Chapter 6

PREFERRED AXIS IN THE CMB PARITY ASYMMETRY

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ABSTRACT

Recent observations, such as the anomalies in the temperature angular distribution of the cosmic microwave background (CMB), indicate a preferred direction in the Universe. However, the foundation of modern cosmology relies on homogeneity and isotropy of the matter distribution on large scales. Here, we considered the preferred axis in the CMB parity violation. We found that this axis coincides with the preferred axes of the CMB quadrupole and octopole, and they all align with the direction of the CMB kinematic dipole, which has not a cosmological origin. The coincidence of these preferred directions hints that these anomalies do have a common origin which is not cosmological or due to a gravitational effect. However, the origin of the CMB anomalies is still an open question in cosmology.

Keywords: cosmic microwave background radiation, cosmological observations, data analysis, statistical methods

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5.1 Anomalies in the CMB temperature distribution

5. 1. 1 Introduction

The cosmic microwave background (CMB) radiation is one of our best cosmological observables. It provides a powerful test of the standard cosmological model, known as the LCDM model + inflation. The most accurate CMB full-sky dataset to date is provided by the Planck satellite, and its consistency with the flat six-parameter standard model is outstanding [1]. Other cosmological observations also support the LCDM model, including the distribution of large-scale structure, Type Ia supernovae, the baryon acoustic oscillation and the cosmic weak lensing. The LCDM cosmology assumes the universe as homogenous and isotropic in large scales (properties known as the cosmological principle), and describes gravity through Einstein's theory of general relativity. Moreover, in the early stages of our universe, quantum fluctuations were stretched by inflation, originating random and nearly gaussian distributed cosmic anisotropies. Despite the great success of the standard model, deviations from isotropy have been reported in CMB temperature data throughout the years. The first anomaly, a low quadrupole amplitude in disagreement with the predicted value, was first reported in the Cosmic Background Explorer (COBE) data [2] and later confirmed by the Wilkinson Microwave Anisotropy Probe (WMAP) observations [3-6]. This was the trigger for other claims regarding violations of isotropy soon announced in WMAP data: the lack of both variance and correlation on the largest angular scales [7-11], the cold spot [12-18], the power asymmetry [20-23], the hemisphere asymmetry [19-20, 22, 24-26], the large-scale quadrant asymmetry [27-29], the alignment of low multipoles [30-39] and parity asymmetry [40-44] (see [45] for a recent review in the CMB anomalies). The origin of these anomalies are not yet well understood and must be studied in detail in order to either confirm the LCDM or search for different explanations from the perspective of new physics.

Understanding the origin of the anomalies in the CMB temperature distribution is of great importance, and it could hint to the physics of the earliest stage of the universe. We know that most of the anomalies found in previous CMB observations were confirmed by Planck data [46-47], suggesting that they are unlikely to be due to systematic effects on the instruments. However, the significance of the CMB anomalies are far from consensus, and their cosmological origin is not confirmed. In the case these anomalies are, in fact, cosmological, the cosmological principle is violated and the standard model of cosmology must be revised. However, other explanations are also possible, including foreground microwave emissions from unpredicted astrophysics objects. A non-cosmological origin implies contaminations that are not being properly removed or considered (possible foreground residuals, for example), and should be taken into account to avoid misleading physical explanations of the universe. Thus, regardless the origin, cosmological or not, the CMB large-scale anomalies must be studied in detail.

Among all these anomalies, several ones are direction dependent. One question arises: Is there a preferred direction in our universe? For instance, the alignment of the CMB low multipoles seems to point to the same direction. Moreover, CMB data shows that there is a significant dominance stored on the power spectrum in the odd multiples over the even one, known as the CMB parity asymmetry [40-44, 48-50], also confirmed in Planck observations recently [46,47]. It was shown in previous works [51-53] that, independent of the statistics, the CMB parity violation favours a preferred direction in the universe. First of all, let's start with a brief review of the directional properties of the CMB parity asymmetry.

5.1.2 An overview of the CMB parity asymmetry

It is well known that CMB temperature fluctuations can be decomposed on a two dimensional sphere as spherical harmonics:

$$\Delta T(\hat{n}) = \sum_{J=2}^{\infty} \sum_{m=-1}^{J} a_{lm} Y_{lm}(\hat{n}), \tag{1}$$

where Y_{lm} and a_{lm} are the spherical harmonics and its correspondent coefficient. Assuming that the CMB temperature field is a Gaussian random field as predicted by the standard inflationary scenario, its statistical properties can be completely described by the second order power spectrum, namely $\langle a_{lm} a^{+}_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$, beingt $\langle ... \rangle$ the average over the statistical ensemble of realizations. However, due to the inability to directly measure the power spectrum, an estimator of it can be constructed. Considering a negligible noise, and a full sky map, the best unbiased estimator of C_l can be written as,

$$C_l = \frac{1}{2l+1} \sum_{m=-1}^{l} a_{lm} a_{lm}^{+} \tag{2}$$

The CMB temperature anisotropy field can be decomposed in symmetric, ΔT^{+} (even parity), and antisymmetric, ΔT^{-} (odd parity), components,

$$\Delta T^{+}(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) \Gamma_l^{+}, \quad \Delta T^{-}(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) \Gamma_l^{-}, \tag{3}$$

being $\Gamma_l^{+} = \cos^2\left(2\pi/2\right)$ and $\Gamma_l^{-} = \sin^2\left(2\pi/2\right)$ Significant power asymmetry between even and odd multipoles may be interpreted as a preference for a particular parity of the anisotropy pattern. Finally, based on the equations described above, we introduce a statistic to quantify the asymmetry in the CMB data:

$$P^{+} = \sum_{l=2}^{l\max} \frac{l(l+1)}{2\pi} C_l \Gamma_l^{+}, \quad P^{-} = \sum_{l=2}^{l\max} \frac{l(l+1)}{2\pi} C_l \Gamma_l^{-}, \tag{4}$$

where P^{+} and P^{-} are the sum of the power spectrum for even and odd multipoles, respectively. The ratio between these two quantities, P^{+}/P^{-}, indicates the parity preference. The analysis of the WMAP data indicates an odd multipole preference for low *ls*, violating the assumption of a random Gaussian field in the CMB temperature distribution, as it can be clearly seen in Figure 1.

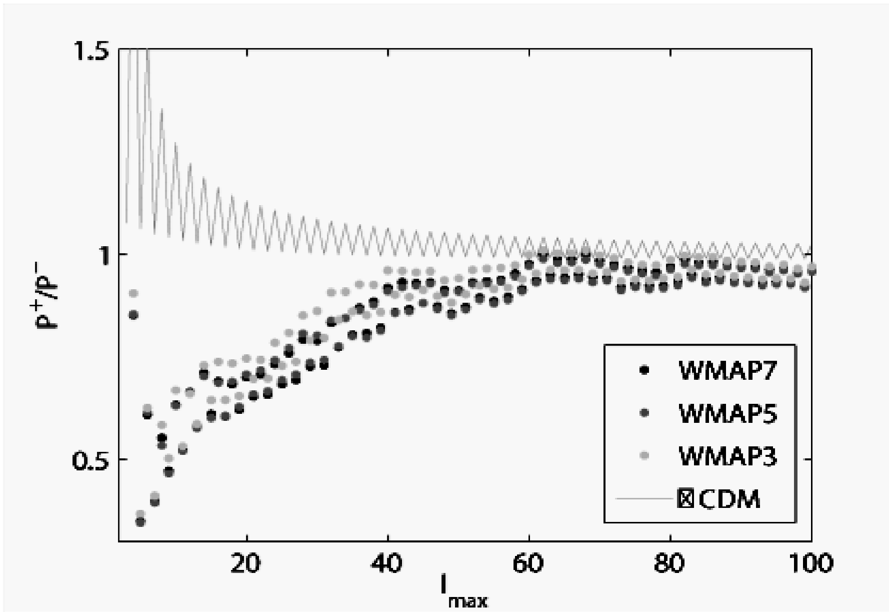

Figure 1. Theoretical values of P^+/P^- compared with WMAP results [54].

5.2 Preferred axis of CMB parity violation

5.2.1 A full sky analysis

In order to investigate the directional properties of the CMB parity asymmetry, we must define a directional dependent statistics G_1, which stands for the amplitude of the original parity parameter P^+/P^-, where a value $G_1 < 1$ and $G_1 > 1$ indicates an odd and even parity preference, respectively.

$$G_1(l;\hat{q}) = \frac{\sum_{l'=2}^{l} l'(l'+1)\hat{D}_{l'}(\hat{q})\Gamma_{\Gamma}^{+}}{\sum_{l'=2}^{l} l'(l'+1)\hat{D}_{l'}(\hat{q})\Gamma_{\Gamma}^{-}} \tag{5}$$

Considering full-sky CMB maps, we replaced the estimator $\hat{C}_l$ by the rotationally variant estimator $\hat{D}_l$, where

$$\hat{D}_l = \frac{1}{2l}\sum_{m=-1}^{l} a_{lm}a_{lm}^{+}\left(1-\delta_{mO}\right),\ a_{lm} = \sum_{m=-1}^{l} a_{lm}D_{mm'}^{l}\left(\psi,\theta,\phi\right), \tag{6}$$

The Kroneker symbol being represented by $\delta_{mm'}$. Considering any coordinate system, the coefficients $a_{lm}\left(\psi,\theta,\phi\right)$ rotated by the Euler angles $\left(\psi,\theta,\phi\right)$ can be calculate using the Wigner rotation matrix $D_{mm'}^{l}\left(\psi,\theta,\phi\right)$ [55]. We consider only two Euler angles, such that $\hat{q} \equiv \left(\theta,\phi\right)$ for $\psi = 0$.

We, finally, compute the directional parity parameter $G_1\left(l;\hat{q}\right)$ for any direction $\hat{q}$ using Planck temperature maps. The results are consistent with the ones previously reported of the odd-parity preference. Moreover, we found that the preferred directions are nearly the same for any maximum multipole l considered (see Table 1, upper values in columns 2 and 3).

Defining other statistics and using different estimators allow us to test the robustness of the results. Therefore, applying six different statistics by means of two different estimators (see, [52] for details), we concluded that the preferred direction in the CMB parity asymmetry is independent of the definition of the statistics.

5.2.2 A partial sky analysis

Foregrounds are unavoidable in CMB experiments, even in satellite surveys, due to the Galactic emission that must be masked out. It is thus important to investigate the case where these contaminated regions are excluded from the analysis. Applying the top-hat mask to the data, we obtain a masked map from which an unbiased estimator for the power spectrum must be calculated properly. Among many methods to perform this non-trivial calculation, we adopted the so-called pseudo C_l estimator method [56-60]. Similar to the previous discussion, we can build an unbiased estimator and finally the direction dependent statistics. Based on the estimator for masked maps, we found that the results are quite similar to the ones without the mask, and that all the directions nearly align to each other (see Table 1, lower values in column 2 and 3). This clearly shows that the preferred directions are independent of the used CMB masks.

5.2.3 The direction of the CMB kinematic dipole, quadrupole and octopole

The first temperature anisotropy reported in CMB data is now known to be due to the peculiar velocity of our local group of galaxies at about 627 ± 22 km/s relative to the CMB rest frame, the Doppler shift. This dipole anisotropy defines a peculiar axis in the universe relative to us, which is $(\theta = 42', \phi = 264'')$ in Galactic coordinate system [46.61]. Moreover, the quadrupole and octopole have preferred directions of $(\theta = 13.4', \phi = 238.5'')$ and $(\theta = 25.7', \phi = 239.0'')$, respectively. The angular difference between these directions is of only 12.3 degrees with a 96.8% confidence level (C.L), being the alignment between them, previously reported by Tegmark et. al, clear. We compare the preferred directions of parity asymmetry and the CMB kinematic dipole, and the cosmological quadrupole and octopole by defining the quantity (on how to evaluate the mean value of the inner product between all pairs of unit vectors corresponding to the considered directions, see):

$$\left\langle \left|\cos\theta_{ij}\right| \right\rangle = \sum_{i,j=1,j\neq i}^{N} \frac{\left|\hat{r}_i \cdot \hat{r}_j\right|}{N(N-1)} \tag{7}$$

where N is the number of investigated directions. First of all, considering the directions correspondent to te kinematic dipole, quadrupole and octopole $(N = 3)$, we found $\left\langle \left|\cos\theta_{ij}\right| \right\rangle = 0.9242$ for the real data and $\left\langle \left|\cos\theta_{ij}\right| \right\rangle = 0.500 \pm 0.167$ for the randomly generated 10^5 CMB simulations. To quantify the significant level of the deviation from random distribution, we define the Δ_c/σ_c quantity, where the Δ_c is the difference between the observed value of $\left\langle \left|\cos\theta_{ij}\right| \right\rangle$ and the mean value of the simulations, and σ_c is the corresponding standard deviation of the simulations. The alignment between these 3 directions is around 2.5σ C.L. If we considere also preferred direction of the CMB parity asymmetry $(N=4)$, we get $\langle|\cos\theta_{ij}|\rangle = 0.500 \pm 0.118$ for the 10^5 CMB random realisations. For every case with $l>3$, we found that $\langle|\cos\theta_{ij}|\rangle$ is close to 0.9 and the correspondent significance of the alignment between these 4 directions increases to $\Delta_c/\sigma_c > 3$ (see table 1, columns 3 and 4). We conclude that the preferred direction of the CMB parity asymmetry is closely aligned with the kinematic dipole and with the preferred axes of the CMB quadrupole and octopole, independently of the choice of the statistics, the CMB map or the mask used in the analysis.

	$\theta[\cdot]$	$\phi[\cdot]$	$\lvert\cos\alpha\rvert$	Δ_c/σ_c
$l_{max}=5$	45.82 45.82	279.73 279.73	0.980 0.980	3.42 3.42
$l_{max}=7$	47.41 48.21	278.00 275.06	0.981 0.985	3.39 3.40
$l_{max}=9$	48.21 49.80	276.47 272.25	0.982 0.985	3.35 3.38
$l_{max}=11$	49.01 49.80	277.17 272.25	0.979 0.985	3.35 3.38
$l_{max}=13$	49.01 49.80	278.58 272.25	0.976 0.985	3.34 3.38
$l_{max}=15$	49.80 49.80	282.10 272.25	0.965 0.985	3.27 3.38
$l_{max}=17$	50.57 49.80	284.21 270.84	0.957 0.987	3.22 3.39
$l_{max}=19$	50.57 49.01	284.21 270.14	0.957 0.990	3.22 3.42

$l_{max}=21$	50.57 49.01	284.21 270.14	0.957 0.990	3.22 3.42

Table 1. The preferred direction (θ,ϕ) for $G(l;\hat{q})$ based on 2015 Planck NILC map for a full sky analysis (upper values) and for a partial sky analysis (lower values), considering different maximum multipole.

5.4 Conclusions

Even though the standard model of cosmology has an extraordinary success in explaining the data, directional anomalies have been reported in various observations: the polarisation distributions of quasars, the large-scale velocity flows, the hardness of spiral galaxies, the anisotropy of cosmic acceleration, the anisotropic evolution of the fine-structure constant, dipole observations with radio galaxies and, finally, the anomalies in the CMB temperature angular distribution (including the parity asymmetry) (see. For example, [62]). A final answer for the origin of these anomalies is still missing. If they are due to cosmological effects, they violate of the cosmological principle, and could be explained, for example, by alternative theories of gravity, a non-trivial topology of the universe, anisotropic dark energy or a particular large-scale fluctuation modes. However, if these anomalies arise from non-cosmological origins, e.g, contaminations, we should properly treat the data to avoid misinterpretation of the universe. Our analysis, regarding the CMB parity anomaly, showed a strong alignment between the parity asymmetry preferred axis and the directions of the kinematic dipole, and the cosmological quadrupole and octopole. The alignment of the CMB dipole (purely kinematic effect) and the other preferred axes strongly suggests a non-cosmological origin of the large scale anomalies, which should be caused by some CMB-dipole related systematics or contaminations.

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