International Journal of Modern Physics D Vol. 16, No. 11 (2007) 1735–1744 © World Scientific Publishing Company



EVOLUTION OF THE MAGNETIC COMPONENT IN YANG–MILLS CONDENSATE DARK ENERGY MODELS

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> Received 21 May 2007 Communicated by I. D. Novikov

The evolution of the electric and magnetic components in an effective Yang–Mills condensate dark energy model is investigated. If the electric field is dominant, the magnetic component disappears with the expansion of the Universe. The total YM condensate tracks the radiation in the earlier Universe, and later it becomes $w_y \sim -1$ and is thus similar to the cosmological constant. So the cosmic coincidence problem can be avoided in this model. However, if the magnetic field is dominant, $w_y > 1/3$ holds for all time, suggesting that it cannot be a candidate for the dark energy in this case.

Keywords: Yang-Mills field; dark energy.

PACS Number(s): 98.80.-k, 98.80.Es, 04.30.-w, 04.62.+v

1. Introduction

Recent observations on the Type Ia supernova,^{1–4} cosmic microwave background radiation^{5–7} and large scale structure^{8–11} all suggest a flat Universe consisting of dark energy (73%), dark matter (23%) and baryon matter (4%). It is important to understand the physics of the mysterious dark energy, which has the equation of state (EOS) w < -1/3 and has caused the recent accelerating expansion of the Universe. The simplest model is the cosmological constant Λ with $\omega_{\Lambda} \equiv -1$, which fits the observation fairly well. However, a number of evidences suggest that the EOS of the dark energy models with a dynamic field. One class of approaches to building the dark energy models with a dynamic field. One class of approaches is to introduce a scalar field, such as the quintessence,^{12–16} the phantom,^{17–21} the k essence^{22–25} and the quintom.^{26–30} Another class of models is based on the conjecture that a vector field can be the origin of the dark energy,^{31–35} which has

different features to those of a scalar field. In Refs. 36–45, it is suggested that the Yang–Mills (YM) field can be a kind of candidate for such a vector field.

Compared with the scalar field, the YM field is the cornerstone of particle physics and the gauge bosons have been observed. There is no room for adjusting the form of effective YM Lagrangian as it is predicted by quantum corrections according to field theory. In the previous works, we have investigated the simplest case with only the electric component and found attractive features: (1) this dark energy can naturally get the EOS of $w_y > -1$ and $w_y < -1$,^{42,43} which is different from the scalar quintessence models; (2) with the expansion of the Universe, the EOS of the YM condensate naturally turns to the critical state of $w_y = -1$,^{42,43} consistent with the observations;⁴⁶ (3) the cosmic coincidence problem is naturally avoided in the YM condensate dark energy models;^{44,45} (4) the EOS of the dark energy can cross -1 in the double-field models or coupled models;^{42,43,45} (5) the big rip is naturally avoided in the models.⁴⁵

In this paper, we discuss the evolution of the YM condensate dark energy with both electric and magnetic components. We find that, if the magnetic component is subdominant in the initial condition, it rapidly decreases to zero with the expansion of the Universe. The states of $w_y > -1$ and $w_y < -1$ can all be realized in the models. In the former case, the state of YM condensate is $w_y \sim 1/3$ in the earlier stage, and later it turns into $w_y \sim -1$, which is similar to the case with only the electric component. So the cosmic coincidence problem is naturally avoided in the models. However, if the magnetic component is dominant in the initial condition, the state of YM condensate keeps $w_y > 1/3$, which cannot be a candidate for dark energy.

2. The Effective Yang–Mills Field Model

The effective Lagrangian density of the YM field up to one-loop order is 4^{7-49}

$$\mathcal{L}_{\text{eff}} = \frac{b}{2} F \ln \left| \frac{F}{e\kappa^2} \right|. \tag{1}$$

Here $b = 11N/24\pi^2$ for the generic gauge group SU(N) is the Callan–Symanzik coefficient,^{50,51} $F = -(1/2)F_{\mu\nu}^a F^{a\mu\nu}$ plays the role of the order parameter of the YM condensate, and κ is the renormalization scale with the dimension of squared mass which is the only model parameter. This effective YM Lagrangian exhibits the features of the gauge invariance, the Lorentz invariance, the correct trace anomaly, and the asymptotic freedom.⁴⁷ With the logarithmic dependence on the field strength, $\mathcal{L}_{\rm eff}$ has a form similar to the Coleman–Weinberg scalar effective potential⁵² and the Parker–Raval effective gravity Lagrangian.⁵³ The effective YM condensate was first put into the expanding Friedmann–Robertson–Walker (FRW) space–time to study inflationary expansion in Refs. 36 and 37 and the dark energy in Refs. 38–41. Following Refs. 42–44, we work in a spatially flat FRW space–time with a metric

$$ds^2 = a^2(\tau)(d\tau^2 - \delta_{ij}dx^i dx^j), \qquad (2)$$

where $\tau = \int (a_0/a)dt$ is the conformal time. Assume that the Universe is filled with the YM condensate. For simplicity we study the SU(2) group. The energy density and pressure are given by

$$\rho_y = \frac{1}{2}b\varepsilon(E^2 + B^2) + \frac{1}{2}b(E^2 - B^2), \tag{3}$$

$$p_y = \frac{1}{6}b\varepsilon(E^2 + B^2) - \frac{1}{2}b(E^2 - B^2), \tag{4}$$

where the dielectric constant is given by $b\varepsilon \equiv b \ln |(E^2 - B^2)/\kappa^2|$. We define two dimensionless quantities, $f \equiv (E^2 - B^2)/\kappa^2$ and $q \equiv (E^2 + B^2)/\kappa^2$. It is easy to find that $q \geq f$, and q = f only if $B^2 = 0$. The energy density and pressure can be rewritten as

$$\rho_y = \frac{1}{2}b\kappa^2(\varepsilon q + f), \quad p_y = \frac{1}{2}b\kappa^2\left(\frac{1}{3}\varepsilon q - f\right).$$
(5)

The energy density of the YM condensate should have the positive value, which follows a constraint of the YM condensate:

$$\varepsilon q + f > 0. \tag{6}$$

The EOS of the YM condensate is

$$w_y = \frac{\varepsilon q - 3f}{3\varepsilon q + 3f}.\tag{7}$$

At the critical point with |f| = 1, one has $\varepsilon = 0$ and $w_y = -1$. Around this critical point, |f| < 1 gives $\varepsilon < 0$ and $\varepsilon < -1$, and |f| > 1 gives $\varepsilon > 0$ and $w_y > -1$. So in the YM field model, the EOS's of $w_y > -1$ and $w_y < -1$ can all be naturally realized.

The effective YM equations are^{42-44}

$$\partial_{\mu}(a^{4}\varepsilon F^{a\mu\nu}) + f^{abc}A^{b}_{\mu}(a^{4}\varepsilon F^{c\mu\nu}) = 0, \qquad (8)$$

which can be reduced to

$$\partial_{\tau}(a^2 \varepsilon E) = 0. \tag{9}$$

At the critical point with $\varepsilon = 0$, this equation is an identity. And when $\varepsilon \neq 0$, this equation has an exact solution,

$$q + f = c \cdot a^{-4} \varepsilon^{-2},\tag{10}$$

where c is the integral constant, and q and f are the variables. The energy conservation equation of the YM condensate is

$$(a^{3}\rho_{y})' = -p_{y},\tag{11}$$

where the prime denotes $d/d(a^3)$. This equation can be reduced to

$$\left(1+\frac{q}{f}\right)f'+\varepsilon q'=-\frac{4}{3}\varepsilon q a^{-3}.$$
(12)

By Eqs. (10) and (12), one can numerically solve the evolution of the EOS of the YM dark energy. It is easily to find that, in the YM condensate models, the conformal

time τ can be entirely replaced by the scale factor a, and the evolution of the YM condensate with the scale factor is independent of the other components in the Universe. In the following discussion, we choose the initial condition at $a = a_i$, where a_i can be at any time, and the initial condition is chosen as

$$q = q_i, \quad f = f_i. \tag{13}$$

So the integral constant c is fixed:

$$c = (q_i + f_i)(\ln f_i)^2.$$
(14)

First, we consider the case of the electric field dominant in the initial condition, which requires an inequality:

$$f_i > 0. \tag{15}$$

The value of w_i is exactly determined by the values of f_i and q_i . Here we consider two kinds of choices of the initial condition, $w_i > -1$ and $w_i < -1$.

The initial condition of $w_i > -1$ requires

$$q_i > f_i > 1. \tag{16}$$

The value of q_i closer to f_i suggests that the density of the electric field is much larger than that of the magnetic field, and $q_i = f_i$ suggests that the YM condensate includes only the electric component. When the values of q_i and f_i are all close to 1, it means that $E^2 \to \kappa^2$ and $B^2 \to 0$. On the contrary, the value of q_i much larger than f_i suggests that the density of the electric field is much closer to that of the magnetic field. Here we consider three different models: **Mod.a1**: $f_i = 50$, $q_i = 100$; **Mod.a2**: $f_i = 5$, $q_i = 100$; **Mod.a3**: $f_i = 5$, $q_i = 10$.

Solving Eqs. (10) and (12), we get the evolution of the EOS of the YM condensate in these models, which are plotted in Fig. 1. We find that the evolution of the EOS is similar in all these models: in the earlier stage, $w_y \sim 1/3$, tracking the evolution of the radiation, and the energy density $\rho_y \propto a^{-4}$. However, at a transition time, w_y rapidly transits from $w_y \sim 1/3$ to an attractor solution of $w_y \sim -1$, similar to the cosmological constant, and the energy density of the YM condensate stays constant. This feature is the same with the simple YM dark energy model with only the electric field.⁴⁴ As is known, an effective theory is a simple representation for an interacting quantum system of many degrees of freedom at and around its respective low energies. Commonly, it applies only in low energies. However, it is interesting to note that the YM condensate model as an effective theory intrinsically incorporates the appropriate states for both high and low temperature. As has been shown above, the same expression in Eq. (7) simultaneously gives $p_y \to -\rho_y$ at low energies and $p_y \to \rho_y/3$ at high energies. Therefore, our model of effective YM condensate can be used even at higher energies than the renormalization scale κ .

Figure 2 plots the evolution of electric and magnetic components in these three models. We find that their evolution processes are different in these models. For the magnetic component, in the earlier stage, $B^2 \propto a^{-4}$, and after the transition



Fig. 1. In models a1, a2, a3, the evolution of the EOS of the YM condensate with the scale factor a.



Fig. 2. In models a1, a2, a3, the evolution of the "electric" and "magnetic" components with the scale factor a.

time (which is also the transition time of the EOS from $w_y \sim 1/3$ to $w_y \sim -1$), the values of B^2 rapidly decrease to zero. For the electric component, in the earlier stage, the value of E^2 is also $\propto a^{-4}$, but after the transition time the value of E^2 stops decreasing and approaches the critical state of $E^2 = \kappa^2$, the renormalization scale. The electric component dominates in the YM condensate for all time. If at the end of the reheating, a very early stage of the Universe, the energy density of the YM condensate is smaller, corresponding to a smaller E^2 , it decreases as $E^2 \propto a^{-4}$ and arrives at the state of $E^2 \sim \kappa^2$ earlier, and the transition time is also earlier. On the contrary, a larger E^2 in the very early Universe leads to a later transition time. So the transition time of the EOS of the YM condensate is directly determined by the choice of initial condition in the very early Universe. However, no matter what initial condition one chooses, the YM condensate must arrive at the attractor solution of $E^2 \to \kappa^2$, $B^2 \to 0$ and $w_y \to -1$. In this solution, the energy density of the YM condensate is

$$\rho_y \to \frac{b\kappa^2}{2},$$
(17)

which is independent of the choice of the initial condition. So the cosmic coincidence problem is naturally avoided. In order to account for the present observational value of the dark energy, one needs to fine-tune the value of the renormalization scale $\kappa \simeq 3.57h \times 10^{-5} \,\mathrm{eV}^2$,⁴⁴ where *h* is the Hubble constant. This energy scale is low compared to typical energy scales in particle physics. So the "fine-tuning" problem is present in these models. From these models, we also find that the EOS of the YM condensate cannot cross -1, which is same with quintessence models,¹²⁻¹⁶ unless the coupling of the YM condensate with the matter is considered.⁴⁵

Now we turn to another case of $w_i < -1$. Equation (6) requires that

$$f_i < q_i < -f_i / \ln f_i, \quad 0 < f_i < 1,$$
(18)

which leads to the constraint of $e^{-1} < f_i < 1$. Here we also consider three different models: **Mod.b1**: $f_i = 0.9$, $q_i = 2.0$; **Mod.b2**: $f_i = 0.9999$, $q_i = 10.0$; **Mod.b3**: $f_i = 0.5$, $q_i = 0.6$.

In Fig. 3, we plot the evolution of the EOS of the YM condensate in these three models, and in Fig. 4 we plot the evolution of the electric and magnetic components. Similar to the previous three models, with the expansion of the Universe, the EOS of the YM condensate runs to the critical state of $w_y = -1$, the density of the electric field approaches the value of κ^2 , which is dominant in the YM condensate for all time, and the energy density of the magnetic field approaches zero. The total energy density of the YM condensate $\rho_y \rightarrow b\kappa^2/2$, and the "fine-tuning" problem also exists. From these models, we also find that the EOS of the YM condensate cannot cross -1, which is same with phantom models.^{17–21}

From this discussion, we get the following conclusion: if the electric component dominates in the initial condition, the YM condensate can have the state of $1/3 > w_y > -1$ or $w_y < -1$, depending on the choice of the initial condition. The former is similar to the quintessence models, and the cosmic coincidence problem is naturally avoided. The latter is similar to the phantom models. In each case, the EOS of the YM condensate approaches -1 in the latter stage, similar to the cosmological constant, which is independent of the choice of the initial condition. The value of E^2 approaches κ^2 , the renormalization scale, and the value of B^2 approaches zero.



Fig. 3. In models b1, b2, b3, the evolution of the EOS of the YM condensate with the scale factor a.



Fig. 4. In models b1, b2, b3, the evolution of the "electric" and "magnetic" components with the scale factor a.

Second, we consider the case of the magnetic field being dominant. In the extreme condition of $E^2 = 0$, the energy density, pressure and EOS of the YM condensate are

$$\rho_y = \frac{1}{2}bB^2(\varepsilon - 1), \quad p_y = \frac{1}{2}bB^2\left(\frac{1}{3}\varepsilon + 1\right), \quad w_y = \frac{\varepsilon + 3}{3\varepsilon - 3},\tag{19}$$

respectively, where $\varepsilon = \ln(B^2/\kappa^2)$. The constraint of $\rho_u > 0$ is reduced to

$$\varepsilon > 1,$$
 (20)

which leads to a constraint of the EOS of the YM condensate:

$$w_y > \frac{1}{3}.\tag{21}$$

This means that this YM condensate cannot get a negative pressure, and be a candidate for dark energy. We turn to the general case with the magnetic component dominant. The energy density, pressure and EOS of the YM condensate are in Eqs. (5) and (7). The constraint of $\rho_y > 0$ yields

$$0 > f > -\varepsilon q, \tag{22}$$

and it follows that $w_y > 1/3$. The YM condensate cannot get a negative EOS. However, is it possible for the YM condensate to evolve from the state of $B^2 > E^2$ to the state of $B^2 < E^2$, and get a negative pressure? If this is possible, in the transition point the YM condensate must have a state of $B^2 = E^2$, where $\varepsilon = \infty$. From the effective YM equation (10), one knows that this occurs only at a = 0. So the transformation from the state of $E^2 > B^2$ to $E^2 < B^2$ cannot be realized. In conclusion, the YM condensate with the magnetic component dominant cannot get a negative pressure, so it cannot be a candidate for dark energy.

3. Summary

We have investigated the evolution of the YM condensate as a candidate for dark energy, which has no free parameters except the value of the present cosmic energy scale, and the cosmic evolution depends entirely on the initial condition. This study shows that the evolution of the electric and magnetic components in the YM condensate is different for the models with different initial conditions. If the electric component is dominant in the initial condition, and $w_i > -1$ is satisfied, $E^2 \propto a^{-4}$ in the earlier stage, and later it turns to the state of $E^2 \to \kappa^2$. For the magnetic component, $B^2 \propto a^{-4}$ in the earlier stage, and later it decreases rapidly to zero. The electric component is dominant in the YM condensate for all time, and the total EOS of the YM condensate transits from the state of $w_y \sim 1/3$ to the state of $w_{y} \sim -1$. So the cosmic coincidence problem is naturally avoided in the models. If in the initial condition the electric component is dominant and $w_i < -1$ is satisfied, the electric component runs to the state of $E^2 \to \kappa^2$ and the magnetic component runs to $B^2 \rightarrow 0$ in the later stage of the Universe. The total EOS of the YM condensate keeps $w_y < -1$, and later it turns to a state of $w_y \rightarrow -1$. So the big rip problem is avoided. However, if the magnetic component is dominant in the initial condition, $w_y > 1/3$ is satisfied for all time. So it cannot be a candidate for dark energy.

Acknowledgment

W. Zhao thanks Yang Zhang for helpful discussion. This work is partly supported by CNSF No. 10703005 and 10775119, the Research Funds launched in ZJUT No. 109001729.

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