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STATEFINDER DIAGNOSTIC FOR THE YANG–MILLS DARK ENERGY MODEL

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We study the statefinder parameters in the Yang–Mills condensate dark energy models, and find that the evolving trajectories of these models are different from those of other dark energy models. We also define two eigenfunctions of the Yang–Mills condensate dark energy models. The values of these eigenfunctions are quite close to zero if the equation of state of the Yang–Mills condensate is not far from −1, which can be used to simply differentiate between the Yang–Mills condensate models and other dark energy models.

Keywords: Dark energy; Yang–Mills condensate; statefinder.

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1. Introduction

Physicists and astronomers have begun to consider the dark energy cosmology seriously and to explore the nature of dark energy actively, since the expansion of our universe has been proven to be accelerating at the present time by the Type Ia supernova observations.[1–4] The analysis of cosmological observations, particularly of the WMAP (Wilkinson Microwave Anisotropy Probe) experiment,[5–7] indicates that dark energy occupies about 70% of the total energy of our universe, and dark matter about 26%. The accelerated expansion of the present universe is attributed to the fact that dark energy is an exotic component with negative pressure, and that the simplest candidate for dark energy is the cosmological constant. However, two difficulties arise from this scenario, namely the fine-tuning problem and the cosmic coincidence problem.[8–10] So the dynamical models are considered by a number of authors, such as the quintessence, phantom, k-essence, and quintom models.[11–22]

The effective Yang–Mills condensate (YMC) as a kind of candidate for dark energy has been discussed in detail in Refs. 23–31. The effective Lagrangian up to one-loop order is[32–34]

$$\mathcal{L}_{\rm eff} = \frac{b}{2} F \ln \left| \frac{F}{e\kappa^2} \right| , \tag{1}$$

where $b = 11N/24\pi^2$ for the generic gauge group SU(N) is the Callan–Symanzik coefficient.[35,36] $F = -(1/2)F^a_{\mu\nu}F^{a\mu\nu}$ plays the role of the order parameter of the YMC, and κ is the renormalization scale, the only model parameter. The attractive features of this effective Lagrangian include the gauge invariance, the Lorentz invariance, the correct trace anomaly, and the asymptotic freedom.[32] The effective YMC was first put into the expanding Friedmann–Robertson–Walker (FRW) space–time to study inflationary expansion[23,24] and dark energy.[25–27] We work in a spatially flat FRW space–time with a metric

$$ds^2 = a^2(\tau)(d\tau^2 - \delta_{ij}dx^i dx^j), \tag{2}$$

where $\tau = \int (a_0/a)dt$ is the conformal time. For simplicity we study the SU(2) group. Compared with the scalar field, the YM field is the cornerstone of particle physics, and the gauge bosons have been observed. There is no room for adjusting the form of the effective YM Lagrangian as it is predicted by quantum corrections according to field theory. In the previous works, we have investigated deeply the one-order YMC models and found attractive features: (a) this dark energy can naturally get the equation of state (EOS) of $w > -1$ and $w < -1$,[28–31] which is different from the scalar quintessence models; (b) in the free field models, with the expansion of the universe, the EOS of the YMC naturally turns to the critical state of $w = -1$,[28–31] consistent with the observations; (c) the cosmic coincidence problem is naturally avoided in the models[28–31]; (d) the EOS of the dark energy can cross -1 in the double-field models or coupled models[28–31]; (e) the big rip is naturally avoided in the models[28–31]; (f) the magnetic component of the YMC naturally decreases to zero with the expansion of the universe.[28–31]

The authors of Ref. 37 have discussed in detail the two-loop YMC dark energy. In this model, the effective Lagrangian is

$$\mathcal{L}_{\rm eff} = \frac{b}{2} F \left[\ln \left| \frac{F}{e\kappa^2} \right| + \eta \ln \left| \ln \left| \frac{F}{e\kappa^2} \right| + \delta \right| \right] , \tag{3}$$

where $\eta \simeq 0.84$ and the dimensionless constant δ is a parameter representing higher order corrections. In this two-loop model, the cosmic coincidence problem is also naturally avoided. This feature is the same with the one-loop models. From the Einstein equation,[37] we can easily find that, in the free field models, the late time attractor exists, which satisfies the relation

$$\beta_c = -\eta \left(\log |\beta_c - 1 + \delta| + \frac{1}{\beta_c - 1 + \delta} \right) , \tag{4}$$

where $\beta = \ln|F/\kappa^2|$. It is easily found that $w = -1$, when $\beta = \beta_c$. So, in these models, the EOS of the YMC also naturally turns to the critical state $w = -1$. However, the cosmological constant crossing is also naturally realized, if we consider the interaction between the YMC and matter.[37] The discussion in Ref. 37 shows that although the two-loop models are much more general and complicated than the one-loop model, most of the physics features are not unchanged. In this paper, we consider only the one-loop models. As in previous works, we consider only the electric case with $B^2 \equiv 0$. The energy density and pressure of the YMC are given by

$$\rho_y = \frac{E^2}{2}(\epsilon + b), \quad p_y = \frac{E^2}{2}\left(\frac{\epsilon}{3} - b\right), \tag{5}$$

where the dielectric constant is

$$\epsilon = b \ln\left|\frac{F}{\kappa^2}\right| \tag{6}$$

and the EOS is obtained:

$$w = \frac{p_y}{\rho_y} = \frac{\beta - 3}{3\beta + 3}, \tag{7}$$

where $\beta \equiv \epsilon/b$. At the critical point with the condensate order parameter $F = \kappa^2$, one has $\beta = 0$ and $w = -1$. Around this critical point, $F < \kappa^2$ gives $\beta < 0$ and $w < -1$, and $F > \kappa^2$ gives $\beta > 0$ and $w > -1$. So, in the YMC model, the EOS of $w > -1$ and $w < -1$ can be naturally realized. When $\beta \gg 1$, the YM field has a state of $w = 1/3$, becoming a radiation component.

In the free field models, the effective YM equations are

$$\partial_\mu(a^4\epsilon F^{a\mu\nu}) + f^{abc}A^b_\mu(a^4\epsilon F^{c\mu\nu}) = 0, \tag{8}$$

which reduce to[28–31]

$$\partial_\tau(a^2\epsilon E) = 0. \tag{9}$$

At the critical point ($\epsilon = 0$), this equation is an identity. When $\epsilon \neq 0$, this equation has an exact solution[28–31]:

$$\beta e^{\beta/2} \propto a^{-2}, \tag{10}$$

where the coefficient of proportionality depends on the initial condition. For a fixed initial condition, we can obtain the evolution of the EOS of the YMC by using the YM equation (10). In the previous works, we found that the free YMC can be separated into two kinds, quintessence-like or phantom-like, which depends only on the choice of the initial condition. In order to differentiate between the YMC dark energy models and other models, a sensitive and robust diagnostic for dark energy models is needed. For this purpose a diagnostic proposal that makes use of the parameter pair $\{r, s\}$, the so-called "statefinder," was introduced by Sahni *et al.*[38] The statefinder probes the expansion dynamics of the universe through higher derivatives of the expansion factor $\dddot{a}$ and is a natural companion to the deceleration parameter q, which depends on $\ddot{a}$. The statefinder pair $\{r, s\}$ is defined thus:

$$r \equiv \frac{\dddot{a}}{aH^3}, \quad s \equiv \frac{r-1}{3(q-1/2)}. \tag{11}$$

The statefinder is a "geometrical" diagnostic, in the sense that it depends on the expansion factor and hence on the metric describing space–time.

Trajectories in the s–r plane corresponding to different cosmological models exhibit qualitatively different behaviors. The spatially flat LCDM (cosmological constant Λ with cold dark matter) scenario corresponds to a fixed point in the diagram $\{s, r\} = \{0, 1\}$. Departure of a given dark energy model from this fixed point provides a good way of establishing the "distance" of this model from LCDM.[38,39] As demonstrated in Refs. 38–44, the statefinder can successfully differentiate between a wide variety of dark energy models, including the cosmological constant, quintessence, the Chaplygin gas, braneworld models and interacting dark energy models. We can clearly identify the "distance" from a given dark energy model to the LCDM scenatio by using the $r(s)$ evolution diagram.

The current location of the parameters s and r in these diagrams can be calculated in models; on the other hand, it can also be extracted from data coming from SNAP (Supernovae Acceleration Probe) type experiments.[45,46] Therefore, the statefinder diagnostic combined with future SNAP observations may possibly be used to discriminate between different dark energy models. In this paper we apply the statefinder diagnostic to YMC dark energy models.

2. Statefinder for YMC Dark Energy

The statefinder parameters r and s in (11) can be rewritten as

$$r = 1 + \frac{9}{2}w(1+w)\Omega_y - \frac{3}{2}w'\Omega_y, \tag{12}$$

$$s = 1 + w - \frac{1}{3}\frac{w'}{w}, \tag{13}$$

where w is the EOS of the YMC, and Ω_y is the fractional energy density of the YMC. A prime denotes derivation with respect to the e-folding time $N \equiv \ln a$. From the previous discussion, we know that

$$w = \frac{\beta - 3}{3\beta + 3}, \quad w' = \beta' \frac{dw}{d\beta}. \tag{14}$$

So the statefinder for the YMC depends only on the evolution of the parameter β, which can be exactly determined by the YM equation (10). From the YM equation, we obtain

$$\beta' = \frac{-4\beta}{2+\beta}, \quad w' = \frac{-16\beta}{3(1+\beta)^2(2+\beta)}, \tag{15}$$

and the statefinder parameters become

$$r = \frac{2 + (3 - 4\Omega_y)\beta + (1 + 2\Omega_y)\beta^2}{2 + 3\beta + \beta^2}, \tag{16}$$

$$s = \frac{4\beta(\beta - 2)}{3(\beta^2 - \beta - 6)}. \tag{17}$$

The deceleration parameter is also obtained:

$$q = \frac{1 + \beta - 3\Omega_y + \beta\Omega_y}{2 + 2\beta}. \tag{18}$$

We first consider the case with $w > -1$, quintessence-like, where $\beta > 0$ is kept for all time. In the very early universe with $\beta \gg 1$ and $\Omega_y \to 0$,[28–31] we obtain

$$r \to 1, \quad s \to \frac{4}{3}, \quad q \to \frac{1}{2}, \tag{19}$$

which is independent of the initial condition, and obviously different from the SCDM (standard cold dark matter) model with $(r, s, q) = (1, 1, 1/2)$. In the later stage of the universe with $\beta \to 0$ and $\Omega_y \to 1$,[28–31] we have

$$r \to 1, \quad s \to 0, \quad q \to -1. \tag{20}$$

The universe approaches an exact de Sitter expansion, which is the same with the late stage of the LCDM model. We also notice that the value of s is infinite when $\beta = 3$, where the EOS of the YMC is $w = 0$. This is also a character of the YMC models.

If the YMC is phantom-like with $w < -1$ and $\beta < 0$. In the late stage of the universe, we have $\beta \to 0$ and $\Omega_y \to 1$,[28–31] and it follows that

$$r \to 1, \quad s \to 0, \quad q \to -1, \tag{21}$$

which is same with the quintessence-like case. Here we should point out that, in the very early universe with $a \to 0$, the YM kinetic equation (10) has no solution for the case with $w_0 < -1$, where w_0 is the present EOS of the YMC. So the free YMC is not applied in the very early universe, where the interaction between the YMC and matter,[28–31,37] or the phase transition of the YMC, must be considered.

The evolution of the parameter β and Ω_y can be obtained through Eq. (8) for a fixed initial condition. In Fig. 1, we show the evolution of the statefinder pair s, r, where the initial conditions of the YMC are $w_0 = -1.1, -0.9, -0.8$, respectively, and the present fraction energy density of the YMC is $\Omega_y = 0.7$. It can be found that the trajectories of these models never cross the LCDM fixed point. However, with the expansion of the universe, they will approach this fixed point, which is independent of the choice of the initial condition. The only difference for the quintessence-like and phantom-like cases is the direction of the trajectories, when the models approach the fixed point. The coordinates of today's points are $(-0.033, 1.113), (0.034, 0.903), (0.070, 0.824)$, respectively, and thus the distance from these models to the LCDM can be easily identified in this diagram.

We also plot the evolution trajectories of the statefinder pair r, q in Fig. 2.

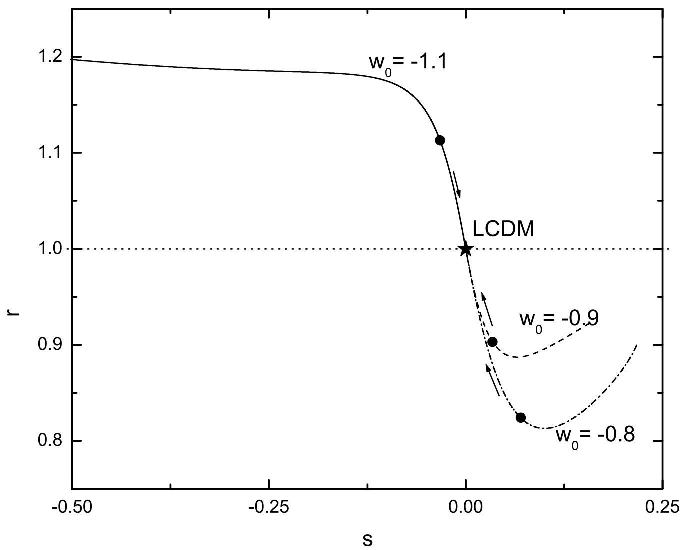

Fig. 1. The $s-r$ diagram of the YMC dark energy models. The dots locate the current values of the statefinder pair $\{s, r\}$, and the arrows denote the evolution direction of the statefinders with expansion of the universe.

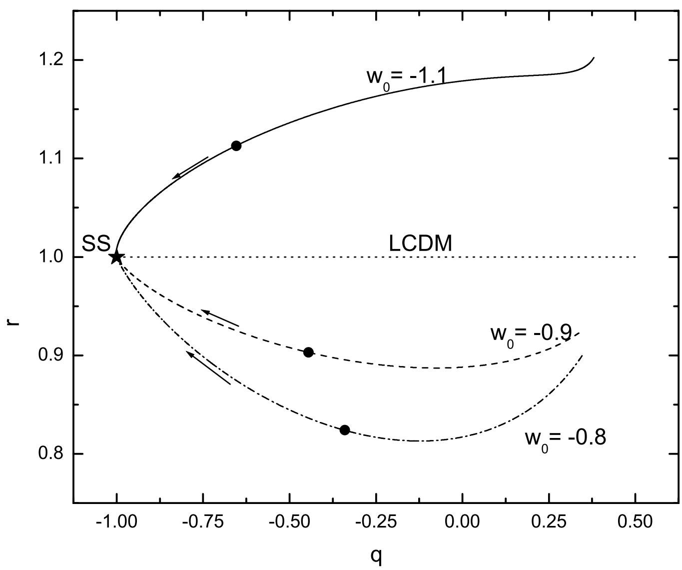

Fig. 2. The $q-r$ diagram of the YMC dark energy models. The dots locate the current values of the statefinder pair $\{q, r\}$, and the arrows denote the evolution direction of the statefinders with expansion of the universe. The point of $(-1, 1)$ corresponds to the steady state models (SS) — the de Sitter expansion.

3. Statefinder of First Order

Cosmological observations show that the EOS of the dark energy is closer to -1. In the YMC dark energy models, from the expression of the EOS of the YMC in (7), we find that $w \to -1$ follows from the fact that $|\beta| \ll 1$. So we can Taylor-expand the EOS and the statefinder of the YMC with the parameter β at the critical state of $w = -1$. Keeping the first order of the smaller quantity β, we can rewrite the EOS of the YMC as

$$w = -1 + \frac{4}{3}\beta + O(\beta^2), \quad w' = -\frac{8}{3}\beta + O(\beta^2). \tag{22}$$

From the expressions (16)–(18), we obtain

$$r = 1 - 2\Omega_y\beta + O(\beta^2), \quad s = \frac{4}{9}\beta + O(\beta^2) \tag{23}$$

and the deceleration parameter

$$q = \left(\frac{1}{2} - \frac{3\Omega_y}{2}\right) + 2\Omega_y\beta + O(\beta^2). \tag{24}$$

These functions depend only on the quantities β and Ω_y, which are all determined by the initial condition, and the initial condition of the YMC directly relates to the present EOS of the YMC.

In order to differentiate between the YMC dark energy models and other models, such as the quintessence, phantom, k-essence, or Chaplygin gas, we can define an eigenfunction of first order for the YMC models:

$$\xi_1 = \frac{2}{9}\frac{r-1}{\Omega_y} + s. \tag{25}$$

From the expressions (23), we find that the value of this eigenfunction is $\xi_1 = 0 + O(\beta^2)$, which is independent of the initial condition of the YM dark energy models. It is easy to find that this feature is not right for other dark energy models. So we can differentiate the YMC dark energy models from other models by the observable quantity ξ_1.

4. Statefinder of Second Order

We also can expand the EOS and the statefinder of the YMC to the second order of β. From the expressions (14), we obtain

$$w = -1 + \frac{4}{3}\beta - \frac{4}{3}\beta^2 + O(\beta^3), \quad w' = -\frac{8}{3}\beta + \frac{20}{3}\beta^2 + O(\beta^3). \tag{26}$$

The statefinder parameters are

$$r = 1 - 2\Omega_y\beta + 4\Omega_y\beta^2 + O(\beta^3), \tag{27}$$

$$s = \frac{4}{9}\beta - \frac{8}{27}\beta^2 + O(\beta^3), \tag{28}$$

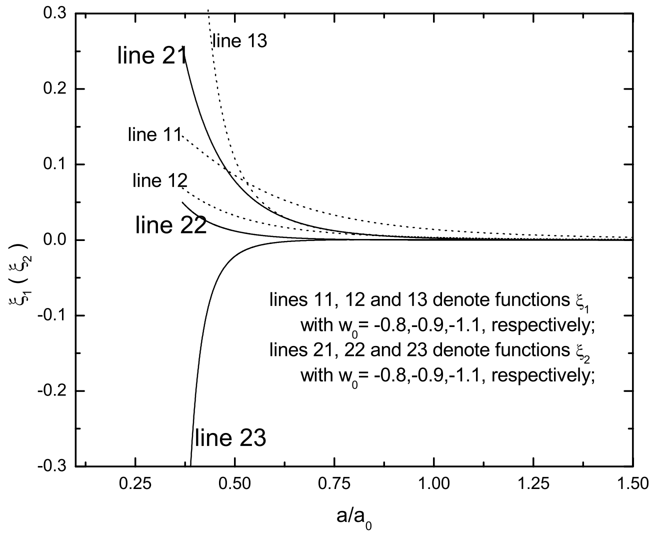

Fig. 3. The evolution of the eigenfunctions ξ_1 and ξ_2 with the scale factor a.

and the deceleration parameter is

$$q = \left(\frac{1}{2} - \frac{3\Omega_y}{2}\right) + 2\Omega_y\beta - 2\Omega_y\beta^2 + O(\beta^3). \tag{29}$$

From these expressions, we can also define an eigenfunction of second order:

$$\xi_2 = -\frac{8}{27}\frac{r-1}{\Omega_y} + \frac{512}{9}(w+1) - 44s. \tag{30}$$

It is easy to find that the value of this eigenfunction $\xi_2 = 0 + O(\beta^3)$. In Fig. 3, we plot the evolution of the eigenfunctions ξ_1 and ξ_2 in the different YMC dark energy models. We find that, in all these models, the values of ξ_1 and ξ_2 are all very closer to 0 if the EOS of the YMC is not very far from -1. From this figure, we also find that the value of ξ_2 is much more closer to 0 than that of ξ_1. The former is a more effective function for affirming the YMC dark energy models. Of course, in the LCDM models, the values of ξ_1 and ξ_2 are all exactly 0, so it is difficult to differentiate between the YMC dark energy model and the LCDM model.

5. Summary

In summary, we have investigated the statefinder of the YMC dark energy models in this paper. We analyzed two cases of the models, i.e. the quintessence-like case and the phantom-like case, and performed a statefinder diagnostic for both cases. It was shown that the evolving trajectory of this scenario in the $s-r$ plane is quite different from those of other models. We also defined two eigenfunctions of the YMC dark energy model. If the EOS of the YMC is not far from -1, the values of the eigenfunctions much very closer to 0, which can be used to simply differentiate between the YMC and other dark energy models.

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