# Quintessence models with an oscillating equation of state and their potentials

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In this paper, we investigate the quintessence models with an oscillating equation of state (EOS) and its potentials. From the constructed potentials, which have an EOS of  $\omega_{\phi} = \omega_0 + \omega_1 \sin z$ , we find that they are all the oscillating functions of the field  $\phi$ , and the oscillating amplitudes decrease (or increase) with  $\phi$ . From the evolutive equation of the field  $\phi$ , we find that this is caused by the expansion of the universe. This also makes it very difficult to build a model whose EOS oscillates forever. However one can build a model with EOS oscillating for a certain period of time. Then we discuss three quintessence models, which are the combinations of the invert power law functions and the oscillating functions of the field  $\phi$ . We find that they all follow the oscillating EOS.

**Keywords:** quintessence, equation of state **PACC:** 9880B

## 1. Introduction

Recent observations on the Type Ia Supernova (SNIa),<sup>[1-4]</sup> Cosmic Microwave Background Radiation  $(CMB)^{[5,6]}$  and Large Scale Structure  $(LSS)^{[7,8]}$  all suggest that the universe mainly comprises dark energy (73%), dark matter (23%) and baryon matter (4%). How to understand the physical essence of the dark energy is an important issue, which has an EOS of  $\omega < -1/3$  and leads to the recent accelerating expansion of the universe. Several scenarios have been put forward as a possible explanation of it. A positive cosmological constant is the simplest candidate, however it needs extreme fine tuning to account for the observations. As an alternative to the cosmological constant, a number of dynamic models have been proposed.<sup>[9-24]</sup> Among them, the quintessence is the most natural model.<sup>[25-29]</sup> in which the dark energy is described by a scalar field  $\phi$  with Lagrangian density  $\mathcal{L}_{\phi} = \dot{\phi}^2/2 - V(\phi)$ . These models can naturally give the EOSs with  $-1 \leq \omega_{\phi} \leq 1$ . Usually, one discusses these models with monotonic potential functions, i.e. the models with the exponential potentials and inverse power law potentials. These models have some interesting characters: for example, some models have late-time attractor solutions with  $\omega_{\phi} < 0$ ,<sup>[30-32]</sup> and some have the track solutions, which can naturally answer the cosmic 'coincidence problem'.<sup>[33,34]</sup> Recently, a number of authors have considered the dark energy

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with an oscillating EOS separately in the quintessence models,<sup>[35]</sup> quintom models,<sup>[36-41]</sup> ideal-liquid models and scalar-tensor dark energy models.<sup>[42]</sup> They have pointed out that this kind of dark energy may naturally answer the 'coincidence problem' and 'fine-tuning problem'. And in some models, it is natural to relate the very early inflation to the recent accelerating expansion. The most interesting is that these models are likely to be marginally suggested by some observations.<sup>[43-45]</sup>

In this paper, we will mainly discuss the quintessence models with an oscillating EOS. First, we construct the potentials from the parametrization  $\omega_{\phi} = \omega_0 + \omega_1 \sin z$ . We find that these potentials are all the oscillating functions and the oscillating amplitudes increase (decrease) with the field  $\phi$ . This character can be analysed from the evolution equation of  $\phi$ . This suggests the way to build the potential functions which can follow an oscillating EOS. Then we discuss three kinds of potentials, which are the combinations of the invert power law functions and the oscillating functions, and find that they indeed give an oscillating EOS.

The rest of this paper is organized as follows: in Section 2, using a parametrized EOS  $\omega_{\phi} = \omega_0 + \omega_1 \sin z$ , we build its corresponding potentials, and investigate the general characters of these potentials by discussing the kinetic equation of the quintessence field; then we build three kinds of models, and dis-

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cuss the evolutions of their potentials, EOS and energy densities in Section 3; finally we draw some conclusions in Section 4.

We use the units of  $\hbar = c = 1$  and adopt the metric convention as (+, -, -, -) throughout this paper.

## 2. Construction of the potentials

First, we study the general characters of the potentials, which can follow an oscillating EOS. We note that many periodic or nonmonotonic potentials have been put forward for dark energy, but rarely give rise to a periodic  $\omega_{\phi}(z)$ . As a well-studied example, the potential for a pseudo-Nambu Goldstone boson (PNGB) field<sup>[46-49]</sup> can be written as  $V(\phi) = V_0[\cos(\phi/f) + 1]$ , clearly periodic, where f is a (an axion) symmetry energy scale. However, unless the field has already rolled through the minimum, the relation  $\omega_{\phi}(z)$  is monotonic and indeed can well described by a usual form of  $\omega_{\phi}(a) = \omega_0 + \omega_1(1-a)$ . Then what kind of potentials can naturally give rise to the oscillating EOS? In Ref.[35], the authors have taken for example a quintessence model, which has a potential  $V(\phi) = V_0 \exp(-\lambda \phi \sqrt{8\pi G}) [1 + A \sin(\nu \phi \sqrt{8\pi G})],$ where  $\lambda$ , A and  $\nu$  are all the constant numbers. They have found that this model can indeed give an oscillating EOS with the appropriate parameters chosen. In this section, we study the general characters of these models by constructing potential functions from a parametrized oscillating EOS. This method has been given by Guo et al in Ref. [50–52], First, we give a brief review of this method.

The Lagrangian density of the quintessence is

$$\mathcal{L}_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi), \qquad (1)$$

and the pressure, energy density and EOS are

$$p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi), \qquad \rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad (2)$$

$$\omega_{\phi} \equiv \frac{p_{\phi}}{\rho_{\phi}} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \tag{3}$$

respectively. When the energy is converted from kinetic energy into potential energy, the value of  $\omega_{\phi}$  is descending, and on the contrary, when the energy is converted from potential energy into kinetic energy, the value of  $\omega_{\phi}$  is ascending. So the evolution of  $\omega_{\phi}$  reflects the energy conversion relation of the quintessence field. This suggests that it is impossible

to acquire an oscillating EOS from the monotonic potentials, where the quintessence fields trend to run to the minimum of their potentials.

Consider the Flat–Robertson–Walker (FRW) universe, which is dominated by the non-relativistic matter and a spatially homogeneous quintessence field  $\phi$ . From the expressions of the pressure and energy density of the quintessence field, we have

$$V(\phi) = \frac{1}{2}(1 - \omega_{\phi})\rho_{\phi}, \qquad (4)$$

$$\frac{1}{2}\dot{\phi}^2 = \frac{1}{2}(1+\omega_{\phi})\rho_{\phi}.$$
(5)

These two equations relate the potential V and field  $\phi$  to the only function  $\rho_{\phi}$ . So the main task below is to build the function form  $\rho_{\phi}(z)$  from the parametrized EOS. This can be realized by the energy conservation equation of the quintessence field

$$\dot{\rho_{\phi}} + 3H(\rho_{\phi} + p_{\phi}) = 0,$$
 (6)

where H is the Hubble parameter, which yields

$$\rho_{\phi}(z) = \rho_{\phi 0} \exp\left[3\int_{0}^{z} (1+\omega_{\phi})d\ln(1+z)\right]$$
$$\equiv \rho_{\phi 0}E(z), \tag{7}$$

where z is the redshift which is given by  $1 + z = a_0/a$ and subscript 0 denotes the value of a quantity at the redshift z = 0 (present). In terms of  $\omega_{\phi}(z)$ , the potential can be written as a function of the redshift z as follows:

$$V(\phi(z)) = \frac{1}{2}(1 - \omega_{\phi})\rho_{\phi 0}E(z).$$
 (8)

With the help of the Friedmann equation

$$H^2 = \frac{\kappa^2}{3} (\rho_{\rm m} + \rho_{\phi}), \qquad (9)$$

where  $\kappa^2 = 8\pi G$  and  $\rho_{\rm m}$  is the matter density, one can have

$$\tilde{V}(\phi) = \frac{1}{2}(1 - \omega_{\phi})E(z),$$
 (10)

$$\frac{\mathrm{d}\tilde{\phi}}{\mathrm{d}z} = \mp \sqrt{3} \frac{1}{(1+z)} \left[ \frac{(1+\omega)E(z)}{r_0(1+z)^3 + E(z)} \right]^{1/2}, \quad (11)$$

where we have defined the dimensionless quantities  $\phi$  and  $\tilde{V}$  as

$$\tilde{\phi} \equiv \kappa \phi, \qquad \tilde{V} \equiv V/\rho_{\phi 0},$$
(12)

and  $r_0 \equiv \Omega_{\rm m0}/\Omega_{\phi 0}$  that is the energy density ratio of matter to quintessence at the present time. The upper (lower) sign in Eq.(11) is taken if  $\dot{\phi} > 0(\dot{\phi} < 0)$ . These two equations relate the quintessence potential  $V(\phi)$ to the equation of state function  $\omega_{\phi}(z)$ . Given an effective equation of state function  $\omega_{\phi}(z)$ , the construction of equations (10) and (11) allows us to construct the quintessence potential  $V(\phi)$ .

Here we consider a most general oscillating EOS

$$\omega_{\phi} = \omega_0 + \omega_1 \sin z, \tag{13}$$

where  $|\omega_0| + |\omega_1| \leq 1$  must be satisfied for a quintessence field. We choose the cosmological parameters as  $\Omega_{\phi 0} = 0.7$ , and  $\Omega_{m0} = 0.3$ . For the initial condition, we choose two different sets of parameters: case 1 with  $\omega_0 = -0.7$ ,  $\omega_1 = 0.2$  and  $\tilde{\phi}_0 = 1.0$ ; case 2 with  $\omega_0 = -0.4$ ,  $\omega_1 = 0.5$  and  $\tilde{\phi}_0 = 1.0$ . We plot them in Fig.1.



**Fig.1.** The parametrized EOS  $\omega_{\phi}(z) = \omega_0 + \omega_1 \sin z$ .

But how is the ' $\mp$ ' sign fixed in Eq.(11)? We choose the initial condition with  $d\tilde{\phi}_0/dz < 0$ , on the assumption that the variation of this sign from '-'to +' exists. And then on the transformation point, for the continuous evolution of the field  $\phi$  we have  $\dot{\phi} = \mathrm{d}\tilde{\phi}/\mathrm{d}z = 0$ , which follows that  $\omega_{\phi} = -1$  on this condition. Since  $\omega_{\phi} > -1$  is always satisfied in these two models that we are considering, there is no transformation of the sign in Eq.(11). So the negative sign holds at all times. In Fig.2, we have plotted the evolution of the potentials of the quintessence models with a redshift, and in Fig.3, we have plotted the constructed potentials. From these figures, one finds that although the potential functions are oscillatory, their amplitudes vary with field. The field always runs from the potential with higher amplitudes to that with lower ones.



**Fig.2.** The evolution of the potentials of the quintessence models with redshift z.



Fig.3. Constructed potential functions.

Now we analyse these strange potential forms. The evolution equation of the quintessence field is

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \qquad (14)$$

where  $V_{,\phi}$  denotes  $dV/d\phi$ . This equation can be rewritten as

$$\ddot{\phi} + V_{,\phi} = -3H\dot{\phi}.\tag{15}$$

If the term on the right-hand side is zero valued, this equation turns into an equation which describes the motion of field  $\phi$  in the potential  $V(\phi)$  in the flat space-time. The term on the right-hand side of this equation is the effect of the expansion of the universe. In order to show clearly its effect on the field, we consider the simplest condition under which  $V(\phi)$  is a constant, if the term on the right-hand side is equal to zero, we obtain that  $\ddot{\phi} = 0$ , and  $\dot{\phi}$  keeps constant, which is the free motion of the field. But if the term on the right-hand side exists, we obtain its solution  $|\dot{\phi}| \propto e^{\int -3Hdt}$ , the speed of the field rapidly decreases with time. So the effect of the cosmic expansion is a kind of resistance to the field, and this force is directly proportional to the speed of the field  $\dot{\phi}$ . In order to overcome this resistance force and keep the kinetic energy unequal to zero, the field must roll from the region with higher amplitudes to that with lower amplitudes. This is the reason why the potential has so strange a form as shown in Figs.2 and 3. Since the field is always running to a relatively smaller value of its potential and the potential cannot be smaller than zero, it is very difficult to build the potential with EOS which oscillates forever if there is no extreme fine-tuning.

#### 3. Three quintessence models

From the previous section, we find the general characters of the potentials which can follow the oscillating EOS. According to these characters, one can find that the potential given in Ref.[35] indeed satisfies this condition. However in that reference, the authors have found that a weak fine-tuning exists in the model for the constraint from the big-bang nucleosynthesis (BBN) observation. And also this model can obviously change the CMB anisotropy power spectrum, compared with the standard  $\Lambda$  cold dark matter ( $\Lambda$ CDM) model. These are because the oscillation of EOS exists at the radiation-dominant stage in that model. Here we build other three kinds of potential functions, which also can generate the oscillating EOSs. First we simplify the evolution equations of the quintessence field. Introduce the following dimensionless variables:

$$x \equiv \frac{\kappa \dot{\phi}}{\sqrt{6}H}, \quad y \equiv \frac{\kappa \sqrt{V}}{\sqrt{3}H},$$
$$z \equiv \frac{\kappa \sqrt{\rho_{\rm m}}}{\sqrt{3}H} \quad \text{and} \quad u \equiv \frac{\sqrt{6}}{\kappa \phi}, \tag{16}$$

then the evolution equations of the matter and quintessence will be rewritten as [30-32]

$$x' = 3x(x^2 + z^2/2 - 1) - f(y, u),$$
(17)

$$y' = 3y(x^2 + z^2/2) + f(y, u)x/y,$$
(18)

$$z' = 3z(x^2 + z^2/2 - 1/2), (19)$$

$$\iota' = -xu^2,\tag{20}$$

where a prime denotes the derivative with respect to the so-called e-folding time  $N \equiv \ln a$ , and the function  $f(y, u) = \frac{\kappa V_{,\phi}}{\sqrt{6}H^2}$ , which has the different forms for different potential functions. In this section, we mainly discuss three simple models, which have the similar potentials to those in Fig.3:

Model 1:  $V(\phi) = V_0(\kappa\phi)^{-2} [\cos(\phi/\phi_c) + 2]$  with  $\kappa\phi_c = 0.1$  and  $f(y, u) = -uy^2 - 5\sqrt{6}y^2 \sin(10\sqrt{6}/u) / [\cos(10\sqrt{6}/u) + 2];$ (21)

Model 2:  $V(\phi) = V_0(\kappa\phi)^{-1}[\cos(\phi/\phi_c) + 2]$  with  $\kappa\phi_c = 0.1$  and

$$f(y,u) = -uy^2/2 - 5\sqrt{6}y^2 \sin(10\sqrt{6}/u) / [\cos(10\sqrt{6}/u) + 2];$$
(22)

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Model 3:  $V(\phi) = V_0[(\kappa \phi)^{-1} + \cos(\phi/\phi_c) + 1]$  with  $\kappa \phi_c = 0.1$  and

$$f(y,u) = -3y^2 [u^2/6 + 10\sin(10\sqrt{6}/u)]/[u + \sqrt{6} + \sqrt{6}\cos(10\sqrt{6}/u)].$$
(23)

These models have been shown in Fig.4, which are all the combinations of the invert power law function and the PNGB field. And  $V(\phi) > 0$  is satisfied at all times. When  $\phi/\phi_c \ll 1$ , they are like the invert power law potential with n = -1 (or -2), and they begin to oscillate when  $\phi > \phi_c$ . The oscillating amplitudes decrease at all times for the first two potentials, and for the last potential, the oscillating amplitudes are nearly constant at  $\phi \gg \phi_c$ . It is interesting that these potentials can be regarded as the invert power law potential  $3V_0(\kappa\phi)^{-1}$  ( $3V_0(\kappa\phi)^{-2}$ ,  $V_0[(\kappa\phi)^{-1} + 2]$ ) with an oscillating amendatory term at  $\phi > \phi_c$ . Here we choose the initial condition (present values)  $\kappa\phi_0 = 0.6$ ,  $\omega_{\phi 0} = -0.9$ ,  $\Omega_{\phi 0} = 0.7$  and  $\Omega_{m 0} = 0.3$ . So at the early stage, the potential functions of the quintessence are monotonic functions, the EOSs are not oscillating at



the early (radiation-dominant) stage, which naturally

overcomes the shortcoming of the model in Ref.[35].

Fig.4. Three kinds of quintessence models.

In Figs.5 and 6, we plot the evolution of EOS and field  $\phi$  in the region  $\ln(a/a_0) = [0, 4]$ . The solid lines are for the model with the first potential, whose EOS has a relatively steady oscillating amplitude. This is for the amplitude in which its potential function rapidly decreases with  $\phi$ . When the field  $\phi$  rolls down to its valley, it has enough kinetic energy to climb up to its adjacent hill and then rolls down again. In each period of its potential, when the field rolls down, the kinetic energy increases, and the potential energy decreases, which makes its EOS raised; on the contrary, when the field climbs up, the kinetic energy decreases, and the potential energy increases, which makes its EOS damped. The minimum value of its EOS never reaches -1, which is because the kinetic energy of the field is never zero valued. This process continues until  $\ln(a/a_0) \simeq 1.7$  ( $\kappa \phi \simeq 2.2$ ), when the field reaches a state with  $\dot{\phi} = 0$  ( $\omega_{\phi} = -1$ ) and has to roll back down to the former valley ( $\dot{\phi} < 0$ ). These can be seen clearly in Fig.6. After this state, the EOS will rapidly run to a steady state with  $\omega_{\phi} = -1$ .

However all these are different between models 2 and 3, which are described with dashed and dotted lines in these figures respectively. When the fields roll down to the valley with  $\kappa \phi \simeq 1$ , they try to climb up to their first hills, but they cannot climb up to the peaks for the large values of their potential functions. When the fields reach a state with  $\dot{\phi} = 0$  (the corresponding EOSs have  $\omega_{\phi} = -1$ ), they have to roll back down to this valley again. This process lasts until the kinetic energy becomes negligible, and the fields stay at the valley with  $\omega_{\phi} = -1$ . The evolution of these fields can be seen clearly in Fig.6.



**Fig.5.** The evolution of the EOS of the quintessence models.



**Fig.6.** The evolution of field  $\phi$  of the quintessence models.

In Fig.7, we plot the evolution of  $\Omega_{\phi}$  in the universe. Although the quintessence is predominant in the universe finally, the values of  $\Omega_{\phi}$  are oscillatory at the evolution stage for all these three quintessence models, which are determined by the evolution of  $\omega_{\phi}$ . When  $\omega_{\phi} > 0$ , the values of  $\Omega_{\phi}$  will decrease, and when  $\omega_{\phi} < 0$ , the values of  $\Omega_{\phi}$  will increase.



**Fig.7.** The evolution of the energy density  $\Omega_{\phi}$  of the quintessence models.

### 4. Conclusions

An understanding of the physical essence of the dark energy is one of the most important missions for modern cosmology. The most effective way is to detect its EOS and the running behaviour by the observations on SNIa, CMB, LSS and so on. There are mild evidences to show that the EOS of the dark energy is an oscillating function, which makes it difficult to build the dark energy models. For the quintessence field dark energy models, it is obvious that this EOS cannot be realized from the monotonic potentials. However, for a simple oscillating potential, it is still difficult to realize.

In this paper, we have discussed the general features of the potentials which can follow an oscillating EOS by constructing the potentials from an oscillating EOS, and found that they are oscillating func-

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tions. However, the oscillating amplitudes increase (decrease) with the field  $\phi$ . And also the field must roll from the region with larger amplitudes to that with smaller amplitudes if the EOS is oscillating. The potentials of this kind are not very difficult to satisfy. However, since the field must roll down to the region with smaller amplitudes if the EOS is oscillating, and also the constraint of  $V(\phi) \geq 0$  must be satisfied at all times, which make it very different to build the quintessence with an oscillating (forever) EOS. In this paper, we have studied three kinds of models:  $V(\phi) = V_0(\kappa\phi)^{-2} [\cos(\phi/\phi_c) + 2], \quad V(\phi) =$  $V_0(\kappa\phi)^{-1}[\cos(\phi/\phi_c) + 2]$  and  $V(\phi) = V_0[(\kappa\phi)^{-1} +$  $\cos(\phi/\phi_{\rm c}) + 1$ ]. They all comprise the invert power law functions and the oscillating functions, and can indeed follow the oscillating EOS. However, this oscillating behaviour can be maintained only for a finite period of time in all these three models.

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