Multicast Throughput for Hybrid Wireless Networks under Gaussian Channel Model

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Abstract—We study the multicast capacity for hybrid wireless networks consisting of ordinary ad hoc nodes and base stations under *Gaussian Channel* model, which generalizes both the unicast capacity and broadcast capacity for hybrid wireless networks. Assume that all ordinary ad hoc nodes transmit at a constant power P, and the power decays along the path, with attenuation exponent $\alpha > 2$. The data rate of a transmission is determined by the SINR (Signal to Interference plus Noise Ratio) at the receiver as $B\log(1+\text{SINR})$. The ordinary ad hoc nodes are placed in the square region $\mathcal{A}(a)$ of area a according to a Poisson point process of intensity n/a. Then, m additional base stations (BSs) acting as the relaying communication gateway are placed regularly in the region $\mathcal{A}(a)$, and are connected by a high-bandwidth wired network. Let $a = n$ and $a = 1$, we construct the *hybrid extended network* (HEN) and *hybrid dense network* (HDN), respectively. We choose randomly and independently n_s ordinary ad hoc nodes to be the sources of multicast sessions. We assume that each multicast session has n_d randomly chosen terminals.
Three broad categories of multicast strategies are proposed. The first one is the *hybrid strategy*, *i.e.*, the multihop scheme with BS-supported, which further consists of two types of strategies called *connectivity strategy* and *percolation strategy* respectively. The second one is the *ordinary ad hoc strategy*, *i.e.*, the multihop scheme without any BS-supported. The third one is the classical BS-based strategy under which any communications between ordinary ad hoc node pairs are relayed by some specific BSs. According to the different scenarios in terms of m, n and n_d, we select the optimal scheme from the three categories of strategies, and derive the achievable multicast throughput based on the optimal decision.

Index Terms—Wireless Hybrid Networks, Wireless Ad Hoc Networks, Multicast Throughput, Random Networks, Multicast Capacity, Gaussian Channel Model

1 Introduction

The asymptotic capacity for wireless ad hoc networks has been intensively studied under difference channel models [2]. Most existing related works are based on two types of channel models. The first is called the *threshold-based channel* model [3] that determines the transmission rate as a binary function. The *protocol interference* model (PrIM) and *physical interference* model (PhIM) [2] both belong to the *threshold-based channel* model. The second one is the *Gaussian Channel* model [4] that determines the transmission rate based on a continuous function of the receiver's SINR (Signal to Interference plus Noise Ratio). The Gaussian Channel model is also called *generalized physical model* [5], it captures better the physical layer of wireless networks than *threshold-based channel* model that is a very crude approximation for wireless networks, under which any communication pair v_i and v_j can establish a direct communication link, over a channel of bandwidth B, of rate $R(v_i, v_j) = B\log(1 + \text{SINR}(v_j))$, *i.e.*, the link achieves Shannon's capacity for a wireless channel with additive Gaussian white noise, see [6], [7].

A *hybrid wireless network* (HN) consists of two types of network terminals: base stations and ordinary ad hoc nodes. Assume that all base stations can communicate with wireless ad hoc nodes, and further assume that each base station is neither a source nor a receiver, it simply serves as a relaying gateway. Intuitively, wireless ad hoc networks and cellular networks can both be regarded as the special cases of the HN, as the number of base stations is adjusted. Thus, the study of the capacity for HN has more generality than that of the wireless ad hoc networks and cellular network, while it was not fully studied. In addition, as we know, multicast capacity can unify the unicast and broadcast capacity, [8], which increases the generality of the research on the multicast capacity for HN. For HNs, there are also generally two channel models as in most existing works for wireless ad hoc networks. To the best of our knowledge, all existing results of multicast capacity for hybrid networks are derived under the threshold-based model, [9], a natural and interesting issue arises: What is the multicast capacity for hybrid networks when the Gaussian channel model is used. This paper aims to derive an achievable multicast throughput for HNs under Gaussian channel model.

We assume that the ordinary ad hoc nodes are placed in the square region $\mathcal{A}(a)$ of area a according to a Poisson point process of intensity n/a. In addition, m additional base stations (BSs) serving as the relaying communication gateway

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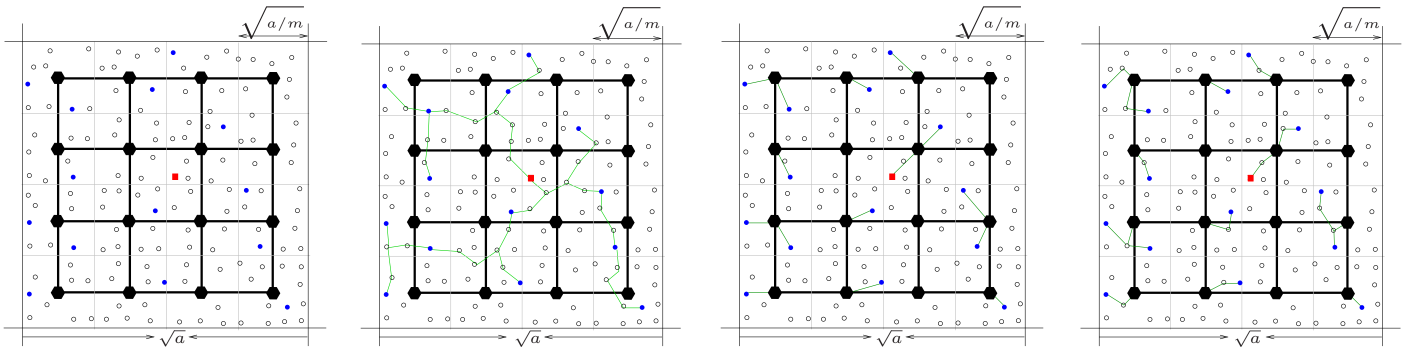


(a) Original network (b) Ordinary ad hoc strategy (c) BS-based strategy (d) Hybrid routing strategy

Fig. 1. Illustrations of three types of multicast routing schemes. The small hexagons represent the base stations that are assumed to be connected via the high-bandwidth wired links. The cases that $a = n$ and $a = 1$ correspond to the *hybrid extended network* and *hybrid dense network*, respectively.

are placed regularly in the region $\mathcal{A}(a)$ and they are connected by the high-bandwidth wired links. Let $a = n$ and $a = 1$, we construct two scaling network models: the *hybrid extended network* (HEN) and *hybrid dense network* (HDN), respectively. There are n_s randomly and independently chosen multicast sessions. Each multicast has n_d randomly chosen terminals. According to different relations among m, n and n_d, we adopt different types of multicast strategies. To be specific, we propose three broad categories of multicast strategies for both HEN and HDN. The first one is called the *hybrid strategy*, *i.e.*, the multihop scheme with BS-supported, which further consists of two types of schemes called *connectivity strategy* and *percolation strategy* respectively. The second one is the *ordinary ad hoc strategy*, *i.e.*, the multihop scheme without any BS-supported. The third one is the classical BS-based strategy, under which any communications between ordinary ad hoc node pairs are relayed by some specific BSs. For different cases in terms of m, n and n_d, we select the optimal strategy from the three categories of strategies, and derive the achievable multicast throughput based on the optimal scheme. To the best of our knowledge, this is the first work that addresses the multicast routing and scheduling strategy in hybrid wireless networks under Gaussian channel model.

The rest paper is structured as follows. In Section 2, we introduce the network model. Main results are presented and discussed in Section 3. We make technical preparations in Section 4. In Section 5, we design the multicast schemes for *hybrid extended networks*. In Section 6, we extend our results to *hybrid dense networks*. In Section 7, we review the related existing literature. In Section 8, we conclude the paper.

2 Network Model

Throughout this paper, we denote the probability of an event E as $\Pr(E)$, and we are mainly concerned with events that take place with high probability (w.h.p.), *i.e.*, with probability 1 as the number of nodes $n \to \infty$.

2.1 Network Topology

For the ordinary ad hoc nodes, we consider two classical random networks, *i.e.*, the *random extended network* (REN) and the *random dense network* (RDN). We construct REN (or RDN) by placing ordinary ad hoc nodes according to a Poisson point process of intensity 1 (or n) on the square $\mathcal{A}(n) = [0, \sqrt{n}] \times [0, \sqrt{n}]$ (or $\mathcal{A}(1) = [0, 1] \times [0, 1]$). By Chebyshev's Inequality, we can easily obtain that the number of ordinary ad hoc nodes in $\mathcal{A}(n)$ (or $\mathcal{A}(1)$) is within $((1-\epsilon)n, (1+\epsilon)n)$, where $\epsilon > 0$ is an arbitrarily small constant. We assume that there are exactly n ordinary ad hoc nodes in $\mathcal{A}(n)$ (or $\mathcal{A}(1)$), which has no impact on our results in order sense [10], [30]. Furthermore, we place regularly m base stations (BSs, with wireless transmitting power P) in $\mathcal{A}(n)$ (or $\mathcal{A}(1)$), which are connected by the high-bandwidth wired links, to construct the *hybrid extended network* (or *hybrid dense network*). Please see the illustration in Fig. 1(b). To be specific, divide $\mathcal{A}(n)$ (or $\mathcal{A}(1)$) into m *subregions* with side length $\frac{\sqrt{n}}{\sqrt{m}}$ (or $\frac{1}{\sqrt{m}}$) and place one BS on the center position of each subregion. We further assume that the number of BSs $m = O(n)$.

2.2 Achievable Multicast Throughput

Now, we give the formal definition of capacity in our model. We assume that $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ is the set of nodes in the network, $\mathcal{S} \subseteq \mathcal{V}$ is the set of sources of multicast, and assume that the number of multicast sessions $|\mathcal{S}| = n_s$. For each source node $v_{\mathcal{S},i} \in \mathcal{S}$, we uniformly select n_d nodes at random from the other nodes to constitute a set $\mathcal{D}_{\mathcal{S},i} = \{v_{\mathcal{S},i_1}, v_{\mathcal{S},i_2}, \cdots, v_{\mathcal{S},i_{n_d}}\}$ as the set of destinations, where obviously $n_d \leq n - 1$. Furthermore, define $\mathcal{U}_{\mathcal{S},i} := \{v_{\mathcal{S},i}\} \cup \mathcal{D}_{\mathcal{S},i}$ as the *spanning set* of the ith multicast sessions.

Denote $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ as the *rate vector* of the multicast data rate of all multicast sessions.

Definition 1 (Feasible Rate Vector): A multicast rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ is *feasible* if there is a spatial and temporal scheme for scheduling transmissions such that by operating the network in a multi-hop fashion and buffering at intermediate nodes when awaiting transmission, the ith source node, denoted as $v_{\mathcal{S},i}$, can deliver data to all its n_d destinations at rate of $\lambda_{\mathcal{S},i}$ bits/second, where $i = 1, 2, \cdots, n_s$. That is, there is a $T < \infty$ such that in every time interval (with unit seconds) $[(j-1) \cdot T, j \cdot T]$, every node $v_{\mathcal{S},i} \in \mathcal{S}$ can send $T \cdot \lambda_{\mathcal{S},i}$ bits to all its n_d destinations.

Considering a multicast rate vector, we define the *total multicast throughput* of such feasible rate vector as $\Lambda^{\mathrm{T}}_{\mathcal{S},n_d}(n) = \Sigma_{i=1}^{n_s}\lambda_{\mathcal{S},i}$, define the *average multicast throughput* as $\Lambda^{\mathrm{A}}_{\mathcal{S},n_d}(n) = \frac{\Sigma_{i=1}^{n_s}\lambda_{\mathcal{S},i}}{n_s}$, and define the *minimum per-session multicast throughput* (also called *per-session multicast throughput* for concise) as $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = \min_{v_{\mathcal{S},i}\in\mathcal{S}}\lambda_{\mathcal{S},i}$.

Definition 2 (Throughput Capacity): An aggregated multicast throughput $\Lambda^{\mathrm{T}}_{\mathcal{S},n_d}(n) = \Sigma_{i=1}^{n_s}\lambda_{\mathcal{S},i}$ is *achievable* for n_s multicast sessions (each session with n_d destinations) if the rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ that is feasible.

Similarly, we can define the achievable average multicast throughput and achievable per-session multicast throughput.

Definition 3 (Multicast Capacity of Random Networks): The *per-session multicast capacity* of a class of random networks is of order $\Theta(g(n))$ bits/sec if there are deterministic constants $c > 0$ and $c < c' < +\infty$ such that

$$\lim_{n\to+\infty} \Pr(\Lambda^{\mathrm{M}}_{\mathcal{S},n_d}(n) = cg(n) \text{ is achievable}) = 1,$$
$$\liminf_{n\to+\infty} \Pr(\Lambda^{\mathrm{M}}_{\mathcal{S},n_d}(n) = c'g(n) \text{ is achievable}) < 1.$$

We can similarly define the aggregated multicast capacity and average multicast capacity for random networks . In this paper, we will only consider the per-session multicast capacity by which the other two types of capacities can be derived straightforwardly. The achievable multicast throughput is a lower bound of the multicast capacity. Without loss of compatibility to most existing works, we assume that $n_s = \Theta(n)$.

2.3 Gaussian Channel Model

Assume that all nodes transmit with a constant power P, and any two nodes can establish a direct communication link over a channel of bandwidth B, of rate

$$R(v_i, v_j) = B\log\left(1 + \frac{P\cdot\ell(v_i, v_j)}{N_0 + \sum_{v_k\in A(i)/v_i} P\cdot\ell(v_k, v_j)}\right),$$

where N_0 is the ambient noise power, $A(i)$ is the set of nodes that transmit when v_i is scheduled. Let d_{ij} denote the Euclidean distance between v_i and v_j. Let the power attenuation function be $\ell(v_i, v_j)$. For HEN, let $\ell(v_i, v_j) := \min\{1,\ d_{ij}^{-\alpha}\}$ with $\alpha > 2$ and $N_0 > 0$; for HDN, let $\ell(v_i, v_j) := d_{ij}^{-\alpha}$ with $\alpha > 2$ and $N_0 \geq 0$, [10], [11].

NOTATIONS: Throughout this paper, for a 2-dimension line segment $L = uv$, $|L|$ represents the Euclidean distance between u and v; for a discrete set $\mathcal{U}$, $|\mathcal{U}|$ represents its cardinality. For a continuous region $\mathcal{A}$, we use $\|\mathcal{A}\|$ to denote its area; for a tree $\mathcal{T}$ (or a forest $\mathcal{F}$) , we use $\|\mathcal{T}\|$ (or $\|\mathcal{F}\|$) to denote its total Euclidean edge length. To simplify the description, let $\theta(n) : [\theta_1(n), \theta_2(n)]$ represent $\theta(n) = \Omega(\theta_1(n))$ and $\theta(n) = O(\theta_2(n))$; and let $\theta(n) : (\theta_1(n), \theta_2(n)]$ represent $\theta(n) = \omega(\theta_1(n))$ and $\theta(n) = O(\theta_2(n))$.

3 MAIN RESULTS

In this paper, for both HEN and HDN, we design three types of strategies, *i.e.*, *hybrid strategy*, *ordinary ad hoc strategy* and *BS-based strategy*. Please see Fig.1 for illustrations.

1) *Ordinary ad hoc strategy* will *not* use any base station, in other words, we treat the hybrid network as a pure ad hoc network assuming there are no base stations.
2) *BS-based strategy* can only allow receivers (or source nodes) to communicate with base stations in corresponding subregion *directly*, *e.g.* we do *not* allow any relay nodes in each subregion.
3) *Hybrid strategy* uses a specific routing and scheduling scheme to let receivers (or source nodes) communicate with central base stations in the corresponding subregion, in particular, we can use the other ordinary ad hoc nodes in same subregion to relay data.

3.1 Optimal Decision based on Three Strategies

According to the different scenarios in terms of m, n and n_d, we select the optimal scheme from the three categories of strategies for HEN and HDN, respectively, and derive the achievable multicast throughput based on the optimal scheme.

3.1.1 Optimal Strategy among Three Strategies for HEN

Theorem 1: Combining three types of multicast strategies, the optimal decision of strategy and the achievable multicast throughput for HEN are made as in Table 1.

3.1.2 Optimal Strategy among Three Strategies for HDN

Theorem 2: Combining three types of multicast strategies, the optimal decision of strategy and the achivable multicast throughput for HDN are made as in Table 2.

3.2 Discussion for Results

3.2.1 Generality of Results

Due to the generality of multicast sessions, that is, unicast and broadcast can be regarded as the special cases of multicast, our results can unify the throughput for unicast and broadcast by letting $n_d = 1$ and $n_d = n - 1$, respectively. However, when we specialize to unicast throughput, *i.e.*, let $n_d = 1$, there is indeed a gap of factor $(\log n)^{-\frac{\alpha}{2}}$ between our results for HEN and those in [12]. In fact, for the routing of [12], the ordinary ad hoc nodes in each subregion access to the corresponding BS via the *connectivity paths* defined in Section 5 of this paper. Unlike in *dense networks*, the *connectivity paths* in *extended networks* can only sustain a rate of order $\Omega((\log n)^{-\frac{\alpha}{2}})$ instead of the constant rate as stated in Lemma 5 of [12]. We believe, the mistake in Lemma 5 of [12] leads to the gap between our results for unicast case and their results.

3.2.2 Analysis of Bottlenecks

As in most existing works for the capacity of hybrid networks, we also assume the links between base stations and ordinary ad hoc nodes (we call such links *B-O links*) have no difference from those among ordinary ad hoc nodes. While, in the analysis of bottlenecks on three types of strategies for both HEN and HDN (Section 5 and Section 6), we find that for most cases in terms of m and n_d, the bottlenecks locate on *B-O links*. Therefore, if the bandwidth of *B-O links* can be increased, the throughput for the whole network will possibly be enhanced.

TABLE 1
Optimal Decision of Strategy and Multicast Throughput for HEN

Range in terms of m	Relations among m, n_d and n	Optimal Strategy	Multicast Throughput
$m : [1, n/\log n]$	If $\begin{cases} n_d : [1, n/(\log n)^{\alpha+1}] \text{ and} \\ m : [\sqrt{nn_d \cdot (\log n)^{\alpha}}, n/\log n] \end{cases}$	Hybrid Strategy	$\Omega(\frac{m}{n \cdot n_d} \cdot (\log n)^{-\frac{\alpha}{2}})$
	Otherwise	Ordinary Ad Hoc Strategy	Theorem 8
$m : [n/\log n, n]$	If $n_d : [1, n/(\log n)^{\alpha+2}]$	BS-based strategy	$\frac{m}{nn_d} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}$
	Else If $\begin{cases} n_d : [n/(\log n)^{\alpha+2}, n/(\log n)^{\alpha+1}] \text{ and} \\ m : [(n^{\alpha+1} \cdot n_d)^{\frac{1}{\alpha+2}}, n] \end{cases}$	BS-based strategy	$\frac{m}{nn_d} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}$
	Else If $\begin{cases} n_d : [n/(\log n)^{\alpha+1}, n/(\log n)^{2}] \text{ and} \\ m : [n \cdot (\log n)^{-\frac{\alpha+1}{\alpha+2}}, n] \end{cases}$	BS-based strategy	$\frac{m}{nn_d} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}$
	Else If $\begin{cases} n_d : [n/(\log n)^{2}, n/\log n] \text{ and} \\ m : [(n^{\alpha+1} \cdot (\log n)^{1-\alpha} \cdot n_d)^{\frac{1}{\alpha+2}}, n] \end{cases}$	BS-based strategy	$\frac{m}{nn_d} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}$
	Else If $\begin{cases} n_d : [n/\log n, m] \text{ and} \\ m : [n \cdot (\log n)^{-\frac{\alpha}{\alpha+2}}, n] \end{cases}$	BS-based strategy	$\frac{m}{nn_d} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}$
	Else If $\begin{cases} n_d : [m, n] \text{ and} \\ m : [(\frac{n}{n_d})^{\frac{2}{\alpha}} \cdot \frac{n}{\log n}, n] \end{cases}$	BS-based strategy	$\frac{1}{n} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}$
	Otherwise	Ordinary Ad Hoc Strategy	Theorem 8

TABLE 2
Optimal Decision of Strategy and Multicast Throughput for HDN

Range in terms of m	Relations among m, n_d and n	Optimal Strategy	Multicast Throughput
$m : [1, n/\log n]$	If $\begin{cases} n_d : [1, n/(\log n)^{3}] \text{ and} \\ m : [\sqrt{nn_d}, n/\log n] \end{cases}$	Hybrid Strategy	$\Omega(\frac{m}{n \cdot n_d})$
	Else If $\begin{cases} n_d : [n/(\log n)^{3}, n/(\log n)^{2}] \text{ and} \\ m : [n \cdot (\log n)^{-\frac{3}{2}}, n/\log n] \end{cases}$	Hybrid Strategy	$\Omega(\frac{m}{n \cdot n_d})$
	Else If $\begin{cases} n_d : [n/(\log n)^{2}, n/(\log n)] \text{ and} \\ m : [\sqrt{nn_d} \cdot (\log n)^{-\frac{1}{2}}, n/\log n] \end{cases}$	Hybrid Strategy	$\Omega(\frac{m}{n \cdot n_d})$
	Otherwise	Ordinary Ad Hoc Strategy	Theorem 15
$m : [n/\log n, n]$		BS-based strategy	Theorem 16

Hence, when we consider the *hybrid strategies*, we designedly derive the throughput without taking the possible bottlenecks on the *B-O links* into account. (Please see details in Theorem 3, Theorem 5, Theorem 10 and Theorem 12.) We deem that these results could be used when some new assumptions are made for the *B-O links*.

3.2.3 Matching Upper Bounds

To the best of our knowledge, even for wireless ad hoc networks, for both *random extended networks* and *random dense networks*, there are still no matching upper bounds and lower bounds for multicast capacity under *Gaussian Channel model*, [3]. For *hybrid networks*, there is no result for such upper bounds. A trivial upper bound is of order $O(1)$, which is the bound *without* interference limitation, [13], [14]. For both HEN and HDN, this bound can be achieved by *BS-based strategy* when $m = \Theta(n)$ and $n_d = \Theta(1)$. Please see Table 1 and Table 2. It is an interesting issue to derive the upper bounds and validate whether the lower bounds proposed in this paper are tight or not for any regime in terms of $m : [1, n]$ and $n_d : [1, n]$.

Moreover, we limit the scope of this paper to *networking-theoretic capacity bounds*, *i.e.*, we assume that the signals received from nodes other than one particular transmitter are simply regarded as noise degrading the communication link, [2], [10]. From the *information-theoretic* perspective [15], the bounds beyond those in this paper can be possibly achieved by introducing some physical layer cooperative strategies, [14].

4 Technical Preparations

4.1 Probability Inequality

Lemma 1 (Chebyshev's Inequality): Let X be a random variable, then

$$\Pr(|X - \mathbf{E}(X)| \geq \epsilon) \leq \mathbf{Var}(X)/\epsilon^2,$$

where $\mathbf{E}(X)$ is the mean of X, $\mathbf{Var}(X)$ is the variance of X, and $\epsilon 0$ is an arbitrary small positive value.

In the following analysis, we often need to prove the uniform convergence of the probability of some events. Vapnik-Chervonenkis Theorem [16] is usually exploited to prove such issue, as in [2], [4], [8]. When the deployment region $\mathcal{A}$ is partitioned into a lattice consisting of subsquares that act as Voronoi cells, the exponent tails of probability bound can be equally used to prove the uniform convergence of some probability [10].

Lemma 2 (Tails of Chernoff bounds, Mitzenmacher [17]): Let X be a Poisson random variable with parameter λ. Then

$$\Pr(X \geq x) \quad \leq \quad \frac{e^{-\lambda} \cdot (e\lambda)^x}{x^x}, \quad \text{for } x > \lambda. \tag{1}$$

$$\Pr(X \leq x) \quad \leq \quad \frac{e^{-\lambda} \cdot (e\lambda)^x}{x^x}, \quad \text{for } 0 < x < \lambda. \tag{2}$$

Proof: The upper tail, (1) has been proved by Franceschetti *et al.* in [10]. Here we concisely prove the lower tail, (2). For $t > 0$ and $0 \le x < \lambda$, by Markov Inequality,

$$\Pr(X \le x) = \Pr(-X \ge -x) \le E(e^{-tX})/e^{-tx}$$

Since $E(e^{-tX}) = \sum_{k=0}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!} \cdot e^{-tk} = e^{\lambda(e^{-t}-1)}$, thus $\Pr(X \le x) \le e^{\lambda(e^{-t}-1)+tx}$. Let $t = \ln(\lambda/x) > 0$, we complete the proof. □

4.2 Euclidean Spanning Tree

Partition the square $\mathcal{A}(a)$ into $\rho \le m$ subsquares while ensuring that there is one base station at the center of each subsquare, where a is the area of the deployment square region. We call those ρ subsquares *subregions*. Note that one subregion may contain more than one base station, but we only need to use the central one in our proposed routing scheme. For each multicast session $\mathcal{M}_k,\ k = 1, 2, \cdots n_s$, we denote the *spanning set* as $\mathcal{U}_k = \{v_k\} \cup \{v_{k_1}, v_{k_2}, \cdots v_{k_{n_d}}\}$, where v_k is the source node and the nodes in the latter set are the destinations of v_k. Let $\mathcal{U}_k^\iota = \{v_{k_1}^\iota, v_{k_2}^\iota, \cdots, v_{k_t}^\iota\}$ denote a subset of $\mathcal{U}_k$ to represent the set of nodes contained in the subregion S_ι, where $\mathcal{U}_k = \bigcup \mathcal{U}_k^\iota$ and $\mathcal{U}_k^{\iota_1} \cap \mathcal{U}_k^{\iota_2} = \emptyset$ for any $\iota_1 \ne \iota_2$. Let $\tilde{\mathcal{U}}_k^\iota = \mathcal{U}_k^\iota \cup \{b_\iota\}$, where b_ι denotes the base station that is placed at the center of subregion S_ι. Then, we can build the Euclidean spanning tree (EST) based on every set $\tilde{\mathcal{U}}_k^\iota$ using the method in [8]. Denote those ESTs as $\mathrm{EST}(\tilde{\mathcal{U}}_k^\iota)$, $1 \le \iota \le \varphi_k$, where φ_k is a random variable representing the number of *occupied* subregions, *i.e.*, those containing at least one ordinary ad hoc node in $\mathcal{U}_k$. We note that for each $\tilde{\mathcal{U}}_k^\iota$ except for that one including v_k (denoted as $\tilde{\mathcal{U}}_k^{\iota_o}$), b_ι acts as the root of EST; for $\tilde{\mathcal{U}}_k^{\iota_o}$, v_k acts as the root of EST.

It is the complement issue of occupancy problem [18], [19] to consider the random variable φ_k, *i.e.*, the number of occupied cells. Suppose that $n_d + 1$ balls are randomly distributed into ρ cells. Assume that each ball has an equal chance of being distributed to each cell. Let $\bar{\varphi}_k$ be the number of cells remaining empty. Hence, $\varphi_k = \rho - \bar{\varphi}_k$. By *occupancy theory* [18], the probability distribution of φ_k is given by

$$\begin{aligned}\Pr(\varphi_k = z) &= \Pr(\bar{\varphi}_k = \rho - z) \\ &= \sum_{i=1}^{z} (-1)^i C_z^i \left(\frac{z-i}{\rho}\right)^{n_d+1}\end{aligned}$$

where C_ρ^z is the binomial coefficient equal to the number of combinations of z items selected from ρ items. We necessarily pursue the uniform bound of $\varphi_k,\ k = 1, 2, \cdots n_s$.

Define the random variables $\varphi_{max} = \max_k\{\varphi_k\}$ and $\varphi_{min} = \min_k\{\varphi_k\}$. Much research has been implemented to the tail bounds for occupancy, [20], [21]. Since we concentrate on the lower bounds on multicast capacity, we only need the following straightforward upper bound on φ_{max} (Lemma 3), while noticing that we should use the tail bounds for occupancy to lowerbound φ_{min} when we study the upper bounds on multicast capacity.

Lemma 3: $\varphi_{max} = \max_k\{\varphi_k\} = O(\min\{n_d, \rho\})$, *w.h.p.*

Next, we recall an result on the total length of the EST based on a given set of nodes.

Algorithm 1 Construction of EST

Input: A set of nodes $\mathcal{U}$ with $|\mathcal{U}| = \mathfrak{u}$ that are distributed into a square region of area a

Output: An Euclidean spanning tree $\mathrm{EST}(\mathcal{U})$.

In the initial state, all nodes in $\mathcal{U}$ are isolated, then there are $\mathfrak{u}$ connected components.
for $i = 1 : \mathfrak{u} - 1$ **do**
  Partition the deployment region $\mathcal{A}(a) = [0, \sqrt{a}]^2$ into at most $\mathfrak{u} - i$ square cells of side length $\sqrt{a}/\lfloor\sqrt{\mathfrak{u} - i}\rfloor$;
  Find a cell that contains more than two nodes of $\mathcal{U}$ belonging to two different connected components. By connecting the pair of nodes, we merge the two connected components.
end for

Lemma 4 ([8]): For any set of nodes, denoted by $\mathcal{U}$, placed in a square of area a, the length of an Euclidean spanning tree (EST) that is obtained by Algorithm 1 with the input $\mathcal{U}$ is at most $2\sqrt{2a} \cdot \sqrt{|\mathcal{U}| - 1}$.

Denote the forest consisting of all $\mathrm{EST}(\tilde{\mathcal{U}}_k^\iota)$ $(1 \le \iota \le \varphi_k)$, as $\mathcal{F}_k$. Then, we have

Lemma 5: The total Euclidean edge length of $\mathcal{F}_k$, *i.e.*, $\|\mathcal{F}_k\|$, is *w.h.p.*of order $O(\frac{\sqrt{a}}{\sqrt{\rho}} \cdot \sqrt{n_d \cdot \min\{n_d, \rho\}})$, for any k, $1 \le k \le n_s$.

Proof: Denote the number of vertexes of $\mathrm{EST}(\mathcal{U}_k^\iota)$ by x_k^ι, and that of $\mathrm{EST}(\tilde{\mathcal{U}}_k^\iota)$ by $\tilde{x}_k^\iota$, where $1 \le \iota \le \varphi_k$ and $1 \le k \le n_s$. Obviously, $\tilde{x}_k^\iota = x_k^\iota + 1$. According to Lemma 4, $\|\mathrm{EST}(\mathcal{U}_k^\iota)\| = O(\sqrt{x_k^\iota - 1} \cdot \frac{\sqrt{a}}{\sqrt{\rho}})$. Hence, there exists a constant κ_1 such that

$$\sum_{\iota=1}^{\varphi_k} \|\mathrm{EST}(\mathcal{U}_k^\iota)\| \le \kappa_1 \cdot \frac{\sqrt{a}}{\sqrt{\rho}} \cdot \sum_{\iota=1}^{\varphi_k} \sqrt{x_k^\iota - 1}$$

By Cauchy-Schwartz Inequality, we have

$$\sum_{\iota=1}^{\varphi_k} \sqrt{x_k^\iota - 1} \le \sqrt{\varphi_k \sum_{\iota=1}^{\varphi_k} (x_k^\iota - 1)} \le \sqrt{\varphi_k (n_d - \varphi_k)}$$

Since $\|\mathrm{EST}(\tilde{\mathcal{U}}_k^\iota)\| \le \|\mathrm{EST}(\mathcal{U}_k^\iota)\| + \frac{\sqrt{2a}}{\sqrt{\rho}}$, there is a constant κ_2 such that $\|\mathcal{F}_k\| \le \frac{\sqrt{a}}{\sqrt{\rho}} \cdot (\kappa_1\sqrt{\varphi_k \cdot (n_d - \varphi_k)} + \kappa_2 \cdot \varphi_k)$. Then, $\|\mathcal{F}_k\| = O(\frac{\sqrt{a}}{\sqrt{\rho}} \cdot \sqrt{n_d \cdot \varphi_k})$. Combining with Lemma 3, we complete the proof. □

4.3 Result on Bond Percolation Model

Let $\mathbb{B}(h, p)$ denote a square lattice composed of $h \times h$ subsquares in which each edge is *open* with the probability p [22]. We call a path consisting of only open edges (bonds) *open path*. For a given constant $\kappa > 0$, we partition the lattice $\mathbb{B}(h, p)$ into horizontal (vertical) rectangle slabs with the horizontal (or vertical) width of h and the vertical (horizontal) width of $\kappa \log h - \epsilon(h)$, denoted by R_i^h (or R_i^v). We can choose ε_h as the smallest value such that the number of rectangle slabs $\frac{h}{\kappa \log h - \epsilon(h)}$ is an integer. It is obvious that $\epsilon(h) = o(1)$ as $h \to \infty$ [10]. Denote the number of edge-disjoint *open paths* in slab R_i^h (or R_i^v) by N_i^h (or N_i^v). Let $N^h = \min_i N_i^h$, $N^v = \min_i N_i^v$. Then, we have

Lemma 6: ([10]) For any constant $\kappa > 0$ and $p \in (\frac{5}{6}, 1)$ satisfying $2 + \kappa \log(6(1-p)) < 0$, there exists a constant $\delta(\kappa, p)$ such that

$$\lim_{h\to\infty} \Pr(N^h \geq \delta \log h) = 1; \lim_{h\to\infty} \Pr(N^v \geq \delta \log h) = 1.$$

4.4 Bottleneck Principle

When the adopted strategy is of hierarchical structure, the final network throughput is determined by the bottleneck in certain phase. That is,

Lemma 7: The achievable multicast throughput derived by the strategy $\Im$ is of $\Lambda = \min\{\Lambda_j;\ j = 1, 2, \cdots, \tau\}$, where we assume that the routing scheme consists of τ phases and let Λ_j denote the throughput in Phase j.

5 Multicast Strategies for HEN

We design three types of multicast strategies, *i.e.*, *hybrid strategy*, *ordinary ad hoc strategy* and *BS-based strategy*, to obtain the achievable multicast throughput for *hybrid extended network* (HEN). A novel technique called *parallel transmission scheduling* [1] is introduced. The assumption is reclaimed that the bottleneck of the whole routing does not locate on the links among BSs, since they are connected by high bandwidth wired network. However, the links between BSs and ordinary ad hoc nodes become possibly, actually often, the bottleneck throughout the whole routing. As mentioned above, for the simplicity of analysis, we partition $\mathcal{A}(n)$ into ρ ($\rho \leq m$) *subregions* of side length $\frac{\sqrt{n}}{\sqrt{\rho}}$, ensuring there is at least one base station contained in each subregion. Note that there may be more than one base station located at same subregion, but we are only interested at the central one. In the following context, we denote the base station located at the center of subregion S_ι by b_ι.

All our strategies are devised based on the cell-partitioned method [4], [8], [10]. For clarify the description of the strategies, we first introduce a notion called *scheme lattice*.

Definition 4 (Scheme Lattice): Divide a square deployment region of side length $\mathfrak{d}$ into a lattice consisting of square cells of side length $\mathfrak{l}$, we call the lattice *scheme lattice* and denote it as $\mathbb{L}(\mathfrak{d}, \mathfrak{l}, \theta)$, where $\theta \in [0, \frac{\pi}{4}]$ is the minimum angle between the edges of the deployment region and those of the cells.

5.1 Hybrid Strategy for HEN

The *hybrid strategies* can be further classified into two optional strategies called *connectivity strategy* and *percolation strategy* respectively.

5.1.1 Connectivity Strategy

We state that the *connectivity strategy* can be applied when $\rho = O(n/\log n)$. We denote *connectivity strategy* by $\bar{\Im}_e$, and the routing and wireless transmission scheduling by $\bar{\Im}_e^r$ and $\bar{\Im}_e^t$ respectively. Divide $\mathcal{A}(n)$ into subsquares with area $\bar{a}_e = 2\theta \cdot \log n$, where θ is a constant with $\theta > \frac{1}{2\log 2 - \log e}$. That is, we design the strategy based on the *scheme lattice* $\mathbb{L}(\sqrt{n}, \sqrt{\bar{a}_e}, 0)$ in which the cells are called *connectivity cells*. Furthermore, we separate each cell into halves horizontally (or vertically) called horizontal (or vertical) *half-cells*. Please see the illustration in Fig.3(b). Then, we have

Lemma 8: With high probability, there are at most $2\theta \cdot \log n$ and at least $\frac{\theta}{2} \cdot \log n$ ordinary ad hoc nodes in every half-cell.

Proof: Define the number of ordinary ad hoc nodes in any half-cell, say c_i, as a random variable μ_i. Then, μ_i follows the Poisson distribution of mean $\bar{a}_e/2$, *i.e.*, $\theta \cdot \log n$. Further, we define the minimum of μ_i for all c_i as $\underline{\chi}$; and define the maximum of μ_i for all c_i as $\overline{\chi}$.

Combining (1) in Lemma 2 and union bounds, we have

$$\begin{aligned} \Pr(\overline{\chi} \geq 2\theta \cdot \log n) &\leq \frac{4n}{\bar{a}_e} \cdot \Pr(\mu_i \geq 2\theta \cdot \log n) \\ &\leq n \cdot (e/4)^{\theta \cdot \log n} \\ &= (e/4)^{(\theta - \frac{1}{2\log 2 - \log e}) \cdot \log n} \end{aligned}$$

Thus, by $\theta > \frac{1}{2\log 2 - \log e}$, we have $\Pr(\overline{\chi} \geq 2\theta \cdot \log n) \to 0$.

Similarly, combining (2) in Lemma 2 and union bounds, we have

$$\begin{aligned} \Pr(\underline{\chi} \leq \frac{\theta}{2} \cdot \log n) &\leq \frac{4n}{\bar{a}_e} \cdot \Pr(\mu_i \leq \frac{\theta}{2} \cdot \log n) \\ &\leq n \cdot (1/2e)^{\frac{\theta}{2} \cdot \log n} \\ &= (1/2e)^{(\theta - \frac{2}{\log 2e}) \cdot \frac{\log n}{2}} \end{aligned}$$

Then, by $\theta > \frac{1}{2\log 2 - \log e}$, we have that $\theta > \frac{2}{\log 2e}$. Hence, $\Pr(\underline{\chi} \leq \frac{\theta}{2} \cdot \log n) \to 0$.

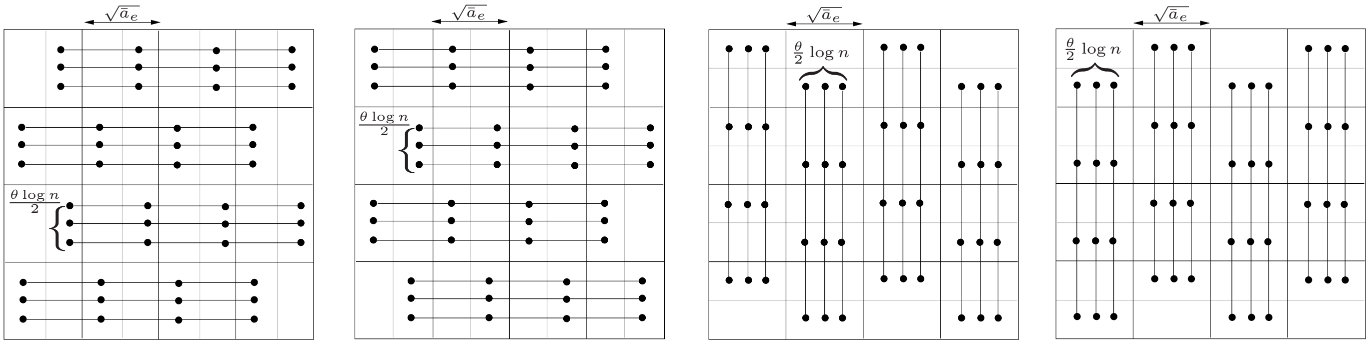
Therefore, for all half-cells, it holds uniform *w.h.p.*, that $\mu_i \in (\frac{\theta}{2} \cdot \log n, 2\theta \cdot \log n)$, which completes the proof. □

Routing scheme $\bar{\Im}_e^r$: We propose Algorithm 2 to construct the multicast routing tree $\mathcal{T}(\mathcal{U}_k)$ for multicast session $\mathcal{M}_k$.

Algorithm 2 Connectivity Routing Scheme $\bar{\Im}_e^r$

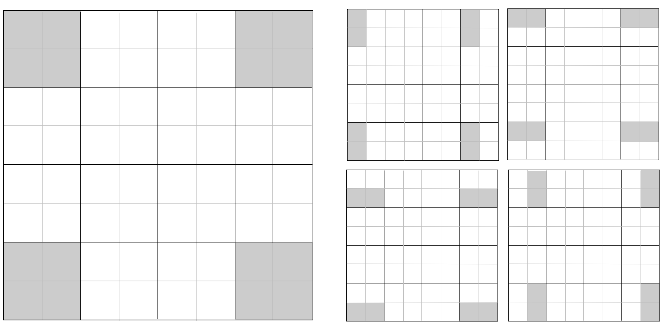
Input: EST($\tilde{\mathcal{U}}_k^\iota$), $1 \leq \iota \leq \varphi_k$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_k)$.
1: **for** each EST($\tilde{\mathcal{U}}_k^\iota$) **do**
2: **for** each link $u_i u_j$ in EST($\tilde{\mathcal{U}}_k^\iota$) **do**
3: Connect u_i and u_j using Manhattan routing: Denote the intersection point of the horizontal line through u_i and the vertical line through u_j as $p_{i,j}$, and denote the nearest node to point $p_{i,j}$ as $u_{i,j}$; choose randomly a node in each *half-cell* passed by $u_i u_{i,j}$ and $u_j u_{i,j}$, and connect alternately those nodes, as illustrated in Fig.2.
4: **end for**
5: Merge the same edges (hops) and remove the circles that have no impact on the connectivity of EST($\tilde{\mathcal{U}}_k^\iota$), we obtain the multicast tree $\mathcal{T}(\mathcal{U}_k^\iota)$.
6: **end for**
7: Based on the forest consisting of the constructed trees, *i.e.*, $\mathcal{T}(\mathcal{U}_k^\iota)$ ($1 \leq \iota \leq \varphi_k$), we obtain the final multicast tree $\mathcal{T}(\mathcal{U}_k)$ by connecting base stations b_ι ($1 \leq \iota \leq \varphi_k$).

For each edge $u_i u_j \in$ EST($\tilde{\mathcal{U}}_k^\iota$), $1 \leq \iota \leq \varphi_k$, we use Manhattan routing to realize it. Note that each hop in Manhattan routing connects two nodes belonging to two adjacent *connectivity cells* but nonadjacent horizontal (or vertical) *half-cells*, which ensures that the Euclidean length of each hop



(a) Horizontal parallel connectivity paths (b) Vertical parallel connectivity paths

Fig. 2. Construction of connectivity paths. There are at least $\theta \log n$ *connectivity paths*, represented by the chains, in each column or row.



(a) 9-TDMA (b) Four subslots

Fig. 3. The shaded cells can be scheduled simultaneously in a 9-TDMA scheme. Each time slot can be further divided into four subslots, and the four *half-cells* in each cell are scheduled one out of four subslots.

is at most $\frac{\sqrt{13}}{2}\sqrt{\bar{a}_e}$ and at least $\frac{1}{2}\sqrt{\bar{a}_e}$. We call such paths *connectivity paths*. According to Lemma 8, there are at least $\frac{\theta}{2}\log n$ *connectivity paths* in each slab of size $\sqrt{\bar{a}_e} \times \sqrt{n}$. Hence, we can allocate the total traffic of each slab to such $\frac{\theta}{2}\log n$ *connectivity paths* averagely. Please see Fig.2 for the illustrations.

Transmission scheduling $\bar{\Im}_e^t$: We adopt a 9-TDMA scheme, and further divide each time slot into 4 equal *subslots* during which we schedule in turn the four *half-cells* of each cell (Fig.3). The main technique called *parallel transmission scheduling* used here is: in each activated subslot, we schedule simultaneously $\frac{\theta}{2} \cdot \log n$ parallel links (the existence guaranteed by Lemma 8) instead of scheduling only one link in most previous works [4], [10]. We further prove the following result.

Lemma 9: By using the *parallel transmission scheduling* $\bar{\Im}_e^t$, the rate along each *connectivity path* can be sustained of order $\Omega((\log n)^{-\frac{\alpha}{2}})$.

Proof: Considering any link in any time slot, since the length of the link is at least $\frac{1}{2}\sqrt{\bar{a}_e}$, we obtain that the sum of interferences to the receivers is bounded by

$$\begin{aligned} I(n) &\leq P \cdot (\tfrac{\theta}{2}\log n - 1) \cdot \ell(\tfrac{1}{2}\sqrt{\bar{a}_e}) \\ &\quad + \sum_{i=1}^{n} 8i \cdot (\tfrac{\theta}{2}\log n) \cdot P \cdot \ell(\tfrac{3i-2}{2}\sqrt{\bar{a}_e}) \\ &\leq P \cdot (\tfrac{2}{\theta})^{\frac{\alpha}{2}-1} \cdot (\log n)^{1-\frac{\alpha}{2}} \cdot \left(1 + \lim_{n\to\infty}\sum_{i=1}^{n} \frac{8i}{(3i-2)^{\alpha}}\right) \end{aligned}$$

The last limitation obviously converges when $\alpha > 2$, thus $I_n = o(1)$. Since the length of every hop is at most $\frac{\sqrt{13\bar{a}_e}}{2}$, we have the signal $S(n)$ at the receiver can be bounded by

$$S(n) \geq (13\theta/2)^{-\frac{\alpha}{2}} \cdot P \cdot (\log n)^{-\frac{\alpha}{2}}.$$

By $\alpha > 2$ and $N_0 > 0$, we have that

$$\frac{S(n)}{N_0 + I(n)} = (\log n)^{-\frac{\alpha}{2}} \to 0.$$

Under the scheme $\bar{\Im}_e^t$, all connectivity paths can be scheduled twice in 4×9 subslots. Hence, each link can sustain a rate of $\Omega((\log n)^{-\frac{\alpha}{2}})$. □

Throughput derived by $\bar{\Im}_e$: Firstly, we consider the relay burden of each *connectivity path*.

Lemma 10: By the routing scheme $\bar{\Im}_e^r$, the relay burden of each *connectivity path* is at most of order

$$\bar{L}_e^r = \begin{cases} O(n_d\sqrt{n}/\sqrt{\rho\log n}) & \text{when } \; n_d : [1,\ \rho] \\ O(\sqrt{nn_d}/\sqrt{\log n}) & \text{when } \; n_d : [\rho,\ n/\log n] \\ O(n_d) & \text{when } \; n_d : [n/\log n,\ n] \end{cases}$$

Proof: Given a node $\bar{v}_t^*$ on a *connectivity path*, define the number of multicast sessions routed through $\bar{v}_t^*$ as a random variable $\bar{\xi}_t$. We finally consider the uniform upper bound $\bar{\xi}$ of $\bar{\xi}_t$ for every node. Define an event $\bar{E}_e^r(k,t)$: The multicast session $\mathcal{M}_k$ passes through $\bar{v}_t^*$. Obviously, if $\bar{E}_e^r(k,t)$ happens then there exists an edge $u_i u_j \in \mathcal{F}_k$ that is routed through $\bar{v}_t^*$, *i.e.*, $u_i u_{i,j}$ or $u_{i,j}u_j$ passes through $\bar{v}_t^*$. Since there exists a constant ϱ_1 such that

$$|u_i u_{i,j}| \leq |u_i p_{i,j}| + \varrho_1 \cdot \sqrt{\bar{a}_e}, \quad |u_{i,j}u_j| \leq |p_{i,j}u_j| + \varrho_1 \cdot \sqrt{\bar{a}_e}$$

and for $|u_i p_{i,j}| + |p_{i,j} u_j| \leq \sqrt{2}|u_i u_j|$, we have

$$\begin{aligned}
&\Pr(\bar{E}_e^r(k,t)) \\
&\leq \frac{1}{\frac{\theta}{2}\cdot\log n}\cdot\frac{\sqrt{\bar{a}_e}}{n}\cdot\sum_{u_iu_j\in\mathcal{F}_k}(|u_iu_{i,j}|+|u_{i,j}u_j|+4\sqrt{\bar{a}_e}) \\
&\leq \frac{2}{\theta\log n}\left(\frac{(4+2\varrho_1)(n_d+\varphi_k)\bar{a}_e}{n}+\frac{\sqrt{2\bar{a}_e}}{n}\cdot\sum_{u_iu_j\in\mathcal{F}_k}|u_iu_j|\right) \\
&\leq \frac{1}{n}\cdot(\kappa_3\cdot n_d+\frac{4}{\sqrt{\theta\cdot\log n}}\cdot\|\mathcal{F}_k\|) \\
&\leq \frac{1}{n}\cdot\left(\kappa_3\cdot n_d+\frac{\kappa_4}{\sqrt{\log n}}\cdot\sqrt{\frac{n\cdot n_d\cdot\min\{n_d,\rho\}}{\rho}}\right)
\end{aligned}$$

where κ_3 and κ_4 are some constants and the last inequality is true according to Lemma 5. Thus, an upper bound of $\bar{\xi}_t$, denoted as $\bar{\eta}_t$, follows Poisson with

$$\bar{\lambda}_e = \frac{n_s}{n}\left(\kappa_3\cdot n_d+\kappa_4\sqrt{\frac{n\cdot n_d\cdot\min\{n_d,\rho\}}{\rho\cdot\log n}}\right).$$

Hence, by union bounds, we have

$$\Pr(\bar{\xi}>\sigma\bar{\lambda}_e)\leq\frac{1}{\bar{a}_e}\cdot\Pr(\bar{\xi}_t>\sigma\bar{\lambda}_e)\leq\frac{n}{2\log n}\Pr(\bar{\eta}_t>\sigma\bar{\lambda}_e)$$

According to Lemma 2, for $\sigma>1$, $\Pr(\bar{\eta}_t>\sigma\bar{\lambda}_e)\leq(\frac{e^{\sigma-1}}{\sigma^\sigma})^{\bar{\lambda}_e}$. Since $n_s=\Theta(n)$ and $\bar{\lambda}_e=\Omega(\log n)$, we can choose σ satisfying $\frac{e^{\sigma-1}}{\sigma^\sigma}<1$ (*e.g.* let $\sigma=e$), by which we get

$$\Pr(\bar{\xi}>\sigma\bar{\lambda}_e)=O(1/\log n)\to 0,\ \text{as } n\to 0.$$

Then, the relay burden of every node on *connectivity paths* is of order $O(\bar{\lambda}_e)$, which completes the proof. □

Combining Lemma 9 and Lemma 10, we can easily obtain Theorem 3.

Theorem 3: When $\rho=O(n/\log n)$, by the strategy $\bar{\Im}_e$ without taking the bottlenecks on BSs into account, the per-session multicast throughput for HEN can be achieved of order

$$\bar{\Lambda}_e^{\bar{r}_b}=\begin{cases}\Omega((\log n)^{\frac{1-\alpha}{2}}\cdot\frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when} \quad n_d:[1,\ \rho]\\ \Omega((\log n)^{\frac{1-\alpha}{2}}\cdot\frac{1}{\sqrt{nn_d}}) & \text{when} \quad n_d:[\rho,\ n/\log n]\\ \Omega((\log n)^{-\frac{\alpha}{2}}\cdot\frac{1}{n_d}) & \text{when} \quad n_d:[n/\log n,\ n]\end{cases}$$

In the following context we will consider the possible bottleneck that may happen on BSs. Under the strategy $\bar{\Im}_e$, all source nodes in some subregion S_ι will send data to the base station b_ι as long as some receiver node(s) falling outside of S_ι. Thus, the base station may become the bottleneck of the network when the number of source nodes exceeds some value. With the increasing number of source nodes inside one subregion, if most of source nodes have some receivers outside the subregion, the base stations may have huge burden, thus become bottlenecks. Using the similar method to Lemma 10, we have the following lemma.

Lemma 11: The maximum load of the links between BSs and ordinary ad hoc nodes is of order

$$\bar{L}_e^{r_b}=\begin{cases}O(n\cdot n_d/\rho) & \text{when} \quad n_d:[1,\ \rho]\\ O(n) & \text{when} \quad n_d:[\rho,\ n]\end{cases}$$

Proof: Define an event $\bar{E}^b(k,t)$: The subregion S_t contains a node belonging to $\mathcal{U}_k$. Then, $\Pr(\bar{E}^b(k,t))\leq\frac{n_d}{\rho}$, for any $t=1,2,\cdots,\rho$. Furthermore, define the load of each subregion as a random variable $\bar{\xi}_t^b$. Then, an upper bound of $\bar{\xi}_t^b$, denoted as $\bar{\eta}_t^b$, follows Poisson with $\bar{\lambda}_e^b=n\cdot\frac{n_d}{\rho}$. Considering the cases $\bar{\lambda}_e^b=O(\log\rho)$ and $\bar{\lambda}_d=\Omega(\log\rho)$ respectively, by using union bounds and Lemma 2, we complete the proof. □

From Lemma 9, the capacity of the links between BSs and ordinary ad hoc nodes is of order $\Omega((\log n)^{-\frac{\alpha}{2}})$. Thus,

Lemma 12: Under the strategy $\bar{\Im}_e$, the throughput along the wireless links via BSs is of order

$$\bar{\Lambda}_e^{r_b}=\begin{cases}\Omega(\frac{\rho}{n\cdot n_d}\cdot(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d:[1,\ \rho]\\ \Omega(\frac{1}{n}\cdot(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d:[\rho,\ n]\end{cases}$$

Combining Theorem 3 and Lemma 12, we conclude that the bottleneck of the whole routing $\bar{\Im}_e^r$ lies on the wireless links via BSs. According to Lemma 7, we obtain the throughput achieved by *connectivity strategy*.

Theorem 4: By the *connectivity strategy* $\bar{\Im}_e$, the per-session multicast throughput for *hybrid extended networks* can be achieved of order:
When $m:[1,n/\log n]$,

$$\bar{\Lambda}_e^{r}=\begin{cases}\Omega(\frac{m}{n\cdot n_d}\cdot(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d:[1,\ m]\\ \Omega(\frac{1}{n}\cdot(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d:[m,\ n]\end{cases}$$

When $m:[n/\log n,n]$,

$$\bar{\Lambda}_e^{r}=\begin{cases}\Omega(\frac{1}{n_d}\cdot(\log n)^{-\frac{\alpha}{2}-1}) & \text{when} \quad n_d:[1,\ \frac{n}{\log n}]\\ \Omega(\frac{1}{n}\cdot(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d:[\frac{n}{\log n},\ n]\end{cases}$$

5.1.2 Percolation Strategy

First of all, we state that the *percolation strategy* applies to the case when $\rho=O(\frac{n}{(\log n)^2})$. We adopt the *percolation strategy* denoted as $\Im_e$. Obviously, the side length of each subregion is of order $\Omega(\log n)$. We divide the region $\mathcal{A}(n)$ into subsquares with area of a constant a_e by inclined lines. That is, we design the strategy based on the *scheme lattice* $\mathbb{L}(\sqrt{n},\sqrt{a_e},\frac{\pi}{4})$ in which the cells are called *percolation cells*. A *percolation cell* is open if it is nonempty (occupied). Obviously, the open probability is $p=1-e^{-a_e}$. Using the same procedure in [10], we can map this model into a bond percolation model $\mathbb{B}(h,p)$ where $h=\sqrt{n}/\sqrt{2a_e}$ and $p=1-e^{-a_e}$. Moreover, we can partition $\mathcal{A}(n)$ into slabs of size $\sqrt{2a_e}(\kappa\log h-\epsilon_h)\times(\sqrt{n}/\sqrt{m})$, where we can make $\frac{\sqrt{n}}{\sqrt{m}\sqrt{2a_e}(\kappa\log h-\epsilon_h)}$ be an integer by adjusting $\epsilon_h=o(1)$. We call those slabs *highway slabs*. Then, by Lemma 6, we have the following lemma.

Lemma 13: For any $\kappa>0$ and $a_e>\log 6+2/\kappa$, there exists a constant $\delta_1(\kappa,a_e)$ such that there are *w.h.p.*at least $\delta_1\log n$ horizontal (vertical) highways in all *highway slabs*.

Based on Lemma 13, we can divide horizontally (or vertically) each *highway slab* into slices of size $\kappa_5\times(\sqrt{n}/\sqrt{\rho})$, where $\kappa_5=\frac{\delta_1}{2\kappa}$ is a constant. Then, we can define a mapping function from the set of highways to the set of slices. In other words, we can ensure that the traffics initiated from each slice are taken charge by a corresponding *highway*, and every highway only bear with the traffic initiated from at most one slice.

Routing scheme $\Im_e^r$: Based on every EST$(\tilde{\mathcal{U}}_k^\iota)$, $1\leq\iota\leq\varphi_k$, we realize the routing of each link $u_iu_j\in$ EST$(\tilde{\mathcal{U}}_k^\iota)$ by two broad phases, *i.e.*, *highway phase* and *connectivity path phase*. By Lemma 8, we can build at least $\frac{\theta}{2}\log n$ disjoint *connectivity paths* in each slab of size $\sqrt{\bar{a}_e}\times(\kappa\cdot\log h-\epsilon_h)$. Thus, similar

Algorithm 3 Percolation Routing Scheme $\Im_e^r$

Input: $\text{EST}(\tilde{\mathcal{U}}_k^\iota)$, $1 \le \iota \le \varphi_k$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_k)$.

```
for each EST(Ũ_k^ι) do
    for each link u_i u_j in EST(Ũ_k^ι) do
        u_i drains the packets into the specific horizontal
        highway along the specific connectivity path.
        Packets are carried along the horizontal highway, and
        are carried along the specific vertical highway.
        Packets are delivered to u_j from the vertical highway
        along the specific connectivity path.
    end for
    Merge the same edges (hops) and remove the circles
    that have no impact on the connectivity of EST(Ũ_k^ι),
    we obtain the multicast tree T(U_k^ι).
end for
By using the similar method as Line 7 in Algorithm 2, we
obtain the final multicast tree T(U_k) based on the forests
consisting of the trees T(U_k^ι) (1 ≤ ι ≤ φ_k).
```

to routing scheme $\bar{\Im}_e^r$, we can allocate averagely the traffics initiated by such slabs to at least $\frac{\theta}{2}\log n$ *connectivity paths*. We propose Algorithm 3 to describe the multicast routing scheme in detail.

Transmission scheduling $\Im_e^t$: We use two independent TDMA schemes to schedule transmissions along *highways* and *connectivity paths*. To be specific, we divide a scheduling period into two sub-periods with the same size, which are called *highway scheduling* $\Im_e^{t_1}$ and *connectivity path scheduling* $\Im_e^{t_2}$, respectively. The two scheduling phases corresponds to the two phases of routing, *i.e.*, *highways phase* $\Im_e^{r_1}$ and *connectivity path phase* $\Im_e^{r_2}$. The scheme $\Im_e^{t_1}$ can be adopted as same as the scheduling of highways in [10]. Then, we have

Lemma 14: By the transmission scheduling $\Im_e^{t_1}$, the rate along *highways* can be achieved of order $\Omega(1)$.

Since we can only ensure that there exists at least one *connectivity path*, instead of *highway*, passing through every BS b_ι, for $1 \le \iota \le \varphi_k$ and $1 \le k \le n_s$, then similar to *connectivity strategy*, we have

Lemma 15: By the strategy $\Im_e$, the throughput along the wireless links via BSs is of order $\Lambda_e^{r_b} = \bar{\Lambda}_e^{r_b}$, where $\bar{\Lambda}_e^{r_b}$ is defined in Lemma 12.

The scheme $\Im_e^{t_2}$ can be adopted as same as $\bar{\Im}_e^t$. Then, according to Lemma 9, we can obtain,

Lemma 16: Under the scheme $\Im_e^{t_2}$, the rate along each *connectivity path* can be achieved of order $\Omega(\frac{1}{(\log n)^{\alpha/2}})$.

Throughput derived by $\Im_e$: Firstly, we analyze the load of the routing paths in the *highway phase* and *connectivity path phase*.

Lemma 17: During *highway phase* $\Im_e^{r_1}$, the maximum relay burden of each node on the *highways* is *w.h.p.* of order

$$L_e^{r_1} = \begin{cases} O(\frac{\sqrt{nn_d}}{\sqrt{\rho}}) & \text{when} \quad n_d : [1,\ \rho] \\ O(\sqrt{nn_d}) & \text{when} \quad n_d : [\rho,\ n/(\log n)^2] \\ O(n_d \log n) & \text{when} \quad n_d : [n/(\log n)^2, n/\log n] \\ O(n) & \text{when} \quad n_d : [n/\log n,\ n] \end{cases}$$

Proof: Given a node v_t^* on the *highways*, define the number of multicast sessions routed through v_t^* in *highway phase* $\Im_e^{r_1}$ as a random variable $\xi_t^{r_1}$, and finally we consider the uniform upper bound ξ^{r_1} of $\xi_t^{r_1}$. Define an Event $E_e^{r_1}(k,t)$: The multicast session $\mathcal{M}_k$ passes through v_t^* in phase $\Im_e^{r_1}$. Obviously, if $E_e^{r_1}(k,t)$ happens then there exists an edge $u_iu_j \in \mathcal{F}_k$ that is routed through v_t^* in phase $\Im_e^{r_1}$, in other words, a vertical (or horizontal) line through v_t^* intersect with the segment $u_iu_{i,j}$ (or $u_{i,j}u_j$). Similar to Lemma 10, we have

$$\begin{aligned} &\Pr(E_e^{r_1}(k,t)) \\ &\le \tfrac{\kappa_5}{n} \cdot \textstyle\sum_{u_iu_j \in \mathcal{F}_k} \left(|u_ip_{i,j}| + |p_{i,j}u_j| + 2\sqrt{2a_e}(\kappa \log h - \epsilon_h)\right) \\ &\le \tfrac{\kappa_6}{n} \cdot (n_d \log n) + \tfrac{\kappa_7}{n} \cdot \|\mathcal{F}_k\| \\ &\le \tfrac{1}{n} \cdot \left(\kappa_6 \cdot n_d \cdot \log n + \kappa_8 \cdot \sqrt{\tfrac{n \cdot n_d \cdot \min\{n_d, \rho\}}{\rho}}\right) \end{aligned}$$

where $\kappa_5 \sim \kappa_8$ are some constants and the last inequality is true according to Lemma 5. Thus, an upper bound of $\bar{\xi}_t$, denoted as η_t, follows a Poisson distribution of mean

$$\lambda_e^{r_1} = \frac{n_s}{n}\left(\kappa_6 \cdot n_d \cdot \log n + \frac{\kappa_8}{\rho} \cdot \sqrt{n \cdot n_d \cdot \min\{n_d, \rho\}}\right)$$

Hence, by the similar procedure of Lemma 10, we obtain that the relay burden of every node on the *highways* in phase $\Im_e^{r_1}$ is of order $O(\lambda_e^{r_1})$, which completes the proof. □

Lemma 18: During *connectivity path phase* $\Im_e^{r_2}$, the maximum relay burden of each node on the *connectivity path* is *w.h.p.*of order $L_e^{r_2} = O(n_d(\log n)^{1/2})$.

Proof: For a given node v_t^{**} on the *connectivity paths*, define the number of multicast sessions routed through v_t^{**} in *connectivity path phase* $\Im_e^{r_2}$ as a random variable $\xi_t^{r_2}$, and finally we consider the uniform upper bound ξ^{r_2} of $\xi_t^{r_2}$. Define an Event $E_e^{r_2}(k,t)$: The multicast session $\mathcal{M}_k$ passes through v_t^{**} in phase $\Im_e^{r_2}$. We can see that if $E_e^{r_2}(k,t)$ happens then there is a node belongs to $\mathcal{U}_k$ and locates in a slab of size $\frac{2\sqrt{a_e}}{\theta \cdot \log n} \times (\kappa \cdot \log h - \epsilon_h)$. Hereafter, using a similar procedure in Lemma 17, we can complete the proof. □

Combining Lemma 14 with 17, we can obtain Lemma 19.

Lemma 19: During phase $\Im_e^{r_1}$, the multicast throughput can be achieved of order

$$\Lambda_e^{r_1} = \begin{cases} \Omega(\frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when} \quad n_d : [1,\ \rho] \\ \Omega(\frac{1}{\sqrt{nn_d}}) & \text{when} \quad n_d : [\rho,\ n/(\log n)^2] \\ \Omega(\frac{1}{n_d \log n}) & \text{when} \quad n_d : [n/(\log n)^2, n/\log n] \\ \Omega(1/n) & \text{when} \quad n_d : [n/\log n,\ n] \end{cases}$$

Furthermore, combining Lemma 16 and Lemma 18, we can obtain the following lemma.

Lemma 20: During phase $\Im_e^{r_2}$, the multicast throughput can be achieved of order $\Lambda_e^{r_2} = \Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{\alpha+1}{2}})$.

Based on Lemma 19 and Lemma 20, and according to Lemma 7, we can obtain Theorem 5.

Theorem 5: When $\rho = O(n/(\log n)^2)$, by the *percolation strategy* $\Im_e$ without taking the bottlenecks on BSs into account, the per-session multicast throughput for HEN can be achieved of order:

When $\rho : [1, (n/(\log n)^{\alpha+1})]$,

$$\Lambda_e^{\bar{r}_b} = \begin{cases} \Omega(\frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when} \quad n_d : [1,\ \rho] \\ \Omega(\frac{1}{\sqrt{nn_d}}) & \text{when} \quad n_d : [\rho,\ \frac{n}{(\log n)^{\alpha+1}}] \\ \Omega(\frac{1}{n_d\cdot(\log n)^{\frac{\alpha+1}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^{\alpha+1}}, n] \end{cases}$$

When $\rho : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{(\log n)^2}]$, $\Lambda_e^{\bar{r}_b} = \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha+1}{2}})$.

Combining Theorem 5 and Lemma 15, we get the following result.

Theorem 6: Under the *percolation strategy* $\Im_e$, the per-session multicast throughput for HEN is achieved of order: When $m : [1, n/\log n]$,

$$\Lambda_e^r = \begin{cases} \Omega(\frac{m}{n\cdot n_d}(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d : [1,\ m] \\ \Omega(\frac{1}{n}(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d : [m,\ \frac{n}{\sqrt{\log n}}] \\ \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha+1}{2}}) & \text{when} \quad n_d : [\frac{n}{\sqrt{\log n}},\ n] \end{cases}$$

When $m : [n/\log n, n]$,

$$\Lambda_e^r = \begin{cases} \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha}{2}-1}) & \text{when} \quad n_d : [1,\ n/\log n] \\ \Omega(\frac{1}{n}(\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d : [n/\log n,\ \frac{n}{\sqrt{\log n}}] \\ \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha+1}{2}}) & \text{when} \quad n_d : [\frac{n}{\sqrt{\log n}},\ n] \end{cases}$$

Furthermore, combining Theorem 4 and Theorem 6, we can get the throughput derived by *hybrid routing strategies*.

Theorem 7: By the *hybrid strategies*, the multicast throughput for HEN is achieved of order Λ_e^r (defined in Theorem 6).

5.2 Ordinary Ad hoc Strategy for HEN

Different from the previous routing strategy, in *ordinary ad hoc strategy*, we will not use any base station but only the ordinary ad hoc nodes. In particular, we treat the network as a ordinary ad hoc network and we construct global multicast trees composed of only ordinary nodes. Similar to the hybrid strategy, the ordinary ad hoc strategy consists of *connectivity strategy* and *percolation strategy*. Indeed, the ordinary ad hoc strategy can be regarded as the special cases of hybrid strategies by removing the technical details about BSs. Then, by using a similar procedure in analysis of the hybrid strategy, we obtain the following result.

Theorem 8: Under the *ordinary ad hoc strategy*, the multicast throughput for HEN is achieved of order

$$\begin{cases} \Omega(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^{\alpha+1}}] \\ \Omega(\frac{1}{n_d(\log n)^{\frac{\alpha+1}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{(\log n)^2}] \\ \Omega(\frac{1}{\sqrt{nn_d}\cdot(\log n)^{\frac{\alpha-1}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Omega(\frac{1}{n_d(\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad n_d : [\frac{n}{\log n}, n] \end{cases}$$

5.3 BS-based Strategy for HEN

Under the classical BS-based strategy, sources deliver data to BSs directly during the uplink phase and BSs deliver received data to destinations directly during the downlink phase. Since in any time slot, all wireless links associate with the BSs, then the *parallel transmission scheduling* is disabled. Denote the BS-based strategy by $\tilde{\Im}_e$; and denote the corresponding routing and transmission scheduling schemes by $\tilde{\Im}_e^r$ and $\tilde{\Im}_e^t$, respectively. Different from the previous partition method, here we simply partition $\mathcal{A}(n)$ into m *subregions* of side length $\frac{\sqrt{n}}{\sqrt{m}}$, and place one base station at the center of each subregion.

Routing scheme $\tilde{\Im}_e^r$: The routing consists of three phases: *uplink phase* $\tilde{\Im}_e^{r_1}$, *BS-to-BS phase* $\tilde{\Im}_e^{r_2}$ and *downlink phase* $\tilde{\Im}_e^{r_3}$. That is,

1) During the uplink phase, source nodes in subregion S_ι, $\iota = 1, 2, \cdots, m$, transmit the packets to BS b_ι.
2) The BS receiving the packets from source v_k, $k = 1, 2, \cdots, n_s$, delivers the packets to those BSs that are placed in the subregions containing the destinations of v_k via BS-to-BS links.
3) During *downlink phase*, each BS b_ι, $\iota = 1, 2, \cdots, m$, broadcasts the packets to the nodes in subregion S_ι.

Transmission scheduling $\tilde{\Im}_e^t$: This transmission scheduling scheme includes three independent phases, denoted by $\tilde{\Im}_e^{t_1}$, $\tilde{\Im}_e^{t_2}$ and $\tilde{\Im}_e^{t_3}$, corresponding to three routing phases. Since the *BS-to-BS phase* is surely not the bottleneck, we only focus on the other two phases. That is,

1) During *uplink phase* $\tilde{\Im}_e^{t_1}$, all BSs b_ι, $\iota = 1, 2, \cdots, m$, receive simultaneously packets from the nodes in S_ι.
2) During *downlink phase* $\tilde{\Im}_e^{t_3}$, all BSs b_ι, $\iota = 1, 2, \cdots, m$, deliver simultaneously packets to the nodes in S_ι.

Lemma 21: Under the scheduling scheme $\tilde{\Im}_e^t$, each *subregion* can sustain a total rate of order $\Omega((n/m)^{-\frac{\alpha}{2}})$ during both the downlink phase and uplink phase.

Proof: Due to the regular location of BSs, for any receiver in a *subregion*, the nearest transmitter outside the *subregion* is faraway in distance of at least $\frac{\sqrt{n}}{2\sqrt{m}}$. Similar to Lemma 9, the sum of interferences to the receivers is bounded by

$$I(n) \leq \sum_{i=1}^{m} 8iP\ell(\frac{2i-1}{2} \cdot \frac{\sqrt{n}}{\sqrt{m}}) \leq (\frac{m}{n})^{\frac{\alpha}{2}} \cdot 2^{\alpha} P \sum_{i=1}^{\infty} \frac{8i}{(2i-1)^{\alpha}}$$

Thus, $I(n) = O((n/m)^{-\frac{\alpha}{2}})$. While, the signal $S(n)$ can be bounded by

$$S(n) \geq P \cdot (\frac{\sqrt{n}}{\sqrt{2m}})^{-\alpha} \geq (\frac{\sqrt{10}}{2})^{-\alpha} \cdot P \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}.$$

Then, $S(n) = \Omega((n/m)^{-\frac{\alpha}{2}})$. For the assumption $m = O(n)$, we have $I(n) = O(1)$ and $S(n) = O(1)$. Hence, we have $\frac{S(n)}{N_0 + I(n)} = O(1)$ and

$$\log(1 + \frac{S(n)}{N_0 + I(n)}) = \Omega((n/m)^{-\frac{\alpha}{2}}),$$

which completes the proof. □

Next, we consider the load of the BSs during the downlink phase and uplink phase. Similar to Lemma 11, we have,

Lemma 22: Under the strategy $\tilde{\Im}_e^r$, the load of each BS is of order $\tilde{L}_e^r = \bar{L}_e^{r_b}$, where $\bar{L}_e^{r_b}$ is defined in Lemma 11.

According to Lemma 21 and Lemma 16, we have

Theorem 9: By the BS-based strategy, the per-session multicast throughput for HEN can be achieved of order

$$\begin{cases} \Omega(\frac{1}{\log m} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}) & \text{when} \quad n_d : [1,\ \frac{m \log m}{n}] \\ \Omega(\frac{m}{n\cdot n_d} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}) & \text{when} \quad n_d : [\frac{m\log m}{n},\ m] \\ \Omega(\frac{1}{n} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}) & \text{when} \quad n_d : [m,\ n] \end{cases}$$

5.4 Integration of Three Types of Strategies

To achieve the optimal multicast throughput, we will select the best strategy according to the different scenarios in terms of m and n_d. Combining Theorem 7, Theorem 8 and Theorem 9, we can obtain the main result in Theorem 1.

6 Multicast Strategies for HDN

In this section, we consider the hybrid dense network (HDN). Corresponding to the hybrid extended network (HEN), we also design the *hybrid strategy*, the *BS-based strategy* and the *ordinary ad hoc strategy*.

6.1 Hybrid Strategy for HDN

As in HEN, the *hybrid strategy* for HDN also consists of *connectivity strategy*, denoted by $\bar{\Im}_d$, and *percolation strategy*, denoted by $\Im_d$. The strategy $\bar{\Im}_d$ can be applied only when $\rho = O(n/\log n)$; the strategy $\Im_d$ can be used when $\rho = O(n/(\log n)^2)$.

6.1.1 Connectivity Strategy $\bar{\Im}_d$

Under the strategy $\bar{\Im}_d$, the routing $\bar{\Im}_d^r$ is built based on the *connectivity paths* (CPs). We construct the CPs based on the *scheme lattice* $\mathbb{L}(1, \sqrt{\bar{a}_e/n}, 0)$. In each column or row of $\mathbb{L}(1, \sqrt{\bar{a}_e/n}, 0)$, we can also construct $\Theta(\log n)$ *connectivity paths* as the case in HEN. However, unlike in HEN, the *parallel transmission scheduling* does not work in HDN, which can be explained in the following lemma.

Lemma 23: The total rate of each *connectivity path* can be achieved of order $\Theta(1/\pi(n))$ when $\pi(n)$ *connectivity paths* are simultaneously scheduled, where $\pi(n) = O(\log n)$.

Proof: For any link in any time slot, since the length of the link is at least $\frac{1}{2}\sqrt{\bar{a}_e/n}$, we obtain that the sum of interferences to the receivers is bounded by

$$\begin{aligned} I(n) &\leq P \cdot (\pi(n) - 1) \cdot \ell(\tfrac{1}{2}\sqrt{\bar{a}_e/n}) \\ &\quad + \sum_{i=1}^{n} 8i \cdot \pi(n) \cdot P \cdot \ell(\tfrac{3i-2}{2}\sqrt{\bar{a}_e/n}) \\ &\leq P \cdot (\tfrac{2}{\theta})^{\frac{\alpha}{2}} \cdot \pi(n) \cdot (\tfrac{n}{\log n})^{\frac{\alpha}{2}} \cdot \left(1 + \lim_{n\to\infty} \sum_{i=1}^{n} \tfrac{8i}{(3i-2)^{\alpha}}\right) \end{aligned}$$

The last limitation obviously converges when $\alpha > 2$, thus $I_n = O(\pi(n) \cdot (\frac{n}{\log n})^{\frac{\alpha}{2}})$. Since the length of every hop is at most $\frac{1}{2}\sqrt{13\bar{a}_e/n}$, we have the signal $S(n)$ at the receiver can be bounded by $S(n) \geq (\frac{13}{2}\cdot\theta)^{-\frac{\alpha}{2}} \cdot P \cdot (\frac{n}{\log n})^{\frac{\alpha}{2}}$. By $N_0 \geq 0$, we have $\frac{S(n)}{N_0 + I(n)} = O(\frac{1}{\pi(n)})$, which completes the proof. □

According to Lemma 23, we construct only one connectivity path in each column or row. Then, by a similar procedure to HEN, we can get the following results.

Theorem 10: When $\rho = O(n/\log n)$, under the strategy $\bar{\Im}_d$, without taking the bottlenecks on BSs into account, the per-session multicast throughput for HDN is achieved of order

$$\bar{\Lambda}_d^{\bar{r}_b} = \begin{cases} \frac{\sqrt{\rho}}{n_d\sqrt{n\log n}}) & \text{when} \quad n_d : [1,\ \rho] \\ \Omega(\frac{1}{\sqrt{nn_d\log n}}) & \text{when} \quad n_d : [\rho,\ n/\log n] \\ \Omega(\frac{1}{n}) & \text{when} \quad n_d : [n/\log n,\ n] \end{cases}$$

Lemma 24: The maximum load on the links between BSs and ordinary ad hoc nodes is of order

$$\bar{L}_d^{r_b} = \begin{cases} O(n \cdot n_d/\rho) & \text{when} \quad n_d : [1,\ \rho] \\ O(n) & \text{when} \quad n_d : [\rho,\ n] \end{cases}$$

On the other hand, similar to Lemma 9, we can prove that the capacity of the links between BSs and ordinary ad hoc nodes (B-O links) is of order $\Omega(1)$. Thus,

Lemma 25: Under the strategy $\bar{\Im}_d$, the throughput along the wireless links via BSs can be achieved of order

$$\bar{\Lambda}_d^{r_b} = \begin{cases} \Omega(\frac{\rho}{n\cdot n_d}) & \text{when} \quad n_d : [1,\ \rho] \\ \Omega(\frac{1}{n}) & \text{when} \quad n_d : [\rho,\ n] \end{cases}$$

Combining Theorem 10 and Lemma 25, we conclude that the bottleneck of the whole routing $\bar{\Im}_d^r$ lies on the B-O links. According to Lemma 7, we obtain the multicast throughput derived by *connectivity strategy*.

Theorem 11: Under the *connectivity strategy* $\bar{\Im}_d$, the per-session multicast throughput for HDN is achieved of order:
When $m : [1, n/\log n]$,

$$\bar{\Lambda}_d^{r} = \begin{cases} \Omega(\frac{m}{n\cdot n_d}) & \text{when} \quad n_d : [1,\ m] \\ \Omega(\frac{1}{n}) & \text{when} \quad n_d : [m,\ n] \end{cases}$$

When $m : [n/\log n, n]$,

$$\bar{\Lambda}_d^{r} = \begin{cases} \Omega(\frac{1}{n_d\log n}) & \text{when} \quad n_d : [1,\ n/\log n] \\ \Omega(\frac{1}{n}) & \text{when} \quad n_d : [n/\log n,\ n] \end{cases}$$

6.1.2 Percolation Strategy $\Im_d$

We design the percolation strategy, denoted by $\Im_d$, based on the *connectivity paths* and *highways*. We build the highways based on the *scheme lattice* $\mathbb{L}(1, \sqrt{a_e/n}, \frac{\pi}{4})$ in which the cells are called *percolation cells*.

The average number of nodes in each *percolation cell* is also the same, namely a_e. Therefore, all the percolation results above still hold for HDN, and we can find as many highways as in HEN. By a similar procedure to HEN, we can obtain the following results.

Lemma 26: Along the *highways*, the multicast throughput can be achieved of order

$$\Lambda_d^{r_1} = \begin{cases} \Omega(\frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when} \quad n_d : [1,\ \rho] \\ \Omega(\frac{1}{\sqrt{nn_d}}) & \text{when} \quad n_d : [\rho,\ n/(\log n)^2] \\ \Omega(\frac{1}{n_d\log n}) & \text{when} \quad n_d : [n/(\log n)^2, n/\log n] \\ \Omega(1/n) & \text{when} \quad n_d : [n/\log n,\ n] \end{cases}$$

Lemma 27: Along the *connectivity paths*, the multicast throughput can be achieved of order $\Lambda_d^{r_2} = \Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{3}{2}})$.

Based on Lemma 26 and Lemma 27, and according to Lemma 7, we can obtain Theorem 12.

Theorem 12: When $\rho = O(n/(\log n)^2)$, by the *percolation strategy* $\Im_d$ without taking the bottlenecks on BSs into account, the per-session multicast throughput for HDN can be achieved of order:
When $\rho : [1, (n/(\log n)^3)]$,

$$\Lambda_d^{\bar{r}_b} = \begin{cases} \Omega(\frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when} \quad n_d : [1,\ \rho] \\ \Omega(\frac{1}{\sqrt{nn_d}}) & \text{when} \quad n_d : [\rho,\ \frac{n}{(\log n)^3}] \\ \Omega(\frac{1}{n_d\cdot(\log n)^{\frac{3}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^3}, n] \end{cases}$$

When $\rho : [\frac{n}{(\log n)^3}, \frac{n}{(\log n)^2}]$, $\Lambda_d^{\bar{r}_b} = \Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{3}{2}})$.

Combining Theorem 12 and Lemma 25, we have the following result.

Theorem 13: Under the *percolation strategy* $\Im_d$, the per-session multicast throughput for HDN is achieved of order:
When $m : [1, n \cdot (\log n)^{-\frac{3}{2}}]$,

$$\Lambda_d^r = \begin{cases} \Omega(\frac{m}{n \cdot n_d}) & \text{when} \quad n_d : [1,\ m] \\ \Omega(\frac{1}{n}) & \text{when} \quad n_d : [m,\ n \cdot (\log n)^{-\frac{3}{2}}] \\ \Omega(\frac{1}{n_d}(\log n)^{-\frac{3}{2}}) & \text{when} \quad n_d : [n \cdot (\log n)^{-\frac{3}{2}},\ n] \end{cases}$$

When $m : [n \cdot (\log n)^{-3/2}, n]$, $\Lambda_d^r = \Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{3}{2}})$.

Furthermore, combining Theorem 11 and Theorem 13, we can get the multicast throughput derived by the hybrid strategy.

Theorem 14: By using the *hybrid strategy*, the multicast throughput for HEN is achieved of order $\bar{\Lambda}_d^r$ that is defined in Theorem 11.

6.2 Ordinary Ad hoc Strategy for HDN

In this case, we treat the network as an ordinary *random dense network*. According to the results in [3], we have

Theorem 15: The achievable per-session multicast throughput for *random dense networks* is of order

$$\begin{cases} \Omega(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^3}] \\ \Omega(\frac{1}{n_d (\log n)^{\frac{3}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^3}, \frac{n}{(\log n)^2}] \\ \Omega(\frac{1}{\sqrt{n n_d \log n}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Omega(\frac{1}{n}) & \text{when} \quad n_d : [\frac{n}{\log n}, n] \end{cases}$$

6.3 BS-based Strategy for HDN

The BS-based strategy for HDN is similar to that for HEN described in Section 5.3. By a similar procedure, we can prove the following theorem.

Theorem 16: Under the strategy $\tilde{\Im}_d$, the achievable multicast throughput for HDN is of order

$$\bar{\Lambda}_d = \begin{cases} O(1/\log n) & \text{when} \quad n_d : [1, \frac{m \cdot \log n}{n}] \\ O(\frac{m}{n \cdot n_d}) & \text{when} \quad n_d : [\frac{m \cdot \log n}{n}, m] \\ O(1/n) & \text{when} \quad n_d : [m, n] \end{cases}$$

6.4 Integration of Three Types of Strategies

To achieve the optimal multicast throughput, we can select the best strategy according to the different scenarios in terms of m and n_d. Combining Theorem 14, Theorem 15 and Theorem 16, we can obtain Theorem 2 as one of main results.

7 Literature Reviews

We review the existing works on the capacity scaling laws of wireless ad hoc networks and hybrid networks under two popular communication models, *i.e.*, the threshold-based channel model and Gaussian Channel model. The former model is simpler, thus more convenient for the analysis for many issues, besides capacity, of wireless networks, such as *localization* [23], [24], *coverage* [25], and *lifetime* [26] problems in wireless sensor networks. Gaussian Channel model captures better the nature of wireless medium, [3], [27].

7.1 Wireless Ad hoc Networks

7.1.1 Under threshold-based channel model

Gupta and Kumar [2] studied the *unicast* capacity in *dense networks*, they showed that a scheme of nearest neighbor communication can achieve a throughput of $\Theta(1/\sqrt{n \log n})$. Keshavarz-Haddad *et al.* [28] studied the broadcast capacity of an arbitrary network, and showed that the per-session broadcast capacity is only of $\Theta(1/n)$. Shakkottai *et al.* [29] designed a novel routing scheme, called *comb scheme*, by which the per-session multicast throughput can be achieved of order $\Omega(\frac{1}{\sqrt{n n_d}})$. Li *et al.* [8] showed that, assuming that $n_s = \Omega(\log n_d \sqrt{n \log n / n_d})$, for *random networks*, the per-session capacity of n_s multicast sessions is $\Theta(1/\sqrt{n_d n \log n})$ when $n_d = O(n/\log n)$, and is $\Theta(1/n)$ when $n_d = \Omega(n/\log n)$.

7.1.2 Under Gaussian Channel model

Franceschetti *et al.* [10] showed the throughput for both *random extended networks* and *random dense networks* can be achieved of order $\Omega(1/\sqrt{n})$. Zheng [30] proved that the broadcast capacity for *random extended networks* is of order $\Theta(\frac{1}{n}(\log n)^{-\frac{\alpha}{2}})$. Li *et al.* [4] showed that, when $n_d = O(\frac{n}{(\log n)^{2\alpha+6}})$ and $n_s = \Omega(n^{\frac{1}{2}+\theta})$, the multicast throughput for *random extended networks* can be achieved of order $\Omega(\frac{\sqrt{n}}{n_s \sqrt{n_d}})$, where $\theta > 0$ is a constant. In [3], such threshold of n_d was improved to $n_d = O(\frac{n}{(\log n)^{\alpha+1}})$, and the corresponding upper bounds were proposed. Keshavarz-Haddad *et al.* [27] proposed a technique called *arena* to study upper bounds of capacity. They [31] devised a scheme and computed the achievable throughput for *random dense networks*.

7.2 Hybrid Wireless Networks

7.2.1 Under threshold-based channel model

Earlier, Liu *et al.* [32] introduced the model based on the *dense network* in which the base stations are regularly placed and the ad hoc nodes are randomly distributed. The case that both base stations and ad hoc nodes are randomly placed in the *dense network* is studied by Kozat and Tassiulas in [33]. Agarwal *et al.* [5] considered the unicast capacity for hybrid networks under PhIM. Recently, Mao *et al.* [9] studied the *multicast capacity* for hybrid networks under *threshold-based channel* model by assuming $m = O(n/\log n)$.

7.2.2 Under Gaussian Channel model

Agarwal and Kumar [5] studied the unicast capacity for *hybrid dense networks*, and they designed the same bounds as that under the threshold-based model. Liu *et al.* [12] studied the achievable unicast throughput for *hybrid extended networks*. They showed that in a two-dimensional square hybrid wireless network with n ordinary ad hoc nodes and m base stations, it is necessary that $m = \Omega(\sqrt{n})$ in order to obtain a linear gain of capacity. Focusing on *hybrid dense networks*, Wang *et al.* [34] derived the achievable multicast throughput under the schemes without introducing the percolation-based routing [10], which leads to poor multicast throughput for some cases in terms of n_d and m.

8 Conclusion

We study the multicast throughput for *hybrid extended networks* and *hybrid dense networks* under Gaussian Channel model. Three types of multicast strategies are devised. Based on the multicast throughputs derived by all strategies, we make the decisions on selecting the optimal strategy according to the different scenarios in terms of m, n and n_d. To the best of our knowledge, this paper is the first work that addresses the multicast routing and scheduling strategy in hybrid wireless networks under Gaussian Channel model. A number of interesting questions remain open: How to derive tight upper bound on the network capacity for hybrid wireless networks? What type of strategy should be implemented if the access links between ordinary ad hoc nodes and base stations are different from those among ordinary ad hoc nodes, *e.g.*, they may have larger bandwidth, or if the links among base stations are not wired and their bandwidth are not arbitrary large?

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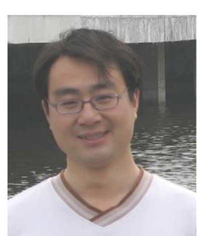

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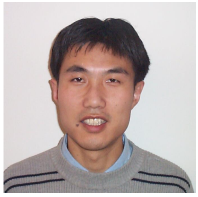

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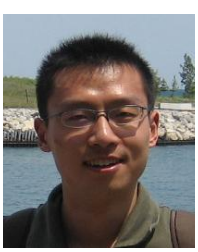

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