Localized Topology Control for Heterogeneous Wireless Ad-hoc Networks

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Abstract—We study topology control in heterogeneous wireless ad hoc networks, where mobile hosts may have different maximum transmission powers and two nodes are connected iff they are within the maximum transmission range of each other. We present several strategies that all wireless nodes self-maintain sparse and power efficient topologies in heterogeneous network environment with low communication cost. The first structure is sparse and can be used for broadcasting. While the second structure keeps the minimum power consumption path, and the third structure is a length and power spanner with a bounded degree. Both the second and third structures are power efficient and can be used for unicast. Here a structure is power efficient if the total power consumption of the least cost path connecting any two nodes in it is no more than a small constant factor of that in the original heterogeneous communication graph. All our methods use at most $O(n)$ total messages, where each message has $O(\log n)$ bits.

Index Terms—Graph theory, wireless ad hoc networks, topology control, heterogeneous networks, power consumption.

I. Introduction

An important requirement of wireless *ad hoc* networks is that they should be self-organizing, i.e., transmission ranges and data paths are dynamically restructured with changing topology. Localized *ad hoc* network topology control scheme is to let each wireless node locally adjust its transmission power and select which neighbors to communicate according to certain strategy, while maintaining a structure that can support energy efficient routing and improve the overall network performance. Hence it can efficiently conserve the transmission energy from soft aspects with low cost. In the past several years, topology control algorithms have drawn significant research interest. Centralized algorithms can achieve optimality or its approximation, which are more applicable to static networks due to the lack of adaptability to topology changes. In contrast, distributed algorithms are more suitable for mobile ad hoc networks since the environment is inherently dynamic and they are adaptive to topology changes at the cost of possible less optimality. Furthermore, these algorithms only attempt to selectively choose some neighbors of each node. The primary distributed topology control algorithms for ad hoc networks aim to maintain network connectivity, optimize network throughput with power-efficient routing, conserve energy and increase the fault tolerance.

Most prior art [1], [2], [3], [4], [5], [6] on network topology control assumed that wireless ad hoc networks are modelled by *unit disk graphs* (UDG), i.e., two mobile hosts can communicate as long as their Euclidean distance is no more than a

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threshold. However, practically, wireless ad hoc networks cannot be perfectly modelled as UDGs: the maximum transmission ranges of wireless devices may vary due to various reasons such as the device differences and the small mechanic/electronic errors during the process of transmitting even the transmission powers of all devices are set the same initially. In [7], [8], the authors extended UDG into a new model, called *quasi unit disk graphs*, which is closer to reality than UDG. In this paper, we study a more generalized model. Each wireless node u may have its own transmission radius r_u. Then heterogeneous wireless networks are modelled by mutual inclusion graphs (MG): two nodes can communicate directly only if they are within the transmission range of each other, i.e., it has a link uv iff $\|uv\| \leq \min(r_u, r_v)$. Clearly UDG is a special case of MG. Few research efforts addressed the topology control for heterogeneous wireless networks.

The main contribution of this paper is as follows. We propose several *localized* strategies for heterogeneous wireless devices to self-form a *globally* sparse power efficient network topology: a power spanner, a sparse structure and a degree-bounded length and power spanner respectively. Here an algorithm is said to construct a topology H *locally* if, every node u can decide which edges uv belong to H using only the information of nodes within a constant number of hops of u. All our algorithms have communication costs $O(n)$, where each message has $O(\log n)$ bits. Notice, to study the topology control in heterogeneous networks, it would be helpful to extend the ideas from the well-studied topologies, such as GG, RNG and Yao, used in homogeneous networks. The topology control for heterogeneous networks is not trivial, since many properties in homogeneous networks disappear in heterogeneous networks.

The rest of the paper is organized as follows. In Section II we introduce the background and review previous methods. Limitations on heterogeneous network topology control are discussed in Section III. We describe a strategy for all nodes forming a sparse structure in Section IV, a sparse power spanner in Section V, and a degree-bounded power and length spanner in Section VI. We also analyze the communication complexities of these methods. Our theoretical results are corroborated in the simulations in Section VII. We conclude our paper in Section VIII with the discussion of future works.

II. Preliminaries

A. Heterogeneous Wireless Network Model

A heterogeneous wireless *ad hoc* network is composed of a set V of n nodes $v_1, v_2, \cdots, v_n$, in which each node v_i has its own maximum transmission power p'_i. Let ϵ_i be the mechanic/electronic error of a node v_i in its power control. Then

the maximum transmission power considered in this paper is actually $p_i = p'_i - \epsilon$. We adopt a common assumption in the literature that the power needed to support the communication between two nodes v_i and v_j is $\|v_i v_j\|^\beta$, where $\beta \in [2, 5]$ is a real number depending on the environment and $\|v_i v_j\|$ is the Euclidean distance between v_i and v_j. Consequently, the signal sent by a node v_i can be received by all nodes v_j with $\|v_i v_j\| \leq r_i$, where $r_i^\beta \leq p_i / p_0$, p_0 is the uniform threshold that a signal with power p_0 can be recognized by a node. Thus, for simplicity, we assume that each mobile host v_i has its own transmission range r_i. The heterogeneous wireless ad hoc network is then modelled by a mutual inclusion graph (MG), where two nodes v_i, v_j are connected iff they are within the transmission range of each other, i.e., $\|v_i v_j\| \leq \min(r_i, r_j)$. Previously, no method is known for topology control when the networks are modelled as mutual inclusion graphs.

B. Current State of Knowledge

Many structures were proposed for topology control in homogeneous wireless ad hoc networks. Due to limited spaces, we will briefly review some of proximity geometric structures. The *relative neighborhood graph* [9] $RNG(V)$ consists of all edges uv such that the intersection of two circles centered at u and v and with radius $\|uv\|$ do not contain any vertex w from V. The *Gabriel graph* [10] $GG(V)$ contains edge uv if and only if $disk(u,v)$ contains no other points of S, where $disk(u,v)$ is the disk with edge uv as a diameter. Both $GG(V)$ and $RNG(V)$ are connected, planar, and contain the Euclidean minimum spanning tree of V. The intersections of $GG(V)$, $RNG(V)$ with a connected $UDG(V)$ are connected. Delaunay triangulation, denoted by $Del(V)$, is also used as underlying structure by several routing protocols. Here a triangle $\triangle uvw$ belongs to $Del(V)$ if its circumcircle does not contain any node inside. It is well known that $RNG(V) \subseteq GG(V) \subseteq Del(V)$. The intersection of $Del(V)$ with a connected $UDG(V)$ has a bounded length spanning ratio [11].

The *Yao graph* [12] with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG}_k(V)$, is defined as follows. At each node u, any k equal-separated rays originated at u define k cones. In each cone, choose the shortest edge uv among all edges from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph is called the *Yao graph*. Let $YG_k(V)$ be the undirected graph by ignoring the direction of each link in $\overrightarrow{YG}_k(V)$. Some researchers used a similar construction named θ-graph [13], the difference is that it chooses the edge which has the shortest projection on the axis of each cone instead of the shortest edge in each cone.

The first effort for topology control in heterogeneous wireless networks was reported in [14] by the same authors of this paper. In [14], we showed how to perform topology control based on Yao structure for heterogeneous wireless networks. The results presented in current paper have been available online since around June 2003. Recently, several structures that extend the relative neighborhood graph and local minimum spanning tree were proposed in [15] for topology control in heterogeneous wireless networks. They build directed network topologies while the methods presented here build undirected

topologies that are beneficial for routing. In addition, as the authors of [15] acknowledged, their original methods cannot preserve the network connectivity. Then new methods were proposed to remedy this in their online version of the paper. Two structures were proposed by them: an extended relative neighborhood graph and the extended local minimum spanning tree. It is unknown if their structures are sparse, power efficient.

C. Spanners and Stretch Factors

When constructing a subgraph of the original communication graph MG, we may need consume more power to connect some nodes since we may disconnect the most power efficient path in MG. Thus, naturally, we would require that the constructed structure approximates MG well in terms of the power consumption for unicast routing. In graph theoretical term, the structure should be a spanner [16], [13]. Let $G = (V, E)$ be a n-vertex weighted connected graph. The distance in G between two vertices $u, v \in V$ is the length of the shortest path between u and v and it is denoted by $d_G(u, v)$. A subgraph $H = (V, E')$, where $E' \subseteq E$, is a *t-spanner* of G if for every $u, v \in V$, $d_H(u, v) \leq t \cdot d_G(u, v)$. The value of t is called the *stretch factor* or *spanning ratio*. When the graph is a geometric graph and the weight is the Euclidean distance between two vertices, the stretch factor t is called the *length stretch factor*, denoted by $\ell_H(G)$. For wireless networks, the mobile devices are usually powered by battery only. We thus pay more attention to the power consumptions. When the weight of a link $uv \in G$ is defined as the power to support the communication of link uv, the stretch factor of H is called the *power stretch factor*, denoted by $\rho_H(G)$ hereafter. The power, denoted by $p_G(u, v)$, needed to support the communication between a link uv in G is often assumed to be $\|uv\|^\beta$, where $2 \leq \beta \leq 5$. Obviously, for any weighted graph G and a subgraph $H \subseteq G$,

Lemma 1: [3] Graph H has stretch factor δ if and only if for any link $uv \in G$, $d_H(u, v) \leq \delta \cdot d_G(u, v)$.

Thus, to generate a spanner H, we only have to make sure that *every link* of G is approximated within a constant factor.

D. Sparseness and Bounded Degree

All well-known proximity graphs ($GG(V)$, $RNG(V)$, $Del(V)$ and $YG(V)$) have been proved to be sparse graphs when network is modeled as a UDG. Recall that a *sparse* graph means the number of edges is linear with the number of nodes. The sparseness of all well-known proximity graphs implies that the average node degree is bounded by a constant. Moreover, we prefer the maximum node degree is bounded by a constant, because wireless nodes have limited resources and the signal interference in wireless communications. Unbounded degree (or in-degree) at a node u will often cause large overhead at u, whereas a bounded degree increases the network throughput. In addition, bounded degree will also give us advantages when apply several routing algorithms. Therefore, it is often imperative to construct a sparse network topology with a bounded node degree while it is still power-efficient. However, Li *et al.* [3] showed that the maximum node degree of RNG, GG and Yao could be as large as $n - 1$. The instance consists of $n - 1$ points

lying on the unit circle centered at a node $u \in V$. Then each edge uv_i belongs to the $RNG(V)$, $GG(V)$ and $\overrightarrow{YG}_k(V)$.

Recently, in homogeneous wireless ad hoc networks, some improved or combined proximity graphs [17], [18] have been proposed to build planar degree-bounded power spanner topology, which meets all preferred properties for unicast. In heterogeneous networks, only a few research efforts [15], [14] are reported so far. In the following, we will first discuss the difficulties and limitations for topology control in heterogeneous networks, then present our localized strategies in detail.

III. LIMITATIONS

In heterogeneous wireless ad hoc networks, the planar topology does not necessarily exist. Figure 1 (a) shows an example, there are four nodes x, y, u and v in the network, where their transmission range $r_x = r_y = \|xy\|$ and $r_u = r_v = \|uv\|$, and node u is out of the transmission range of node x and y, while node v is in the transmission range of node y and out of the range of x. The transmission ranges of x and y are illustrated by the dotted circles. According to the definition of MG, there are only three edges xy, vy and uv in the graph. Hence any topology control method can not make the topology planar while keeping the communication graph connected. On the other hand, it is worth to think whether we can design a new routing protocol on some pseudo-planar topologies. As will see later, the pseudo-planar topology GG(MG) and RNG(MG) proposed in this section has some special properties which are different from other general non-planar topologies. For instance, two intersecting triangles can not share a common edge. We leave it as a future work.
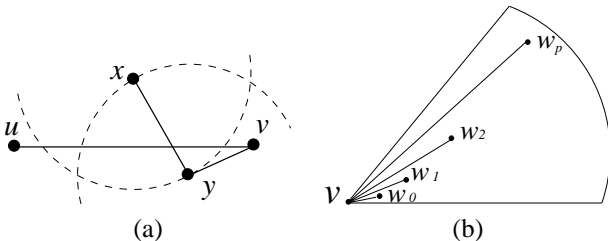

Fig. 1. Limitations on heterogeneous networks: (a) Planar topology does not exist. (b) Degree of node v can not be bounded by constant.

We also can show that the node degree in heterogeneous networks can not be bounded by a constant if the radius ratio is unbounded. Figure 1 (b) shows such an example. In the example, a node v has $p + 1$ incoming neighbors w_i, $0 \leq i \leq p$. Assume that each node w_i has a transmission radius $r_{w_i} = r_v/3^{p-i}$ and $\|vw_i\| = r_{w_i}$. Obviously, $\|w_i w_j\| > \min(r_{w_i}, r_{w_j})$, i.e., any two nodes w_i, w_j are not directly connected in MG.

Obviously, none of those edges incident on v can be deleted, hence there is no topology control method to bound the degree by a constant without violating connectivity. Consider the example illustrated by Figure 1 (b), edges vw_i, $0 \leq i \leq p$, are all possible communication links. Thus, node v in any connected spanning graph has degree $p + 1$. On the other hand, we will show in section VI, in the worst case, any connected MG graph has degree $O(\log_2 \gamma)$ where $\gamma = \max_{v \in V} \max_{w \in I(v)} \frac{r_v}{r_w}$. Here, $I(v) = \{w \mid wv \in MG\}$. In the example, recall

that $3^p r_{w_0} = r_v$, hence γ equals to 3^p. Thus, v has degree $\log_3 \gamma + 1 = O(\log_2 \gamma)$. In the paper, we always assume γ is a constant. It is practical, since it is trivial that two wireless devices in same network have unbounded radius ratio.

IV. HETEROGENEOUS SPARSE STRUCTURE

In this section, we propose a strategy for all nodes to self-form a sparse structure, called $RNG(MG)$, based on the relative neighborhood graph structure, whose total number of links is $O(n)$. We add a link $uv \in MG$ to $RNG(MG)$ if there is *no* another node w inside $lune(u,v)$ and *both* links uw and wv are in MG. Here $lune(u,v)$ is the intersection of $disk(u, \|uv\|)$ and $disk(v, \|uv\|)$. The algorithm will be similar to Algorithm 2, thus we omit it here. Notice that the total communication cost of constructing $RNG(MG)$ is $O(n \log n)$ bits, assuming that the radius and ID information of a node can be represented in $O(\log n)$ bits. In addition, the structure $RNG(MG)$ is symmetric: if a node u keeps a link uv, node v will also keep the link uv. Thus, a node u does not have to tell its neighbor v whether it keeps a link uv or not.

It is not difficult to prove that structure $RNG(MG)$ is connected by induction. On the other hand, same as the case in homogeneous networks (i.e., UDG mode), $RNG(MG)$ does not have a bounded length stretch factor, nor constant bounded power stretch factor, and does not have bounded node degree. In this paper, we will show that $RNG(MG)$ is a *sparse* graph: it has at most $6n$ links.

In the following, we define a new structure, called $ERNG(MG)$. Assume that each node v knows its maximum transmission radius r_v. Let $B(u) = \{v \mid r_v \geq r_u\}$. A node u processes a link uv from MG if $r_v \geq r_u$, i.e., $v \in B(u)$. Node u removes a link uv, where $v \in B(u)$, if there is another node $w \in B(u)$ inside $lune(u,v)$ with both links uw and wv are in MG. All the links uv kept by all nodes form the final structure $ERNG(MG)$.

Algorithm 1: Constructing-ERNG
1) Each node u initiates sets $E_{MG}(u)$ and $E_{ERNG}(u)$ to be empty. Here $E_{MG}(u)$ is the set of links of MG known to u so far and $E_{ERNG}(u)$ is the set of links of ERNG known to u so far. Then, each node u locally broadcasts a HELLO message with ID_u, r_u and its position (x_u, y_u) to all nodes in its transmission range. Note that $r_u = p_u^{1/\beta}$ is its maximum transmission range.
2) At the same time, each node u processes the incoming messages. Assume that node u gets a message from some node v. If $\|vu\| \leq \min\{r_u, r_v\}$, then node u adds a link uv to $E_{MG}(u)$. If $r_v \geq r_u$, then node u performs the following procedures. Node u checks if there is another link $uw \in E_{MG}(u)$ with the following additional properties: 1) $w \in lune(u,v)$, 2) $r_w \geq r_u$, and 3) $\|wv\| \leq \min\{r_w, r_v\}$. If no such link uw, then add uv to $E_{ERNG}(u)$. For any link $uw \in E_{ERNG}(u)$, node u checks if the following conditions hold: 1) $v \in lune(u,w)$, and 2) $\|wv\| \leq \min\{r_w, r_v\}$. If the conditions hold, then remove link uw from $E_{ERNG}(u)$.
3) Node u repeats the above steps until no new HELLO messages received.

4) For each link $uv \in E_{ERNG}(u)$, node u informs node v to add link uv.

5) All links uv in $E_{ERNG}(u)$ are the final links in $ERNG(MG)$ incident on u.

We then prove that the structure ERNG has at most $6n$ links.

Lemma 2: Structure ERNG(MG) has at most $6n$ links.

Proof: Consider any node u. We will show that u keeps at most 6 directed links emanated from u. Assume that u keeps more than 6 directed links. Obviously, there are two links uw and uv such that $\angle wuv < \pi/3$. Thus, vw is not the longest link in triangle $\triangle uvw$. Without loss of generality, we assume that $\|uw\|$ is the longest in triangle $\triangle uvw$. Notice that the existence of link uw implies that $\|uw\| \leq \min(r_u, r_w) = r_u$. Consequently, $\|vw\| \leq \|uw\| \leq \min(r_u, r_w)$. Thus, from the fact that $r_u \leq r_v$, we know $\|vw\| \leq \min(r_v, r_w)$. Hence, link vw does exist in the original communication graph MG, it implies that link uw cannot be selected to ERNG. ∎

Similar to Lemma 2, we can prove the following lemma.

Lemma 3: Structure RNG(MG) has at most $6n$ links.

Proof: Imagine that each link uv has a direction as follows: $\overrightarrow{uv}$ if $r_u \leq r_v$. Then similar to Lemma 2, we can prove that each node u only keeps at most 6 such imagined direct links. Thus, total links are at most $6n$. ∎

Similarly, we can define a structure $EGG(MG)$, which contains an edge uv if $r_u \leq r_v$ and there is no node w with the following properties: 1) $r_u \leq r_w$, 2) w is inside the disk $disk(u, v)$. However, we cannot prove that $EGG(MG)$ has a linear number of links.

V. HETEROGENEOUS POWER SPANNER

Then, we give a strategy for all nodes to self-form a power spanner structure, called $GG(MG)$, based on the Gabriel graph. We add a link $uv \in MG$ to $GG(MG)$ if there is *no* another node w inside $disk(u, v)$ and both links uw and wv are in MG. Our localized construction method works as follows.

Algorithm 2: Constructing-GG

1) Let $E_{MG}(u)$ and $E_{GG}(u)$ are the set of links known to u from MG and GG respectively. Each node u initiates both $E_{MG}(u)$ and $E_{GG}(u)$ as empty. Then, each node u locally broadcasts a HELLO message with ID_u, r_u and its position (x_u, y_u) to all nodes in its transmission range.

2) At the same time, each node u processes the incoming messages. Assume that node u gets a message from some node v. If $\|vu\| \leq \min\{r_u, r_v\}$, then node u adds a link uv to $E_{MG}(u)$. Node u checks if there is another link $uw \in E_{MG}(u)$ with the following two additional properties: 1) $w \in disk(u, v)$, and 2) $\|wv\| \leq \min\{r_w, r_v\}$. If no such link uw, add uv to $E_{GG}(u)$. For any link $uw \in E_{GG}(u)$, node u checks if the following two properties hold: 1) $v \in disk(u, w)$, and 2) $\|wv\| \leq \min\{r_w, r_v\}$. If they hold, remove link uw from $E_{GG}(u)$.

3) Node u repeats the above steps until no new HELLO messages received.

4) All links uv in $E_{GG}(u)$ are the final links in $GG(MG)$ incident on u.

We first show that Algorithm 2 builds the structure $GG(MG)$ correctly. For any link $uv \in GG(MG)$, clearly, we cannot remove them in Algorithm 2. For a link $uv \notin GG(MG)$, assume

that a node w is inside $disk(u, v)$ and both links uw and wv belong to MG. If node u gets the message from w first, and then gets message from v, clearly, uv cannot be added to $E_{GG}(u)$. If node u gets the message from v first, then u will remove uv from $E_{GG}(u)$ (if it is there) when u gets the information of w.

It is not difficult to prove that structure $GG(MG)$ is connected by induction. In addition, since we remove a link uv only if there are two links uw and wv with w inside $disk(u, v)$, it is easy to show that the power stretch factor of $GG(MG)$ is 1. In other words, the minimum power consumption path for any two nodes v_i and v_j in MG is still kept in $GG(MG)$. Remember that here we assume the power needed to support a link uv is $\|uv\|^\beta$, for $\beta \in [2, 5]$.

On the other hand, same as the case in homogeneous networks (i.e., UDG mode), $GG(MG)$ is not a length spanner, and does not have bounded node degree. Furthermore, it is unknown whether $GG(MG)$ is a *sparse* graph. Recently, it was proven in [19] that $GG(MG)$ has at most $O(n^{8/5} \log \gamma)$ edges where $\gamma = \max r_u/r_v$.

Notice that, the extension from Gabriel graph is non-trivial. In [19], two structures defined as follows even cannot guarantee the connectivity. In the first structure, called $LGG_0(MG)$, they remove a link $uv \in MG$ if there is another node w inside $disk(u, v)$. In the second structure, called $LGG_1(MG)$, they remove a link $uv \in MG$ if there is another node w inside $disk(u, v)$, and either link uw or link wv is in MG.

VI. HETEROGENEOUS DEGREE-BOUNDED SPANNER

Undoubtedly, as described in preliminaries, we always prefer a structure has more nice properties, such as degree-bounded (stronger than sparse), power spanner etc. Naturally, we could extend the previous known degree-bounded spanner, such as the Yao related structures, from homogeneous networks to heterogeneous networks. Unfortunately, a simple extension of the Yao structure from UDG to MG even does not guarantee the connectivity. Figure 2 (a) illustrates such an example. Here $r_u = r_v = \|uv\|$, $r_w = \|uw\|$, $r_x = \|vx\|$, and $\|uw\| < \|uv\|$, $\|uw\| < \|vw\|$, $\|vx\| < \|uv\|$, and $\|vx\| < \|ux\|$. In addition, v and w are in the same cone of node u, and nodes x and u are in the same cone of node v. Thus, the original MG graph contains links uv, uw and vx only and is connected. However, when applying Yao structure on all nodes, node u will only have information of node v and w and it will keep link uw. Similarly, node w keeps link uw; node v keeps link vx; and node x keeps link xv. In other words, only link xv and uw are kept by Yao method. Thus applying Yao structure disconnects node v, x from the other two nodes u and w. Consequently, we need more sophisticated extensions of the Yao structure to MG to guarantee the connectivity of the structure.

A. Extended Yao Graph

Algorithm 3: Constructing-EYG

1) Initially, each node u divides the disk $disk(u, r_u)$ centered at u with radius r_u by k equal-sized cones centered at u. We generally assume that the cone is half open and half-close. Let $\mathbb{C}_i(u)$, $1 \leq i \leq k$, be the k cones partitioned. Let $C_i(u)$, $1 \leq i \leq k$, be the set of nodes v inside
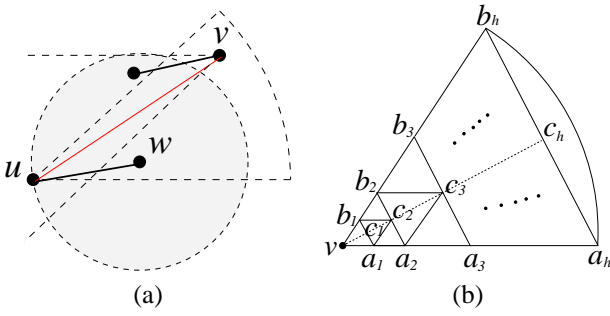
Fig. 2. Extend Yao structure on heterogeneous networks: (a) Simple extension of Yao structure does not guarantee the connectivity. (b) Further space partition in each cone to bound in-degree.

the ith cone $\mathbb{C}_i(u)$ with a *larger or equal*[1] radius than u. In other words,

$$C_i(u) = \{v \mid v \in \mathbb{C}_i(u), \ and \ r_v \geq r_u\}.$$

Initially, $C_i(u)$ is empty.

2) Each node u broadcasts a HELLO message with ID_u, r_u and its position (x_u, y_u) to all nodes in its transmission range.

3) At the same time, each node u processes the incoming broadcast messages. Once it gets a HELLO message from some node v, it sets $C_i(u) = C_i(u) \bigcup \{v\}$, if node v is inside the ith cone $\mathbb{C}_i(u)$ of node u and $r_v \geq r_u$.

4) Node u chooses a node v from each cone $C_i(u)$ so that the link uv has the smallest $ID(uv)$ among all links uv_j with v_j in $C_i(u)$, if there is any.

5) Finally, each node u informs all 1-hop neighbors of its chosen links through a broadcast message. Let $\overrightarrow{EYG}_k(MG)$ be the union of all chosen links.

Since the symmetric communications are required, let $EYG_k(MG)$ be the undirected graph by ignoring the direction of each link in $\overrightarrow{EYG}_k(MG)$. Graph $EYG_k(MG)$ is the final network topology. Since node u chooses a node $v \in disk(u, r_u)$ with $r_v \geq r_u$, link uv is indeed a bidirectional link, i.e., u and v are within the transmission range of each other. Additionally, this strategy could avoid the possible disconnection by simple Yao extension we mentioned before.

Obviously, each node only broadcasts twice: one for broadcasting its ID, radius and position; and the other for broadcasting the selected neighbors. Remember that it selects at most k neighbors. Thus, each node sends messages at most $O((k+1) \cdot \log n)$ bits. Here, we assume that the node ID and its position can be represented using $O(\log n)$ bits for a network with n wireless nodes.

Before we study the properties of this structure, we have to define some terms first. Assume that each node v_i of MG has a unique identification number $ID_{v_i} = i$. The identity of a bidirectional link uv is defined as $ID(uv) = (\|uv\|, ID_u, ID_v)$ where $ID_u > ID_v$. Note that we use the bidirectional links instead of the directional links in the final topology to guarantee connectivity. In other words, we require that both node u and

node v can communicate with each other through this link. In this paper, all proofs about connectivity or stretch factors take the notation uv and vu as same, which is meaningful. Only in the topology construction algorithm or proofs about bounded-degree, uv is different than vu: the former is initiated and built by u, whereas the latter is by node v. Sometimes we denote a directional link from v to u as $\overrightarrow{vu}$ if necessary. Then we can order all bidirectional links (at most $n(n-1)$ such links) in an increasing order of their identities. Here the identities of two links are ordered based on the following rule: $ID(uv) > ID(pq)$ if (1) $\|uv\| > \|pq\|$ or (2) $\|uv\| = \|pq\|$ and $ID_u > ID_p$ or (3) $\|uv\| = \|pq\|$, $u = p$ and $ID_v > ID_q$.

Correspondingly, the rank of each link uv, denoted by $rank(uv)$, is its order in sorted bidirectional links. Notice that, we actually only have to consider the links in MG. We then show that the constructed network topology is a length and power spanner.

Theorem 4: The length stretch factor of $EYG_k(MG)$, $k > 6$, is at most $\ell = \frac{1}{1 - 2\sin(\frac{\pi}{k})}$.

Proof: Notice it is sufficient to show that for any nodes u and v with $\|uv\| \leq \min(r_u, r_v)$, i.e. $uv \in MG$, there is a path connecting u and v in $EYG_k(MG)$ with length at most $\ell\|uv\|$. We construct a path $u \rightsquigarrow v$ connecting u and v in $EYG_k(MG)$ as follows.

Assume that $r_u \leq r_v$. If link $uv \in EYG_k(MG)$, then set the path $u \rightsquigarrow v$ as the link uv. Otherwise, consider the $disk(u, r_u)$ of node u. Clearly, node u will get information of v from v and node v will be selected to some $C_i(u)$ since $r_v \geq r_u$. Thus, from $uv \notin EYG_k(MG)$, there must exist another node w in the same cone as v, which is a neighbor of u in $EYG_k(MG)$. Then set $u \rightsquigarrow v$ as the concatenation of the link uw and the path $w \rightsquigarrow v$. Here the existence of path $w \rightsquigarrow v$ can be easily proved by induction on the distance of two nodes. Notice that the angle θ of each cone section is $\frac{2\pi}{k}$. When $k > 6$, then $\theta < \frac{\pi}{3}$. It is easy to show that $\|wv\| < \|uv\|$. Consequently, the path $u \rightsquigarrow v$ is a simple path, i.e., each node appears at most once.

We then prove by induction that the path $u \rightsquigarrow v$ has total length at most $\ell\|uv\|$.

Obviously, if there is only one edge in $u \rightsquigarrow v$, $d(u \rightsquigarrow v) = \|uv\| < \ell\|uv\|$. Assume that the claim is true for any path with l edges. Then consider a path $u \rightsquigarrow v$ with $l+1$ edges, which is the concatenation of edge uw and the path[2] $w \rightsquigarrow v$ with l edges, as shown in Figure 3 where $\|wv\| = \|xv\|$.
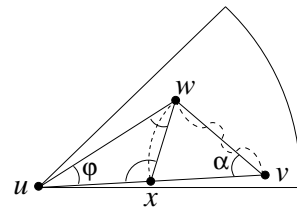

Fig. 3. The length stretch factor of $EYG_k(MG)$ is at most $\frac{1}{1 - 2\sin(\frac{\pi}{k})}$.

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[1] This is the main difference between this algorithm and the simple extension of Yao structure discussed before, in which it considers all nodes v that u can get signal from.

[2] In the procedure of induction, if $r_w \leq r_v$ then we induct on path $w \rightsquigarrow v$, otherwise we induct on path $v \rightsquigarrow w$. In fact, here $w \rightsquigarrow v$ is same as $v \rightsquigarrow w$ since the path is bidirectional for communication. Directional link is only considered in building process and is meaningless when we talk about the path. This induction rule is applied throughout the remainder of the paper.

Notice, from induction, $d(w \leftrightsquigarrow v) \leq \ell\|wv\|$. Then, let $\varphi = \angle wuv$ and $\alpha = \angle uvw$, we have

$$\frac{\|uw\|}{\|ux\|} = \frac{\sin(\angle uxw)}{\sin(\angle xwu)} = \frac{\sin(\frac{\pi}{2} + \frac{\alpha}{2})}{\sin(\frac{\pi}{2} + \frac{\alpha}{2} + \varphi)}$$

$$= \frac{1}{\cos\varphi - \sin\varphi\tan\frac{\alpha}{2}} \leq \frac{\cos(\frac{\pi}{4} - \frac{\varphi}{4})}{\cos(\frac{\pi}{4} + \frac{3}{4}\varphi)} \leq \frac{1}{1 - 2\sin(\frac{\pi}{k})}$$

The first inequality is because $0 \leq \alpha \leq \frac{\pi}{2} - \frac{\varphi}{2}$ and the second inequality is because $0 \leq \varphi \leq \frac{2\pi}{k}$. Consequently, $d(u \leftrightsquigarrow v) = \|uw\| + d(w \leftrightsquigarrow v) < \ell\|ux\| + \ell\|wv\| = \ell\|uv\|$, where $\ell = \frac{1}{1-2\sin(\frac{\pi}{k})}$. That is to say, the claim is also true for the path $u \leftrightsquigarrow v$ with $l + 1$ edges.

Thus, the length stretch factor of $EYG_k(MG)$ is at most $\ell = \frac{1}{1-2\sin(\frac{\pi}{k})}$. This finishes the proof. ∎

Theorem 5: The power stretch factor of the extended Yao graph $EYG_k(MG)$, $k > 6$, is at most $\rho = \frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$.

Proof: The proof is similar to that in UDG [3], [4] except the induction procedure. We show by induction, on the number of its edges, that the path $u \leftrightsquigarrow v$ constructed in theorem 4 has power cost, denoted by $p(u \leftrightsquigarrow v)$, at most $\rho\|uv\|^\beta$. ∎

B. Novel Space Partition

Partitioning the space surrounding a node into k equal-sized cones enables us to bound the node out-degree using the Yao structure. Using the same space partition, Yao-Yao structure [3], [4] produces a topology with bounded in-degree when the networks are modeled by UDG. Yao-Yao structure (for UDG) is generated as follows: a node u collects all its incoming neighbors v (i.e., $\overrightarrow{vu} \in \overrightarrow{YG_k}(V)$), and then selects the closest neighbor v in each cone $\mathbb{C}_i(u)$. Clearly, Yao-Yao has bounded degree at most k. They also showed that another structure YaoSink [3], [4] has not only the bounded node degree but also a constant bounded stretch factor. The network topology with a bounded degree can increase the communication efficiency. However, these methods [3], [4] may fail when the networks are modeled by MG: they cannot even guarantee the connectivity, which is verified by following discussions.

Assume that we already construct a connected directed structure $\overrightarrow{EYG_k}(MG)$. Let $I(v) = \{w \mid \overrightarrow{wv} \in \overrightarrow{EYG_k}(MG)\}$. In other words, $I(v)$ is the set of nodes that have directed links to v in $\overrightarrow{EYG_k}(MG)$. Let $I_i(v) = I(v) \cap \mathbb{C}_i(u)$, i.e., the nodes in $I(v)$ located inside the ith cone $\mathbb{C}_i(v)$. Yao-Yao structures will pick the closest node w in $I_i(v)$ and add undirected link wv to Yao-Yao structure. Previous example in Figure 1 (b) also illustrates the situation that Yao-Yao structure is not connected. In the example, a node v has $p + 1$ incoming neighbors w_i, $0 \leq i \leq p$. Assume that each node w_i has a transmission radius $r_{w_i} = r_v/3^{p-i}$ and $\|vw_i\| = r_{w_i}$. Obviously, $\|w_i w_j\| > \min(r_{w_i}, r_{w_j})$, i.e., any two nodes w_i, w_j are not directly connected in MG. It is easy to show that the Yao structure $\overrightarrow{EYG_k}(MG)$ only has directed links $\overrightarrow{w_i v}$. Obviously, node v will only select the closest neighbor w_0 to the Yao-Yao structure, which disconnects the network. This same example can also show that the structure based on Yao-Sink [3], [4] is also not connected for heterogeneous wireless ad hoc networks.

Thus, selecting the closest incoming neighbor in each cone $\mathbb{C}_i$ is too aggressive to guarantee the connectivity. Observe that, in Figure 1 (b), to guarantee the connectivity, when we delete a directed link $\overrightarrow{w_i v}$, we need to keep *some* link, say $w_j v$, such that $w_i w_j$ is a link in MG. Thus, we want to further partition the cone into a limited number of smaller *regions* and we will keep *only* one node in each region, e.g., the closest node. Clearly, to guarantee that other nodes in the same region are still connected to v, we have to make sure that any two nodes $w_i, w_j \in I(v)$ that co-exist in a same small region are directly connected in MG. Consequently, if the number of regions is bounded by a constant, a degree-bounded structure could be generated. In the remainder of this subsection, we will introduce a novel space partition strategy satisfying the above requirement.

Method 1: Partition-EYG

For each node v, let $\gamma_v = \max_{w \in I(v)} \frac{r_v}{r_w}$. Remember that all nodes in $I(v)$ have transmission radius at most r_v. Let h be the positive integer satisfying $2^{h-2} < \gamma_v \leq 2^{h-1}$. We then discuss in detail our partition strategy of the cones, which is illustrated by Figure 2 (b). Each node v divides each cone centered at v into limited number of triangles and caps, where $\|va_i\| = \|vb_i\| = \frac{1}{2^{h-i}}r_v$ and c_i is the mid-point of the segment $a_i b_i$, for $1 \leq i \leq h$. Notice that this partition can be conducted by node v locally since it can collect the transmission radius information of nodes in $I(v)$. The triangles $\triangle va_1 b_1$, $\triangle a_i b_i c_{i+1}$, $\triangle a_i a_{i+1} c_{i+1}$, $\triangle b_i b_{i+1} c_{i+1}$, for $1 \leq i \leq h-1$, and the cap $\overset{\frown}{a_n b_n}$ form the final space partition of each cone. For simplicity, we call such a triangle or the cap as a *region*. We then prove that this partition indeed guarantees that any two nodes in any same region are connected in MG.
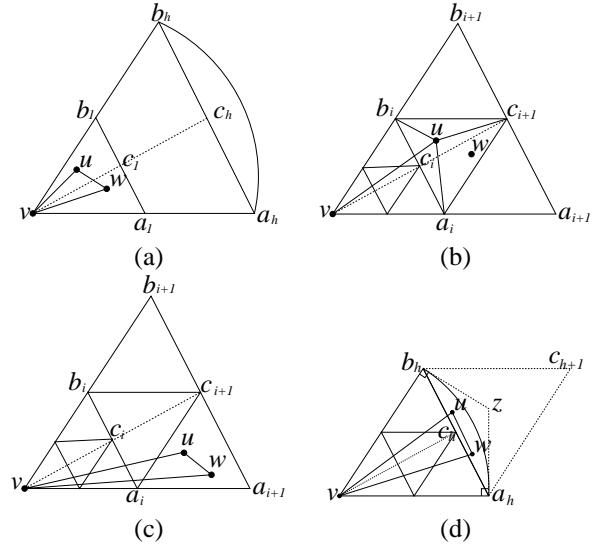

Fig. 4. (a) Two nodes are in triangle $\triangle va_1 b_1$. (b) Two nodes are in triangle $\triangle a_i b_i c_{i+1}$. (c) Two nodes are in triangle $\triangle a_i a_{i+1} c_{i+1}$. (d) Two nodes are inside cap $\overset{\frown}{a_h b_h}$.

Lemma 6: Assume that $k \geq 6$. Any two nodes $u, w \in I(v)$ that co-exist in any one of the generated regions are directly connected in MG, i.e., $\|uw\| < \min(r_u, r_w)$.

Proof: There are four different cases.

1) Two nodes are in $\triangle va_1 b_1$, as shown in Figure 4 (a).

Remember that all nodes in $I(v)$ have transmission ra-

dius at least $\|va_1\| = \frac{1}{2^{(h-1)}}r_v$. We have $\min(r_u, r_w) \geq \|va_1\| = \|vb_1\|$ and $\|a_1b_1\| \leq \|va_1\|$. In addition, since uw is a segment inside $\triangle va_1b_1$, we have $\|uw\| \leq \max(\|a_1b_1\|, \|va_1\|, \|vb_1\|)$. Consequently, $\|uw\| < \min(r_u, r_w)$, i.e. $uw \in MG$.

2) Two nodes are in $\triangle a_ib_ic_{i+1}$, as shown in Figure 4 (b). In this case, we have

   a) $\|vu\| > \|uc_{i+1}\|$, since a_ib_i is the perpendicular bisector of vc_{i+1} and u is at the same side of line a_ib_i as c_{i+1}.
   b) $\|vu\| > \|ua_i\|$, because $\angle va_iu > \frac{\pi}{3} > \angle uva_i$.
   c) $\|vu\| > \|ub_i\|$, because $\angle vb_iu > \frac{\pi}{3} > \angle uvb_i$.
   d) $\|uw\| < \max(\|uc_{i+1}\|, \|ua_i\|, \|ub_i\|)$, because node w must be inside one of the triangles $\triangle a_ib_iu$, $\triangle a_ic_{i+1}u$ and $\triangle b_ic_{i+1}u$.

   Thus, $\|uw\| < \|uv\|$. Similarly, $\|uw\| < \|wv\|$. Consequently, $uw \in MG$ from

   $$\|uw\| < \min(\|uv\|, \|wv\|) < \min(r_u, r_w).$$

3) Two nodes are in $\triangle a_ia_{i+1}c_{i+1}$, as shown in Figure 4 (c). We have $\min(r_u, r_w) \geq \|va_i\| = \|a_ia_{i+1}\| = \|a_ic_{i+1}\| > \|a_{i+1}c_{i+1}\|$. Since uw is a segment inside $\triangle a_ia_{i+1}c_{i+1}$, $\|uw\| < \max(\|a_ia_{i+1}\|, \|a_ic_{i+1}\|, \|a_{i+1}c_{i+1}\|) \leq \min(r_u, r_w)$, i.e. $uw \in MG$. Triangle $\triangle b_ib_{i+1}c_{i+1}$ is the symmetric case with triangle $\triangle a_ia_{i+1}c_{i+1}$, so the claim holds similarly.

4) Two nodes are inside the cap $\overparen{a_hb_h}$, as shown in Figure 4 (d), where a_hz and b_hz is the tangent of arc $\overparen{a_hb_h}$ at point a_h and b_h respectively.
   Since $\angle a_hvb_h < \frac{2\pi}{k}$, $k \geq 6$, we have

   $$\angle vb_hz = \frac{\pi}{2} < \pi - \angle a_hvb_h = \angle vb_hc_{h+1}.$$

   Similarly, $\angle va_hz < \angle va_hc_{h+1}$. This means $\overparen{a_hb_h}$ is inside $\triangle a_hb_hc_{h+1}$. The remaining of the proof directly follows from the proof for the case of $\triangle a_ib_ic_{i+1}$. ∎

C. Extended Yao-Yao Graph

Using the space partition discussed in Section VI-B, we present our method to locally build a sparse network topology with bounded degree for heterogeneous wireless ad hoc network. Here we assume that $\gamma = \max_{v \in V} \gamma_v$ is bounded, where $\gamma_v = \max_{w \in I(v)} \frac{r_v}{r_w}$, and $I(v) = \{w \mid \overrightarrow{wv} \in \overrightarrow{EYG}_k(MG)\}$.

Algorithm 4: Constructing-EYY

1) Each node finds the incident edges in the Extended Yao graph $\overrightarrow{EYG}_k(MG)$, as described in Algorithm 3.
2) Each node v partitions the k cones centered at v using the partitioning method described in Method 1. Notice that for partitioning, node v uses parameter γ_v in Method 1, which can be easily calculated from local information. Figure 5 (a) illustrates such a partition.
3) Each node v chooses a node u from each generated region so that the link $\overrightarrow{uv}$ has the smallest $ID(uv)$ among all directed links toward to v computed in the first step in

the partition. Figure 5 (b) illustrates such a selection of incoming links.

4) Finally, for each link uv selected by v, node v informs node u of keeping link uv.

The union of all chosen links is the final network topology, denoted by $\overrightarrow{EYY}_k(MG)$. We call it *extended Yao-Yao* graph. Let $EYY_k(MG)$ be the undirected graph by ignoring the direction of each link in $\overrightarrow{EYY}_k(MG)$.

Theorem 7: The out-degree of each node v in $\overrightarrow{EYY}_k(MG)$, $k \geq 6$, is bounded by k and the in-degree is bounded by $(3\lceil \log_2 \gamma_v \rceil + 2)k$, where $\gamma_v = \max_{w \in I(v)}(\frac{r_v}{r_w})$.

Proof: It is obvious that the out-degree of a node v is bounded by k because the out-degree bound of $\overrightarrow{EYG}_k(MG)$ is k and this algorithm does not add any directed link.

For the in-degree bound, as shown in Figure 2 (b), obviously, the number of triangle regions in each cone is $3h - 2$. Remember that $2^{h-2} < \gamma_v \leq 2^{h-1}$, which implies $h = 1 + \lceil \log_2 \gamma_v \rceil$. Thus, considering the cap region also, the in-degree of node v is at most $(3\lceil \log_2 \gamma_v \rceil + 2)k$. ∎

Let $\gamma = \max_v \gamma_v$. Obviously, the maximum node degree in graph $EYY_k(MG)$ is bounded by $(3\lceil \log_2 \gamma \rceil + 3)k$.

Notice that the extended Yao-Yao graph $EYY_k(MG)$ is a subgraph of the extended Yao graph $EYG_k(MG)$, thus, there are at most $k \cdot n$ edges in $EYY_k(MG)$. Thus, the total communications of Algorithm 4 is at most $O(k \cdot n)$, where each message has $O(\log n)$ bits. It is interesting to see that the communication complexity does not depend on γ at all.
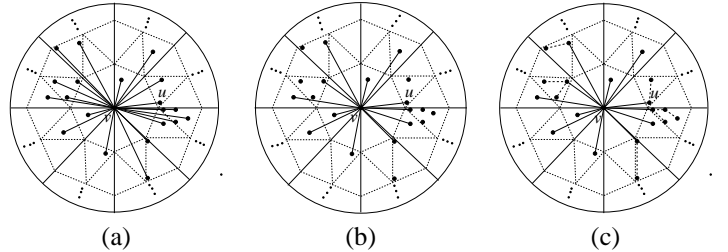

Fig. 5. (a) In $EYG_k(MG)$, star formed by links toward to v. (b) Node v chooses the shortest link in $EYG_k(MG)$ toward itself from each region to produce $EYY_k(MG)$. (c) The sink structure at v in $EYY^*_k(MG)$.

Theorem 8: The graph $EYY_k(MG)$, $k \geq 6$, is connected if MG is connected .

Proof: Notice that it is sufficient to show that there is a path from u to v for any two nodes with $uv \in MG$. Remember the graph $EYG_k(MG)$ is connected, therefore, we only have to show that $\forall uv \in EYG_k(MG)$, there is a path connecting u and v in $EYY_k(MG)$. We prove this claim by induction on the ranks of all links in $EYG_k(MG)$.

If the link uv has the smallest rank among all links of $EYG_k(MG)$, then uv will obviously survive after the second step. So the claim is true for the smallest rank.

Assume that the claim is true for all links in $EYG_k(MG)$ with rank at most r. Then consider a link uv in $EYG_k(V)$ with $rank(uv) = r+1$ in $EYG_k(MG)$. If uv survives in Algorithm 4, then the claim holds. Otherwise, assume that $r_u < r_v$. Then directed edge vu cannot belong to $\overrightarrow{EYG}_k(MG)$ from Algorithm 3. Thus, directed edge uv is in $\overrightarrow{EYG}_k(MG)$. In Algorithm 4, directed edge uv can only be removed by node v due to

the existence of another directed link wv with a smaller identity and w is in the same region as u. In addition, the angle $\angle wvu$ is less than $\theta = \frac{2\pi}{k}(k \geq 6)$. Therefore we have $\|wu\| < \|uv\|$. Notice that here wu is guaranteed to be a link in MG, but it is not guaranteed to be in $EYG_k(MG)$. We then prove by induction that there is a path connecting w and u in $EYY_k(MG)$. Assume $r_w \leq r_u$. The scenario $r_w > r_u$ can be proved similarly. There are two cases here.

Case 1: the link wu is in $EYG_k(MG)$. Notice that rank of wu is less than the rank of uv. Then by induction, there is a path $w \leftrightsquigarrow u$ connecting w and u in $EYY_k(MG)$. Consequently, there is a path (concatenation of the undirected path $w \leftrightsquigarrow u$ and the link wv) between u and v.

Case 2: the link wu is not in $EYG_k(MG)$. Then, from proof of Theorem 4, we know that there is a path $\Pi_{EYG_k}(w, u) = q_1 q_2 \cdots q_m$ from w to u in $EYG_k(MG)$, where $q_1 = w$ and $q_m = u$. Additionally, we can show that each link $q_i q_{i+1}$, $1 \leq i < m$, has a smaller rank than wu, which is at most r. Here $rank(q_1 q_2 = wq_2) < rank(w, u)$ because the selection method in Algorithm 3. And $rank(q_i q_{i+1}) < rank(w, u)$, $1 < i < m$, because

$$\|q_i q_{i+1}\| \leq \|q_i u\| < \|q_{i-1} u\| < \cdots < \|q_1 u\| = \|wu\|.$$

Then, by induction, for each link $q_i q_{i+1}$, there is a path $q_i \leftrightsquigarrow q_{i+1}$ survived in $EYY_k(MG)$ after Algorithm 4. The concatenation of all such paths $q_i \leftrightsquigarrow q_{i+1}$, $1 \leq i < m$, and the link wv forms a path from u to v in $EYY_k(MG)$. ■

Although $EYY_k(MG)$ is a connected structure, it is unknown whether it is a power or length spanner. We leave it as a future work.

D. Extended Yao-Sink Graph

In [3], [4], the authors applied the technique in [16] to construct a sparse network topology in UDG, *Yao and sink graph*, which has a bounded degree and a bounded stretch factor. The technique is to replace the directed star consisting of all links toward a node v by a directed tree $T(v)$ with v as the sink. Tree $T(v)$ is constructed recursively. To apply this technique on MG, we need extend it by a more sophisticated way. In the remainder of this section, we discuss how to locally construct a bounded degree structure with bounded power stretch factor for heterogeneous wireless ad hoc networks. Our method works as follows.

Algorithm 5: Constructing-EYG*

1) Each node finds the incident edges in the Extended Yao graph $\overrightarrow{EYG_k}(MG)$, as described in Algorithm 3. Each node v will have a set of incoming nodes $I(v) = \{u \mid \overrightarrow{uv} \in \overrightarrow{EYG_k}(MG)\}$.

2) Each node v partitions the k cones centered at v using the partitioning method described in Method 1. Notice that for partitioning, node v uses parameter γ_v in Method 1, which can be easily calculated from local information. Figure 5(a) illustrates such a partition.

3) Each node v chooses a node u from each region Ω. Let $\Omega_u(v)$ be the region Ω partitioned by node v with node u inside, so that the link uv has the smallest $ID(uv)$ among all links computed in the first step in the region $\Omega_u(v)$. In other words, in this step, it constructs $\overrightarrow{EYY_k}(MG)$.

4) For each region $\Omega_u(v)$ and the selected node u, let $S_\Omega(u) = \{w \mid w \neq u, \ w \in \Omega_u(v) \cap I(v)\}$, i.e., the set of incoming neighbors of v (other than u) in the same region as u. For each node u, node v uses the following function Tree($u,S_\Omega(u)$) (described in Algorithm 6) to build a tree $T(u)$ rooted at u. We call $T(u)$ a *sink tree* and call the union of all links chosen by node v the *sink structure* at v. Figure 5(c) illustrates a sink structure at v, which is composed of all trees $T(u)$ for u selected in the previous step.

5) Finally, node v informs nodes x and y for each selected link xy in the sink structure rooted at v.

The union of all chosen links is the final network topology, denoted by $EYG_k^*(MG)$. We call such structure as the *Extended Yao-Sink* graph. Notice that, sink node v, not u, constructs the tree $T(u)$ and then informs the end-nodes of the selected links to keep such links if already exist or add such links otherwise.

Algorithm 6: Constructing-Tree Tree($u,S_\Omega(u)$)

1) If $S_\Omega(u)$ is empty, then return.

2) Otherwise, partition the disk centered at u by k equal-sized cones: $\mathbb{C}_1(u), \mathbb{C}_2(u), \cdots, \mathbb{C}_k(u)$.

3) Find the node $w_i \in S_\Omega(u) \cap \mathbb{C}_i(u)$, $1 \leq i \leq k$, with the smallest $ID(w_i u)$, if there is any. Link $w_i u$ is added to $T(u, S_\Omega(u))$ and node w_i is removed from $S_\Omega(u)$.

4) For each node w_i, call Tree($w_i, S_\Omega(u) \cap \mathbb{C}_i(u)$) and add the created edges to $T(u, S_\Omega(u))$.

Notice that the above Algorithm 6 is only performed by a node v. We then prove that the constructed structure $\overrightarrow{EYG_k^*}(MG)$ indeed has bounded degree (thus sparse), and is power efficient.

Theorem 9: The maximum node degree of the graph $\overrightarrow{EYG_k^*}(MG)$ is at most $k^2 + 3k + 3k \cdot \lceil \log_2 \gamma \rceil$.

Proof: Initially, each node has at most k out-degrees after constructing graph $EYG_k(MG)$. In the algorithm, each node v initiates only one sink structure, which will introduce at most $(3\lceil \log_2 \gamma \rceil + 2) \cdot k$ in-degrees. Additionally, each node x will be involved in Algorithm 6 for at most k sink trees (once for each directed link $xy \in EYG_k(MG)$). For each sink tree involvement, Algorithm 6 assigns at most k links incident on x. Thus, at most k^2 new degrees could be introduced here. Then the theorem follows. ■

Since the total number of edges is at most $(k^2 + 3k + 3k \cdot \lceil \log_2 \gamma \rceil) \cdot n$, the total communication cost of our method is $O(\log_2 \gamma \cdot n)$. Here each message has $O(\log n)$ bits.

Theorem 10: The length stretch factor of $EYG_k^*(MG)$, $k > 6$, is at most $(\frac{1}{1-2\sin(\frac{\pi}{k})})^2$.

Proof: We have proved that $EYG_k(MG)$ has length stretch factor at most $\frac{1}{1-2\sin(\frac{\pi}{k})}$. We thus have only to prove that, for each link $vw \in EYG_k(MG)$, there is a path connecting them in $EYG_k^*(MG)$ with length at most $\frac{1}{1-2\sin(\frac{\pi}{k})}\|vw\|$. If link vw is kept in $EYG_k^*(MG)$, then this is obvious. Otherwise, assume $r_w \leq r_v$, then directed link wv belongs to $\overrightarrow{EYG_k}(MG)$. Then, there must exist a node u in the same

region (partitioned by node v) as node w. Using the same argument as Theorem 4, we can prove that there is a path connecting v and w in $T(u)$ with length at most $\frac{1}{1-2\sin(\frac{\pi}{k})}\|vw\|$. It implies that the length stretch factor of $EYG_k^*(MG)$ is at most $(\frac{1}{1-2\sin(\frac{\pi}{k})})^2$. ∎

Similarly, we have:

Theorem 11: The power stretch factor of the graph $EYG_k^*(MG)$, $k > 6$, is at most $(\frac{1}{1-(2\sin\frac{\pi}{k})^\beta})^2$.

VII. SIMULATIONS

In this section we measure the performance of the proposed heterogeneous network topologies by conducting extensive simulations. In our simulations, we randomly generate a set V of n wireless nodes with random transmission range for each node. We then construct the mutual inclusion communication graph $MG(V)$, and test the connectivity of $MG(V)$. If it is connected, we construct different localized topologies: $GG(MG)$, $EGG(MG)$, $RNG(MG)$, $ERNG(MG)$, $EYG_k(MG)$, $EYY_k(MG)$ and $EYG_k^*(MG)$. Then we measure the sparseness (the average node degree), the power efficiency and the communication cost of building these topologies. In the simulation results presented here, the wireless nodes are distributed in a $400m \times 400m$ square field. Each wireless node has a transmission radius randomly selected from $[60m, 260m]$. The number of wireless nodes is $30i$, where i is varied from 1 to 10. For each $1 \le i \le 10$, we randomly generate 100 sets of $30i$ nodes. All structures proposed in this paper are generated for each set of nodes. The number of cones is set to 7 for $EYG_k(MG)$, $EYY_k(MG)$ and $EYG_k^*(MG)$.

A. Node Degree

First of all, we want to test the sparseness of each network topology proposed in this paper. Notice that, we have theoretically proved that $RNG(MG)$ and $ERNG(MG)$ have at most $6n$ links; $EYG_k(MG)$ has at most $k \cdot n$ links, where $k \ge 7$ is the number of cones divided; $EYY_k(MG)$ also has at most $k \cdot n$ links since $EYY_k(MG) \subseteq EYG_k(MG)$; $EYG_k^*(MG)$ also has at most $k \cdot n$ links since the sink structure for each node u has exactly the number of links as the links toward u in the directed structure $EYG_k(MG)$. We do not know how many links $GG(MG)$ and $EGG(MG)$ could have.

Although almost all proposed structures are sparse theoretically, all of them could have unbounded node degree. The node degree of the wireless networks should not be too large. Otherwise a node with a large degree has to communicate with many nodes directly. This potentially increases the signal interference and the overhead at this node. Figure 6 (a) illustrates the average node degree of different topologies. Notice that graph $RNG(MG)$ always has the smallest average node degree in our simulations and structure $EYG_k^*(MG)$ always has the largest average node degree. We also found that the average node degree becomes almost stable when the number of nodes increases, i.e., the network becomes denser.

Figure 6 (b), as proved in Theorem 9, confirms that the maximum node degree of Yao-based structure $EYG_k^*(MG)$ is bounded by $3k \cdot \log_2 \gamma + k^2 + 3k$, where $\gamma = \max_{uv \in MG} \frac{r_u}{r_v}$.

The most upper curve in Figure 6 (b) represents the maximum $3k \cdot \log_2 \gamma$. This figure also shows that $EYG_k(MG)$ generally will have a larger maximum node degree than $EYG_k^*(MG)$ and $EYY_k(MG)$. It is interesting to see that the maximum degree of $EYG_k^*(MG)$ and $EYY_k(MG)$ almost have the same curve when network density changes. Given the size of the network $n = 30i$, we take the average of the maximums of all 100 random networks with n nodes we generated as the final maximum value for n plotted here.

B. Spanning ratio

We proved that $GG(MG)$ and $EGG(MG)$ have power spanning ratio exactly one; $EYG_k(MG)$ and $EYG_k^*(MG)$ both have bounded length and power spanning ratios. Notice that $RNG(MG)$ and $ERNG(MG)$ could have power and length spanning ratios as large as $n-1$ for a network of n nodes; and the length spanning ratios of $GG(MG)$ and $EGG(MG)$ could be $\sqrt{n-1}$ even when all nodes have the same transmission range. It is unknown whether $EYY_k(MG)$ has a bounded length or power spanning ratio even for wireless networks modelled by UDG. We then conduct extensive simulations to study how good these structures are for heterogeneous networks when the nodes' transmission ranges are randomly set.

Figure 6 (c) illustrates the length spanning ratio of these structures. As the theoretical results suggest, we found that $RNG(MG)$ has a much larger length spanning ratio compared with other structures. It is surprising to see that $ERNG(MG)$ also has a much smaller spanning ratio than $RNG(MG)$. We know that $ERNG(MG)$ has a smaller spanning ratio than $RNG(MG)$ since $ERNG(MG) \supseteq RNG(MG)$. Also notice that $EYG_k(MG)$, as the theoretical results suggest, has the smallest spanning ratio among all structures proposed here.

For wireless ad hoc networks, we want to keep as less links as possible while still keep relatively power efficient paths for every pair of nodes. Figure 6 (d) illustrates the power spanning ratio of these structures. Here we assume that the power needed to support a link uv is $\|uv\|^2$. As we expected, structures $GG(MG)$ and $EGG(MG)$ keep the most power efficient path for every pair of nodes, i.e., their power spanning ratios are exactly one. We found that all structures have power spanning ratio almost one, and again $RNG(MG)$ and $ERNG(MG)$ do have the largest power spanning ratios in our simulations.

C. Communication Cost of Construction

It is not difficult to see that $GG(MG)$, $RNG(MG)$, and $EYG_k(MG)$ can be constructed using only n messages by assuming that each node can tell its neighbors its maximum transmission range, and its geometry position information in one single message. Each node u can uniquely determine all the links uv in these three structures after knowing all its one hop neighbors in MG. Structures $EYG_k(MG)$, and $EYG_k^*(MG)$ can be constructed using only $k \cdot n + n$ messages since the final structures have at most kn links. Similarly, $ERNG(MG)$ can be constructed using at most $7n$ messages. We do not know any theoretical bound about the number of messages needed to construct $EGG(MG)$ since each node u has to inform its
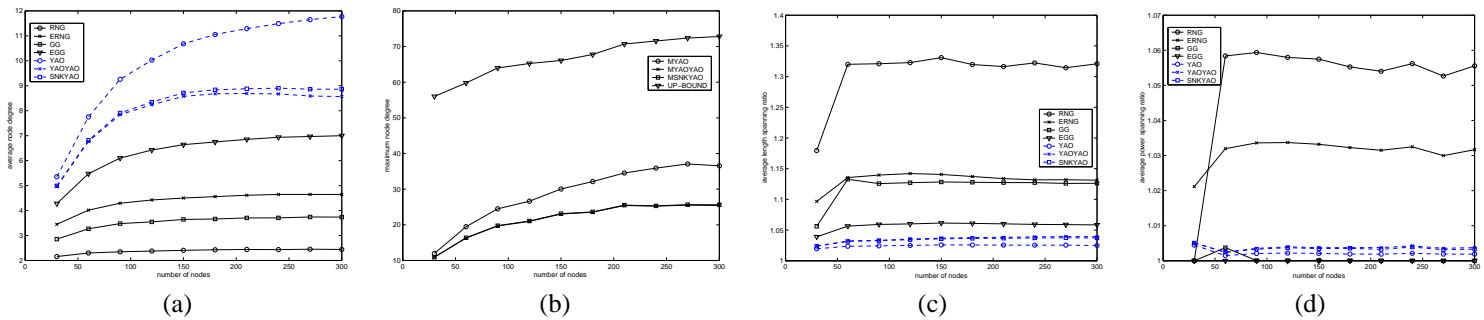
Fig. 6. (a) Average node degree of different topologies. (b) Maximum node degree of Yao-based structures. (c) Average length spanning ratio of different topologies. (d) Average power spanning ratio of different topologies.

neighbors the links selected by u for $EGG(MG)$. We measured the actual average number of messages needed to construct these structures. We only measure the average number of messages per wireless node for $EGG(MG)$, $ERNG(MG)$, $EYG_k(MG)$, and $EYG_k^*(MG)$ (since every node only has to spend one message for other three structures $GG(MG)$, $RNG(MG$, and $EYG_k(MG)$). Figure 7 illustrates the communication cost. We found that structure $EYG_k^*(MG)$ is the
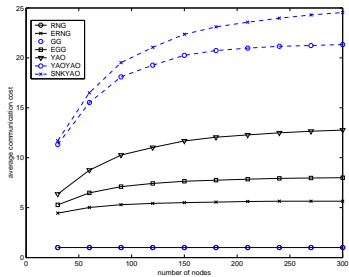

Fig. 7. Average communication cost of building different topologies.

most expensive one to construct although it has the most favorable properties theoretically (bounded length, power spanning ratio and bounded node degree). Constructing $EYG_k^*(MG)$ is almost as expensive as constructing $EYG_k(MG)$.

VIII. CONCLUSION

In this paper, we studied topology control in heterogeneous wireless ad hoc networks, where each mobile host has different maximum transmission power and two nodes are connected iff they are within the maximum transmission range of each other. We presented several strategies for all wireless nodes self-maintaining sparse and power efficient topologies in heterogeneous network environment with low communication cost. All structures $GG(MG)$, $RNG(MG)$, $EYG_k(MG)$, $EYY_k(MG)$, and $EYG_k^*(MG)$ are connected if MG is connected, while $EYG_k(MG)$ and $EYG_k^*(MG)$ have constant bounded power and length stretch factors. Additionally, we showed that $EYY_k(MG)$ and $EYG_k^*(MG)$ have bounded node degrees $O(\log_2 \gamma)$, where $\gamma = \max_{v \in V} \max_{w \in I(v)} \left(\frac{r_v}{r_w}\right)$. In the worst cast any connected graph will have degree at least $O(\log_2 \gamma)$ for heterogeneous wireless ad hoc networks. In other words, the structures constructed by our method are almost optimum. Our algorithms are all localized and have communication cost at most $O(n)$, where each message has $O(\log n)$ bits.

It remains an open problem whether graph $EYY_k(MG)$ is a length or power spanner. It is also unknown how many links $GG(MG)$ could have in the worst case although we show that it is definitely less than $O(n^{8/5} \log_2 \gamma)$. Some other future works are what are the conditions that we can build a structure with some other properties for MG, such as planar or low weight. Notice that it is easy to show we cannot build a planar topology for an arbitrary heterogeneous wireless ad hoc network.

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