]:

$$\ddot{h}_k + 2\frac{\dot{a}}{a}\dot{h}_k + \left[k^2 - 24f(1 - K(0))\left(\frac{\dot{a}}{a}\right)^2\right]h_k = 0. \quad (5)$$

When $f = 0$, this equation returns to the evolution equation of gravitational waves in the vacuum, $\ddot{h}_k + 2\frac{\dot{a}}{a}\dot{h}_k + k^2 h_k = 0$ [21–23], which only depends on the evolution of the scale factor $a(\eta)$. Eq. (5) has been solved by perturbations, yielding the full analytic solution $h_k(\eta)$, from the inflation up to the present accelerating stage [14,19], and it has been found that the relativistic free-streaming gas causes a damping of h_k by $\sim 20\%$ in the frequency range $\nu \simeq (10^{-16}, 10^{-10})$ Hz.

The RGWs can generate the CMB temperature and polarization anisotropies power spectra $C_\ell^{XX'}$ ($XX' = TT, TE, EE, BB$), by the Sachs-Wolfe effect [13,14,24–32]. As shown in Ref. [14], the mode functions $h_k(\eta_d)$ and $\dot{h}_k(\eta_d)$ at the photon decoupling time η_d , i.e. $z \sim 1100$, appear in the integral expressions of the spectra of CMB temperature and polarization anisotropies. In Fig. 1, we plot the quantities $|h_k(\eta_d)|$ and $|\dot{h}_k(\eta_d)|$ as a function of $k\eta_0$, where η_0 is the present conformal time. The conformal wavenumber k is related to the frequency by $\nu = k/2\pi$, by setting the present scale factor $a(\eta_0) = 1$. We find that the neutrino free-streaming shifts the peaks of $h_k(\eta_d)$ and $\dot{h}_k(\eta_d)$ to the right side. In addition, when $k\eta_0 > 200$, the amplitudes of $h_k(\eta_d)$ and $\dot{h}_k(\eta_d)$ are obviously reduced by $\sim 20\%$, due to the existence of the neutrino free-streaming.

The modifications on $h_k(\eta_d)$ and $\dot{h}_k(\eta_d)$ by this relativistic free-streaming gas leave observable imprints in the spectra of CMB. To demonstrate this, the spectra C_ℓ^{BB} with and without neutrino free-streaming gas are plotted in Fig. 2. The $\ell < 200$ portion of the spectra is not affected much by neutrino free-streaming gas. Only on the scales of $\ell > 200$, the spectra are modified effectively, i.e. the reduction of amplitude of C_ℓ^{BB} by neutrino free-streaming gas is noticeable only starting from the second peak. Given the current precision level of observations on CMB, these small modifications caused by neutrino free-streaming gas are difficult to detect. However, as will be shown in the next section, this modification is expected to be detected by the future CMB experiments, such as the CMBPol project [33], which are sensitive for the CMB polarization observations.

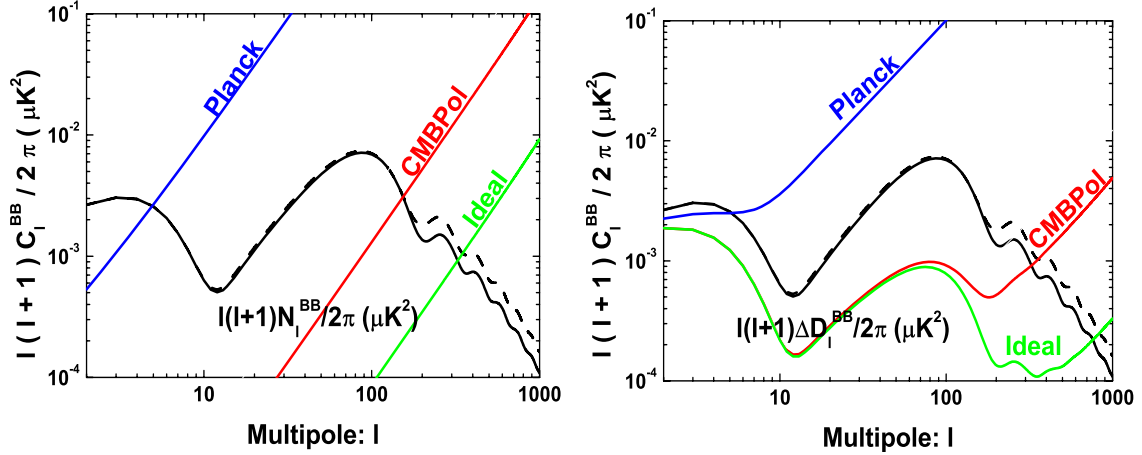


Fig. 2. In both panels, the black solid lines denote the CMB BB power spectrum with $f = 0.41$ and $r = 0.1$, and the black dashed lines denote the BB spectrum with $f = 0$ and $r = 0.1$. In the left panel, we also plot the noise power spectra N_ℓ^{BB} , and in the right panel, we also plot the quantity ΔD_ℓ^{BB} .

3. Constraint on the relativistic free-streaming gas

As mentioned, in addition to the decoupled neutrino, there may be other relativistic free-streaming gases in the early Universe, which may also modify the RGWs and CMB power spectra. So by the observations of the CMB power spectra, especially the B -polarization power spectrum (which is only generated by RGWs), we can constrain the fraction energy density of all the relativistic free-streaming gases, which is helpful to understand the various components in the Universe.

If all the CMB fields are Gaussian random, the power and cross spectra of the CMB temperature and polarization anisotropies quantify all the information contained in the observation [34]. We can use the Fisher information matrix techniques to compare and contrast the precision, to which various surveys can determine the parameters underlying the power spectra.

The Fisher matrix is a measure of the curvature of the likelihood function around its maximum in a space spanned by the parameters, such that the statistical error on a given parameter p_i is: $\Delta p_i \simeq (\mathbf{F}^{-1})_{ii}^{1/2}$ [35,36]. Here we consider the simplest case, only the fraction density of the relativistic free-streaming gas, f , is taken as the free parameter, and only the CMB B -polarization power spectrum is employed to constrain f . The other cosmological parameters are assumed to be well determined by the CMB power spectra C_ℓ^{TT} , C_ℓ^{TE} and C_ℓ^{EE} by the future CMB observations, so they will be fixed as their fiducial choices in the data analysis. Thus the Fisher matrix $\Delta p_i = 1/\sqrt{\mathbf{F}_{ii}}$ for $p_i \equiv f$ can be written as [35]

$$\Delta f = \left[\sum_\ell \left(\frac{\partial C_\ell^{BB}}{\partial f} \frac{1}{\Delta D_\ell^{BB}} \right)^2 \right]^{-1/2}. \quad (6)$$

Here ΔD_ℓ^{BB} is the standard deviation of the estimator D_ℓ^{BB} [34], which is calculable by

$$\Delta D_\ell^{BB} = \sqrt{\frac{2}{(2\ell+1)f_{\text{sky}}}} (C_\ell^{BB} + N_\ell^{BB}), \quad (7)$$

where f_{sky} is the cut sky factor. For a special experiment, the noise power spectrum is calculated by

$$N_\ell^{BB} = (\Delta_p)^2 \exp \left[\frac{\ell(\ell+1)\theta_F^2}{8 \ln 2} \right], \quad (8)$$

where Δ_p is the constant noise per multipole and θ_F is the full width at a half maximum beam in radians. We shall discuss three

Table 1

Instrumental parameters of the CMB experiments.

	Planck	CMBPol	Ideal
f_{sky}	0.8	0.8	0.8
θ_F (arcmin)	7.1	5	2
Δ_p ($\mu\text{K arcmin}$)	81.2	3.1	0.8
Δf (for $r = 0.1$)	...	0.046	0.008

kinds of future CMB experiments: the Planck satellite, the planned CMBPol experiment, and an ideal CMB experiment. Reference sensitivity for representative CMB polarization experiments are given in Table 1 [33,37]. In the ideal case, we have only considered the reduced lensed B -polarization spectrum as the contamination of C_ℓ^{BB} , which approximately corresponds to a noise with $\Delta_p \simeq 0.8 \mu\text{K arcmin}$ [38].

In Fig. 2, we have plotted the noise power spectra N_ℓ^{BB} and the uncertainty ΔD_ℓ^{BB} compared with the signal C_ℓ^{BB} in the model with the ratio $r = 0.1$, where we have taken our fiducial choice of the cosmological parameters as below: $\Omega_b = 0.0456$, $\Omega_c = 0.228$, $\Omega_\Lambda = 0.726$, $\Omega_k = 0$, $h = 0.705$, $f = 0.41$. The perturbation parameters are adopted as follows: $A_s = 2.445 \times 10^{-9}$, $n_s = 0.96$, $\alpha_s = 0$, $n_t = 0$.

Fig. 2 shows that the modification of C_ℓ^{BB} by the relativistic free-streaming gas is noticeable only at $\ell > 200$. Since the amplitude of C_ℓ^{BB} is very small in this range, only the very sensitive CMB experiments are expectable to be able to detect this modification. Fig. 2 also shows that, Planck mission is only sensitive for the reionization peak of C_ℓ^{BB} , i.e. $\ell < 10$. So it will be not expected to be able to constrain on the relativistic free-streaming gas in the Universe. However, for the CMBPol experiment, the signal C_ℓ^{BB} is larger than ΔD_ℓ^{BB} when $\ell < 300$, and a detection of this modification due to the relativistic free-streaming gas becomes possible. By solving Eq. (6), we obtain $\Delta f = 0.046$ for the model with $r = 0.1$, and this uncertainty reduced to $\Delta f = 0.008$ for the ideal experiment.

As expected, the value of Δf sensitively depends on the value of tensor-to-scalar ratio r . In Fig. 3, we plot the value of Δf as a function of r for the CMBPol and ideal experiments. It is seen that, with the increasing of r , the value of Δf becomes smaller. For $r = 0.3$, one has $\Delta f = 0.020$ for CMBPol experiment, and $\Delta f = 0.005$ for ideal experiment. However, when $r = 0.01$, one has $\Delta f = 0.233$ for CMBPol experiment, and $\Delta f = 0.030$ for the ideal experiment.

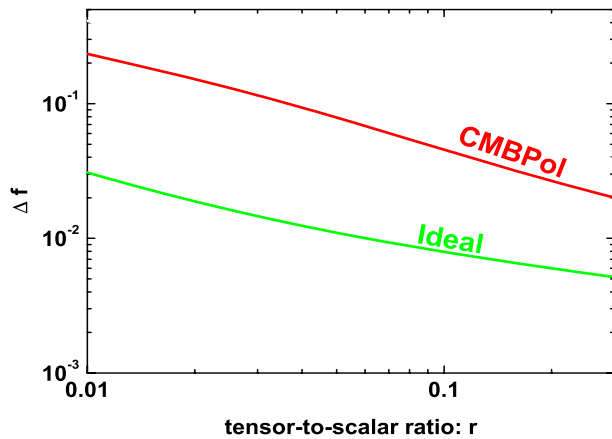


Fig. 3. The value of Δf depends on the tensor-to-scalar ratio r .

4. Conclusions

The relativistic free-streaming gas can modify the spectrum of RGWs and consequently reduce the CMB B -polarization power spectra at the scale $\ell > 200$. In this Letter, by taking into account the noise power spectra of the future CMB experiments, we have presented a constraint on the fraction density f of the relativistic free-streaming gas among all the relativistic components during the radiation-dominant stage. We find the value of Δf strongly depends on the noise of the experiments and the amplitude of the RGWs. CMBPol experiment is expected to obtain $\Delta f = 0.046$ for the model with $r = 0.1$, and $\Delta f = 0.020$ for the model with $r = 0.3$. For an ideal experiment, where only the B -polarization contamination by the reduced cosmic lensing effect is included, we expect to have $\Delta f = 0.008$ for the model with $r = 0.1$, and $\Delta f = 0.005$ for the model with $r = 0.3$. Our result shows that the experiments, like CMBPol, can provide a great chance to study the relativistic components in the early Universe.

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References

[1] E. Komatsu, et al., *Astrophys. J. Suppl.* 180 (2009) 330.

[2] M. Tegmark, et al., *Phys. Rev. D* 74 (2006) 123507.
 [3] M. Kowalski, et al., *Astrophys. J.* 686 (2008) 749.
 [4] L.P. Grishchuk, *Zh. Eksp. Teor. Fiz.* 67 (1974) 825;
 L.P. Grishchuk, *Ann. N. Y. Acad. Sci.* 302 (1977) 439;
 L.P. Grishchuk, *Pis'ma Zh. Eksp. Teor. Fiz.* 23 (1976) 326;
 L.P. Grishchuk, *Usp. Fiz. Nauk* 121 (1977) 629.
 [5] I. Zlatev, L. Wang, P.J. Steinhardt, *Phys. Rev. Lett.* 82 (1999) 896;
 P.J. Steinhardt, L. Wang, I. Zlatev, *Phys. Rev. D* 59 (1999) 123504.
 [6] W. Zhao, Y. Zhang, *Phys. Lett. B* 640 (2006) 69.
 [7] W. Zhao, Y. Zhang, *Class. Quantum Grav.* 23 (2006) 3405;
 Y. Zhang, T.Y. Xia, W. Zhao, *Class. Quantum Grav.* 24 (2007) 3309;
 T.Y. Xia, Y. Zhang, *Phys. Lett. B* 656 (2007) 19.
 [8] M. Cirelli, G. Marandella, A. Strumia, F. Vissani, *Nucl. Phys. B* 708 (2005) 215;
 M. Cirelli, *astro-ph/0410122*.
 [9] R.H. Cyburt, B.D. Fields, K.A. Olive, E. Skillman, *Astropart. Phys.* 23 (2005) 313.
 [10] J.R. Bond, A.S. Szalay, *Astrophys. J.* 274 (1983) 443;
 S. Bashinsky, *astro-ph/0411013*;
 S. Bashinsky, U. Seljak, *Phys. Rev. D* 69 (2004) 083002.
 [11] T.L. Smith, E. Pierpaoli, M. Kamionkowski, *Phys. Rev. Lett.* 97 (2006) 021301.
 [12] K. Ichikawa, T. Sekiguchi, T. Takahashi, *Phys. Rev. D* 78 (2008) 083526.
 [13] J. Pritchard, M. Kamionkowski, *Ann. Phys. (N.Y.)* 318 (2005) 2.
 [14] T.Y. Xia, Y. Zhang, *Phys. Rev. D* 78 (2008) 123005.
 [15] L.A. Boyle, P.J. Steinhardt, *Phys. Rev. D* 77 (2008) 063504.
 [16] S. Weinberg, *Phys. Rev. D* 69 (2004) 023503.
 [17] D.A. Dicus, W.W. Repko, *Phys. Rev. D* 72 (2005) 088302.
 [18] Y. Watandabe, E. Komatsu, *Phys. Rev. D* 73 (2006) 123515.
 [19] H.X. Miao, Y. Zhang, *Phys. Rev. D* 75 (2007) 104009.
 [20] J.F. Beacom, N.F. Bell, S. Dodelson, *Phys. Rev. Lett.* 93 (2004) 121302;
 X. Zhang, *hep-ph/0510072*.
 [21] L.P. Grishchuk, *Phys. Usp.* 48 (2005) 1235.
 [22] Y. Zhang, Y.F. Yuan, W. Zhao, Y.T. Chen, *Class. Quantum Grav.* 22 (2005) 1383.
 [23] W. Zhao, Y. Zhang, *Phys. Rev. D* 74 (2006) 043503;
 Y. Zhang, X.Z. Er, T.Y. Xia, W. Zhao, H.X. Miao, *Class. Quantum Grav.* 23 (2006) 3783.
 [24] A.G. Polnarev, *Sov. Astron.* 29 (1985) 607.
 [25] L.P. Grishchuk, *Phys. Rev. D* 48 (1993) 3513;
 L.P. Grishchuk, *Phys. Rev. Lett.* 70 (1993) 2371.
 [26] U. Seljak, M. Zaldarriaga, *Phys. Rev. Lett.* 78 (1997) 2054.
 [27] M. Kamionkowski, A. Kosowsky, A. Stebbins, *Phys. Rev. Lett.* 78 (1997) 2058.
 [28] W. Zhao, Y. Zhang, *Phys. Rev. D* 74 (2006) 083006;
 W. Zhao, *Phys. Rev. D* 79 (2009) 063003;
 W. Zhao, D. Baskaran, *Phys. Rev. D* 79 (2009) 083003.
 [29] D. Baskaran, L.P. Grishchuk, A.G. Polnarev, *Phys. Rev. D* 74 (2006) 083008.
 [30] R. Flauger, S. Weinberg, *Phys. Rev. D* 75 (2007) 123505.
 [31] B.G. Keating, A.G. Polnarev, N.J. Miller, D. Baskaran, *Int. J. Mod. Phys. A* 21 (2006) 2459.
 [32] Y. Zhang, W. Zhao, T.Y. Xia, X.Z. Er, H.X. Miao, *Int. J. Mod. Phys. D* 17 (2008) 1105;
 T.Y. Xia, Y. Zhang, *Phys. Rev. D* 79 (2009) 083002.
 [33] D. Baumann, et al., *arXiv:0811.3919*.
 [34] W. Zhao, D. Baskaran, L.P. Grishchuk, *Phys. Rev. D* 79 (2009) 023002.
 [35] M. Tegmark, A. Taylor, A. Heavens, *Astrophys. J.* 480 (1997) 22.
 [36] M. Zaldarriaga, D. Spergel, U. Seljak, *Astrophys. J.* 488 (1997) 1.
 [37] Planck Collaboration, *astro-ph/0604069*.
 [38] U. Seljak, C.M. Hirata, *Phys. Rev. D* 69 (2004) 043005.