Nash Equilibria and Dominant Strategies in Routing

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Abstract. Nash equilibria and dominant strategies are two of the major approaches to deal with selfishness in an automated system (AS), where each agent is a selfish entity.
In this paper, we consider the scenario when the receiver(s) and the relay links are both selfish, which generalizes the previous scenario in which either the relay links are selfish or the receivers are selfish. This also advances all previous studying in routing by taking into account the budget balance ratio. We prove that no mechanism can achieve budget balance ratio greater than $\frac{1}{n}$ when truthful revealing is a dominant strategy for each of the relay links and receivers. Here, n is the number of vertices in the network. In the meanwhile, we also present a mechanism that achieves the budget balance ratio $\frac{1}{n}$ and is truthful for both the receivers and relay links, which closes the bounds. When we relax the truthful revealing requirement to Nash Equilibrium for relay links, we present a mechanism that achieves an asymptotically optimal budget balance ratio.

1 Introduction

More and more research effort has been done to study the non-cooperative games recently. Among various forms of games, the unicast/multicast routing game [11, 4] and multicast cost sharing game [6, 2] have received a considerable amount of attentions over the past few years due to its applications in the Internet. However, both unicast/multicast routing game and multicast cost sharing game are one folded: the unicast/multicast routing game does not treat the receivers as selfish while the multicast cost sharing game does not treat the links as selfish. In this paper, we study the scenario, which we called *multicast system*, in which both the links and the receivers could be selfish.

In the first part, we study the α-*stable* multicast system that satisfies the following main properties: (1) strategyproofness for both the links and receivers; and (2) α-budget-balance. To illustrate our approaches, we first focus on the

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unicast system which is a special case of multicast system. We prove that if we use the least cost path for unicast routing, then there does not exist an α-stable unicast system such that $\alpha > \frac{1}{n}$, where n is the number of the nodes in the graph. On the other side, we present an $\frac{1}{n}$-stable unicast system, and further extend this idea to construct an $\frac{1}{r \cdot n}$-stable multicast system where r is the number of receivers in a multicast game.

In the second part, we relax the dominant strategy requirement to Nash Equilibrium for the links and study the performance of the Nash Equilibria for a multicast system. Again, we first study the unicast scenario and propose a unicast system that achieves $\frac{1}{2}$-budget-balance factor under any Nash Equilibrium. We then extend this to the multicast game which results in a multicast system with budget balance factor $\frac{1}{2r}$ under *any* Nash Equilibrium for the links.

2 Technical Preliminaries

2.1 Mechanism Design

A standard model for mechanism design is as follows. There are n agents $1, \dots, n$. Each agent i has some private information t_i, called its *type*, only known to itself. The agent' types define the *type vector* $t = (t_1, t_2, \dots, t_n)$. Each agent i has a set of strategies A_i from which it can choose. For each strategy vector $\mathbf{a} = (a_1, \dots, a_n)$ where agent i plays strategy $a_i \in A_i$, the *mechanism* $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ computes an *output* $o = \mathcal{O}(\mathbf{a})$ and a *payment* vector $\mathcal{P}(\mathbf{a}) = (\mathcal{P}_1(\mathbf{a}), \dots, \mathcal{P}_n(\mathbf{a}))$. A *valuation* function $v(t, o)$ assigns a monetary amount to agent i for each possible output o and t. Let $u_i(t, o)$ denote the *utility* of agent i at the output o and type vector t. Here, following a common assumption in the literature, we assume the utility for agent i is quasi-linear, i.e., $u_i(t, o) = v(t, o) + \mathcal{P}_i(\mathbf{a})$. Let $\mathbf{a}|^i b$ denote that every agent j, except i, plays strategy a_j, and agent i plays the strategy b. Let $\mathbf{a}_{-i}$ denote the strategies played by all agents other than i.

A strategy vector $a^\star$ is a *Nash Equilibrium* if it maximizes the utility of each agent i when the strategies of all the other agents are fixed as $a^\star_{-i}$, i.e., $u_i(t, \mathcal{O}(a^\star)) \geq u_i(t, \mathcal{O}(a^\star_{-i}|^i a'_i))$ for all i and all $a'_i \neq a^\star_i$. A strategy a_i is called a *dominant* strategy for agent i if it maximizes agent i's utility for all possible strategies of the other agents. If $\mathbf{a}$ is a dominant strategy vector for agents, then $\mathbf{a}$ is also a Nash Equilibrium.

A *direct-revelation* mechanism is a mechanism in which the only actions available to each agent are to report its private type. A direct-revelation mechanism is *incentive compatible* (IC) if reporting valuation truthfully is a dominant strategy. A direct-revelation mechanism satisfies *individual rationality* (IR) if the agent's utility of participating in the output of the mechanism is at least its utility if it did not participate the game at all. A direct-revelation mechanism is *truthful* or *strategyproof* if it satisfies both IC and IR properties.

A *binary demand game* is a game $\mathcal{G}$ such that (1) the range of the output method $\mathcal{O}$ is $\{0, 1\}^n$; (2) the valuation of the agents are not *correlated.* Binary demand game has been studied extensively [5, 1, 7, 4] and the type t_i of agent i

could be expressed as the cost c_i in many applications. Here, if agent i provides a certain service, then its cost $c_i \geq 0$; if agent i requires a certain service, then its cost $c_i \leq 0$. It is generally known that [5, 1, 7, 4] if a mechanism $\mathcal{M}$ is strategyproof, then $\mathcal{O}$ should satisfy a certain *monotonicity* property: for every agent i, if it is selected when it has a cost c_i, then it is still selected when it has a cost $c_i' < c_i$. If $\mathcal{O}_i(\mathbf{c}) = 0$, we require that $\mathcal{P}_i(\mathbf{c}) = 0$, which is known as *normalization*. If $\mathcal{O}$ is monotonic and the payment scheme is normalized, then the *only* strategyproof mechanism based on $\mathcal{O}$ is to pay $\kappa_i(\mathbf{c})$ to agent i if it is selected and 0 otherwise, where $\kappa_i(\mathbf{c})$ is the threshold cost of i being selected.

2.2 Multicast Payment Sharing Mechanism

In this paper, we model a network by a link weighted graph $G = (V, E, c)$, where V is the set of all nodes and c is the cost vector of the set E of links. For a multicast session, let Q denote the set of all receivers. In game theoretical networking literatures, usually there are two models for the multicast cost/payment sharing.

Axiom Model (AM) All receivers must receive the service, or equivalently, each receiver has an infinity valuation [3]. In this model, we are interested in a sharing method ξ that computes how much each receiver should pay when the receiver set is R and cost vector is $\mathbf{c}$.

Valuation Model (VM) There is a set $Q = \{q_1, q_2, \cdots, q_r\}$ of r possible receivers. Each receiver $q_i \in Q$ has a valuation η_i for receiving the service. Let $\eta = (\eta_1, \eta_2, \ldots, \eta_r)$ be the valuation vector and η_R be the valuation vector of a subset $R \subseteq Q$ of receivers. In this model, we are interested in a sharing mechanism $\mathcal{S}$ consisting of a *selection scheme* $\sigma(\eta, \mathbf{c})$ and a *sharing method* $\xi(\eta, \mathbf{c})$. Here $\sigma_i(\eta, \mathbf{c}) = 1$ (or 0) denotes that receiver i receives (or does not receive) the service, and $\xi_i(\eta, \mathbf{c})$ computes how much the receiver q_i should pay for the multicast service. Let $\mathbb{P}(\eta, \mathbf{c})$ be the total payment for providing the service to the receiver set. For the notational consistency, we denote the sharing method and total payment under AM as $\xi(\eta_R^{=\infty}, \mathbf{c})$ and $\mathbb{P}(\eta_R^{=\infty}, \mathbf{c})$, where $\eta_R^{=\infty}$ denotes a valuation vector where each individual valuation is infinity. The utility of a receiver i is denoted as $u_i(\eta, \mathbf{c})$.

In the valuation model, a receiver who is willing to receive the service is not guaranteed to receive the service. For notational simplicity, we abuse the notations by letting $\sigma(\eta, \mathbf{c})$ be the set of actual receivers decided by the selection method σ. Under the Valuation Model, we need to find a sharing mechanism that is *fair* according to the following criteria.

1. **Budget Balance** (BB): For the receiver set $R = \sigma(\eta, \mathbf{c})$, $\mathbb{P}(\eta, \mathbf{c}) = \sum_{q_i \in Q} \xi_i(\eta, \mathbf{c})$. If $\alpha \cdot \mathbb{P}(\eta, \mathbf{c}) \leq \sum_{i \in R} \xi_i(\eta, \mathbf{c}) \leq \mathbb{P}(\eta, \mathbf{c})$, for some given parameter $0 < \alpha \leq 1$, then $\mathcal{S} = (\sigma, \xi)$ is called α-budget-balance. If budget balance is not achievable, then a sharing scheme $\mathcal{S}$ may need to be α-budget-balance.
2. **No Positive Transfer** (NPT): Any receiver q_i's sharing should not be negative. In other words, we don't pay the receiver to receive.
3. **Free Leaving** (FR): The potential receivers who do not receive the service should not pay anything, *i.e.*, if $\sigma_i(\eta, \mathbf{c}) = 0$, then $\xi_i(\eta, \mathbf{c}) = 0$.

4. **Consumer Sovereignty** (CS): For any receiver q_i, if η_i is sufficiently large, then q_i is guaranteed to be an actual receiver. In other words, fix any η_{-i}, there must exist a valuation x for q_i such that $\forall y \geq x$, $\sigma_i((y, \eta_{-i}), \mathbf{c}) = 1$.
5. **Group-Strategyproof** (GS): Assume that η is the valuation vector and $\eta' \neq \eta$. If $u_i(\eta', \mathbf{c}) \geq u_i(\eta, \mathbf{c})$ for each $q_i \in \eta$, then $u_i(\eta', \mathbf{c}) = u_i(\eta, \mathbf{c})$.

A sharing mechanism $\mathcal{S}$ that is α-budget-balance and satisfies the remaining criteria (*i.e.*, NPT, FR, CS, GS) is *α-fair*. It is generally known that if a sharing method ξ satisfies cross-monotonicity and NPT under Axiom Model, one can explicitly construct a fair sharing mechanism $\widetilde{\mathcal{S}} = (\widetilde{\sigma}, \widetilde{\xi})$ as shown in [6, 2].

2.3 Problem Statement

Assume there is a graph $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$. Let $\mathbf{c}$ be the cost vector of the links, *i.e.*, $c(e_i) = c_i$ and $\mathbf{c} = (c_1, c_2, \ldots, c_m)$. Given a source s and a set of receiver Q, multicast first chooses a receiver set $R \subseteq Q$, and then constructs a tree rooted at s that spans the receivers set R. In this paper, we focus on the *least cost path tree* (LCPT), which is the union of the least cost paths from the source to each receiver, due to the reason that LCPT is most widely used in practice. Let $\mathsf{LCPT}(R, \mathbf{c})$ be the LCPT when the cost vector is $\mathbf{c}$ and the actual receiver set is R. We are interested in a *multicast system* $\Psi = (\mathcal{M}, \mathcal{S})$ consisting of a mechanism $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ and a sharing scheme $\mathcal{S} = (\sigma, \xi)$. A multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ is *α-stable* if it satisfies that (1) $\mathcal{M}$ is strategyproof (2) $\mathcal{S} = (\sigma, \xi)$ is α-fair for some α. For any Nash Equilibrium (NE) $\widetilde{\mathbf{b}}$ for links, if $\alpha \cdot \mathbb{P}(\eta, \widetilde{\mathbf{b}}) \leq \sum_{q_i \in Q} \xi_i(\eta, \widetilde{\mathbf{b}}) \leq \mathbb{P}(\eta, \widetilde{\mathbf{b}})$, for some given parameter $\alpha \leq 1$, then $\mathcal{S} = (\sigma, \xi)$ is α-NE-budget-balance. Comparing with definition of α-budget-balance, we replace the actual cost vector $\mathbf{c}$ with any NE $\widetilde{\mathbf{b}}$ for the links. Similarly, we have the definition Nash Equilibrium Consumer Sovereignty (NE-CS).

A sharing scheme $\mathcal{S}$ is *NE-strategyproof* if $\eta_i - \xi_i(\eta, \widetilde{\mathbf{b}}) \geq \eta_i' - \xi_i(\eta|^i\eta_i', \widetilde{\mathbf{b}}')$ for any receiver i, any valuation $\eta_i' = \eta_i$, and any NE $\widetilde{\mathbf{b}}$ for the links under η and any NE $\widetilde{\mathbf{b}}'$ for the links under $\eta|^i\eta_i'$. In other words, receiver q_i can not increase its utility by falsely declaring its valuation to affect the NE of the links under any circumstance. $\mathcal{S}$ is α-NE-fair if it is NE-strategyproof and satisfies NE-CS, NPT and FR. A multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ is α-NE-stable if it satisfies that (1) there exists a NE for the links; (2) $\mathcal{S}$ is α-NE-fair under any NE $\widetilde{\mathbf{b}}$. If there is only one receiver, which we assume to be q_1, then it is a *unicast system*, which is a special case of multicast system.

Following we present some notations that are used in this paper.

Notations: The path with the lowest cost between two odes s and t is denoted as $\mathsf{LCP}(s, t, \mathbf{c})$, and its cost is dented as $|\mathsf{LCP}(s, t, \mathbf{c})|$. Given a simple path P in the graph G with cost vector $\mathbf{c}$, the sum of the cost of links on path P is denoted as $|\mathsf{P}(\mathbf{c})|$. For a simple path $P = v_i \rightsquigarrow v_j$, if $\mathsf{LCP}(s, t, \mathbf{c}) \bigcap P = \{v_i, v_j\}$, then P is called a *bridge* over $\mathsf{LCP}(s, t, \mathbf{c})$. This bridge P *covers* link e_k if $e_k \in \mathsf{LCP}(v_i, v_j, \mathbf{c})$. Given a link $e_i \in \mathsf{LCP}(s, t, \mathbf{c})$, the path with the minimum cost that covers e_i is denoted as $\mathrm{B}_{\min}(e_i, \mathbf{c})$. We call the bridge $\mathrm{B}_{mm}(s,t,\mathbf{c}) = \max_{e_i \in \mathsf{LCP}(s,t,\mathbf{c})} \mathrm{B}_{\min}(e_i, \mathbf{c})$ the *max-min cover* of the path $\mathsf{LCP}(s,t,\mathbf{c})$.

A bridge set $\mathscr{B}$ is a *bridge cover* for $\mathsf{LCP}(s,t,\mathbf{c})$, if for every link $e_i \in \mathsf{LCP}(s,t,\mathbf{c})$, there exists a bridge $\mathrm{B} \in \mathscr{B}$ such that $e_i \in \mathsf{LCP}(v_{s(\mathrm{B})}, v_{t(\mathrm{B})}, \mathbf{c})$. The *weight* of a bridge cover $\mathscr{B}(s,t,\mathbf{c})$ is defined as $|\mathscr{B}(s,t,\mathbf{c})| = \sum_{B \in \mathscr{B}(s,t,\mathbf{c})} \sum_{e_i \in B} c_i$. Notice that a link may be counted multiple times here. A bridge cover $\mathscr{B}$ is a *minimal bridge cover* (MBC), if for each bridge $B \in \mathscr{B}$, $\mathscr{B} - B$ is not a bridge cover. A bridge cover is a *least bridge cover* (LB), denoted by $\mathbb{LB}(s,t,\mathbf{c})$, if it has the smallest weight among all bridge covers that cover $\mathsf{LCP}(s,t,\mathbf{c})$.

3 Dominant Strategies and Multicast Systems

In this section, we study how to design a multicast system that is α-stable with large α. We present some results on both the negative and positive sides.

3.1 α-stable Unicast System

Unicast routing [9] may be one of the introductory problems that bring algorithm mechanism design to the attention of the computer scientists. Fortunately, the unicast routing problem is solved by using the celebrated VCG mechanism in the seminal paper [9] by Nisan and Ronen. However, one important question that has not been addressed in any previous literatures is who is going to pay the payments to the agents. By assuming that the unicast routing is receiver-driving, the very naive way is that the receiver should pay the payment.

We use $\mathcal{P}^{\mathsf{UVCG}}$ to denote the VCG payment for unicast under AM. The payment to a link $e_k \in \mathsf{LCP}(s, q_1, \mathbf{c})$ according to VCG mechanism is $\mathcal{P}_k^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}) = |\mathsf{LCP}(s, q_1, \mathbf{c}|^k\infty)| - |\mathsf{LCP}(s, q_1, \mathbf{c}|^k 0)|$. The payment to link that is not on the LCP is 0. Simply applying the sharing scheme $\widetilde{\mathcal{S}}$ obtains the unicast system Ψ^{VCG} as follows: first computing $\mathbb{P}^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{d})$, q_1 is charged $\mathbb{P}^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{d})$ and receives the service if $\eta_1 \geq \mathbb{P}^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{d})$; q_1 is charged 0 and does not receive the service otherwise. Each link receives its VCG payment if q_1 receives the service and 0 otherwise. Regarding the unicast system Ψ^{VCG}, we have:

Theorem 1. *For unicast system $\Psi^{\mathsf{VCG}} = (\mathcal{M}^{\mathsf{VCG}}, \mathcal{S}^{\mathsf{VCG}})$, $\mathcal{S}^{\mathsf{VCG}}$ is fair. However, $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof.*

Proof. Obviously, $\mathcal{S}^{\mathsf{VCG}}$ is fair. We then show that $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof by giving a counter example in Figure 1. Consider the graph in Figure 1 in which $c_i = 1$ for $1 \leq i \leq k$, $c_{k+1} = a \cdot k$ and the valuation for receiver q_1 is $a \cdot k + 1$. The VCG payment to link e_i is $a \cdot k - k + 1$ for $1 \leq i \leq k$ and the total payment to all links is $k(a \cdot k - k + 1)$. Thus, if every link e_i reveals its true cost, the receiver will reject the service. Consequently, every link receives payment 0 and has a utility 0. Consider the scenario when link e_1 reports its cost as $a \cdot k + 1 - \epsilon$ for a small positive ϵ. The total payment to all links is $a \cdot k + 1$ when $\epsilon = \frac{1}{k-1}$. Then, receiver q_1 accepts the service and pay $a \cdot k + 1$. Consequently, link e_1 receives a payment $k + 1$, and its utility is k. This violates the IC property. Thus, $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof.

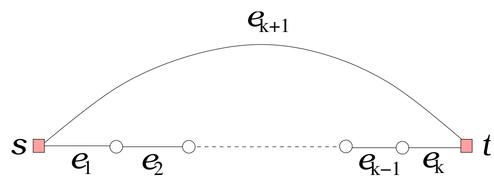

Fig. 1. $\mathcal{M}^{\mathsf{VCG}}$ is not strategyproof: link e_1 can lie to increase its utility.

If we take both the links and the receiver q_1 into account as agents, then the unicast system is still a binary demand game. Thus, we have the following lemma which is a simple application of the binary demand game [5, 1, 7, 4].

Lemma 1. *If $\Psi = (\mathcal{M}, \mathcal{S})$ is a unicast system that is α-stable, then*

1. *There exists a function $\zeta(\mathbf{c})$ such that (a) $\sigma_1(\eta_1, \mathbf{c}) = 1$ if and only if $\eta_1 \geq \zeta(\mathbf{c})$, (b) $\xi_1(\eta_1, \mathbf{c}) = \zeta(\mathbf{c})$ when $\sigma_1(\eta_1, \mathbf{c}) = 1$,*
2. *If $c_j < c'_j < \mathcal{P}_j^{\mathsf{UVCG}}(q_1, c)$, then $\zeta(c) \leq \zeta(c|^j c'_j)$.*

The proof of this lemma is omitted due to space limit. Based on Lemma 1, the following theorem reveals a negative result of α-stable unicast system.

Theorem 2. *If $\Psi = (\mathcal{M}, \mathcal{S})$ is an α-stable unicast system, then $\alpha \leq \frac{1}{n}$.*

Proof. We prove it by presenting an example graph in Figure 1. Consider the cost vector $c^{(1)} = c|^1(ak - k + 1 - \epsilon)$. From Lemma 1, the sharing by q_1 is $\xi_1(\eta_1, \mathbf{c}) = \zeta(\mathbf{c})$ and

$$\zeta(c) \leq \zeta(c^{(1)}) \tag{1}$$

Recall that for any valuation $\eta_1 \geq \zeta(c^{(1)})$, $\sigma_1(\eta_1, \mathbf{c}^{(1)}) = 1$, *i.e.*, the LCP between $\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)})$ is selected. Since mechanism $\mathcal{M}$ is strategyproof, the payment to link e_i is a threshold value [4] $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)})$ which does not depend on c_i. Now we prove by contradiction that $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) \leq \mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, c^{(1)})$. For the sake of contradiction, assume that $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) > \mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}^{(1)})$. Recall that $\mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}^{(1)}) = |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i\infty)| - |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i 0)|$. However, when we set the cost of e_i as $\hat{\mathbf{c}}_i = |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i\infty)| - |\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)}|^i 0)| + \delta$ for a sufficiently small positive value $\delta < \kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) - \mathcal{P}_i^{\mathsf{UVCG}}(\eta_1^{=\infty}, \mathbf{c}^{(1)})$, e_i is still on path $\mathsf{LCP}(s, q_1, \mathbf{c}^{(1)})$, which is a contradiction. Thus, for graph shown in Figure 1, $\kappa_i(\eta_1, \mathbf{c}_{-i}^{(1)}) \leq 1 + \epsilon$ for $2 \leq i \leq k$ and $\kappa_1(\eta_1, \mathbf{c}_{-1}^{(1)}) = a \cdot k - k + 1$. Since Ψ is a binary demand game, $\mathcal{P}_i(\eta_1, c^{(1)}) = \kappa_i(\eta_1, c_{-i}^{(1)})$. Thus, the total payment to all links is $\mathbb{P}(\eta_1, \mathbf{c}^{(1)}) = \sum_{e_i} \mathcal{P}_i(\eta_1, \mathbf{c}^{(1)}) \leq a \cdot k - k + 1 + (k-1) \cdot (1 + \epsilon) = a \cdot k + (k-1) \cdot \epsilon$. Recall that $\mathcal{S}$ is α-budget-balance, then $\zeta(\mathbf{c}^{(1)}) \leq \mathbb{P}(\eta_1, \mathbf{c}^{(1)}) \leq a \cdot k + (k-1) \cdot \epsilon$. By combining Inequality (1) and the above inequality, we have $\zeta(c) \leq \zeta(c^{(1)}) \leq a \cdot k + (k-1) \cdot \epsilon$. Similarly, let cost vector $\mathbf{c}^{(i)}$ be $\mathbf{c}|^i(a \cdot k + 1 - \epsilon)$ for $1 \leq i \leq k$. Let χ be a large positive number such that $\chi \geq \max_{1 \leq i \leq k} \xi(\mathbf{c}^{(i)})$. Consider the cost vector c and receiver valuation χ, we argue that $\kappa_i(\chi, \mathbf{c}_{-i}) \geq ak - k + 1 - \epsilon$

for any $1 \leq i \leq k$. Considering any link e_i, $1 \leq i \leq k$, if it reports its cost as $ak - k + 1 - \epsilon$, then the cost vector is $\mathbf{c}^{(i)}$. From the way we choose the valuation χ, the receiver q_1 will receive the service. Thus, e_i is also selected. From IR, $\kappa_i(\chi, \mathbf{c}_{-i}) = \mathcal{P}_j(\chi, \mathbf{c}) = \mathcal{P}_j(\chi, \mathbf{c}^{(i)}) \geq ak - k + 1 - \epsilon$. Therefore, $\mathbb{P}(\chi, \mathbf{c}) = \sum_{e_i} \mathcal{P}_i(\chi, \mathbf{c}) \geq k \cdot (ak - k + 1 - \epsilon)$. This obtains that

$$\alpha \leq \frac{\zeta(c)}{\mathbb{P}(\chi, \mathbf{c})} \leq \frac{ak + (k-1) \cdot \epsilon}{k \cdot (ak - k + 1 - \epsilon)}.$$

Let $\epsilon \to 0$, $a \to \infty$ and $k = n$, then $\alpha \leq \frac{1}{n}$. This finishes our proof.

Theorem 2 gives an upper bound for α for any α-stable unicast system Ψ. It is not difficult to observe that even the receiver q_1 is cooperative, Theorem 2 still holds. Following we present an $\frac{1}{n}$-stable unicast system that is based on the max-min cover of the LCP.

Algorithm 1 An $\frac{1}{n}$-stable unicast system $\Psi^{\mathsf{DU}} = (\mathcal{M}^{\mathsf{DU}}, \mathcal{S}^{\mathsf{DU}})$.

1: Compute $\mathsf{LCP}(s, q_1, \mathbf{d})$, and set $\phi = \omega(\mathrm{B}_{mm}(s, t, \mathbf{d}), \mathbf{d})$.
2: **if** $\eta_1 \geq \phi$ **then**
3:     Each link $e_k \in \mathsf{LCP}(s, q_1, \mathbf{d})$ is selected and receives a payment $\mathcal{P}_k^{\mathsf{UVCG}}(\eta_1^{=\infty}, c)$; all other links are not selected and get a payment 0.
4:     Receiver q_1 is granted the service and charged ϕ.
5: **else**
6:     All links are not selected and each link receives a payment 0.
7:     Receiver is not granted the service and is charged 0.

Theorem 3. *Unicast system* $\Psi^{DU} = (\mathcal{M}^{DU}, \mathcal{S}^{DU})$ *is* $\frac{1}{n}$*-stable.*

The proof is omitted here due to space limit. Theorem 3 closes the gap between the upper and lower bound by presenting a tight bound $\frac{1}{n}$ for the budget balance factor α for unicast.

3.2 Multicast System

In Section 3.1, we consider how to construct a unicast system $\Psi = (\mathcal{M}, \mathcal{S})$ such that $\mathcal{M}$ is strategyproof and $\mathcal{S}$ is α-budget-balance with a large budge balance factor α. In this section, we discuss how to construct a multicast system. Under Axiom Model, Wang *et al.* [11] gave a strategyproof multicast mechanism $\mathcal{M}^{\mathsf{LCPT}} = (\mathcal{O}^{\mathsf{LCPT}}, \mathcal{P}^{\mathsf{LCPT}})$. For a link $e_k \in \mathsf{LCPT}(R, \mathbf{c})$, they compute an intermediate payment $\mathcal{P}_k^{\mathsf{UVCG}}(\eta_i^{=\infty}, \mathbf{c}) = |\mathsf{LCP}(s, q_i, \mathbf{c}|^k\infty)| - |\mathsf{LCP}(s, q_i, \mathbf{c}|^k 0)|$ to link e_k based on each downstream receiver q_i of e_k on the LCPT tree. The final payment to a link $e_k \in \mathsf{LCPT}(R, \mathbf{c})$ is $\mathcal{P}_i^{\mathsf{LCPT}}(\eta_R^{=\infty}, \mathbf{c}) = \max_{q_j \in R} \mathcal{P}_i^{\mathsf{UVCG}}(\eta_j^{=\infty}, \mathbf{c})$, where $\eta_R^{=\infty}$ is the valuation vector such that $\eta_i = \infty$ if $q_i \in R$ and 0 otherwise. They also present a payment sharing scheme based on $\mathcal{M}^{\mathsf{LCPT}}$ that is reasonable

[10]. By generalizing the unicast system Ψ^{DU}, we present a multicast system Ψ^{DM} (illustrated in Algorithm 2) based on the tree LCPT. Here, DM stands for the multicast system with dominant strategy requirement for the links.

Algorithm 2 Multicast system $\Psi^{\mathsf{DM}} = (\mathcal{M}^{\mathsf{DM}}, \mathcal{S}^{\mathsf{DM}})$ based on tree LCPT.

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Compute path LCP(s, q_j, d) and set φ_j = ω(B_mm(s, q_j, d), d) / r for every q_j ∈ Q.
Set O_i^DM(η, d) = 0 and P_i^DM(η, d) = 0 for each link e_i ∉ LCP(s, q_j, d).
for each receiver q_j do
  if η_j ≥ φ_j then
    Receiver q_j is granted the service and charged ξ_j^DM(η, d), set R = R ∪ q_j.
  else
    Receiver q_j is not granted the service and is charged 0.
Set O_i^DM(η, d) = 1 and P_i^DM(η, d) = P_i^LCPT(η_R̄^=∞, d) for each link e_i ∈ LCPT(R, d).
```

Where $\phi_j = \frac{\omega(\mathrm{B}_{mm}(s,q_j,\mathbf{d}),\mathbf{d})}{r}$, and in the last step $\mathcal{O}_i^{\mathsf{DM}}(\eta,\mathbf{d}) = 1$ and $\mathcal{P}_i^{\mathsf{DM}}(\eta,\mathbf{d}) = \mathcal{P}_i^{\mathsf{LCPT}}(\eta_{\overline{R}}^{=\infty},\mathbf{d})$ for each link $e_i \in \mathsf{LCPT}(R,\mathbf{d})$.

Theorem 4. *The multicast system $\Psi^{DM} = (\mathcal{M}^{DM}, \mathcal{S}^{DM})$ is $\frac{1}{r \cdot n}$-stable, where r is the number of receivers.*

The proof of this theorem is omitted here. Recall that the unicast system is a special case of multicast system. Thus, for any multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ that is α-stable, the budget balance factor is at most $\frac{1}{n}$. In this section, we propose a multicast system Ψ^{DM} that achieves a budge balance factor $\frac{1}{r \cdot n}$. It is of interests to find some multicast system $\Psi = (\mathcal{M}, \mathcal{S})$ that achieves a larger budget balance factor while $\mathcal{M}$ is strategyproof and $\mathcal{S}$ is α-fair. Our conjecture is that the upper bound of α is also $\Theta(\frac{1}{rn})$.

4 Nash Equilibrium and Multicast Systems

In light of the inefficiency of the multicast/unicast mechanism that is strategyproof for both links and receivers, it is natural to relax the dominant strategy to a weaker requirement – Nash Equilibrium. In this section, we study how to design multicast/unicast system that is α-NE-stable with a small additive ϵ.

4.1 Unicast Auction in Axiom Model

In this section, we disregard the receiver valuation and show how to design a mechanism that can induce some Nash Equilibria for links that can pay comparably smaller than the strategyproof mechanism does. Notice that, in [8], Immorlica *et al.* showed that if we simply pay whatever the link reports, which is known as the *first price auction*, there does not exist Nash Equilibrium. Due to the non-existence of the Nash Equilibrium, they propose a modified first price auction that can achieve ϵ-Nash Equilibrium with a small additive value. With further modification of the auction rules, we obtain a unicast auction that induces efficient Nash Equilibria. The high level idea of our unicast auction is as follows. We require the agents to bid two bids instead of one: the first bid vector $\mathbf{b}$ is used to find the LCP, the second bid vector $\mathbf{b}'$ is used to determine the payment. In the meanwhile, we also give a small "bonus" to all links such that each link e_i gets the maximum bonus when it reports its true cost, *i.e.*, $b_i = c_i$.

Algorithm 3 FPA Mechanism $\mathcal{M}^{\mathsf{AUC}} = (\mathcal{O}^{\mathsf{AUC}}, \mathcal{P}^{\mathsf{AUC}})$.

1: **for** each link $e_i \in G$ **do**
2: Set $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = f_i(s, q_1, \mathbf{b})$, where $f_i(s, q_1, \mathbf{b}) = \tau \cdot \left[b_u \cdot (n \cdot b_u - \sum_{e_j \in G - e_i} b_j) - \frac{h_i^2}{2}\right]$. Here, b_u is the maximum cost any link can declare.
3: Every link sends a unit size dummy packet and $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}})$ for every link $e_i \in G$ $\rho = \tau \cdot (n \cdot b_u - \sum_{e_i \in G} b_i)$.
4: Compute the unique path $\mathsf{LCP}(s, q_1, \mathbf{b}')$ by applying certain fixed tie-breaking rule consistently.
5: **for** each link e_i **do**
6: **if** $e_i \in \mathsf{LCP}(s, q_1, \mathbf{b}')$ **then**
7: Set $\mathcal{O}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = 1$ and $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = b'_i$.
8: **else**
9: Set $\mathcal{P}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) = {}_i^{\mathsf{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) - \gamma \cdot (b_i - b'_i)^2$.

Algorithm 3 sends the data along $\mathsf{LCP}(s, q_1, \mathbf{b})$ and broadcasts something to the network with a probability ρ. Following theorem (its proof is omitted here) reveals the existence and several properties of Nash Equilibria.

Theorem 5. *There exists NE for FPA mechanism $\mathcal{M}^{\mathsf{AUC}}$ and for any NE $\widetilde{\mathbf{b}} = \langle \mathbf{b}, \mathbf{b}' \rangle$ we have (1) $\mathbf{b} = \mathbf{c}$; (2) $\mathsf{LCP}(s, q_1, \mathbf{c}) = \mathsf{LCP}(s, q_1, \mathbf{b}')$; (3) For any $e_i \in \mathsf{LCP}(s, q_1, \mathbf{c})$, $|\mathsf{LCP}(s, q_1, \mathbf{b}')| = |\mathsf{LCP}(s, q_1, \mathbf{b}'|^i \infty)|$.*

Theorem 6. *Assume that $\widetilde{\mathbf{b}} = \langle \mathbf{b}, \mathbf{b}' \rangle$ is a NE of $\mathcal{M}^{\mathsf{AUC}}$ and ϵ is a fixed positive value, then by properly setting the parameter τ, $|\mathbb{LB}(s, q_1, \mathbf{c})| + \epsilon > \mathbb{P}^{\mathrm{AUC}}(\eta_1^{=\infty}, \widetilde{\mathbf{b}}) \geq \frac{|\mathbb{LB}(s, q_1, \mathbf{c})|}{2}$. Moreover, the inequalities are tight.*

4.2 Unicast and Multicast System in Valuation Model

Based on the Auction Mechanism Ψ^{AUC}, we design a unicast system that is $\frac{1}{2}$-NE-stable with small additive ϵ as follows: (1) Execute Line $1 - 2$ in Algorithm 3; (2) Compute $\mathbb{LB}(s, q_1, \mathbf{b})$, and set $\phi = \frac{|\mathbb{LB}(s, q_1, \mathbf{b})|}{2}$; (3) If $\phi \leq \eta_1$ then set $\sigma_1^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) = 1$ and $\xi_1^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) = \phi$. Every relay link on LCP is selected and receives an extra payment b'_i. (4) Set $\mathcal{P}_i^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) = \mathcal{P}_i^{\mathsf{AU}}(\eta_1, \widetilde{\mathbf{b}}) - \gamma \cdot (b'_i - b_i)^2$ for each link $e_i \notin \mathsf{LCP}(s, q_1, \mathbf{b}')$.

Theorem 7. *The unicast system $\Psi^{\mathsf{AU}} = (\mathcal{M}^{\mathsf{AU}}, \mathcal{S}^{\mathsf{AU}})$ has Nash Equilibria for links, and Ψ^{AU} is $\frac{1}{2}$-NE-stable with ϵ additive, for any given ϵ.*

With the unicast system Ψ^{AUC}, we can simply extend the unicast system to a multicast system by treating each receiver as a separate receiver and applying the similar process as in the unicast system Ψ^{AU}. Notice that the bid vector is $\widetilde{\mathbf{b}} = (\mathbf{b}, \mathbf{b}^{(1)\prime}, \mathbf{b}^{(2)\prime}, \ldots, \mathbf{b}^{(r)\prime})$. The details are omitted here due to the space limit. For more details, please refer [12].

5 Conclusion

In this paper, we study the multicast system in networks consisting of selfish, non-cooperative relay links and receivers. We first prove that no unicast system can achieve α-stable when $\alpha > \frac{1}{n}$. We then present a unicast system that is $\frac{1}{n}$-stable, which closes the bounds of the budget balance factor. We extend this idea to a multicast system that is $\frac{1}{rn}$-stable where r is the number of the receivers. We also consider how to relax the strategyproofness requirement for the links and propose the FPA mechanism in Axiom Model that provably reduces the inevitable overpayment by achieving Nash equilibrium for the relay links. Based on the FPA mechanism, we propose a unicast system and a multicast system that are $\frac{1}{2}$-NE-stable with ϵ additive and $\frac{1}{2r}$-NE-stable with ϵ respectively.

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