INFORMATION DISSEMINATION DELAY IN VEHICLE-TO-VEHICLE COMMUNICATION NETWORKS IN A TRAFFIC STREAM

LiLi Du
Department of Civil, Architectural, and Environmental Engineering
Illinois Institute of Technology
3300 South Federal Street, Chicago, IL 60616, USA
ldu3@iit.edu

Hoang Dao
Department of Civil, Architectural, and Environmental Engineering
Illinois Institute of Technology
3300 South Federal Street, Chicago, IL 60616, USA
hdao@hawk.iit.edu

Xiang-Yang Li
Department of Computer Science
Illinois Institute of Technology
10 West 31st Street, Chicago, IL 60616, USA
xli@cs.iit.edu

***Abstract*—Vehicle-to-vehicle (V2V) communication networks, as one of the core components in connected vehicle systems, have been granted many promising applications to address traffic mobility, safety and sustainability. However, only a limited amount of work has been completed to understand the fundamental properties of information propagation in such systems, while comprehensively considers traffic and communication reality. Motivated by this view, this proposed research develops analytical formulations to estimate information propagation time delay via a V2V communication network formed on a one-way or two-way road segment with multiple lanes. Distinguished to previous efforts, the proposed study carefully involves several critical communication and traffic flow features in reality, such as wireless communication interference, intermittent information transmission, and dynamic traffic flow. Moreover, this study elaborately analyzes the interactions between information and traffic flow under sparse and congested traffic flow conditions. The numerical experiments based on Next-Generation Simulation (NGSIM) field data illustrate that the proposed analytical formulations are able to provide very good estimation, with the relative error less than 5%, for the information propagation time delay on a one-way or two-way road segment under various traffic conditions. The proposed work can be further extended to characterize information propagation time delay and coverage over local transportation networks.**

Index Terms—***vehicle-to-vehicle communication, time delay, dynamic traffic flow***

I. INTRODUCTION

Connected vehicle systems (CVS), as a new generation of intelligent transportation system, is consisted of smart vehicles and roadside infrastructure equipped with wireless communication facilities, which enable vehicle-to-vehicle (V2V) and vehicle-to-infrastructure traffic information exchange. As one of the key components, V2V communication networks (i.e. vehicular ad hoc networks (VANET) in literature) have been granted many promising applications in traffic safety, mobility, and sustainability. For example: [1][2][3] demonstrate that V2V will allow drivers to aware of other cars' speed, acceleration or deceleration so that keep away from accidents such as road departure and collisions. Thus, V2V communication networks can be used to develop driver assistance systems which avoid traffic crash and improve driving safety. Alsabaan et al. [4] propose employing vehicle-to-vehicle as well as traffic-light-signal-to-vehicle communication technologies to enable drivers to adaptively adjust their driving speed so that the objectives to promote fuel conservation and emission reduction within transportation systems can be achieved. Yamaha, et al [5] shows that vehicle-to-vehicle communication network may detect road condition such as road surface status in snowing weather, and then informs traffic management center to adjust traffic control strategies and improve traffic mobility. In the field of traffic routing, V2V informs drivers traffic condition information such as work zone, accident ahead, closing lane, etc; so they can change route to avoid the waiting time. The community based online navigation system developed by Waze [6] has indicated a great potential of developing on-line routing guidance based on V2V communication technologies. At the meantime, many national and international projects such as PATH [7]; CarTalk [8]; FleetNet [9], have been dedicated to test the applicability of V2V technologies in various transportation scenarios.

Even though many dramatic applications have been proposed, it is noticed that the availability of real-time traffic information propagating via V2V communication is one of the critical bottlenecks to limit their implementation in practice. For example, forward collision warning applications based on V2V need the information regarding how fast a warning can be propagated to a vehicle; accident warning applications need the information to reach as many vehicles as possible in the local transportation network. Without knowing the information propagation characteristics, such as connectivity, transmission distance, time delay, coverage, it is hard to successfully implement those applications based on V2V technologies. This requirement has spurred plenty of studies in both transportation and wireless communication communities. Research from wireless communication field mainly focuses on developing advanced communication protocols to ensure efficient information transmission among smart vehicles once the physical connections between smart vehicles are built, which is not the focus on this study. The proposed study shares common interests with previous studies in transportation community, which seeks to capture the information propagation benchmarks associated with traffic stream features. Accordingly, the assumptions on successful wireless transmission conditions and dynamic traffic flow along with their methodologies shaped the characteristics of most previous studies. Thus, this study reviews the existing literature from these two aspects, which also differentiate the proposed study and highlight our main contributions.

Plenty of research explores instantaneous information propagation in a traffic stream, considering that information propagation is instantaneous compared to vehicle movement. Usually, a successful communication is simply identified by the condition stating the geometric distance between two devices (equipped on smart vehicles or roadside sensors) less than a pre-defined transmission range (with 1 kilometer being the maximum). Representative research in this category studied the topics covering connectivity, inter-vehicle communication system, information transmission distance, and probability of success for information propagation. For example, Yang and Recker [10] built a simulation framework to test the information dissemination efficiency (coverage, speed) over various traffic conditions as information propagating through inter-vehicle communication; Jin and Recker [11] computed the probability of a successful instantaneous information transmission between two vehicles in uniform and general traffic streams; Wang [12] provided the mean and variance of information propagation distance considering equipped vehicle density and transmission range. Both Ukkusuri and Du [13] and Jin and Recker [14] developed analytical formulations to predict multi-hop connectivity of inter-vehicle communication network assuming stationary traffic stream, but different mathematical models are used. Chen et al. [15] evaluated the performance of multi-hop broadcast communication (information propagation distance, and throughput of information package to be received for a given distance) with vehicles following shockwave mobility pattern which mixes free flow and congested flow traffic. Wang et al. [16] and Yin et al. [17] estimated the expectation, variance and probability distribution of instantaneous information propagation distance assuming that vehicles' headway follows Gamma, Poisson, or Log-normal distribution. Clearly, the deficiencies of this group of research are in two aspects: 1) traffic flow dynamics is not fully considered. Since information transmission time is omitted, and message propagation is studied at a snapshot, traffic flow is treated as static flow; 2) oversimplify the wireless communication constrains, ignoring background noise and interference. These two deficiencies will be addressed in the proposed study.

Some researchers recognized that information propagation in V2V communication is significantly impacted by traffic dynamics. For example Schonhof et al. [18] considered dynamic communication link in a dynamic traffic flow on a two-way freeway traffic stream, and then investigated how the smart vehicle density impacts information propagation speed and efficiency. Agarwal [19] studied delay tolerant message propagation in V2V and developed upper and lower bounds for information propagation rate as the functions of traffic density, vehicle speed and transmission range. Wu et al. [20] indicated that information propagation distance and speed depend on relative vehicle movement and other traffic characteristics such as vehicle density and average vehicle speed; both one- and two-way vehicle traffic scenarios are considered. Wang et al. [21] used traffic flow theories such as car-following models to capture the vehicle mobility and applied a Monte Carlo simulation model to evaluate the impacts of traffic flow, transmission range on the throughput of a VANET. Wang [22] modeled information propagation in VANET as a relay process, and provided the mean and variance of information propagation distance as well as its distribution in VANET, but considering the presence of equipped vehicles follows an independent homogeneous Poisson process, which is usually denied in actual traffic flow condition. Utilizing the information propagation model proposed in Wang [22] to evaluate information travel times on the individual arcs, Ng and Waller [23] provided the lower and upper bounds of information propagation delay between two nodes in a network, where traffic flow characteristics are evaluated by a static traffic assignment model. Wang et al. [24] proposed an analytical model to estimate the expected information propagation speed in the early stage of deploying V2V communication network, which implies very low smart vehicle penetration. Du and Ukkusuri [25] modeled information propagation along a one-way road segment as a time-expanded network and provided a closed-form formulation to capture the network connectivity over a time period (reachability) for VANET. The above review noticed that many studies explored information propagation distance or speed, but a limited amount of work studies information delay (which is the focus of this study); in addition, all the aforementioned study applied simplified successful transmission condition, which will be substituted by a more comprehensive condition relevant to communication interference.

Overall, the above brief survey demonstrates that previous research has significantly promoted the understanding of information propagation in a traffic stream from different angles by enriching realistic traffic flow feature in the modeling process, nevertheless, the consideration of the communication side is relatively weak, which degrades the value of the research in practical application. In addition, there is not sufficient study focusing on the interactions between traffic flow movement and information propagation under various traffic conditions on two-way roads. Motivated by the above points, this research proposes mathematical description models to strengthen previous research from the following aspects. i) The proposed approach identifies successfully wireless communication by Signal Interference plus Noise Ratio (SINR) condition rather than only factoring the transmission range and Euclidean distance between communication devices. SINR considers multiple impact factors in wireless communication such as transmission power and interference between concurrent transmissions. This improvement will make our description model more realistic from wireless communication perspective. ii) This study takes account of information communication time to provide the applicability of the proposed formulations to the research of network level information dissemination. It is realized that the communication time is ignorable for measuring the delay on a road segment, but its accumulation effect is significant for the information delay over the network. iii) This research elaborately considers the interaction of traffic flow and information propagation. Namely, various traffic condition (e.g. free flow, mild congested traffic flow, congested traffic flow), different moving directions of traffic flow and information flow (i.e. information and traffic flows in same or opposite direction on one-way or two-way roads) as well as their combinations are fully covered. Therefore, the proposed research comprehensively considers the realistic in both transportation and communication sides. It will improve our understanding of real-time traffic information delay in V2V communication and promotes reliable applications in practices.

The whole of the paper is organized by the structure below: section I introduces research background and motivations; section II presents the problem formulations including traffic flow model, information flow model and successful wireless communication condition; section III proposes our methodology to develop mathematical estimation for information propagation time delay in V2V communication networks on a road segment. Various traffic conditions are considered. The proposed formulations are validated by numerical experiment tests presented in section IV; and the conclusion of this study is given in section V.

II. Problem Formulations

The proposed research is dedicated to exploring mathematical formulations to estimate information propagation time delay in a V2V communication network, running on a road segment. This study first conduct analyses on traffic flow, information flow, and successful communication condition, which all together serviced as the basis to develop rigorous analysis in the proposed study.

A. Road Segment and Traffic Flow

Without loss of generality, this study works on a road segment (either one-way or a two-way) only with an exit and an entrance at each end of the road segment. Namely, no vehicles exit or enter in the middle of the road segment. The road segment is with length L and its traffic stream is composed of n vehicles including both smart vehicles and non-smart vehicles moving on either same or opposite directions. Non-smart vehicles do not have wireless communication capability; they only impact overall traffic flow features and do not influence information propagation directly, thus they are not counted. Throughout the paper, unless noted otherwise vehicle and smart vehicle are equally used. Smart vehicles are numbered from left to right according to their position in a traffic stream. As shown in Fig. 1, x_{ij} represents the distance between smart vehicles i and j. As, vehicles i and j are consecutive; x_{ij} represents the corresponding space headway.

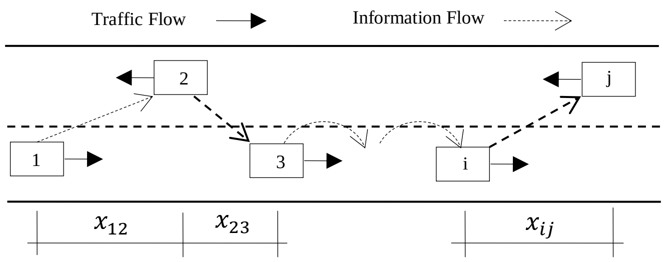


Fig. 1. Information propagation process

B. Information Flow Model

The information flow in this study is modeled by specifying the following aspects. (i) Information always propagates from one vehicle to its first nearest neighbor (from vehicle 1 to 2, 2 to 3, until it arrives at the end of the road segment), where information transmission has the highest opportunity to success according to the successful transmission condition introduced in Part C of this section. (ii) Information propagating from one end of the road segment to the other along the direction that the road extends is studied. The curvature of the road is ignored. (iii) Physical dimensions of smart vehicle are ignored. Smart vehicle is represented by a small rectangle without considering its physical dimensions.

Information may flow in either the same or the opposite direction to the traffic flow. Thus, four possible cases only with smart vehicles illustrated in Fig. 2 are in consideration. More exactly, Case (a) or Case (b) represents the situation where information flows in same or opposite direction as the traffic flow on a one-way (possibly multiple-lane) road. Case (c) and Case (d) are essentially the same, representing a situation where the information flows opposite to one of the traffic flows on a two-way road. The proposed study will cover all these four cases.

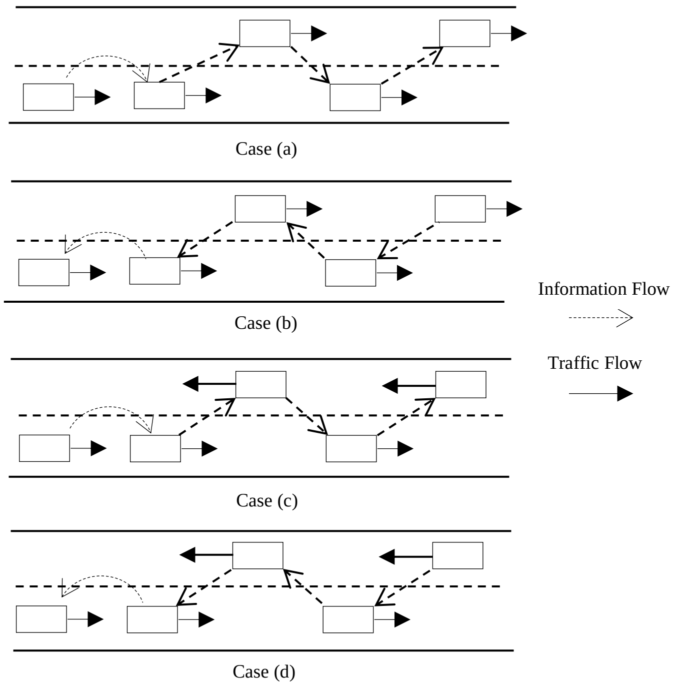


Fig. 2. Traffic flow and information flow on a segment

C. Successful Condition

This study assumes that the information transmission between smart vehicles applies a dedicated short-range communications (DSRC) radio tuned to the 5.9 GHz frequency, allocated by the Federal Communications Commission for transportation safety and mobility applications on vehicle and infrastructure. The successful commination between two consecutive vehicles is identified by SINR condition, whose standard formulation is shown in Eq. 1, in which vehicle w is the transmitter and vehicle i is the receiver. It indicates that vehicle w will successfully transmit information to vehicle i, if SINR value at the receiver i is greater than a threshold value.

$$SINR = \frac{P_w \left(x_{wi}\right)^{-\alpha}}{N + I} \geq \beta \tag{1}$$

Where, P_w represents transmission power of node w; x_{wi} represents the distance between transmitter vehicle w and receiver vehicle i; α is the signal power decay, typically $2 \leq \alpha \leq 6$; N represents the background noise on the frequent channel utilized by network; β is the threshold which depends on the designing modulation and code rate (values which indicate the data transmission rate during a wireless connection) of wireless network. β = 0.15 is recommended for V2V communication [26]; $I = \sum_{j=1, j\neq w}^{n} e_j p_j (x_{ji})^{-\alpha}$ represents the sum of interference power from other vehicles except vehicle w to receiver vehicle i. $e_j = 1$, if vehicle j is in transmission status, otherwise, $e_j = 0$. SINR is a physical model to determine successful reception of a transmission over one hop in wireless network. It considers many environment factors: the distance between two nodes, path loss of signal and wireless interference. Thus, using SINR will make our formulations capture more communication reality.

The standard SINR formulation can be further simplified by considering the communication features in V2V wireless communication network. First, smart vehicles in V2V usually apply broadcast protocol, thus, we have $e_j = 1,\ j = 1, \dots, n$. Second, existing literature [27] shows that the background noise in V2V communication usually follows normal distribution with zero mean. Accordingly, this study applies $N = 0$ and $\alpha = 2$ corresponding to free space information propagation [28]. Last, assuming all smart vehicles adapt the same transmission power (this assumptions have been widely used in literature such as [29]), we have $P_w = p, \forall\, w$. With the above four features holding in V2V communication network, the standard SINR formulation in Eq. 1 is transformed to Eq. 2, and further processed to obtain the relationships in Eq. 3.

$$SINR = \frac{\left(x_{wi}\right)^{-2}}{\sum_{j=1, j\neq w}^{n} \left(x_{ji}\right)^{-2}} \geq \beta \tag{2}$$

$$\Leftrightarrow (\beta+1) \times \left(x_{wi}\right)^{-2} \geq \beta \times \left(\sum_{j=1, j\neq w}^{n} \left(x_{ji}\right)^{-2} + \left(x_{wi}\right)^{-2} \right)$$

$$\Leftrightarrow \left(x_{wi}\right)^2 \leq \frac{\beta+1}{\beta} \times \frac{1}{\Lambda_i} \tag{3}$$

Where, $\Lambda_i = \sum_{j=1}^{n} 1/\left(x_{ji}\right)^2$. Eq. 3 represents the condition of a successful transmission between two smart vehicles. It not only factors the distance between the transmitter and receiver, but also the distribution of all other vehicles around them (represented by Λ_i), reflecting the instantaneous traffic condition. However, Λ_i varies with the location of the transmitter and receiver. Thus, Eq. 3 is the formulation of individuality, which implies micro-level vehicle distribution information is needed. It is not a proper formulation to be used. The proposed study then explores a uniform formulation (a pseudo transmission range derived from SINR condition), which can be used to identify successful information transmission by known aggregated traffic information. We present our method below.

D. Pseudo Transmission Range

To develop the uniform successful transmission condition from SINR, this study labels smart vehicles from the left to right by number $0, 1, \dots, n$, as shown in Fig. 3. Next, the spacing x_{ij} is approximated by $x_{ij} = (j - i)h$, where h is the expected spacing between two adjacent smart vehicles. Accordingly, Λ_i can be approximated by Eq. 4, where i is the label of the receiver.

$$\Lambda_i \approx \frac{1}{h^2} \times \left(\sum_{m=1}^{i} \frac{1}{m^2} + \sum_{m=1}^{n-i} \frac{1}{m^2} \right) \tag{4}$$

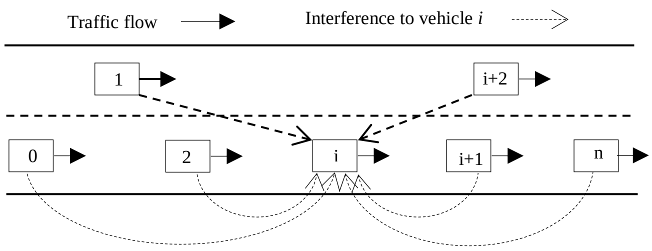


Fig. 3. Interference at vehicle *i*

Note that $i = 1$ implies the information is transmitted from vehicle 0 to vehicle 1; the case $i = 0$ is not considered since information starts from vehicle 0 and it will only be a transmitter rather than a receiver in this study; case $i = n$ indicates that information arrives at the last vehicle so there is no interference coming from its right side.

This study further works on Eq. 4 and obtains the approximation for $\sum_{m=1}^{\infty} \frac{1}{m^2}$ in Eq. 5 according to [30],

$$\sum_{m=1}^{\infty} \frac{1}{m^2} = \frac{\pi^2}{6} \tag{5}$$

More, we know that the two lower bounds shown in Eq. 6 and Eq. 7 exist, according to [31].

$$\sum_{m=1}^{i} \frac{1}{m^2} \geq \sum_{m=1}^{\infty} \frac{1}{m^2} - \int_{i}^{\infty} \frac{1}{m^2} dm = \sum_{m=1}^{\infty} \frac{1}{m^2} - \frac{1}{i} = \frac{\pi^2}{6} - \frac{1}{i} \tag{6}$$

$$\begin{aligned} \sum_{m=1}^{n-i} \frac{1}{m^2} &\geq \sum_{m=1}^{\infty} \frac{1}{m^2} - \int_{n-i}^{\infty} \frac{1}{m^2} dm \\ &= \sum_{m=1}^{\infty} \frac{1}{m^2} - \frac{1}{n-i} = \frac{\pi^2}{6} - \frac{1}{n-i} \end{aligned} \tag{7}$$

Plugging Eq. 6 and Eq. 7 into Eq. 4, the lower bound for Λ_i is given in Eq. 8.

$$\Lambda_i = \frac{1}{h^2}\left(\sum_{m=i}^{i}\frac{1}{m^2} + \sum_{m=i}^{n-i}\frac{1}{m^2}\right) \geq \frac{1}{h^2}\left(\frac{\pi^2}{3} - \frac{n}{i(n-i)}\right) \quad (1 \leq i \leq n-1) \tag{8.a}$$

$$\Lambda_i = \frac{1}{h^2}\sum_{m=1}^{i}\frac{1}{m^2} \geq \frac{1}{h^2}\left(\frac{\pi^2}{6} - \frac{1}{i}\right), i = n \tag{8.b}$$

According to Appendix A1 and A2, it is observed that the lower bound in Eq. 8.a is a tight bound to Λ_i. For example, Equation 8.a is with the maximum relative error equal to 14% as $n = 10$ and 13% as $n = 20$, which happens at boundary points; as $1 < i < n-1$, the relative error of Eq. 8.a is significantly reduced (less than 2%). Equation 8.b ($i = n$) is with a very small relative error (0.31% as $n = 10$ and 0.08% as $n = 20$). In addition, it is observed that a larger n value leads to a smaller relative error and tighter lower bound. Substituting Λ_i in Eq. 3 by the lower bounds given above, the SINR condition is transformed to Eq. 9.

$$x_{wi} \leq \sqrt{\frac{\beta+1}{\beta} \times \frac{1}{\Lambda_i}} \leq \begin{cases} r_i = h \times \sqrt{\frac{\beta+1}{\beta} \times \frac{1}{\frac{\pi^2}{3} - \frac{n}{i(n-i)}}}, 1 \leq i < n \\ r_n = h \times \sqrt{\frac{\beta+1}{\beta} \times \frac{1}{\frac{\pi^2}{6} - \frac{1}{i}}}, i = n \end{cases} \tag{9}$$

Observing $\pi^2/6 - 1/n \leq \pi^2/3 - n/i(n-i)$, we know $r_i < r_n$ in Eq. 9. In addition, it is recognized that r_i reaches to the minimum value at $i = n/2$. By applying the tightest bound for r_i , SINR condition is led to the format in Eq. 10.

$$x_{ji} \leq r = \min\{r_i\}_{i=1}^{n} = h \times \sqrt{\frac{\beta+1}{\beta} \times \frac{1}{\frac{\pi^2}{3} - \frac{4}{n}}} \tag{10}$$

Where, r is considered as a pseudo transmission range which limits the successful transmission. Note that here r is derived from SINR condition. It reflects the traffic flow influence by factoring space headway between vehicles as well as the vehicle distribution around the receiver on the road. It is different to the fixed transmission range specified by transmission power and frequency.

III. Methodology

This section presents our methodology to capture the time delay of information spreading on a road segment in Case (a). Namely, the information delay on a one-way road is first studied, considering information flows in the same direction as traffic flow. Furthermore, we demonstrate that the proposed methodology is applicable to Cases (b), (c) and (d).

A. Time Delay of Intermittent Transmission

Due to the effect of traffic flow dynamics on wireless communication connection, it has been recognized that intermittent communication represents a general information transmission fashion, in which wireless connection is intermittently connected (leading to instantaneous transmission) and broken (leading to ferry transmission) due to relative movement between vehicles. Intermittent transmission usually happens in a traffic flow with mild congestion. Pure ferry and instantaneous communication are two extremes of intermittent transmission. They usually happen in very sparse traffic and highly congested traffic flow, where the wireless connection between two vehicles happens rarely or constantly. Thus, the methodology proposed below will focus on intermittent communication.

Fig. 4 provides an illustration about the information propagation in Case (a) during a time interval. An intermittent transmission is considered to follow a pattern in which instantaneous transmission and ferry transmission alternatively happen until the information arrives at the end of the road segment. As an instantaneous transmission occurs, several vehicles are well connected and information is smoothly transmitted, such as from node i to node $i+k$ in Fig. 4. The corresponding time delay is calculated by: $t_1 = k \times \tau$, where τ represents the transmission time and k represents the number of the hops. Note that the communication time delay is taken into account to make the proposed model applicable to capture the time delay of information dissemination over a large scale network, where the accumulated effects of a huge number of instantaneous communication time delay shows impact, even though it is ignorable on a short road segment. As traffic is very sparse, ferry transmission happens. Namely, the information will be ferried by a vehicle until it meets another vehicle, such as the information propagation at node $(i+k)$ is broken, and then ferried by node $(i+k)$ until it reaches to node j at another time in Fig. 4. The corresponding time delay is calculated by $t_2 = y/v_i$, where y represents the carrying-on distance and v_i represents the average speed of the vehicle carrying information. Ignoring the boundary case in which instantaneous transmission or ferry transmission happens one more times than the other, the expected time delay of a piece of information traversing on a road segment is estimated by the totally time delay for one intermittent transmission (i.e. an instantaneous transmission followed by a ferry transmission and vice versa) multiple by expected times of the intermittent transmission happens. Mathematically, this idea is presented by Eq. 11 below.

$$\begin{aligned} E(T) &= \left(E(t_1) + E(t_2)\right) \times \frac{L}{E(x) + E(y)} \\ &= \left(\bar{k} \times \tau + \frac{E(y)}{v_i}\right) \times \frac{L}{E(x) + E(y)} \end{aligned} \tag{11}$$

Where, x and y represent the information propagation distance following instantaneous and ferry transmission respectively; $E(x)$and $E(y)$ represent their expected values; $\bar{k}$ represents the expected number of hops in instantaneous transmission; L is the length of the road segment. Eq. 11 covers various traffic flow conditions. $E(x)$ dominates the time delay in congested traffic condition, but $E(y)$ mainly accounts for the time delay in sparse traffic condition; $E(x)$ and $E(y)$ together captures the feature in the intermediate congested traffic condition. The follows of this paper further present our approaches to develop the formulations for $E(x)$, $E(y)$ and $\bar{k}$ incorporating traffic flow features and communication limits.

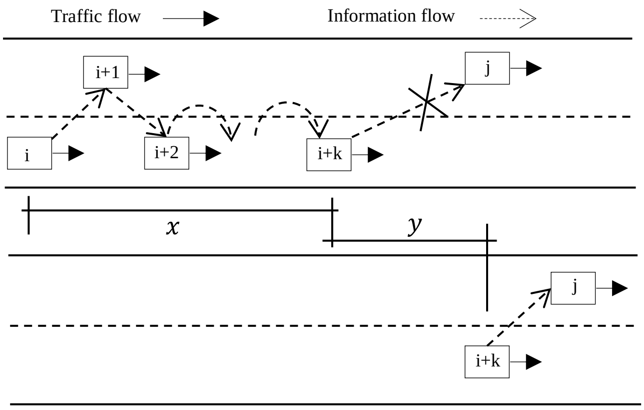


Fig. 4. Information propagation on one-way road

B. Expected Information Propagation Distance in an Instantaneous Transmission

As the space headway between a transmitter and receiver satisfy $S < r$, information propagates by instantaneous transmission. Considering information always propagates to its nearest neighbor, the conditional random variable $S|S \le r$ represents the space headway in instantaneous transmission. Given an instantaneous transmission includes $\bar{k}$ hops on average, we obtain the formulation for $E(x)$ below.

$$E(x) = \bar{k} \times E(S|S < r) \tag{12}$$

Where, $E(S|S < r)$ is the expected space headway in instantaneous transmission; $E(S|S < r)$ can be calculated by Eq. 13, which is derived by the mathematical process given in Eq. 14 & Eq. 15, given the spacing distribution $f(s)$ is known.

$$E(S \mid S < r) = \frac{\int_0^r sf(s)ds}{\int_0^r f(u)du} \tag{13}$$

$$\begin{aligned} F(S|S < r) &= P(0 < S < s|0 < S < r) \\ &= \frac{P(0 < S < s, 0 < S < r)}{P(0 < S < r)} \\ &= \frac{\int_0^s f(u)du}{\int_0^r f(u)du} \end{aligned} \tag{14}$$

$$f(S \mid S < r) = \frac{dF(S \mid S < r)}{ds} = \frac{f(s)}{\int_0^r f(u)du} \tag{15}$$

This study noticed that in reality, one transmitter may successfully transmit one piece of information to multiple receivers (referred to as scenario II) rather than only to the nearest neighbor (referred to as scenario I). However, we focus on scenario I in this study for the three reasons. (i) It is hard to decide how many vehicles one transmission will cover. It depends on the vehicle distribution around a transmitter. As the traffic distribution is not uniform, this number is uncertain and becomes very difficult to decide. (ii) This study observed that the information propagates the same distance along the road segment under these two scenarios given we ignore the distance between vehicles vertical to direction that the road extends. Fig. 5 provides examples to illustrate this observation, where a piece of information propagates on a road segment with same vehicle distribution, but the instantaneous transmission Fig. 5 (a) follows scenario I, i.e. a transmitter only reach to the nearest neighbor; the instantaneous transmission in Fig. 5 (b) or (c) follows scenario II, i.e. a transmitter reaches to the nearest two or three neighbors respectively. Then, if vehicle 5 fails to "reach" its nearest neighbor, vehicle 6, then we know that vehicle 5 cannot reach other vehicle further in Fig. 5 (b) and (c) according to our successful transmission condition given in Eq. 10. In this context, vehicle 4 will fail to "reach" vehicle 6 in both Fig. 5 (b) and (c) since is more difficult to building up connection between vehicle 4 and 6 than vehicle 5 and 6. Following the same thought, vehicle 3 cannot connect to vehicle 6 either in Fig. 5 (c). As a result, the information propagates the same distances from vehicle 1 to vehicle 5 under the three examples in Fig. 5, even though one transmitter only connects the nearest neighbor in scenario I, two or three receivers in scenario II. This is a good quality for the proposed methodology in Eq. 11. (iii) The instantaneous transmission time is very small (micro seconds), so the time delay difference between Scenario I and Scenario II is small and negligible. Hence, this study focus on Scenario I to study the information propagation time delay on road segment.

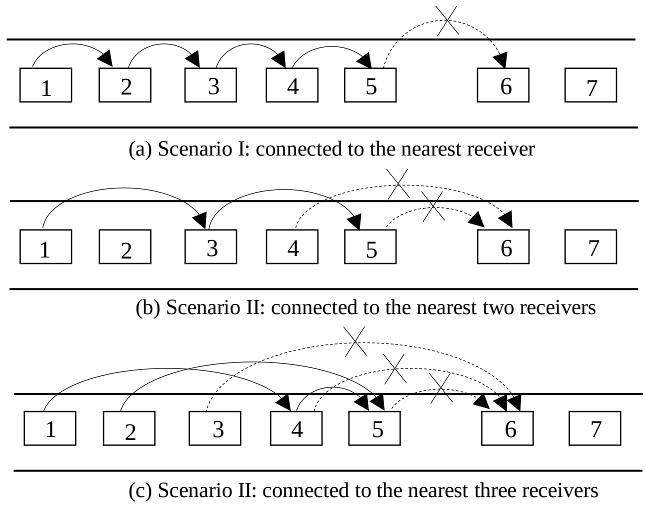


Fig. 5. One transmitter has one, two or three receivers

C. Expected Information Propagation Distance in a Ferry Transmission

As the space headway between a transmitter and a receiver (two consecutive vehicles) satisfy $S > r$, information will be spread by a ferry transmission. A ferry transmission will stop as the spacing between this transmitter and a receiver satisfies $S < r$. Therefore, the expected information propagation distance by a ferry transmission, $E(y)$, can be calculated by Eq. 16.

$$E(y) = v_i \times \frac{E(S \mid S > r) - r}{v_{ij}} \tag{16}$$

Where, $E(S|S > r)$ represents the expected spacing given a ferry transmission happens; v_{ij} is the average relative speed between two vehicles *i* and *j*. Considering $S|S > r$ as a conditional random variable, $E(S|S>r)$ can be calculated by Eq. 17, which is derived by the cumulative distribution and probability density formulations for $S|S>r$ given by Eq. 18 and Eq. 19 respectively.

$$E(S \mid S>r)=\frac{\int_r^{+\infty} sf(s)ds}{\int_r^{+\infty} f(u)du}=\frac{\int_r^{+\infty} sf(s)ds}{1-\int_0^r f(u)du} \tag{17}$$

$$F(S|S>r)=P(0<S<s|r<S<+\infty) \\ =\frac{P(r<S<s)}{P(r<S<+\infty)}=\frac{\int_r^s f(u)du}{\int_r^{+\infty} f(u)du} \tag{18}$$

$$f(S \mid S>r)=\frac{dF(S \mid S>r)}{db}=\frac{f(s)}{\int_r^{+\infty} f(u)du} \tag{19}$$

D. Expected Hops in an Instantaneous Transmission

A piece of information may propagate multiple hops in an instantaneous transmission along the well-connected vehicle network on a road segment, until the communication link is broken and the information propagation turns to ferry transmission. To develop the formulation for $\bar{k}$ (the expected number of hops in an instantaneous transmission), we consider there are $n+1$ number of vehicles running on the road segment, and label the vehicles from left to right with the number from *0* to n as we did before. Fig. 6 shows an example. Based on that, we consider k as a random variable and explore its expectation by the events defined below.

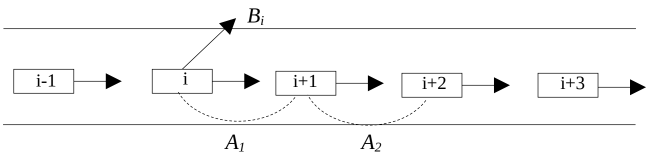


Fig. 6. Instantaneous transmission on a segment

(1) Event B_i represents an instantaneous transmission starting at the i^{th} vehicle, $i \in [0,n]$. Considering an instantaneous communication may start at any individual vehicles evenly, we have $P(B_i)=1/_{n+1}$.
(2) Event A_c, $1 \le c \le k$ represents the c^{th} hop in an instantaneous communication. For instance: if the instantaneous transmission starts at 2^{nd} vehicle, A_1 represents transmission between 2^{nd} and 3^{rd} vehicle, A_2 represents transmission between 3^{rd} and 4^{th} vehicle, etc. With the pseudo transmission range r (derived from section II.D), we calculate $P(A_c)$, the probability of a successful information transmission from any vehicles to its nearest neighbor by Eq. 20.

$$P(A_c)=P(0\le S\le r)=\int_0^r f(s)ds=P \tag{20}$$

(3) Event P_k represents k hops of successive transmission, $k \in [0,n]$. Using the same notation for the event and its probability, we have $P_k=P(A_1\cap A_2\cap \ldots\cap A_k)$.
(4) Event $P_k\cap B_i$ represents an instantaneous transmission starts at vehicle i and only successively propagates k hops. Then, we have $P(P_k\cap B_i)=P(P_k|B_i)P(B_i)$. It is noticed that for a given i, $k\in[0,n-i]$. Namely, if an instantaneous transmission starts from the i^{th} vehicle, its maximum number of successive hops is $(n-i)$. Table 1 provides the calculations for all possible $P(P_k|B_i)$.
(5) Event $g(k)$ represents an instantaneous transmission with only k hops. $g(k)=\cup_{i=0}^{n} P_k\cap B_i$.

Table 1 below demonstrates the calculations of $P(P_k|B_i)$ and $g(k)$ as $k=0,\ldots,n,$ and $i=0,\ldots,n$. For example, $P(P_k|B_i)=P_2(1-P)$ when $k=2$ and $i=2$. It indicates that the instantaneous transmission starts from vehicle 2, propagates two hops, and then the connection is broken. Each row in Table 1 provides the probabilities that an instantaneous transmission spreads k hops given this instantaneous transmission starts at any vehicle $i, i=0,\ldots,n$. By summing $P(P_k|B_i)$ in each row, we obtain the general formulation: $g(k)=\left((n-k)P_k(1-P)+P_k\right)/(n+1)$ (note that the same notation is used for event $g(k)$ and its probability).

With the solution given in Table 1, $\bar{k}$ can be calculated by Eq. 21 below:

$$\bar{k}=\sum_1^n kg(k)=g(1)+2g(2)+\ldots+ng(n) \\ g(k)=\frac{(n-k)P_k(1-P)+P_k}{n+1} \tag{21}$$

Clearly, P_k is the key component to calculate $\bar{k}$. The following study investigates the formulation for P_k as well as $\bar{k}$ under different traffic conditions.

1) Free flow traffic condition

Under free flow condition, the large spacing between vehicles guarantees vehicle's movement freedom without concerning safety. This implies the independence of the spacing and successful information transmission between any two consecutive vehicles so that $P_k=P(A_1\cap A_2\cap\ldots\cap A_k)=P^k$, and then $g(k)$ can be calculated by Eq. 22.

$$g(k)=\frac{(n-k)P^k(1-P)+P^k}{n+1} \tag{22}$$

Based on that, it can be proved that the sum of the last column in Table 1 equals 1, thus the correctness of the probability distribution in Table 1 is verified. In addition, we obtain the closed-form formulation to predict the expected hops of an instantaneous transmission in Eq. 23.

$$\bar{k}=\sum\nolimits_1^n kg(k)=g(1)+2g(2)+\cdots+ng(n) \\ =\sum\nolimits_1^n k\frac{(n-k)P^k(1-P)+P^k}{n+1} \tag{23}$$

2) Congested flow

Under congested traffic condition, the spacing between two vehicles is relatively small. The movement of a following vehicle needs to consider the movement of the leading vehicle in front to keep safety, therefore, the spacing between any two consecutive vehicles is dependent. Accordingly, it brings the difficulty to accurately calculate $P_k=P(A_1\cap A_2\cap\ldots\cap A_k)$ and the associated $\bar{k}$ in Eq. 21. As a compromise, this study develops the lower and upper bounds of P_k, which further lead to lower and upper bounds of $\bar{k}$. Below presents our methods. According to Bonferroni bound [32] and Caen bound [33], the lower and upper bounds for P_k are given in Eq. 24 and Eq. 25, respectively.

$$P_k = P(A_1 \cap A_2 \cap \ldots \cap A_k) \geq 1 - \sum_{c=1}^{k} P(\bar{A}_c) = k\,P - (k-1) \quad (24)$$

$$P_k = P(A_1 \cap A_2 \cap \ldots \cap A_k) = 1 - P\left(\bigcup_{c=1}^{k} \overline{A_c}\right)$$
$$\leq 1 - \sum_{c=1}^{k} \frac{P(\overline{A_c})^2}{\sum_{b=1}^{k} P(\overline{A_c} \cap \overline{A_b})} \quad (25)$$

By observing $\sum_{b=1}^{k} P(\overline{A_c} \cap \overline{A_b}) \leq k$, Eq. 25 provides a new upper bound for P_k in Eq. 26 below.

$$P_k = P(A_1 \cap \ldots \cap A_k) \leq 1 - \frac{1}{k}\sum_{c=1}^{k} P(\overline{A_c})^2 = 2P - P^2 \quad (26)$$

Accordingly, we obtain the lower and upper bound for $\bar{k}$ in Eq. 27 and Eq. 28 respectively.

$$\bar{k} \geq \sum_{1}^{n} k \frac{(n-k)\big(kP-(k-1)\big)(1-P) + kP - (k-1)}{n+1} \quad (27)$$

$$\bar{k} \leq \sum_{1}^{n} k \frac{(n-k)(2P-P^2)(1-P) + 2P - P^2}{n+1} \quad (28)$$

At this point, by combining Eq. 11, Eq. 12, Eq. 16, Eq. 23, Eq. 27, and Eq. 28, we are ready to calculate the expected time delay of information propagation along a road segment in Case (a).

TABLE I. THE PROBABILITY THAT AN INSTANTANEOUS TRANSMISSION PROPAGATES K HOPS GIVEN IT STARTS AT I[TH] VEHICLE

Hops	**$P(P_k \mid B_i)$**									**$g(k)$**
	B_0	**B_1**	**B_2**	**…**	**B_{n-k}**	**…**	**B_{n-2}**	**B_{n-1}**	**B_n**	
0	$1-P$	$1-P$	$1-P$		$1-P$		$1-P$	$1-P$	1	$\frac{(1-P)n+1}{n+1}$
1	$P_1(1-P)$	$P_1(1-P)$	$P_1(1-P)$		$P_1(1-P)$		$P_1(1-P)$	P_1	0	$\frac{(n-1)P_1(1-P)+P_1}{n+1}$
2	$P_2(1-P)$	$P_2(1-P)$	$P_2(1-P)$		$P_2(1-P)$		P_2	0	0	$\frac{(n-2)P_2(1-P)+P_2}{n+1}$
⋮	⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮
k	$P_k(1-P)$	$P_k(1-P)$	$P_k(1-P)$		P_k	0	0	0	0	$\frac{(n-k)P_k(1-P)+P_k}{n+1}$
⋮	⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮
n-2	$P_{n-2}(1-P)$	$P_{n-2}(1-P)$	P_{n-2}	0	0		0	0	0	$\frac{2P_{n-2}(1-P)+P_{n-2}}{n+1}$
n-1	$P_{n-1}(1-p)$	P_{n-1}	0		0		0	0	0	$\frac{P_{n-1}(1-P)+P_{n-1}}{n+1}$
n	P_n	0	0		0		0	0	0	$\frac{P_n}{n+1}$

E. Extension to Other Cases

This study next demonstrates the applicability of the proposed approach to case (b), case (c) and case (d) in Fig. 2.

1) Case (b): One-way road segment with traffic and information flowing in the opposite direction

Case (b) represents a situation that information flow spreads in an opposite direction to traffic flow. It is observed that information propagation in case (b) also follows the same pattern, alternatively presenting instantaneous transmission and ferry transmission. More specifically, as information spreads by instantaneous transmission, we may ignore the movement of smart vehicles since it is much slower than wireless information spread. Accordingly, the time delay resulting from instantaneous transmission in case (b) can be measured by $t_1 = \tau \times \bar{k}$. However, as a ferry transmission happens in case (b), the vehicles conducting ferry transmission may carry information backward to the information propagation direction for time $E(y)/v_i$. Hence, the distance that a piece of information being spread forward in one cycle of the transmission pattern (an instantaneous transmission followed by a ferry transmission or vice versa) equals to $E(x) - E(y)$. Accordingly, the time delay of the information propagation along a road segment under case (b) will be calculated by Eq. 29.

$$E(T) = \left(\frac{E(y)}{v_i} + \bar{k} \times \tau\right) \times \frac{L}{E(x) - E(y)} \quad (29)$$

Note that (i) as $E(x) - E(y) \leq 0$, the information will never reach to the end of the road segment since it is always carried back by ferry transmission; (ii) Eq. 29 is a variant of Eq. 11; all the elements such as $E(x), E(y)$, and $\bar{k}$ can be measured by the formulations proposed in previous sections. Therefore, our approach also works for case (b).

2) Case (c) and (d): Two-way road

Case (c) and Case (d) are essentially the same. They both illustrate a situation that information spreads in a same direction to one of the traffic flows (such as in East Bound (EB)) but opposite to the other way (such as in West Bound (WB)). The time delay formulation developed for instantaneous transmission in Case (a) still works for these two cases. This study next provides more discussions about ferry transmission based on the examples in Fig. 7 and Fig. 8, where S' represents the spacing distance between vehicles on EB direction and vehicles on WB direction; S_{EB} and S_{WB} represent the spacing distance between vehicles on EB and WB direction respectively. It is observed that there are two possible scenarios for a ferry transmission.

Scenario (1). The example shown in Fig. 7 indicates that a previous multi-hops instantaneous transmission stops at vehicle 1, with the last hop from vehicle 2 in WB direction occurring. Thus a ferry transmission starts on a ferry vehicle (vehicle 1) carries information forward (i.e. given $S_{EB} > r$, and $S^{'} > r$, vehicle 1 carries information in EB direction). This ferry transmission will be stopped by an instantaneous transmission between the ferry vehicle and the other vehicle in the same way (such as vehicle 4) and implies $\frac{S_{EB}-r}{v_{EB}} \leq \frac{S'-r}{v_{EW}}$, or in the other way (such as vehicle 3) and implies $\frac{S_{EB}-r}{v_{EB}} \geq \frac{S'-r}{v_{EW}}$. These two possible ferry transmissions are denoted as f_1 and f_2 with probability p_1 and p_2; they lead to the expected forward ferry transmission distance $E(y_{f_1})$ and $E(y_{f_2})$ respectively.

Scenario (2). The example shown in Fig. 8 indicates that a previous multi-hops instantaneous transmission stops at vehicle 2 with the last hop from vehicle 1 in EB direction. Thus, the ferry transmission starts on a ferry vehicle (such as vehicle 2) carrying information backward (i.e. given $S_{WB} > r$, and $S^{'} > r$, vehicle 2 carries information in WB direction). This ferry may stop at a vehicle behind itself on the same way (such as vehicle 3), but not on the other way (such as vehicle 4) due to the opposite moving direction. However, as this backward carrying happens, information is still possible to move forward since it is noticed that vehicle 1 will carry information and move forward. This study next performs more elaborate discussions for this scenario, which may include two other situations. (a) If vehicle 3 "meets" vehicle 1 ($x_{13} \leq r$) before it "meets" vehicle 2 (i.e. $x_{23} \leq r$), then information is carried forward by vehicle 1. The ferry transmission conducted by vehicle 1 is similar to the forward ferry f_2 we discussed in Scenario (1). The only difference is that the forward ferry here will cancel the propagation of information resulting from previous instantaneous information transmission from vehicle 1 to vehicle 2. Considering it is hard to measure the information propagation distance from vehicle 1 to vehicle 2 under this situation, this study ignores this details and considers it as the forward ferry f_2. Note that the backward ferry conducted by vehicle 1 under this situation does not have an effect on information propagating forward. (b) If vehicle 3 "meets" vehicle 2 before it "meets" vehicle 1, then information is carried backward by vehicle 2 before the instantaneous transmission happens between vehicle 2 and vehicle 3. We denote this backward ferry as f_b happening with probability p_b. Accordingly, it results in the expected backward ferry transmission distance $E(y_{\mathrm{b}})$. Clearly, to identify this backward ferry, we need to recognize the distance between vehicle 1 and vehicle 3 and it is very difficult to get. To address this issue, we observed that if there are many other vehicles between vehicle 1 and vehicle 2, then vehicle 1 is very likely relatively far away, the chance that vehicle 1 "meets" vehicle 3 earlier than vehicle 2 is low; then we are sure about the backward ferry when $S_{SB} > r$; on the other hand if vehicle is almost adjacent to vehicle 2, then the average distance between vehicle 1 and vehicle 3 can be approximated by S'. In this context, we think that the backward ferry happens as $\frac{S'-r}{v_{EW}} \geq \frac{S_{WB}-r}{v_{WB}}$ and $S_{WB} > r$, and $S' > r$.

Clearly, $E(y_{f_1})$, $E(y_{f_2})$ and $E(y_b)$ can be calculated by the formulations given in Eq. 16 and Eq. 17. Note that the spacing and relative speed used to calculate $E(y_{f_2})$ should be measured for vehicles moving in opposite directions.

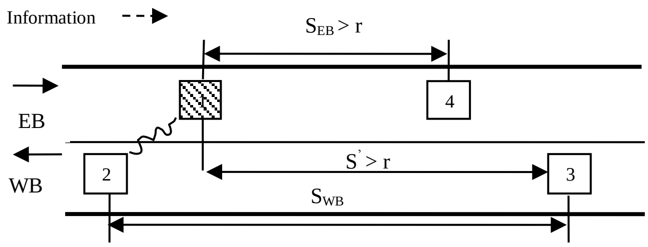


Fig. 7. Ferry transmission on vehicle 1 in EB direction on two-way segment

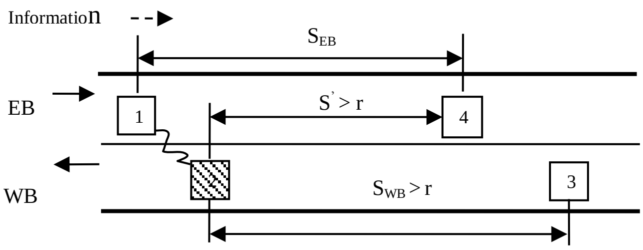


Fig. 8. Ferry transmission on vehicle 2 in WB direction on a two-way segment

This study next develops the formulations for p_1, p_2 and p_b based on the examples in Fig. 7 and Fig. 8. It is observed that the probability that a ferry transmission starts (the last hop of the previous instantaneous transmission stops) on the EB direction or the WB direction depends on the spacing of vehicles on these two directions. Namely, if the vehicles on the EB directions are sparser than on the WB direction, then the chance that a ferry transmission starts from EB direction is higher than from the WB direction. With this observation, this study estimates the probability that a ferry transmission happens on the WB or the EB direction by $\frac{S_{WB}}{S_{WB}+S_{EB}}$ and $\frac{S_{EB}}{S_{WB}+S_{EB}}$. Furthermore, we develop the formulations for p_1, p_2 and p_b based on the above discussions for f_1, f_2 and f_b.

$$\begin{aligned} p_1 &= P(f_1 \mid f) = \frac{P(f_1)}{P(f)} \\ &= \frac{S_{EB}}{S_{WB}+S_{EB}} . P\left(S^{'} \geq r \cap S_{EB} \geq r \cap \frac{S_{EB}-r}{v_{EB}} \leq \frac{S^{'}-r}{v_{EW}} \right) \frac{1}{P(f)} \end{aligned} \tag{30}$$

$$\begin{aligned} p_2 &= P(f_2 \mid f) = \frac{P(f_2)}{P(f)} \\ &= \frac{S_{EB}}{S_{WB}+S_{EB}} . P\left(S^{'} \geq r \cap S_{EB} \geq r \cap \frac{S_{EB}-r}{v_{EB}} \geq \frac{S^{'}-r}{v_{EW}} \right) \frac{1}{P(f)} \end{aligned} \tag{31}$$

$$\begin{aligned} p_b &= P(f_b \mid f) = \frac{P(f_b)}{P(f)} \\ &= \frac{S_{WB}}{S_{WB}+S_{EB}} . P\left(S^{'} \geq r \cap S_{WB} \geq r \cap \frac{S_{WB}-r}{v_{WB}} \leq \frac{S^{'}-r}{v_{EW}} \right) \frac{1}{P(f)} \end{aligned} \tag{32}$$

Where, $P(f_1)$, $P(f_2)$, and $P(f_b)$ represent the corresponding probabilities of forward ferry transmission f_1, f_2 and backward ferry transmission f_b; $P(f)$ represents the probability of ferry transmission; v_{EB} represents the relative speed on the lanes in EB direction. v_{EW} represents the relative speed between the lane on EB direction and the lane on WB direction.

Overall, a piece of information may spread on a two-way road segment through instantaneous and ferry transmissions. By following the ideas to develop the time delay formulation for Case (a) and Case (b) (i.e., Eq. 11 and Eq. 29), we develop Eq. 33 to estimate the expected information propagation time delay time delay for Case (c) and Case (d).

$$E(T)=\left(\bar{k}\times\tau+p_1\frac{E(y_{f1})}{v_{f1}}+p_2\frac{E(y_{f2})}{v_{f2}}+p_b\frac{E(y_b)}{v_b}\right)\times\frac{L}{E(x)+p_1\times E(y_{f1})+p_2\times E(y_{f2})-p_b\times E(y_b)} \quad (33)$$

Where, v_f and v_b are the average vehicle speed in a traffic flow with the same and opposite direction to information flow respectively. Note that Eq. 33 implies that the distance that a piece of information moving forward in one cycle including an instantaneous transmission and a ferry transmission is calculated by the difference between forward transmission (through instantaneous transmission and forward ferry) and backward transmission (through backward ferry). The expected ferry transmission distance is calculated by the weighted average of forward ferry transmission and backward ferry transmission. Mathematically, this difference is calculated by $E(x)+p_1E(y_{f_1})+p_2E(y_{f_2})-p_bE(y_b)$.

Clearly, Eq. 33 presents the same underline logic as Eq. 11 and Eq. 29). So far, we claim that the proposed approaches cover all the four cases in Fig. 1. To validate the proposed formulations, this study conducts the numerical experiments in next section.

IV. Numerical Experiments

This section presents the numerical experiments to validate the proposed mathematical formulations.

A. Test-bed and Input Data

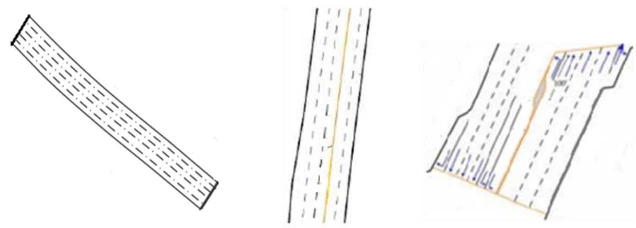
The field data collected by Next-Generation Simulation (NGSIM) is used to validate the proposed methodology and formulations. The data set provides vehicle trajectory data including the attributes: vehicle ID, frame ID, total frames, global time, local X, local Y, global X, global Y, vehicle length, vehicle width, vehicle class, vehicle velocity, vehicle acceleration, lane ID, preceding vehicle ID, following vehicle ID, space headway (in same lane) and time headway (also in same lane). Three test-beds are selected so that the experiments cover one-way or two-way road segment as well as free flow and congested traffic conditions. Below provides details for the experiments which are setup on a one-way and two-way road segment.

1) One-way traffic flow

The validation experiments for one-way traffic flow was conducted on a road segment of US Highway 101 in Los Angeles, CA (see Fig. 9 (a)). The study area was a one-way segment with 2100-feet long, and five lanes on which vehicle moving from North to South throughout the section. Traffic data was collected during two 15-minutes periods on June 15th 2005. Given the speed limit of the road segment is 55 mph, the data set collected during (7:50 a.m. – 8:05 a.m.) represents free flow traffic condition with a flow rate equal to 9500 vph and average speed equal to 48 mph; the data set collected during (8:20 a.m. - 8:35 a.m.) represents an intermediate congested traffic flow with average flow rate equal to 7800 vph and average speed equal to 25 mph.

2) Two-way traffic flow

The validation experiments for two-way traffic flow were conducted on two road segments: Peachtree Street, Atlanta (GA) (see Fig. 9 (b)) for free flow and Lankershim Blvd, Los Angeles (CA) (see Fig. 9 (c)) for congested flow. The segment of Peachtree Street is 650 feet long with five lanes. Traffic data is collected during (12:50 p.m. – 1:00 p.m.) on November 8th 2006. Given the speed limit of 55 mph, the data indicates free flow traffic condition with the average speed equal to 50 mph. The segment of Lankershim Blvd is 600 feet long with six lanes. Traffic data is collected during (8:50 a.m. – 9:00 a.m.) on June 16th 2005. Given speed limit of 55 mph, the collected data indicates a congested flow with average speed equal to 23 mph.



(a) US Highway 101 (b) Peachtree St (c) Lankershim Blvd

Fig. 9. Test-beds

B. Experiment Design

This section designs the experiments to evaluate the performance of the proposed mathematical estimation for the time delay that a piece of information propagates through a road segment. Considering the expected number of hops in an instantaneous transmission ($\bar{k}$), is one of the key components in those mathematical estimation formulations, we also check the accuracy of its formulation.

The overall ideas of the experiment are presented first. Based on the field data collected from the selected test-beds, this study first measures field information propagation time delay (T^g) as well as the expected number of hops in an instantaneous transmission (k^g), and then we calculate the corresponding mathematical estimations for the time delay (T^M) and the expected number of hops (k^M), respectively. After that, we compare (k^g) to (k^M) and (T^g) to (T^M), and demonstrate the accuracy by root-mean-square error (RMSE) and relative error (e).

The field information propagation time delay, T^g is defined as the time interval that a piece of information propagates through a road segment, given a successful transmission between any every two vehicles is identified by SINR condition in Eq. 1. T^g is considered as the ground truth in this study. Mathematical estimation T^M, is calculated by the proposed formulations, given the needed distribution and parameters are obtained from the field data. These experiments select Log-normal distribution to represent the space headway distribution vehicles on one-way or two-way road segment after it was calibrated by the field data. But, the applicability of the proposed approaches does not depend on the distribution selection. Along the process to calculated T^g and T^M, k^g and k^M are also calculated through field counts and proposed mathematical estimation formulations, respectively. k^g represents the ground truth.

Next, we provide the experimental procedure. For every $\Delta t(= 6$ or 5) seconds, the experiment starts to track a piece of information just launching on the start of the road segment until it reaches to the end of the road segment. Accordingly, T^g (and k^g) and T^M (and k^M) are checked very Δt seconds. According to the field dataset, 151 (or 121) pieces of information in total are tracked for one-way (or two-way) test-beds. The detailed experiments steps to track information propagation are given below.

Step 1: at time t_0, a piece of information launches on the start of the road segment.

Step 2: track instantaneous (or ferry) information propagation until it is broken; record k.

Step 3: check if the information reaches to the end of the road segment.

1. Yes, record current time t_e; $\mathrm{T^g} = t_e - t_0$, k^g= average (k), calculated T^M and k^M; go to Step 4
2. No, change transmission scenario to ferry (instantaneous), go back to step 2

Step 4: if all data examined, stop, otherwise, $t_0 = t_0 + \Delta t$, go to Step 1.

The accuracy of T^M is evaluated by Root-Mean-Square-Error (RMSE) and relative error (e) given in Eq. 34 and Eq. 35 below. RMSE demonstrates the average difference between $T^M$and T^g over all tracked information. Relative error (e) measures the percentage of the error between T^M and T^g to T^g, thus gives us the idea how significant the error is.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}\left(T_i^g - T_i^M\right)^2}{N}} \tag{34}$$

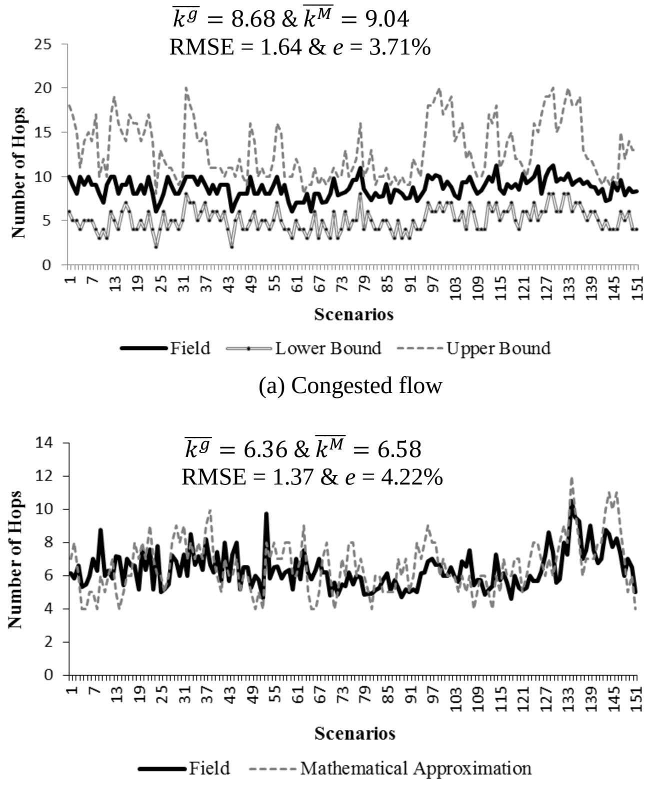
$$\mathrm{e} = \frac{1}{N}\sum_{i=1}^{N}\frac{T_i^M - T_i^g}{T_i^g} \times 100\% \tag{35}$$

Where, N represents number of scenarios, T_i^grepresents the field time delay in a scenario i, T_i^Mrepresents corresponding mathematical estimation in free flow (the average of the upper and lower bounds in congested flow). A negative e value indicates an underestimation over all experimental scenarios and a positive e value means the other way around. The same evaluation work will be conducted for k^M.

C. Experiment Results and Insights

1) One-way segment

This section presents our numerical experiment results and the insights we obtained for one-way road segment. The results given in Fig. 10 indicate that k^g is well bounded by our mathematical lower and upper bounds (calculated by Eq. 27 and Eq. 28) in congested flow, and accurately estimated by the mathematical model (Eq. 23) in free flow. More exactly, RMSE values for k^g under congested and free flow are 1.64 and 1.37 respectively, and the relative errors are 3.71% and 4.22% respectively. In addition, we see that on average $\overline{k^g}$ =8.68 and $\overline{k^M}$ =9.04 in the tested congested flow, and $\overline{k^g}$ =6.36 and $\overline{k^M}$ =6.58 in the tested free flow; they are very close. Thus, our mathematical formulations provide reliable estimations for $\bar{k}$ under both congested and free flow on one-way road segments.

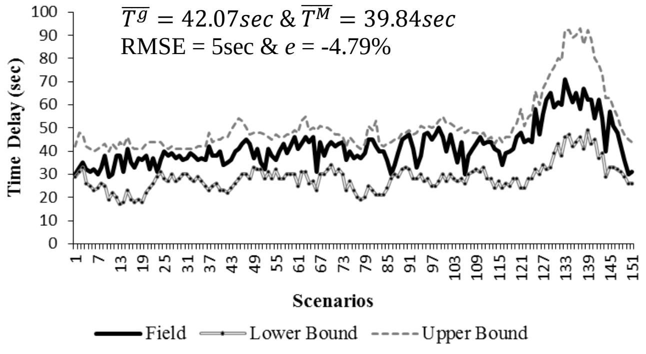


(a) Congested flow

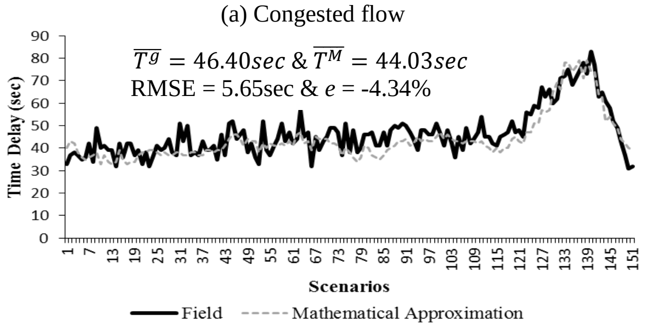
(b) Free flow

Fig. 10. Comparison of number of hops between field and mathematical estimation on one-way segment

The results for evaluating the mathematical formulation (Eq.11) to estimate the information propagation time delay on one-way road segment are given in Fig. 11. Fig. 11(a) shows that field time delay T^g in congested flow is well bounded by our mathematical bounds. The relative error -4.79% and RMSE equal to 5 seconds; and Fig. 11(b) also indicates that the mathematical formulation can estimate the field time delay very well in the free flow case; the relative error is -4.34% and RMSE equals to 5.65 seconds. The negative sign of e indicates that on the average, the proposed mathematical formulation underestimates the time delay. In addition, the results show that $\overline{T^g}$ =42.07 seconds and $\overline{T^M}$ =39.84 seconds in the tested congested flow; $\overline{T^g}$ =46.40 seconds and $\overline{T^M}$ =44.03 seconds. Clearly, the average of the field time delay $\overline{T^g}$ is very close to the average of the estimated time delay $\overline{T^M}$. Moreover, in both cases, the relative error is around 4%, so our mathematical formulations work well.



(a) Congested flow



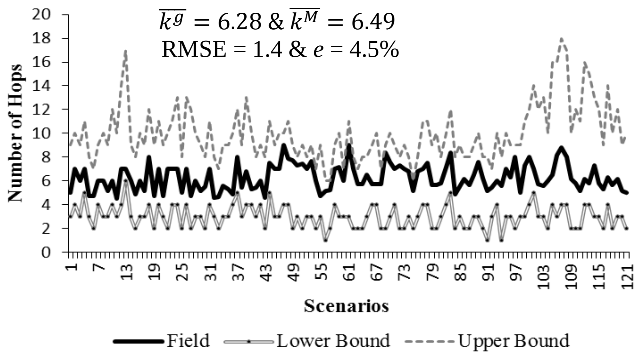
(b) Free flow

Fig. 11. Comparison of time delay between field and mathematical estimation on one-way segment

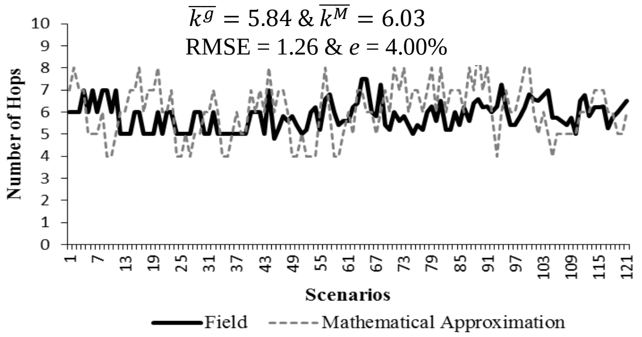
2) Two-way segment

The performances of the proposed approaches (Eq.33 and all the related equations) on two-way road are also evaluated by the same way that we did for the one-way segment. The results in Fig. 12 and Fig. 13 show that the field vales (time delay or the expected number of hops) under congested flow are well bounded by the mathematical bounds, and they are accurately estimated by the mathematical formulations for free flow. More exactly, the relative error for k^g under free flow (or congested flow) is 4% (or 4.5%) with RMSE equal to 1.26 (or 1.4), implying the average difference between our estimation and the field value for $\bar{k}$ is about 1. In addition, the results show that on average $\overline{k^g}$ =6.28 and $\overline{k^M}$ =6.49 in the tested congested flow, and $\overline{k^g}$ =5.84 and $\overline{k^M}$ =6.03 in free flow. Clearly, they are very close. The relative error for T^g under free flow (or congested flow) is 4.36% (or 4.15%) with RMSE equal to 3.11 seconds (or 3.24 seconds). More, the results show that on average $\overline{T^g} = 25.4$ seconds and $\overline{T^M} =$ 26.02 seconds in the tested congested flow, and $\overline{T^g}$ =17.8 seconds and $\overline{T^M}$ =18.48 seconds in the tested free flow. Again, they are very close. The experimental results indicate small estimation errors. Thus, we claim that our mathematical estimation formulations for two-way road also perform well.

Overall, the numerical experiments indicate that the proposed approaches perform pretty well under both free flow and congested traffic flow on a one-way segment and two-way segment. In more details, our mathematical estimations perform a bit better for free flow traffic condition than for congested traffic flow condition on a one-way road segment and two-way road segment, due to probabilistic bounds rather than closed form formulations are developed to estimate $\bar{k}$ in congested flow.

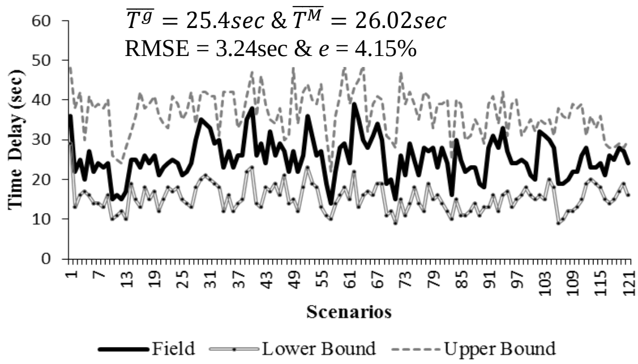


(a) Congested flow

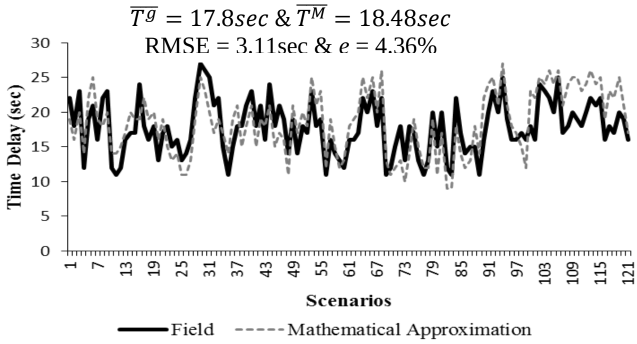


(b) Free flow

Fig. 12. Comparison of number of hops between field and mathematical estimation on two-way segment



(a) Congested flow



(b) Free flow

Fig. 13. Comparison of time delay between field and mathematical estimation on two-way segment

V. Conclusion and Future Work

Vehicle-to-vehicle communication holds promising future applications to improve traffic safety, sustainability, and mobility. However, to successfully implement these applications and cash these benefits, practitioners are still lack of reliable formulations to estimate the information propagation time delay based on traffic flow characteristics and communication limitations. Even though plenty of research worked on this issue in the literature, oversimplified communication or traffic flow assumptions weaken their applicability in practice. Motivated by the above view, the proposed research aims to develop more reliable formulations to estimate the time delay of a piece of information propagating through a traffic stream, considering more traffic flow scenarios, such as on one-way or two-way road segments under either free or congested traffic flow, and realistic wireless communication constraints, such as interference, information flow direction, instantaneous, and ferry transmission.

Stochastic and probabilistic models combined with analytical approaches are adopted to develop the pseudo transmission range from SINR condition, to estimate the expected number of hops in instantaneous transmission, and to estimate the expected transmission distance under instantaneous and ferry transmission individually. Based upon these analytical formulations and elaborate analyses, this study further proposes closed-form analytical formulations to estimate exact time delay value for information propagating through free flow, and provides analytical solutions to estimate the upper and lower bound time delay for information propagating through congested flow. Numerical experiments are conducted to validate our approaches, based on the field data collected by NGSIM for one-way segment on US101 (congested and free flow), and two-way segments on Peachtree Street (free flow) and Lankershim Blvd (congested flow). The experiment results indicate that the proposed mathematical formulations provide reliable estimation to information propagation time delay, with the relative error about 4% under various traffic conditions on a one-way or two-way road segment. The ground true time delays in congested flow on either one-way or two-way road segment are well-bounded inside the analytical upper and lower bounds.

There is some potential future research stemmed from this study. First, the presented study can be extended to network level since it counts the time delay resulting from instantaneous transmission, which is negligible in single road segment but significant as information spreads over a large scale of network. Information propagation at intersection is another interested related research issue in this context. Second, the proposed methodology can be further extended to establish information propagation dynamics overtopping traffic flow dynamics. Information flow throughput over both temporal and spatial dimensions can be explored. Clearly, the proposed analytical formulations, capturing the time delay of information propagating through a road segment, provide a firm base to investigate these advanced topics. This research team will work on them in the near future.

Appendix

Appendix A1. Relative error of the lower bounds for Λ_i (with n=10)

$1 \le i \le n-1$	$a_1 = \sum_{m=1}^{i} \frac{1}{m^2}$	$a_2 = \sum_{m=1}^{n-i} \frac{1}{m^2}$	$a_3 = \frac{\pi^2}{3} - \frac{n}{i(n-i)}$	$a_1 + a_2 - a_3$	Relative Error (%)
1	1.0000	1.5398	2.1796	0.3602	0.1418
2	1.2500	1.5274	2.6657	0.1117	0.0402
3	1.3611	1.5118	2.8145	0.0584	0.0203
4	1.4236	1.4914	2.8741	0.0409	0.0140
5	1.4636	1.4636	2.8907	0.0365	0.0125
6	1.4914	1.4236	2.8741	0.0409	0.0140
7	1.5118	1.3611	2.8145	0.0584	0.0203
8	1.5274	1.2500	2.6657	0.1117	0.0402
9	1.5398	1.0000	2.1796	0.3602	0.1418
$i = n$	$a_1 = \sum_{m=1}^{i} \frac{1}{m^2}$	$a_2 = 0$	$a_3 = \frac{\pi^2}{6} - \frac{1}{i}$	$a_1 - a_3$	Relative Error(%)
10	1.5498		1.5449	0.0049	0.0031
a_3 is the estimation of $(a_1 + a_2)$, which is one of the components in Λ_i; Relative Error = $100 \times \frac{(a_1+a_2)-a_3}{(a_1+a_2)}$					

Appendix A2. Relative error of the lower bounds for Λ_i (with n=20)

$1 \le i \le n-1$	$a_1 = \sum_{m=1}^{i} \frac{1}{m^2}$	$a_2 = \sum_{m=1}^{n-i} \frac{1}{m^2}$	$a_3 = \frac{\pi^2}{3} - \frac{n}{i(n-i)}$	$a_1 + a_2 - a_3$	Relative error (%)
1	1.0000	1.5937	2.2381	0.3556	0.1371
2	1.2500	1.5909	2.7352	0.1057	0.0372
3	1.3611	1.5878	2.8986	0.0504	0.0171
4	1.4236	1.5843	2.9782	0.0297	0.0099
5	1.4636	1.5804	3.0241	0.0200	0.0066

6	1.4914	1.5760	3.0526	0.0148	0.0048
7	1.5118	1.5709	3.0709	0.0117	0.0038
8	1.5274	1.5650	3.0824	0.0100	0.0032
9	1.5398	1.5580	3.0887	0.0091	0.0029
10	1.5498	1.5498	3.0907	0.0088	0.0028
11	1.5580	1.5398	3.0887	0.0091	0.0029
12	1.5650	1.5274	3.0824	0.0100	0.0032
13	1.5709	1.5118	3.0709	0.0117	0.0038
14	1.5760	1.4914	3.0526	0.0148	0.0048
15	1.5804	1.4636	3.0241	0.0200	0.0066
16	1.5843	1.4236	2.9782	0.0297	0.0099
17	1.5878	1.3611	2.8986	0.0504	0.0171
18	1.5909	1.2500	2.7352	0.1057	0.0372
19	1.5937	1.0000	2.2381	0.3556	0.1371
$i = n$	$a_1 = \sum_{m=1}^{i} \frac{1}{m^2}$	$a_2 = 0$	$a_3 = \frac{\pi^2}{6} - \frac{1}{i}$	$a_1 - a_3$	Relative Error (%)
20	1.5962		1.5949	0.0013	0.0008
a_3 is the estimation of $(a_1 + a_2)$, which is one of the components in Λ_i; Relative Error = $100 \times \frac{(a_1+a_2)-a_3}{(a_1+a_2)}$					

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Lili Du received the B.S. degree in Mechanical Engineering from Xi'An JiaoTong University, Xi'an, China in 1998, the M.S. degree in Industrial Engineering from Tsinghua University, Beijing, China in 2003, and Ph.D. degree from Rensselaer Polytechnic Institute, Troy, USA in 2008.

She is currently an assistant professor in the Department of Civil, Architectural, and Environmental Engineering at the Illinois Institute of Technology, Chicago. Her work has appeared in ACM Workshop on Vehicular Ad Hoc Networks, Transportation Research Part B: Methodological, Transportation Research Part C: Emerging Technologies, Networks and Spatial Economics, Renewable Energy, International Journal of Production Research, among others. Her current research interests include vehicle-to-vehicle communication network, connected vehicle, intelligent transportation systems, real-time traffic sensing, and transportation network design and system analysis.

Dr. Du is a member of Transportation Research Board Transportation Network Modeling Committee (ADB30).

Hoang Dao received the B.S. degree in Civil Engineering from Hanoi University of Civil Engineering, Hanoi, Vietnam in 2009 and the M.S. degree in Civil Engineering from Illinois Institute of Technology, Chicago, USA in 2012.

He is currently a Ph.D. student in Department of Civil, Architectural, and Environmental Engineering at Illinois Institute of Technology, Chicago, USA. His current research interests include intelligent transportation systems and vehicle-to-vehicle communication networks.

Xiang-Yang Li received the M.S. (2000) and Ph.D. (2001) degree at Department of Computer Science from University of Illinois at Urbana-Champaign, a Bachelor degree at Department of Computer Science and a Bachelor degree at Department of Business Management from Tsinghua University, P. R. China, both in 1995.

He is currently a professor in the Department of Computer Science at the Illinois Institute of Technology, Chicago; He holds EMC-Endowed Visiting Chair Professorship at Tsinghua University; a distinguished visiting professor at Xi'An JiaoTong University, and University of Science and Technology of China. His research interests include mobile computing, cyber physical systems, wireless networks, security and privacy, and algorithms.

Dr. Li is a senior member of Institute of Electrical and Electronics Engineers (IEEE) and a member of Association for Computing Machinery (ACM).