Polarizing primordial gravitational waves by parity violation

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We study primordial gravitational waves in the Horava-Lifshitz theory of quantum gravity, in which high-order spatial derivative operators, including the ones violating parity, generically appear in order for the theory to be power-counting renormalizable and ultraviolet complete. Because of both parity violation and the nonadiabatic evolution of the modes due to a modified dispersion relationship, a large polarization of primordial gravitational waves becomes possible, and it could be well within the range of detection of the BB, TB and EB power spectra of the forthcoming cosmic microwave background observation, where T, E, and B stand, respectively, the different components of the polarization of the cosmic microwave background radiation.

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I. INTRODUCTION

With the arrival of the precision era of cosmological measurements, temperature and polarization maps of the cosmic microwave background (CMB) with unprecedented accuracy will soon become available [1], and shall provide a wealth of data concerning the physics of the early universe, including inflation [2]—a dominant paradigm, according to which primordial density and primordial gravitational waves (PGWs) were created from quantum fluctuations in the very early universe. The former grows to produce the observed large-scale structure, and meanwhile creates CMB temperature anisotropy, which was already detected by the Cosmic Background Explorer almost two decades ago [3]. PGWs, on the other hand, produce not only temperature anisotropy, but also a distinguishable signature in CMB polarization. In particular, by decomposing the polarization into two modes—one is curl-free, the E-mode, and the other is divergence-free, the B-mode—one finds that the B-mode pattern cannot be produced by density fluctuations. Thus, its detection would provide a unique signature for the existence of PGWs [4]. PGWs normally produce the TT, EE, BB and TE spectra of CMB, but the spectra of TB and EB vanish when the parity of the PGWs is conserved [1]. However, if the theory is chiral, the power spectra of right-hand and left-hand PGWs can have different amplitudes, which then induces nonvanishing TB and EB correlations on large scales [5]. This provides the opportunity to directly detect the chiral asymmetry of the theory by observations [5–7].

With the above motivations, the studies of PGWs have attracted a great deal of attention, and various aspects have been explored [1]. Current 7-year Wilkinson Microwave Anisotropy Probe (WMAP) observations give the constraint on the tensor-to-scalar ratio $r < 0.36$ [8], and the 9-year data give a similar result, $r < 0.38$ [9]. However, if combined with other cosmological observations, the recent 9-year WMAP gives the tightest constraint, $r < 0.13$ at 95% confidence level [9], which corresponds to the amplitude of the PGWs $\Delta_h^2 < 3.03 \times 10^{-10}$. It should be noted that they impose no constraint on their chirality [6].

In this paper, we investigate the possibility of detecting the chirality of PGWs through three information channels—BB, TB and EB of the CMB—in the recently proposed Horava-Lifshitz (HL) theory of quantum gravity, in which parity violation happens generically [10]. Such a detection allows the theory to be tested directly, and provides a smoking gun for its parity violation in the early universe. This represents one of the few observations/experiments that one can construct currently as well as in the near future, considering the quantum nature of the theory.

The HL theory is power-counting renormalizable because of the presence of high-order spatial derivative operators. The exclusion of high-order time-derivative operators, on the other hand, renders the theory unitary, whereby it is expected to be ultraviolet (UV) complete. In the infrared (IR), the low-order derivative operators take over [cf. Eq. (3)] and presumably provide a healthy IR limit [10]. When applying it to cosmology, various remarkable features were found [11]. In particular, the higher-order spatial-curvature terms can give rise to a bouncing universe [12], which may ameliorate the flatness problem [13] and lead to caustic avoidance [14]. The anisotropic scaling provides a solution to the horizon problem and the generation of scale-invariant perturbations with [15] or without inflation [16]. It also provides a new mechanism for the generation of a primordial magnetic seed field [17], and a modification of the spectrum of the gravitational wave background via a peculiar scaling of the radiation energy density [18]. With the projectability condition, the lack of a local Hamiltonian constraint leads to ''dark matter as an integration constant'' [19]. The dark sector can also have its purely geometric origins [20]. A large non-Gaussianity is possible for both scalar [21] and tensor [22] perturbations even with a single slow-roll scalar field because of the presence of high-order derivative terms, and so on.

Despite all of these remarkable features, the theory also faces some challenging questions, such as instability and strong coupling. To answer these questions, various models have been proposed, including the ones with an additional local U(1) symmetry [23,24], in which the problems—such as instability, ghosts, strong coupling, and different speeds in the gravitational sector,—can be avoided. In all of those models, the tensor perturbations are almost the same [25], so without loss of generality we shall work with the model proposed in Ref. [24].

The rest of the paper is organized as follows. In Sec. II we specify the model that accounts for the polarization of PGWs, while in Sec. III we consider the polarization of PGWs in the de Sitter background. In Sec. IV, we discuss their detectability for the Planck satellite and forthcoming observations. The paper is ended with Sec. V, in which we derive our main conclusions.

It should be noted that, although our motivation to study the polarization of PGWs is the HL theory, our final conclusions are applicable to any theory in which the dispersion relation of the PGWs is described by Eq. (5).

In addition, the effects of the chirality of gravitons on CMB was first studied in Ref. [5] in Einstein's theory of gravity, and lately in Ref. [26] in the HL theory, but our model is fundamentally different from theirs. In particular, the model studied in Ref. [26] produces a negligible polarization, and is not detectable within current and near future observations, as will be shown below. To have a sizable effect, we find that the existence of a non-WKB region in the dispersion relation that leads to nonadiabatic evolution of the modes is essential, a feature that is absent in the model of Ref. [26]. In addition, the chirality of gravitons was also considered in Ref. [27], but this model is not power-counting renormalizable, and cannot be considered as a viable candidate of quantum gravity.

II. THE MODEL

In Ref. [24], the parity was assumed by setting all the fifth- and third-order spatial derivative operators,

$$\Delta\mathcal{L}_V = (\alpha_0 K_{ij}R^{ij} + \alpha_2\epsilon^{ijk}R_{il}\nabla_j R^l_k)/M_*^3 + \alpha_1\omega_3(\Gamma)/M_* + \text{``}\ldots\text{''}, \tag{1}$$

to zero, where α_i's are dimensionless coupling constants, ϵ^{ijk} is the total antisymmetric tensor, and K_{ij} and R_{ij} denote, respectively, the extrinsic curvature and the three-dimensional Ricci tensor built out of the three-metric g_{ij}. ∇_i denotes the covariant derivative with respect to g_{ij}, $\omega_3(\Gamma)$ the three-dimensional gravitational Chern-Simons term, M_* the energy scale above which the high-order derivative operators become important, and "…" is the part proportional to a_i, which vanishes for tensor perturbations, where $a_i \equiv N_{,i}/N$ with N being the lapse function [28]. For details, we refer readers to Ref. [24]. However, once the radiative corrections are taken into account, these terms are expected to be present generically [29]. Therefore, in this paper we shall add these terms into the action of Ref. [24], and show that it is exactly because of their presence that the PGWs will get polarized. Depending on the strength of these terms, the polarization of PGWs can be well within the range of detectability of the forthcoming observations of CMB, as will be shown explicitly below.

In the Friedmann-Robertson-Walker flat universe, the background is given by $\hat{N} = a(\eta)$, $\hat{g}_{ij} = a^2(\eta)\delta_{ij}$ and $\hat{N}^i = \hat{\varphi} = \hat{A} = 0$, where $\hat{N}^i$ denotes the shift vector [28], and $\hat{\varphi}$ and $\hat{A}$ are, respectively, the Newtonian prepotential and U(1) gauge field [23]. Consider the tensor perturbations, $\delta g_{ij} = a^2 h_{ij}(\eta, \mathbf{x})$, where h_{ij} is transverse and traceless, $\partial^i h_{ij} = 0 = h^i{}_i$. Then, the quadratic part of the total action can be cast in the form

$$\begin{aligned} S^{(2)}_{\text{total,g}} = \zeta^2 \int d\eta d^3x \Big\{ & \frac{a^2}{4}(h'_{ij})^2 - \frac{1}{4}a^2(\partial_k h_{ij})^2 \\ & - \frac{\hat{\gamma}_3}{4M_*^2}(\partial^2 h_{ij})^2 - \frac{\hat{\gamma}_5}{4M_*^4 a^2}(\partial^2\partial_k h_{ij})^2 \\ & - \frac{\alpha_1 a e^{ijk}}{2M_*}(\partial_l h^m_i \partial_m\partial_j h^l_k - \partial_l h_{im}\partial^l\partial_j h^m_k) \\ & - \frac{\alpha_2 e^{ijk}}{4M_*^3 a}\partial^2 h_{il}(\partial^2 h^l_k)_{,j} - \frac{3\alpha_0\mathcal{H}}{8M_*^3 a}(\partial_k h_{ij})^2 \Big\}, \end{aligned} \tag{2}$$

where $\partial^2 = \delta^{ij}\partial_i\partial_j$, $\mathcal{H} = a'/a$, $\epsilon^{ijk} \equiv e^{ijk}/\sqrt{g}$, and $\gamma_3 = \hat{\gamma}_3\zeta^2/M_*^2$, $\gamma_5 = \hat{\gamma}_5\zeta^4/M_*^4$, with $\zeta^2 = M_{\text{pl}}^2/2$ and $e^{123} = 1$. $\gamma_{3,5}$ are the dimensionless coupling constants of the theory [24], similar to $\alpha_{0,1,3}$. To avoid fine-tuning, α_n and $\hat{\gamma}_n$ are expected to be of the same order. Then, the field equations for h_{ij} read

$$\begin{aligned} & h''_{ij} + 2\mathcal{H}h'_{ij} - \alpha^2\partial^2 h_{ij} + \frac{\hat{\gamma}_3}{a^2M_*^2}\partial^4 h_{ij} - \frac{\hat{\gamma}_5}{a^4M_*^4}\partial^6 h_{ij} \\ & + e_i^{lk}\Big(\frac{2\alpha_1}{M_* a} + \frac{\alpha_2}{M_*^3 a^3}\partial^2\Big)(\partial^2 h_{jk})_{,l} = 0, \end{aligned} \tag{3}$$

where a prime denotes a derivative with respect to η, and $\alpha^2 \equiv 1 + 3\alpha_0\mathcal{H}/(2M_*^3 a)$.

III. POLARIZATION OF PGWS IN THE DE SITTER BACKGROUND

When the background is de Sitter, we have $a = -1/H\eta$, where H is the Hubble constant. To study the evolution of h_{ij}, we first expand it over spatial Fourier harmonics as

$$h_{ij}(\eta, \mathbf{x}) = \sum_{A=R,L}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\psi^A_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}P^A_{ij}(\hat{\mathbf{k}}),$$

where $P^A_{ij}(\hat{\mathbf{k}})$ are the circular polarization tensors and satisfy the relations $ik_s e^{rsj}P^A_{ij} = k\rho^A P^{rA}_i$ with $\rho^R = 1$, $\rho^L = -1$, and $P^{*iA}_j P^{jA'}_i = \delta^{AA'}$ [26]. Substituting it into Eq. (3), we obtain

$$\phi^A_{k,yy} + (\omega^2_{\rm A} - 2y^{-2})\phi^A_k = 0, \qquad (4)$$

where $\phi^A_k \equiv \sqrt{\alpha k} a \psi^A_k$, $y \equiv -\alpha k \eta$, and

$$\omega^2_{\rm A} = 1 + \rho^A(\delta_1 y + \delta_3 y^3) + \delta_2 y^2 + \delta_4 y^4, \qquad (5)$$

with $\delta_1 \equiv -2(\alpha_1/\alpha^3)\varepsilon_{\rm HL}$, $\delta_2 \equiv (\hat{\gamma}_3/\alpha^4)\varepsilon^2_{\rm HL}$, $\delta_3 \equiv (\alpha_2/\alpha^5)\varepsilon^3_{\rm HL}$, $\delta_4 \equiv (\hat{\gamma}_5/\alpha^6)\varepsilon^4_{\rm HL}$, where $\epsilon_{\rm HL} \equiv H/M_* \ll 1$. Note the difference between $\epsilon_{\rm HL}$ used in this paper and the one defined in [21,22]. Note that the unitarity of the theory in the UV requires $\hat{\gamma}_5 > 0$, while a healthy IR limit requires $\alpha^2 \simeq 1$. Thus, without loss of generality, in the following we set $\alpha = 1$, or equivalently $\alpha_0 = 0$. However, α_1, α_2 and $\hat{\gamma}_3$ have no such restrictions, as long as $\omega^2_A > 0$ holds. Following Ref. [30], we choose the initial conditions at $\eta = \eta_i$ as

$$\phi^A_k(\eta_i) = \sqrt{\frac{1}{2\omega_A}}, \qquad \frac{d\phi^A_k(\eta_i)}{dy} = i\sqrt{\frac{\omega_A}{2}}, \qquad (6)$$

which minimizes the energy density of the field.

Before proceeding further, let us note that the case studied in Ref. [26] corresponds to the choice $\beta = \delta_2$, $\gamma = \delta_3/(2\delta_2)$, $\delta_1 = 0$ and $\delta_4 = \delta_3^2/(4\delta_2)$, where β and γ are parameters introduced in Ref. [26]. To obtain a sizable polarization of PGWs for future observations, Ref. [26] showed that β and γ should be of order one, which implies $\hat{\gamma}_3 = \mathcal{O}(\varepsilon^{-2}_{\rm HL})$, $\alpha_2 = \mathcal{O}(\varepsilon^{-3}_{\rm HL})$, and $\hat{\gamma}_5 = \mathcal{O}(\varepsilon^{-4}_{\rm HL})$. Clearly, this represents fine-tuning.

To see the effects of the parity-violated terms, let us first consider two representative cases: (i) $\delta_2 = \delta_3 = 0$, and (ii) $\delta_1 = \delta_2 = 0$. In each of them, we can obtain $\omega_{R,\rm ph}$ from $\omega_{L,\rm ph}$ by flipping the sign of δ_1 or δ_3, where $\omega^2_{\rm A,ph} \equiv \alpha^2 k^2_{\rm ph}\omega^2_A$, and $k_{\rm ph} \equiv k/a$. Thus, without loss of generality, we assume that $\delta_{1,3} \geq 0$. Then, from Eq. (5) we can see that $\omega^2_{\rm R}$ is a monotonic function of y, and the equation $\omega_{R,\rm ph} = H$ has only one real positive root, as shown by curve (a) in Fig. 1. Then, the WKB approximations are applicable in the region $\omega_{R,\rm ph} > H$, and the mode function $\phi^R_k = \sqrt{\alpha k} v^R_k$ of Eq. (4) is given by

$$v^R_k = \begin{cases} \frac{1}{\sqrt{2\omega_R}} e^{-i\int^{\eta}_{\eta_i}\omega_R(k,\eta')d\eta'}, & \omega_{R,\rm ph} > H, \\ D_+ a(\eta) + D_- a(\eta)\int^{\eta}_{\eta_3}\frac{d\eta'}{a^2(\eta')}, & \omega_{R,\rm ph} < H, \end{cases} \qquad (7)$$

where $D_\pm(k)$ are uniquely determined by the boundary conditions at the horizon crossing $\omega_R = a(\eta_3)H$, which require v^R_k and its first-order time derivative to be continuous across the boundary.

On the other hand, $\omega_{L,\rm ph} = H$ can have one, two or three real positive roots, depending on the ratio δ_1/δ_4 or δ_3/δ_4. If it has only one root, as shown by curve (b), the WKB approximations are applicable in the region $\omega_{L,\rm ph} > H$, and v^L_k is given by Eq. (7), where $\phi^L_k = \sqrt{\alpha k} v^L_k$. In the case with two real roots, we have $\eta_1 = \eta_2$, and Region II in Fig. 1 does not exist. As a result, v^L_k is also given by

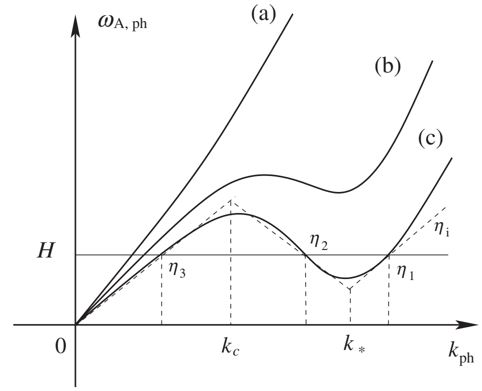

FIG. 1. The evolution of $\omega_{\rm A,ph}$ vs $k_{\rm ph}$ for (i) $\delta_2 = \delta_3 = 0$ or (ii) $\delta_1 = \delta_2 = 0$, where $k_{ph} \equiv k/a$. Region I: $\eta \in (\eta_i, \eta_1)$. Region II: $\eta \in (\eta_1, \eta_2)$. Region III: $\eta \in (\eta_2, \eta_3)$. Region IV: $\eta \in (\eta_3, 0)$.

Eq. (7). But, when it has three real roots, as shown by curve (c), the WKB approximations are not applicable in Region II, and the evolution becomes nonadiabatic. Then, the mode function $v^L_k [= \phi^L_k/\sqrt{\alpha k}]$ of Eq. (4) is given by

$$v^L_k = \begin{cases} \frac{1}{\sqrt{2\omega_L}} e^{-i\int^{\eta}_{\eta_i}\omega_L(k,\eta')d\eta'}, & \text{Region I}, \\ C_+ a(\eta) + C_- a(\eta)\int^{\eta}_{\eta_1}\frac{d\eta'}{a^2(\eta')}, & \text{Region II}, \\ \frac{\alpha_k e^{-i\Theta^L_2(k,\eta)} + \beta_k e^{i\Theta^L_2(k,\eta)}}{\sqrt{2\omega_L(k,\eta)}}, & \text{Region III}, \\ D_+ a(\eta) + D_- a(\eta)\int^{\eta}_{\eta_3}\frac{d\eta'}{a^2(\eta')}, & \text{Region IV}, \end{cases} \qquad (8)$$

where $\Theta^A_n(k,\eta) \equiv \int^{\eta}_{\eta_n}\omega_A(k,\eta')d\eta'$. The coefficients $C_\pm(k)$, $D_\pm(k)$, α_k, β_k are uniquely determined by requiring that v^L_k and its first-order time derivative be continuous across the boundaries that separate these regions [31]. Note that due to the nonadiabaticity of the evolution in Region II, particles are created, where their occupation number n_k is given by $n_k = |\beta_k|^2$. To have the energy density of such particles be smaller than that of the background, one must require [32] $|\beta_k|^2 < (M_{\rm pl}/M_*)^2\varepsilon^2_{\rm HL}$. In the case without the U(1) symmetry, the study of the parametrized post-Newtonian corrections requires $M_* \leq 10^{15}$ GeV [33]. In the case with the U(1) symmetry, the spin-0 gravitons are not present, and the gravitational sector has the same degree of freedom as that in general relativity. So, some softer constraint on the values of M_* is expected, although such considerations have been carried out so far only for the static spherical case [34], in which the Eddington-Robertson-Schiff parameters were calculated, and it was found that they are consistent with observations and do not impose any constraint on M_*. But, it is expected that other considerations, such as frame effects, will impose some constraints on M_*. With some anticipation, in this paper we simply assume $M_* \leq M_{\rm pl}$. Then, for $\varepsilon_{\rm HL} \simeq M_*/M_{\rm pl}$, we have $|\beta_k|^2 \simeq \mathcal{O}(1)$ [35].

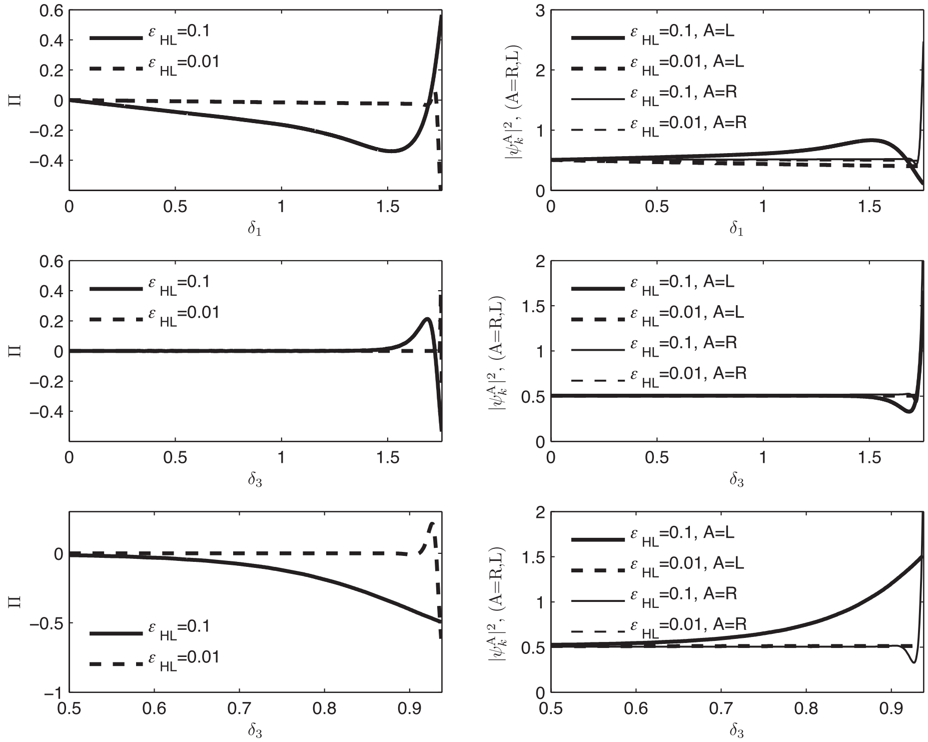

FIG. 2. (a) Top panel: $\delta_1 \neq 0$, $\delta_2 = \delta_3 = 0$. (b) Middle panel: $\delta_3 \neq 0$, $\delta_1 = \delta_2 = 0$. (c) Low panel: $\delta_1 \neq 0$, $\delta_2 = -\varepsilon_{HL}^2$. In all the plots, we have set $\delta_4 = \varepsilon_{HL}^4$.

Assuming that all the above conditions hold, we can see that in both cases (i) and (ii) there are only two distinguishable combinations, described, respectively, by curves (a) + (b) and curves (a) + (c) in Fig. 1. In the former, the power spectrum of PGWs and the circular polarization are given by

$$\Delta_h^2 \equiv \frac{k^3(|\psi_k^R|^2 + |\psi_k^L|^2)}{(2\pi)^2} = \frac{H^2}{4\pi^2}(1 + 21\alpha_1^2\varepsilon_{HL}^2 + \mathcal{O}(\varepsilon_{HL}^3)),$$
$$\Pi \equiv \frac{|\psi_k^R|^2 - |\psi_k^L|^2}{|\psi_k^R|^2 + |\psi_k^L|^2} = 3\alpha_1\varepsilon_{HL} + (17\alpha_1^3 - 3\alpha_2)\varepsilon_{HL}^3/2 + \mathcal{O}(\varepsilon_{HL}^5). \quad (9)$$

Therefore, in this case the polarization of PGWs is negligible for physically reasonable values of α_1 and α_2. Note that the case studied in Ref. [26] belongs to this case (with $\alpha_1 = 0$) [cf. Fig. 2(b)].

For the combination of curves (a) + (c), we find

$$\Delta_h^2 = \frac{H^2}{4\pi^2}\Big[1 + \Delta_k^L - 3\alpha_1\Delta_k^L\varepsilon_{HL} + \frac{21}{2}(1 + \Delta_k^L)\alpha_1^2\varepsilon_{HL}^2 + \mathcal{O}(\varepsilon_{HL}^3)\Big],$$
$$\Pi = -\frac{\Delta_k^L}{1 + \Delta_k^L} + \frac{3(1 + 2\Delta_k^L)\alpha_1}{(1 + \Delta_k^L)^2}\varepsilon_{HL} + \frac{9\alpha_1^2\Delta_k^L(1 + 2\Delta_k^L)}{(1 + \Delta_k^L)^3}\varepsilon_{HL}^2 + \mathcal{O}(\varepsilon_{HL}^3), \quad (10)$$

where $\Delta_k^A \equiv |\beta_k^A|^2 + \mathrm{Re}(\alpha_k^A\beta_k^{A*}e^{-2i\Theta_{23}^A})$, and $\Theta_{nm}^A = \Theta_n^A(k, \eta_m)$. Thus, in the present case a large Π becomes possible. Figure 2(a) shows such possibilities.

In addition to the two specific cases above, when $\delta_2 \neq 0$, there is another possibility in which both ω_{ph}^R and ω_{ph}^L are given by curve (c). Then, we find

$$\Delta_h^2 = \frac{H^2}{4\pi^2}\Big[1 + \Delta_k^+ + 3\alpha_1\Delta_k^-\varepsilon_{HL} + \frac{3}{2}(7\alpha_1^2 - \hat{\gamma}_3)(1 + \Delta_k^+)\varepsilon_{HL}^2 + \mathcal{O}(\varepsilon_{HL}^3)\Big],$$
$$\Pi = \frac{\Delta_k^-}{1 + \Delta_k^+} + \frac{3\alpha_1(1 + 2\Delta_k^R)(1 + 2\Delta_k^L)}{(1 + \Delta_k^+)^2}\varepsilon_{HL} + \frac{9\alpha_1^2\Delta_k^-(1 + 2\Delta_k^R)(1 + 2\Delta_k^L)}{(1 + \Delta_k^+)^3}\varepsilon_{HL}^2 + \mathcal{O}(\varepsilon_{HL}^3), \quad (11)$$

where $\Delta_k^\pm \equiv \Delta_k^R \pm \Delta_k^L$. Again, since $\Delta_k^A(A = R, L)$ can be as large as of order one, a large Π now also becomes possible, as shown by Fig. 2(c).

IV. DETECTABILITY OF PGWS

In the case of the two-point statistics, the CMB temperature and polarization anisotropies are completely specified by six (TT, EE, BB, TE, TB, EB) power spectra. Usually, the PGWs produce the TT, EE, BB and TE spectra, but the spectra of TB and EB should vanish due to the parity consideration of the PGWs. However, if the linearized gravity is chiral—as in the current case—the power spectra of right-hand and left-hand PGWs can have different amplitudes, and thus induce nonvanishing TB and EB correlations on large scales [5]. This provides the opportunity to directly detect the chiral asymmetry of gravity by observations, which has been discussed in some detail in Refs. [5–7]. However, we differ from them in that here we consider three information channels—BB, TB and EB—to contain the chiral PGWs by determining both parameters, r and Π. In Fig. 3 (top panel), we show the CMB power spectra produced by PGWs with $r = 0.1$ and $\Pi = 1$, from which one can see that the PGWs are more easily detected in the smaller BB channel than in the larger TB one. The main reason is, as stated in Ref. [7], the uncertainties of the TB and EB channels are much larger than those of the BB channel, especially the effect of TT and EE power spectra generated by density perturbations. So, the determination of r is mainly from the BB channel, but not from the TB or EB ones. Note that the background cosmological parameters are chosen as 9-year WMAP best-fit values [9], and $n_t = 0$ is fixed throughout this paper.

In order to determine the uncertainties of the parameters by the potential observations, we use the Fisher matrix technique to avoid the Monte Carlo simulations. The Fisher matrix is

$$F_{ij} = \sum_l \sum_{XX',YY'} \frac{\partial C_l^{XX'}}{\partial p_i} \mathrm{C}^{-1}(D_l^{XX'}, D_l^{YY'}) \frac{\partial C_l^{YY'}}{\partial p_j},$$

where $C_l^{XX'}$ are the CMB power spectra and $D_l^{XX'}$ the corresponding estimators. p_i are the parameters to be determined, which are r and Π in the present case. The covariance matrix of the estimators is given by

$$\mathrm{C}(D_l^{XX'}, D_l^{YY'}) = \frac{\mathcal{C}_l^{XY}\mathcal{C}_l^{X'Y'} + \mathcal{C}_l^{XY'}\mathcal{C}_l^{X'Y}}{(2l+1)f_{\rm sky}},$$

where $\mathcal{C}_l^{XY} = C_l^{XY} + N_l^{XY}$, and the noise power spectra N_l^{XY} include the instrumental noises, lensed B-mode polarization, and the CMB power spectra generated by density perturbations. $f_{\rm sky}$ is the sky-cut factor, which will be taken as $f_{\rm sky} = 0.65$ for Planck [37], 0.8 for CMBPol [38] and ideal experiments, and 1.0 for the cosmic variance limit. Note that for Planck and CMBPol, we have ignored the contaminations from Galactic radiations, especially the synchrotron and dust-dust radiations, which are expected to be well controlled and become subdominant by the multiband observations [39]. In the ideal case, only the reduced lensed B-mode polarization is considered as the contamination [40], and in the cosmic variance limit, we assume that all the contaminations can be well removed. Throughout our calculations, we choose $l_{\rm max} = 2000$. Once the Fisher matrix is calculated, the uncertainties of the parameters can be evaluated by $\Delta p_i = \sqrt{F_{ii}^{-1}}$.

As mentioned in Ref. [7], the determination of r is mainly from the observation of the BB information channel, but not from the TB or EB channels, even if the PGWs are completely chiral. In order to quantify the detection abilities of the experiments, we define the signal-to-noise ratio $S/N \equiv r/\Delta r$. Similar to the discussions in previous works [41], we find that if the condition $S/N > 3$ is required, i.e., a definite detection, $r > 0.03$ is needed for the Planck satellite, while the CMBPol mission can detect the signal if $r > 0.4 \times 10^{-3}$, and the ideal experiment can detect it if $r > 0.8 \times 10^{-5}$ [42].

The constraint on Π is mainly from the TB and EB channels, where the cosmic variances caused by TT, EE and BB are dominant. So, the values of $\Delta\Pi$ are nearly independent of Π in the fiducial model. In Fig. 3 (lower panel), we present the uncertainties $\Delta\Pi$ as a function of r for the four measurements, where we have set $\Pi = 1$ in the fiducial model. The results are quite close to those presented in Refs. [6,7]. It also shows that if $\Delta\Pi < 0.3$, the value of r should be larger than 0.3 for Planck, and $r > 0.12$ for the CMBPol mission, which have nearly been excluded by the current observations [9]. However, for the ideal measurement, it requires $r > 0.09$. Even if we consider the extreme cosmic variance limit, $r > 0.04$ is still required. So, we conclude that, if $\Pi < 0.3$, the determination of the chirality of PGWs is quite difficult, unless the tensor-to-scalar ratio r is large enough. However, if the PGWs are fully chiral, i.e., $\Pi \sim \pm 1$, the detection becomes much easier. We find that to get $\Delta\Pi < 1$, we only need $r > 0.05$ for Planck, $r > 0.014$ for CMBPol,

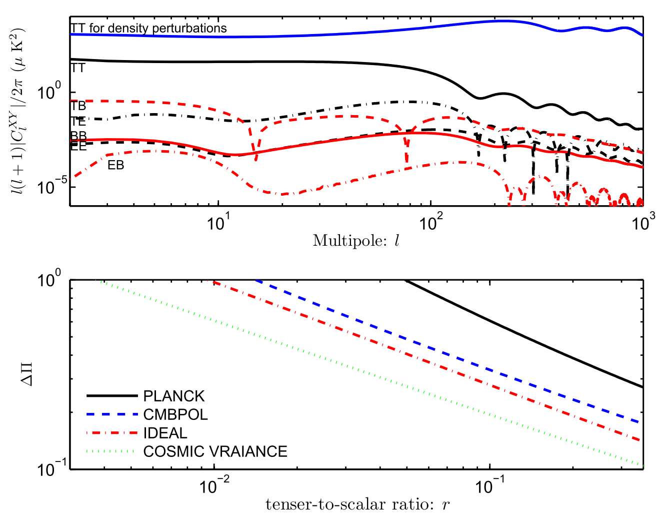

FIG. 3 (color online). Top panel: The CMB power spectra generated by polarized PGWs with $r = 0.1$ and $\Pi = 1$, and the TT power spectrum generated by density perturbations for comparison. Bottom panel: The uncertainties $\Delta\Pi$ as a function of r for potential observations.

$r > 0.01$ for the ideal experiment, and $r > 0.004$ for the cosmic variance limit.

V. CONCLUSIONS

In this paper, we have studied the evolution of PGWs, described by the dispersion relation (5), obtained from the HL theory of quantum gravity [24]. From the analytical results given by Eqs. (9)–(11), one can see that the polarization of PGWs is precisely due to the parity violation and nonadiabatic evolution of the mode function in Region II of Fig. 1, in which particles are created, where their occupation number is given by $n_k = |\beta_k|^2$. Figure 2, on the other hand, shows clearly that the polarization is considerably enhanced by the third- and fourth-order spatial derivative terms of Eq. (5). The effects of the fifth-order terms were studied in Ref. [26], and it has been shown explicitly in the present paper that their contributions to the polarization are sub-dominant, and will be quite difficult to detect in the near future. The detectability of the polarization caused by other terms, on the other hand, seems very optimistic, as shown in Fig. 3 (lower panel).

It should be noted that, although the dispersion relation (5) is obtained from the HL theory [24,25], our results are actually applicable to any theory where the mode function of PGWs are described by Eqs. (4) and (5), including the trans-Planck physics [30]. In addition, the chirality of PGWs could also be detected by the potential lensing observations [43].

The new released results of Planck mission give a slightly tighter constraint $r < 0.11$ [44], which is based on the observations on the CMB temperature anisotropies. In the near future, the polarization data of Planck mission, especially the B-mode data, will be released, which might give some direct hint of PGWs. In particular, if PGWs are fully chiral, i.e., $\Pi \sim \pm 1$, the detection of this chirality would also be expectable.

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