Applications of k-Local MST for Topology Control and Broadcasting in Wireless Ad Hoc Networks

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Abstract

In this paper, we propose a family of structures, namely, k-localized minimum spanning tree (LMST_k) for topology control and broadcasting in wireless ad hoc networks. We give an efficient localized method to construct LMST_k using only $O(n)$ messages under the local-broadcast communication model, i.e., the signal sent by each node will be received by all nodes within the node's transmission range. We also analytically prove that the node degree of the structure LMST_k is at most 6, LMST_k is connected and planar, and more importantly, the total edge length of the LMST_k is within a constant factor of that of the minimum spanning tree when $k \geq 2$ (called *low weighted* hereafter). We then propose another structure, called *Incident MST and RNG Graph* (IMRG), that can be locally constructed using at most $13n$ messages under the local broadcast communication model. Test results are corroborated in the simulation study. We study the performance of our structures in terms of the total power consumption for broadcasting, the maximum node power needed to maintain the network connectivity. We theoretically prove that our structures are asymptotically the best possible for broadcasting among all locally constructed structures. Our simulations show that our new structures outperform previous locally constructed structures in terms of the broadcasting and power assignment for connectivity.

Keywords

Localized algorithms, broadcasting, topology control, minimum spanning tree, wireless ad hoc networks.

I. Introduction

We consider a wireless ad hoc network composed of n wireless devices (called *nodes* hereafter) distributed in a two-dimensional plane. Assume that all wireless nodes have distinctive identities, quasi-static, and each wireless node knows its geometry position information either through a low-power Global Position System (GPS) receiver or through some other way. More specifically, it is enough for our protocols when each node knows the distance to each of its one-hop neighbors, which can be estimated by the *strength of signal*. We assume that each wireless node has an omni-directional antenna and a single transmission of a node can be received by *any* node within its vicinity which, under a common assumption in the literature, is a unit disk centered at this node. A wireless node can receive the signal from another node if it is within the transmission range of the sender. Otherwise, they communicate through multi-hop wireless links by using intermediate nodes to relay the messages. Consequently, each node in the wireless ad hoc network also acts as a router, forwarding data packets for other nodes. By one-hop broadcasting, each node u can gather the location information of all nodes within its transmission range. Consequently, all wireless nodes together define a unit-disk graph (UDG), which has an edge uv iff the Euclidean distance $\|uv\|$ is less than one unit.

A wireless ad hoc network needs some special treatment as it intrinsically has its own special characteristics and some unavoidable limitations compared with wired networks. For example, wireless nodes are often powered by batteries only and they often have limited memories. So wireless ad hoc networks prefer localized algorithms and power-efficient network topologies. Unlike the wired networks, a transmission by a wireless device will be received by all nodes within its vicinity. Thus, we model the communication characteristics as *broadcasting* by assuming that the message sent by a node will *always* be received by *all* nodes within its transmission range. We can utilize this broadcasting property to save the communications needed to send some information. Throughout this paper, a *local broadcast* by a node means it sends the message to all nodes within its transmission range; a *global broadcast* by a node means it tries to send the message to all nodes in the network by the possible relaying of other nodes.

Due to the limited power and memory, a wireless node prefers to only maintain the information of a subset of neighbors it can communicate, which is called *topology control*. In recent years, there is a substantial amount of research on topology control for wireless ad hoc networks [1], [2], [3], [4], [5]. These algorithms are designed for different objectives: minimizing the maximum link length while maintaining the network connectivity [3]; bounding the node degree [5]; bounding the spanning ratio [1], [2]; constructing planar spanner locally [1]. Here a structure H is a spanner of UDG if, for any two nodes, the length of the shortest-path connecting them in H is no more than a constant factor of the length of the shortest-path connecting them in the original UDG. Planar structures are used by several localized routing algorithms [6]. In [7], Wang and Li proposed the first localized algorithm to construct a bounded degree planar spanner.

Recently, Li, Hou and Sha [8] proposed a novel MST-based method for topology control and broadcasting. Each node u uses its one-hop neighbors to build a *local* minimum spanning tree and an edge uv is kept if it belongs to this local minimum spanning tree. They proved that the final graph, called *local minimum spanning tree* (LMST), is connected, and has a bounded degree 6. However,we will show that LMST is not a low weight structure and the broadcasting based on it can still consume power $O(n^2)$ times of the minimum in the worst case.

Minimum-energy broadcast/multicast routing in ad hoc networking environment has been addressed in [9], [10]. Three centralized greedy heuristics algorithms were presented in [10]:

MST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). Wan *et al.* [11] showed that the approximation ratio of the MST-based approach is between 6 and 12 by assuming that the power needed to support a link uv is $\|uv\|^\beta$, where $\|uv\|$ is the Euclidean distance between u and v, β is a real constant between 2 and 5 dependent on the wireless transmission environment. The best distributed algorithm [12] can compute MST in $O(n)$ rounds using $O(m + n \log n)$ communications for a general graph with m edges and n nodes. Obviously, MST cannot be constructed in a localized manner, i.e., each node cannot determine which edge is in the defined structure by purely using the information of the nodes within some constant hops. Thus, several localized structures, such as RNG [13], have been used for broadcasting. As shown in [14], the total energy used by RNG based approach could be about $O(n^\beta)$ times optimum.

The main contributions of this paper are as follows. Firstly, we propose a family of structures, namely, k-localized minimum spanning tree (LMST$_k$) for topology control and broadcasting in wireless ad hoc networks. We analytically prove that the node degree of the structure LMST$_k$ is at most 6, LMST$_k$ is connected and planar, and more importantly, the total edge length of the LMST$_k$ is within a constant factor of that of the minimum spanning tree when $k \geq 2$. We give an efficient localized method to construct the LMST$_k$ using only $O(n)$ messages under a local broadcast communication model, i.e., the message sent by a node is received by all nodes within its transmission range. Secondly, we propose another structure, called *Incident MST and RNG Graph* (IMRG), that can be constructed using at most $13n$ messages under the local broadcast communication model. Every node only uses its partial two-hop information to construct the structure IMRG. Notice that it was shown in [14] that some two-hop information is necessary to construct any low-weighted structure for UDG. Thirdly, we study the application of these structures for efficient broadcasting in wireless ad hoc networks. Notice that Wan *et al.* [11] proved that the broadcasting based on the MST consumes energy within a constant factor of the optimum when *only* consider the energy consumed by the senders. However, in practice, the receiver node also consumes energy to receive the signal. In this paper, we adopt the later model and assume that the energy consumed by the receiver node is *no more than* the energy consumed by the sender. We then prove that the approximation ratio of the MST-based approach is still a constant when this more practical energy model is used. Since it is expensive to construct

MST in a distributed way, we will use our newly proposed structures LMST$_k$ and IMRG to approximate it. Although a low-weighted structure cannot guarantee that the broadcasting based on it consumes energy within a constant factor of the optimum in the worst case, the energy consumptions using our new structures LMST$_k$ ($k \geq 2$), and IMRG are within $O(n^{\beta-1})$ of the optimum theoretically in the worst case. This improves the previously known "lightest" structure RNG and LMST by $O(n)$ factor. We show that these structures are asymptotically optimum for broadcasting among all locally constructed structures. Test results are corroborated in the simulation study. Our extensive simulations show that the energy consumption of broadcasting based on these structures is within a small constant factor of that based on the MST for randomly deployed wireless networks.

The rest of the paper is organized as follows. In Section II, we review the related works on network topology control and minimum energy broadcasting. In Section III, we present our communication and computation efficient localized methods that can construct connected, planar, bounded degree, low-weighted structures LMST$_k$ and IMRG. The total communication costs of our methods are $O(n)$ (at most $13n$ for IMRG). We then study the applications of our structures in broadcasting and topology control by comparing the performances of these structures with previously best-known structures in Section V. We conclude our paper in Section VI.

II. Related Work

Before reviewing the related works, we first introduce the formal definition of *low weight*. Given a geometric structure G over a set of points, let $\omega(G)$ be the total length of the links in G and $\omega_\beta(G) = \sum_{uv \in G} \|uv\|^\beta$. Then, a structure G is called *low weight* if $\omega(G)$ is within a constant factor of $\omega(MST)$.

A. Topology Control

Recently, topology control for wireless ad hoc networks has attracted considerable attentions [3], [15], [17], [18], [19], [20]. Rajaraman [21] conducted an excellent survey. Several geometrical structures have been used in topology control, and broadcasting in wireless ad hoc networks, whose definitions are reviewed as follows.

A disk centered at a point x with a radius r, denoted by $disk(x, r)$, is the set of points whose distance to x is at most r. Let $lune(u,v)$ defined by two points u and v be the intersection of two disks with radius $\|uv\|$ and centered at u and v respectively, i.e., $lune(u,v) = disk(u, \|uv\|) \cap disk(v, \|uv\|)$. Let $disk(u,v)$ be the disk with diameter uv. The *relative neighborhood graph* [22], denoted by RNG, consists of all edges uv such that the *interior* of $lune(u,v)$ contains no node $w \in V$. The *Gabriel graph* (GG) [23] contains an edge uv if and only if $disk(u,v)$ contains no other node w inside. It is easy to show that RNG is a subgraph of the Gabriel graph. For unit disk graph, the relative neighborhood graph and the Gabriel graph only contain the edges in UDG and satisfying the respective definitions.

Notice that, traditionally, the relative neighborhood graph will always select an edge uv even if there is some node on the boundary of $lune(u,v)$. Thus, RNG may have unbounded node degree, e.g., considering $n-1$ points equally distributed on the circle centered at the nth point v, the degree of v is $n-1$. Notice that for the sake of lowing the weight of a structure, the structure should contain as less edges as possible without breaking the connectivity. Li [14] then extended the traditional definition of RNG as follows.

The *modified relative neighborhood graph* consists of all edges uv such that (1) the *interior* of $lune(u,v)$ contains no point $w \in V$ and, (2) there is no point $w \in V$ with $ID(w) < ID(v)$ on the boundary of $lune(u,v)$ and $\|wv\| < \|uv\|$, and (3) there is no point $w \in V$ with $ID(w) < ID(u)$ on the boundary of $lune(u,v)$ and $\|wu\| < \|uv\|$, and (4) there is no point $w \in V$ on the boundary of $lune(u,v)$ with $ID(w) < ID(u)$, $ID(w) < ID(v)$, and $\|wu\| = \|uv\|$. See Figure 1 for an illustration when an edge uv is *not* included in the modified relative neighborhood graph. Li called such structure RNG'. Obviously, RNG' is a subgraph of RNG and still can be

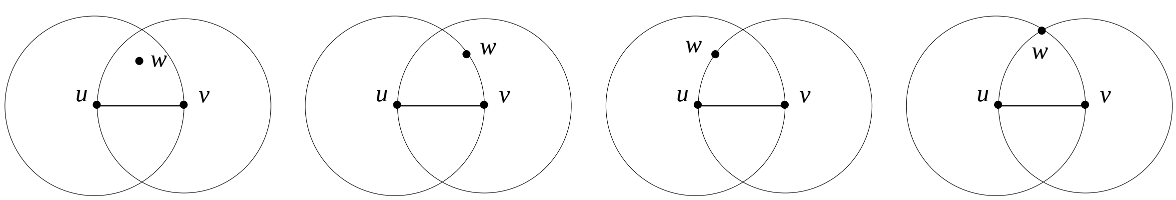


Fig. 1. Four cases when edges are not in the modified RNG.

constructed using n messages. It was proved in [14] that RNG' has a maximum node degree 6 and still contains a MST as a subgraph.

The *Yao graph* with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG_k}$, is defined as follows. At each node u, any k equally-separated rays originated at u define k cones. In each cone, choose the shortest edge uv, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by the smallest ID. The resulting directed graph is called the Yao graph. Some researchers used a similar construction named θ-graph [24]. Recently, the Yao structure has been re-discovered by several researchers for topology control in wireless ad hoc networks of directional antennas.

Li *et al.* [25] extended the definitions of these structures on top of any given graph G. Wattenhofer *et al.* [20] also proposed a two-phased approach that consists of a variation of the Yao graph followed by a variation of the Gabriel graph.

Li *et al.* [18] proposed a structure that is similar to the Yao structure for topology control. Each node u finds a power $p_{u,\alpha}$ such that in every cone of degree α surrounding u, there is some node that u can reach with power $p_{u,\alpha}$. Notice that the number of cones to be considered in the traditional Yao structure is a constant k. However, unlike the Yao structure, for each node u, the number of cones needed to be considered in the method proposed in [18] is about $2n$, where each node v could contribute two cones on both side of segment uv. Then the graph G_α contains all edges uv such that u can communicate with v using power $p_{u,\alpha}$. They proved that, if $\alpha \leq \frac{5\pi}{6}$ and the UDG is connected, then G_α is a connected graph. On the other hand, if $\alpha > \frac{5\pi}{6}$, they showed that the connectivity of G_α is not guaranteed by giving some counter-example [18]. Unlike the Yao structure, the final topology G_α is not necessarily a bounded degree graph.

Li *et al.* [25] also proposed another structure called *YaoYao graph* $\overrightarrow{YY}_k$ by applying a *reverse* Yao structure on $\overrightarrow{YG}_k$. They proved that the directed graph $\overrightarrow{YY}_k$ is strongly connected if UDG is connected and $k > 6$. In [5], Wang *et al.* considered another undirected structure, called *symmetric Yao graph* YS_k. An edge uv is selected if and only if both directed edges $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are in the $\overrightarrow{YG}_k$. Then it is obvious that its maximum node degree is k. They showed that the graph YS_k is strongly connected if UDG is connected and $k \geq 6$.

Recently, Li, Hou and Sha [8] proposed an MST-based method for topology control. Each node u first collects its one-hop neighbors $N_1(u)$. Node u then computes its minimum spanning tree $MST(N_1(u))$ of the induced unit disk graph on its one-hop neighbors $N_1(u)$. Node u keeps a directed edge uv if and only if uv is an edge in $MST(N_1(u))$. They called the union of all directed edges the *local minimum spanning tree*, denoted by G_0. If only symmetric edges are kept, then the graph is called G_0^-, i.e., it has an edge uv iff both directed edge uv and directed edge vu exist. If ignoring the directions of the edges in G_0, the graph is called G_0^+, i.e., it has an edge uv iff either directed edge uv or directed edge vu exists. They proved that the graph is connected, and has bounded degree 6.

Here, we prove that graph G_0^- is also planar. For the sake of contradiction, assume that G_0^- is not planar and two edges uv and xy intersect each other. Assume that the clockwise order of these four nodes are u, y, v, x. Obviously, one of the four angles $\angle uxv$, $\angle xvy$, $\angle vyu$, and $\angle yux$ is at least $\pi/2$. Without loss of generality, assume that $\angle uxv \geq \pi/2$. Then, edge uv is the longest edge among triangle $\triangle uvx$. Thus, in the local minimum spanning tree $MST(N_1(u))$, edge uv cannot appear since there is already a path uxv whose edges are all shorter than uv. Similarly, graph G_0^+ is a planar graph (by replacing the undirected edges with directed edges in the above proof).

We then construct an example such that the structures G_0^- and G_0^+ are not low-weighted. Figure 2 illustrates such an example. Since it uses only one-hop information, at every node, the algorithm only knows that there are a sequence of nodes evenly distributed with small separation, and another node which is one-unit away from current node. It is easy to show that the final structure G_0^+ is exactly illustrated in Figure 2. The minimum spanning tree will only use one horizontal link while LMST has $n/2$ horizontal links. It is easy to show that the total edge length of G_0 is $O(n)$ times of that of MST for this example.

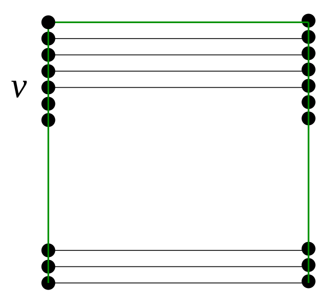


Fig. 2. G_0 could consumes arbitrarily large power for broadcasting compared with the optimum.

Inspired by the local minimum spanning tree structure in [8], in this paper, we propose a sequence of structures called k-local minimum spanning tree (LMST$_k$). To improve the communication cost, we further propose another structure, called IMRG. Our structures have an additional property: they are low-weighted. We also show that our structures are always subgraphs of the structures G_0^+ and G_0^- constructed in [8]. Locally constructed low-weighted structure was first proposed by us in [14]. We will show that our new structures are subgraphs of that structure and our structures have less computational cost. We do rely on a main theorem proved in [14] to show that our structures are low-weighted.

B. Power Assignment

A transmission power assignment on the vertices in V is a function $\mathcal{P}$ from V into real numbers representing the node power. The *directed* (or called *asymmetric* by some researchers) *communication graph*, denoted by $\overrightarrow{G}_{\mathcal{P}}$, induced by a transmission power assignment $\mathcal{P}$, is a directed graph with V as its vertices and has a directed edge $\overrightarrow{v_i v_j}$ if and only if $||v_i v_j||^\beta \leq \mathcal{P}(v_i)$. The *undirected* (or called *symmetric* by some researchers) *communication graph*, denoted by $G_{\mathcal{P}}$, induced by a transmission power assignment $\mathcal{P}$, is a undirected graph with V as its vertices and has an edge $v_i v_j$ if and only if $||v_i v_j||^\beta \leq \mathcal{P}(v_i)$ and $||v_i v_j||^\beta \leq \mathcal{P}(v_j)$. Given a graph $H = (V, E)$, we say the power assignment $\mathcal{P}$ is induced by H, denoted by $\mathcal{P}_H$, if $\mathcal{P}(v) = \max_{(v,u)\in E} ||vu||^\beta$. In other words, the power assigned to a node v is the largest power needed to reach all neighbors of v in H. The *maximum-cost* (and *total-cost*) of a transmission power assignment $\mathcal{P}$ is defined as $mc(\mathcal{P}) = \max_{v_i \in V} \mathcal{P}(v_i)$ (and $sc(\mathcal{P}) = \sum_{v_i \in V} \mathcal{P}(v_i)$ respectively). The min-max assignment (and min-total assignment) problem is to find a transmission power assignment $\mathcal{P}$ whose cost $mc(\mathcal{P})$ (and $sc(\mathcal{P})$ respectively) is minimized while the induced communication graph is connected.

Let EMST(V) be the Euclidean minimum spanning tree over a point set V. Both [3] and [26] use the power assignment induced by EMST(V). It was proved in [3] that power assignment induced by EMST(V) is optimum for the min-max assignment problem. Using the fact that RNG, GG and YG_k have $O(n)$ edges and contain EMST as a subgraph, min-max assignment problem can be solved in $O(n \log n)$ time complexity by a centralized algorithm and solved using $O(n \log n)$ messages in a distributed manner.

Kiroustis *et al.* [27] first proved that the min-total assignment problem is *NP-hard* when the mobile nodes are deployed in a three-dimensional space. A simple 2-approximation algorithm based on the Euclidean minimum spanning tree was also given in [27]. The algorithm guarantees the same approximation ratio in any dimensions. Clementi *et al.* [28], [29] proved that the min-total assignment problem is still NP-hard when nodes are deployed in a two dimensional space.

For the symmetric communication, several methods also guarantee a good performance. It is easy to show that the minimum spanning tree method still gives the optimum solution for the min-max assignment and a 2-approximation for the min-total assignment. Recently, Călinescu *et al.* [30] gave a method that achieves better approximation ratio $\frac{5}{3}$ by using idea from the minimum Steiner tree. Like the minimum spanning tree method, it works for any power definition.

Since it is expensive to construct the Euclidean MST in a distributed manner, we would like to approximate the Euclidean MST efficiently in a distributed way. We thus will study the performance of our structures for power assignment. Notice that our structures do approximate the total edge length of the Euclidean minimum spanning tree. Our simulations show that our locally constructed structures outperform the previous structures in terms of both the maximum assigned power and the total assigned power while guarantee the network connectivity.

C. Minimum Energy Broadcasting

Minimum-energy broadcast/multicast routing in a simple ad hoc networking environment has been addressed in [9], [10]. Any broadcast routing is viewed as an arborescence (a directed tree) T, rooted at the source node of the broadcasting, which spans all nodes. Let $\mathcal{P}_T(v)$ denote the transmission power of the node v required by the tree T. For any leaf node v of T, $\mathcal{P}_T(v) = 0$. For any internal node v of T, let $\mathcal{P}_T(v)$ denote the minimum power needed to reach its farthest children in T. The total energy required by T is $\sum_{v\in V}\mathcal{P}_T(v)$. It is known [31] that the minimum-energy broadcast routing problem cannot be solved in polynomial time if $P \neq NP$. Three greedy heuristics were proposed in [10] for the minimum-energy broadcast routing problem: MST, SPT, and BIP. By assuming that the power needed to support a link uv is $\|uv\|^{\beta}$. It was proved in [11] that, for any point set V in the plane, the total energy required by any broadcasting among V is at least $\omega_\beta(MST)/C_{mst}$, where $6 \leq C_{mst} \leq 12$ is a constant related to the Euclidean minimum spanning tree. In addition, they [11] showed that the approximation ratio of MST based approach is between 6 and 12 and the approximation ratio of BIP based approach is between $\frac{13}{3}$ and 12; on the other hand, the approximation ratio of SPT is at least $\frac{n}{2}$, where n is the number of nodes.

Unfortunately, all these structures cannot be constructed locally. Thus, several locally constructed structures have been proposed for broadcasting in wireless ad hoc networks, such as RNG [13]. The ratio of the weight in RNG over the weight of MST could be $O(n)$ for n points set [25]. By assuming that the power needed to support a link uv is $\|uv\|^{\beta}$, an example was given in [14] to show that the total energy used by broadcasting on RNG could be about $O(n^{\beta})$ times of the minimum-energy used by an optimum method. The same example can be used to show that the structure G_0 [8] could consumes power $O(n^{\beta})$ times of the optimum for broadcasting. On the other hand, we will prove that $\omega_{\beta}(IMRG) \leq O(n^{\beta-1}) \cdot \omega_{\beta}(MST)$, and $\omega_{\beta}(LMST_k) \leq O(n^{\beta-1}) \cdot \omega_{\beta}(MST)$ for $k \geq 2$. In other words, the power consumption for broadcasting based on our newly proposed structures are only $O(n^{\beta-1})$ times of the optimum in the worst case, which improves the previously known structure RNG by $O(n)$ factor. When we assume that the receivers do consume power for receiving signal, all the statements still hold.

III. k-LOCAL MINIMUM SPANNING TREE (LMST$_k$)

In this section, we define a sequence of structures, namely, k-local minimum spanning tree (LMST$_k$), which can be constructed locally using only $O(n)$ messages. All these structures are connected, low-weighted (when $k \geq 2$), planar and have a bounded degree.

A. k-local Minimum Spanning Tree (LMST$_k$)

We define a sequence of structures k-local minimum spanning tree (LMST$_k$) as follows. Let $N_k(u)$ be the set of nodes that are within k hops of node u in UDG. Here $N_k(u)$ includes node u itself for the simplicity of notation later.

Definition 1: The k-local minimum spanning tree (LMST$_k$) contains a *directed* edge $\overrightarrow{uv}$ if edge uv belongs to $MST(N_k(u))$. We further define two undirected variations LMST$_k^-$, and LMST$_k^+$. Structure LMST$_k^-$ contains an edge uv if both directed edge $\overrightarrow{uv}$ and directed edge $\overrightarrow{vu}$ belong to LMST$_k$. Structure LMST$_k^+$ contains an edge uv if either $\overrightarrow{uv}$ or $\overrightarrow{vu}$ belongs to LMST$_k$.

Notice that one way to construct MST is to add edges in the order of their lengths if it does not create a cycle with previously added edges. If there are two edges with the same length, we break the tie by comparing the larger ID of the two end-points then comparing the smaller ID of the two-end points. We label an edge uv by $(\|uv\|, \max(ID(u), ID(v)), \min(ID(u), ID(v)))$, and an edge uv is ordered before an edge xy if the lexicographic order of the label of uv is less than that of xy. In this paper, we only consider the minimum spanning tree constructed using the above edge ordering.

Before we present our communication efficient method to construct them, we first study their properties. First of all, it is easy to prove the following monotone property of the structures.

Lemma 1: $LMST_{k+1} \subseteq LMST_k$, $LMST^+_{k+1} \subseteq LMST^+_k$, and $LMST^-_{k+1} \subseteq LMST^-_k$.

Lemma 2: LMST^+_k is a subgraph of RNG, so does LMST^-_k.

Proof. We prove it by contradiction. Assume that a node u adds an edge $uv \notin RNG$ to LMST_k. Since edge $uv \notin RNG$, there is a node w inside the lune defined by segment uv. Remember that the minimum spanning tree of the node set $N_1(u)$ can be constructed by adding edges in ascending order whenever it does not create a cycle with previously added edges. Clearly, when we process the edge uv, there is already a path connecting u and w and a path connecting w and v since uw and wv are not longer than uv. It implies that node u cannot add the edge uv to its $MST(N_k(u))$. Consequently, both graphs LMST^+_k and LMST^-_k are subgraphs of RNG. ◻

Actually we can enhance Lemma 2 by showing that LMST^+_k is a subgraph of RNG'. The above lemma immediately implies that the structures LMST_k, LMST^+_k and LMST^-_k are planar. Remember that k-local minimum spanning tree LMST_k is proposed to approximate Euclidean minimum spanning tree MST. We then show that MST is a subgraph of LMST_k for any k.

Lemma 3: Euclidean minimum spanning tree MST is a subgraph of LMST_k for any k.

Proof. Consider any edge uv from MST. Assume that we add edges in ascending order of their lengths to MST. Clearly, when we decide whether to add the edge uv, there is no path connecting u and v using edges added before uv. Obviously, this property still holds when node u decide whether to add edge uv to the minimum spanning tree $MST(N_k(u))$ of its k-hop neighbors $N_k(u)$. It implies that edge uv belongs to $MST(N_k(u))$, and $MST(N_k(v))$. Consequently, MST is a subgraph of all structures LMST_k, LMST^+_k and LMST^-_k for any k. ◻

The above lemma immediately implies that all these k-localized minimum spanning trees are connected when the original communication graph UDG is connected.

Since every node in the Euclidean minimum spanning tree has a degree at most 6, the out-degree of every node u in LMST_k is at most 6. Consequently, the degree of every node u in LMST^-_k is although at most 6 since we keep an edge uv if both directed edges $\overrightarrow{uv}$ and $\overrightarrow{vu}$ belong to LMST_k. We then show that the degree of every node in LMST^+_k is also at most 6.

Lemma 4: Each node in LMST_k has at most 6 neighbors in LMST^+_k.

Proof. We prove it by contradiction. Assume that one node v has more than 6 total in-neighbors and out-neighbors. From the pigeonhole principle, there must have two neighbors, say u_1 and u_2, of v such that $\angle u_1 v u_2 < \pi/3$. There are three cases: 1) both u_1 and u_2 are in-neighbors; 2) both u_1 and u_2 are out-neighbors; 3) one is out-neighbor and one is in-neighbor.

We first consider the case that both u_1 and u_2 are in-neighbors. Obviously, $\angle u_1 v u_2$ cannot be the largest angle in the triangle $u_1 v u_2$. Assume that $\angle v u_1 u_2$ is the largest, i.e., $u_2 v$ is the longest edge in triangle $u_1 v u_2$. Thus, node u_2 cannot have $u_2 v$ in its minimum spanning tree $MST(N_k(u_2))$ since there is already a path (using node $u_1 \in N_1(u_2)$) connecting u_2 and v when we try to add edge $u_2 v$. It is a contradiction to the fact that u_2 is an in-coming neighbor of v. Similarly, we can prove that the other two cases are also impossible. This finishes the proof. ◻

The above lemma immediately implies that every node in graphs LMST$_k^+$ and LMST$_k^-$ has a degree at most 6. To show that the final structures LMST$_k$, LMST$_k^+$ and LMST$_k^-$ are low weighted when $k \geq 2$, we first review a result proved in [14].

Lemma 5 ([14]) A subgraph G of RNG' is low-weighted if for any two edges $uv \in G$ and $xy \in G$, neither uv nor xy is the longest edge of the quadrilateral $uvyx$.

We then prove the main result of this paper.

Lemma 6: All structures LMST$_k^+$ are low weighted when $k \geq 2$.

Proof. Since we showed that LMST$_k^+$ is a subgraph of modified RNG for any k, we will only need prove that there are no two edges $uv \in LMST_k^+$ and $xy \in LMST_k^+$, such that one of them is the longest edge of the quadrilateral $uvyx$. We prove this by contradiction. Assume that we have two edges $uv \in LMST_k^+$ and $xy \in LMST_k^+$, and uv is the longest edge of the quadrilateral $uvyx$. Clearly, x, v and y are at most 2-hops away from u in the unit disk graph. Then when we decide whether to add edge uv to the minimum spanning tree $MST(N_k(u))$ of the k-hop neighbors $N_k(u)$ for $k \geq 2$, edges xu, xy, and yv have already been processed, i.e., there are paths using shorter edges to connect u to x, x to y, and y to v. Thus, the edge uv will not be added to $MST(N_k(u))$ when $k \geq 2$. It is a contradiction to $uv \in LMST_k^+$. This finishes the proof. ◻

B. Efficient Construction of k-local Minimum Spanning Tree (LMST$_k$)

We then discuss in detail how to construct the k-local Minimum Spanning Tree (LMST$_k$) efficiently, i.e., using only $O(n)$ messages under the local broadcasting model. Since LMST$_2$ is already a low weighted structure, we will only describe our method for constructing LMST$_2$ although the same method works for general LMST$_k$.

Algorithm 1: Construct $LMST_2$ Locally

1. Every node u collects the location information of $N_2(u)$ based on an efficient method described in [32] (reviewed in detail later).
2. Every node u computes the Euclidean minimum spanning tree $MST(N_2(u))$ of its 2-hop neighbors $N_2(u)$, including u itself.
3. A node u proposes to add a directed edge $\overrightarrow{uv}$ if $uv \in MST(N_2(u))$ and $||uv|| \leq 1$.
4. If $LMST_2^+$ is needed, node u keeps an edge uv when either u or v proposed to add it. If $LMST_2^-$ is needed, node u keeps an edge uv when both u and v proposed to add it.

We then review the communication efficient method proposed in [32] to collect $N_2(u)$ for every node u when the geometry information is known. Computing the set of 1-hop neighbors with $O(n)$ messages is trivial: every node broadcasts a message announcing its ID. Computing the 2-hop neighborhood is not trivial, as the UDG can be dense. The approach in [32] is based on the specific connected dominating set introduced in [33], which again is based on a maximal independent set (MIS). In the algorithm, each node uses its adjacent node(s) in the MIS to broadcast over a larger area relevant information. Listening to the information about other nodes broadcast by the MIS nodes enables a node to compute its 2-hop neighborhood. The algorithm uses heavily the nodes in the connected dominating set, an example in [32] shows that overloading certain nodes might be unavoidable.

We start from the moment the virtual backbone is already constructed, and every node knows the ID and the position of its neighbors. The idea of the algorithm is for every node to efficiently announce its ID and position to a subset of nodes which includes its 2-hop neighbors. The responsibility for announcing the ID and position of a node v is taken by the MIS nodes adjacent to v. Each such MIS node assembles a packet containing: $<$ID; *position*; *counter*$>$, with the ID and position of v, and a counter variable being set to 2. The MIS node then broadcasts the packet.

A connector node is used to establish a link in between several pairs of virtually-adjacent MIS nodes, and will not retransmit packets which do not travel in between these pairs of MIS nodes. Here two MIS nodes are said to be virtually-adjacent if they are within 2 or 3 hops of each other. The connector node will rebroadcast packets with nonzero counter originated by one of the nodes in a pair of virtually-adjacent MIS nodes, thus making sure the packet advances towards the other MIS node in the pair. Recall that the path in between a pair of virtually-adjacent MIS nodes has one or two connector nodes.

When receiving a packet of type <ID; *position*; *counter*>, an MIS node checks whether this is the first message with this ID, and if yes decreases the counter variable and rebroadcasts the packet. A node listens to the packets broadcast by all the adjacent MIS nodes and, using its internal list of 1-hop neighbors, checks if the node announced in the packet is a 2-hop neighbor or not - thus constructing the list of 2-hop neighbors.

The above approach can be extended to find the k-hop neighbors of every node using total $O(n)$ communications: the initial counter is set to k. The total communications used by this approach is at most $(6k+3)^2 \cdot n$ after a backbone based on MIS is constructed [4].

IV. Structures with Improved Communication Cost

In the previous section, we defined a sequence of structures that are guaranteed to be low weighted and can be constructed in a localized manner using only $O(n)$ messages. However, the hidden constant in the communication cost could be large although it is a constant. In this section, we define several structures that can be constructed using at most $13n$ messages. All these structures are connected, low-weighted, bounded degree, planar graphs.

A. *Sparse Structure From RNG'*

In [14], Li gave the first localized method to construct a structure LRNG with weight $O(\omega(MST))$ using total $O(n)$ local-broadcast messages, but the computation at each node is expensive. For the completeness of presentation, we first review the localized algorithm given in [14] that constructs a low-weighted structure using only some two hops information.

Algorithm 2: [14] Construct Low Weighted Sparse Structure LRNG

1. All nodes together construct the graph RNG' in a localized manner.
2. Each node u locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node u listens to the messages from its one-hop neighbors.
3. Assume node u received a message informing the existence of an edge xy from its neighbor x. For each edge uv in RNG', if uv is the longest among uv, xy, ux, and vy, node u removes the edge uv. Ties are broken by the label of the edges. Here we assume that $uvyx$ is the convex hull of u, v, x, and y.

4. Let LRNG denote the final structure formed by all remaining edges in RNG'.

Obviously, if an edge uv is kept by node u, then it is also kept by node v, i.e., the edges kept by all nodes are symmetric. It was shown in [14] that the structure LRNG has total edge length $\Theta(\omega(MST))$.

Clearly, the communication cost of Algorithm 2 is at most $7n$: initially each node spends one message to tell its one-hop neighbors its position information, then each node u tells its one-hop neighbors all its incident edges $uv \in RNG'$ (there are at most total $6n$ such messages since RNG' has at most $3n$ edges). The computational cost of Algorithm 2 could be high since for each link $uv \in RNG'$, node u has to test whether there is an edge $xy \in RNG'$ and $x \in N_1(u)$ such that uv is the longest among uv, xy, ux, and vy. We continue to present our new algorithms that improve the computational complexity of each node while still maintain low communication costs.

B. Incident MST and RNG Graph (IMRG)

Although the structures $LMST_2^-$ and $LMST_2^+$ have several nice properties such as bounded degree, planar, and low-weighted, the communication cost of constructing them could be very large to save the computational cost of each node compared with structure LRNG. The large communication costs are from collecting the two hop neighbors information $N_2(u)$ for each node u, although the total communication of the protocol described in [32] is $O(n)$, the hidden constant is large.

We could improve the communication cost by using a subset of two hop information without sacrificing any properties. For any node u, we define the partial two hop of u as

$$N_2^{RNG'}(u) = \{w \mid vw \in RNG' \text{ and } v \in N_1(u)\} \cup N_1(u).$$

Definition 2: The Incident MST and RNG Graph (IMRG) contains a *directed* edge $\overrightarrow{uv}$ if edge uv belongs to $MST(N_2^{RNG'}(u))$, the Euclidean minimum spanning tree of nodes $N_2^{RNG'}(u)$. We further define two undirected variations IMRG$^-$, and IMRG$^+$. Structure IMRG$^-$ contains an edge uv if both directed edge $\overrightarrow{uv}$ and directed edge $\overrightarrow{vu}$ belong to IMRG. Structure IMRG$^+$ contains an edge uv if either $\overrightarrow{uv}$ or $\overrightarrow{vu}$ belongs to IMRG.

We then describe a communication efficient algorithm to build these structures as follows.

Algorithm 3: Construct Low Weighted Structure IMRG

1. Each node u tells its position information to its one-hop neighbors $N_1(u)$ using a local broadcast model. All nodes together construct the graph RNG' in a localized manner.
2. Each node u locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node u listens to the messages from its one-hop neighbors.
3. Each node u collects $N_2^{RNG'}(u)$ and computes the Euclidean minimum spanning tree, denoted by $MST(N_2^{RNG'}(u))$, of all nodes $N_2^{RNG'}(u)$, including u itself.
4. Node u proposes to add an edge $uv \in MST(N_2^{RNG'}(u))$ if $\|uv\| \leq 1$.
5. If IMRG$^-$ is needed, node u keeps an edge uv if both node u and node v proposed to add edge uv. If IMRG$^+$ is needed, node u keeps an edge uv if either node u or node v proposed to add edge uv.

As will seen later (Lemma 7), the constructed structures are subgraphs of the modified RNG graph. Thus, these structures are planar and have at most $3n$ edges. In addition, the total communication cost of Algorithm 3 is at most $13n$ when either structure IMRG$^-$ or IMRG$^+$ is needed; the total communication cost is at most $7n$ if the directed structure IMRG is needed. We first show that these two structures IMRG$^+$ and IMRG$^-$ are still planar, bounded degree, and low-weighted.

Lemma 7: Structure IMRG is a subgraph of modified RNG.

Proof. Consider any edge $uv \notin RNG'$. We show that node u will not propose uv. From the definition of RNG', we know that there is a node w inside the lune defined by segment uv and edge uw and wv has a label less than uv. Considering the process of constructing $MST(N_2^{RNG'}(u))$, when we decide whether to add edge uv after processing edges with smaller labels, there is already a path connecting u and w, and a path connecting w and v. Thus, edge uv cannot be added by node u to $MST(N_2^{RNG'}(u))$. This finishes the proof. □

The above lemma immediately implies that all structures IMRG$^+$ and IMRG$^-$ are planar , and have a bounded node degree at most 6. We then show that IMRG$^+$ and IMRG$^-$ are connected by proving the following lemma.

Lemma 8: MST is a subgraph of IMRG$^+$ and IMRG$^-$.

Proof. We prove this by induction on the length of the edges from MST.

Consider the shortest edge uv in the original unit disk graph. Clearly, the edge uv belongs to

MST, and uv belongs to $MST(N_2^{RNG'}(u))$ and $MST(N_2^{RNG'}(v))$. Thus, uv belongs to IMRG$^-$.

Assume that the first kth shortest edges from MST are in IMRG$^-$. Then consider the $(k+1)$th shortest edge uv from MST. For the sake of contradiction, assume that node u removes edge uv since $uv \notin MST(N_2^{RNG'}(u))$. Consequently, there is a path in the unit disk graph formed on $N_2^{RNG'}(u)$ connecting u and v using edges with length at most $\|uv\|$ (ties are broken by rank). It is a contradiction to the fact that uv belongs to MST. Thus, edge uv is also kept IMRG$^-$. Therefore, MST is a subgraph of IMRG$^-$ and MST is a subgraph of IMRG$^+$. $\square$

We then show that the structures IMRG$^-$ and IMRG$^+$ are low-weighted.

Lemma 9: The structures IMRG$^-$ and IMRG$^+$ are low-weighted.

Proof. The proof is similar to the proof that LMST$_k$ is low weighted. We can show that there are no two edges uv and xy from IMRG such that one of them is the longest edge in the quadrilateral $uvyx$, which can be proved easily by contradiction. Notice that we already proved that IMRG$^-$ and IMRG$^+$ are subgraphs of RNG'. Thus, we can use Lemma 5. $\square$

We then summarize the properties of the structure IMRG by the following theorem.

Theorem 10: Algorithm 3 constructs structures IMRG$^-$ and/or IMRG$^+$ using at most $13n$ messages. The structures IMRG$^-$ or IMRG$^+$ are connected, planar, bounded degree (at most 6), and low-weighted.

It is easy to show that the structure LMST$_2$ is always a subgraph of IMRG since IMRG uses only a partial information to construct the minimum spanning tree. If an edge uv is removed from $MST(N_2^{RNG'}(u))$, it means that there is a path connecting u and v using shorter edges when we process uv. By a simple induction, we can show that there is also a path connecting u and v when we process uv in constructing $MST(N_2(u))$. We further show that the structure IMRG is a subgraph of LMST$_1$. Consider any directed edge $\overrightarrow{uv}$ that is not proposed by node u in constructing $MST(N_1(u))$. It means that there is a path connecting u and v in the induced unit disk graph on $N_1(u)$, whose edges have length less than $\|uv\|$. Clearly, this path is still in the induced unit disk graph on $N_2^{RNG'}(u)$ since $N_1(u) \subset N_2^{RNG'}(u)$. Consequently, edge uv cannot appear in the Euclidean minimum spanning tree $MST(N_2^{RNG'}(u))$. It then implies that the structure IMRG is always a subgraph of LMST$_1$. Consequently, the structure IMRG$^+$ is always a subgraph of the structure G_0^+ and the structure IMRG$^-$ is always a subgraph of the structure G_0^- constructed in [8].

Lemma 11: Structure IMRG is a subgraph of LMST_1 and a supergraph of LMST_2.

C. Fault-Tolerance

We have presented algorithms to build structures that are connected, planar, low-weighted and have a bounded node degree. However, none of these structures are fault-tolerant in the worst case. Here we say that a structure is node fault-tolerant if the graph is still connected when one node breaks down. In [34], Li *et al.* discussed how to build a k-fault-tolerant structure such that each node has a degree at most $6k$ and is a spanner. In this subsection, we present a method that transforms any structure into a fault-tolerant structure by at most doubling the total edge length. Notice that also this method has been used previously for various purposes [15], [16], we will show that it keeps the low-weight and bounded degree properties. Assume that we are given a topology structure G that is connected.

Algorithm 4: Transform Structure G to Fault-Tolerant

1. Each node u collects all incident edges $uv \in G$.

2. Node u sorts all its incident neighbors from G in a clockwise order and let $v_1, v_2 \cdots, v_d$ be its neighbors. Node u informs node v_i to add links $v_{i-1}v_i$ and v_iv_{i+1}. Here $v_0 = v_d$ and $v_{d+1} = v_1$. Let $F(G)$ be the final structure formed by all edges, including the edges from G.

Lemma 12: If structure G has bounded degree Δ, then graph $F(G)$ has degree at most 3Δ.

Proof. Consider any node v_i. Notice that, only the neighbors of node v_i can add edges $v_{i-1}v_i$ and v_iv_{i+1} incident on v_i. Node v_i has at most Δ neighbors in G. Thus, there are at most 2Δ newly added edges to node v_i. Considering the previous incident edges (at most Δ), the total number of edges incident on v_i is at most 3Δ. □

Lemma 13: If structure G has low weight, then graph $F(G)$ has low weight.

Proof. We show that $\omega(F(G)) \leq 3\omega(G)$. Consider any node u and the added edges v_iv_{i+1}. Clearly, $\|v_iv_{i+1}\| \leq \|uv_i\| + \|uv_{i+1}\|$. Thus, $\sum_{i=1}^{d} \|v_iv_{i+1}\| \leq 2\sum_{i=1}^{d} \|uv_i\|$. Clearly, $\omega(F(G))$ is at most $\omega(G)$ plus the summation of all newly added edges v_iv_{i+1}, which is at most $2\omega(G)$. Thus, $\omega(F(G)) \leq 3\omega(G)$. □

Lemma 14: Structure $F(G)$ is fault-tolerant.

Proof. Consider any path that uses node u and assume that node u breaks down. Assume that v_iu and uv_j are the two links in that path. Then we can use the path $v_iv_{i+1}\cdots v_j$ to connect v_i

and v_j. Thus, there is still another path connecting the source and the target without node u. This finishes the proof that $F(G)$ is fault-tolerant. ◻

It is not difficult to show that the total communications of transforming a structure into a fault-tolerant one uses messages at most $2m$, where m is the number of edges in the original structure G. Since the structures discussed in this paper all have at most $3n$ edges, the total communication cost of this transforming is at most $6n$. The price of this transforming is that the new structure $F(G)$ is not guaranteed to be a planar graph even if the original graph G is planar.

Lemma 15: Structures F(LMST$_k$) and F(IMRG) have bounded node degree at most 18, have total edge length at most $O(\omega(EMST))$, are connected, fault-tolerant, and can be constructed using $O(n)$ messages under local broadcast communication model. Structure F(IMRG) can be constructed using at most $19n$ messages. Each of the messages has at most $2\log n$ bits.

Notice that, here we implicitly assumed that the maximum transmission power of each node can support the additional links added by Algorithm 4. In addition, instead of connecting the neighbors of a node u in a clockwise order, we can connect the neighbors of each node u using the minimum spanning tree of these nodes, which will further decrease the total edge length of the final structure.

D. Impossibility Results

Power assignment and topology control have been well studied recently by various researchers. Although most questions can be solved exactly or approximated within a constant factor using a centralized approach, it is still unknown whether we can solve or approximate some questions using localized approaches. For example, using centralized methods, we can minimize the maximum transmission power while the resulting network topology has some properties that can be tested in polynomial time. Such property includes the network is connected, or the network is k-connected, or the network topology is a spanner of the original communication graph UDG. In addition, using centralized methods, we can approximate the minimum total transmission power of all nodes within a constant factor, while the resulting network is connected, or k-connected, or consumes the minimum energy for broadcasting. However, centralized methods are expensive to implement in wireless ad hoc networks due to their possible massive communications. Thus, it is natural to ask what kind of questions we can approximate within a constant factor using localized approaches, and what kind of questions we cannot.

We have shown that we can construct a bounded degree planar spanner, or a bounded degree planar low-weighted structure, or a bounded degree k-fault tolerant spanner, or a bounded degree fault tolerant low-weighted structure, in a localized manner using only $O(n)$ messages. In the following, we will show that several questions in wireless ad hoc networks cannot be approximated within a constant factor in a localized manner at all.

The first such example is the min-max assignment problem. It was proved in [3] that the longest edge of the Euclidean minimum spanning tree EMST(V) is always the optimum solution to the min-max assignment problem. Since it is communication expensive to construct MST in a distributed manner, we would like to know whether we can construct a structure in a localized manner such that the longest edge of this structure is within a constant factor of that of MST. We show by example that there is *no* such *deterministic* localized algorithm unfortunately. Assume that there is such a deterministic localized algorithm $\mathcal{A}$ that uses k-hop information. Figure 3 illustrates an example that algorithm $\mathcal{A}$ cannot approximate the longest edge of the MST within a constant factor. In the example, $||ux|| > k$ and $\|uv\| = 1$. Then algorithm $\mathcal{A}$ will have the

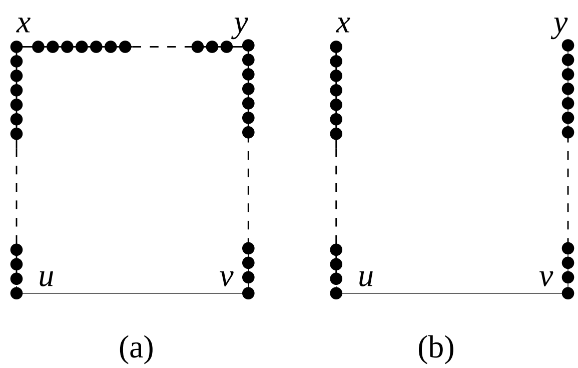


Fig. 3. No localized algorithm approximates the minimum of the maximum node power while the resulting structure is connected.

same information at the node u for both configurations illustrated in Figure 3 (a) and (b). If $\mathcal{A}$ decides to keep edge uv, then the longest edge kept by $\mathcal{A}$ could be arbitrarily larger than that of MST for configuration (a). If $\mathcal{A}$ decides not to keep edge uv, then the structure constructed by $\mathcal{A}$ is not connected for configuration (b).

Figure 3 also shows that there is *no* deterministic localized algorithm that can find a structure that approximates the total energy consumption of broadcasting within a constant factor[1] of the

[1]We actually can show that no deterministic localized algorithm can find a structure such that the energy consumed by broad-

optimum, or that approximates the total node power within a constant factor of the optimum while the network topology is connected. Similarly, we can show that there is *no* deterministic localized algorithm that can find a structure minimizing the total node power while the structure is node fault-tolerant.

V. Applications of Our Structures in Broadcasting and Topology Control

After we proved some properties of our structures, we then study how our structures can be used to improve the performances of broadcasting and topology control compared with some previously developed structures.

A. Worst Case Performances

We first assume that the energy needed to support the communication between a link uv is $\|uv\|^{\beta}$. Li proved in [14] that, if H is a low-weighted structure, then $\omega_{\beta}(H) \leq O(n^{\beta-1}) \cdot \omega_{\beta}(MST)$. Here $\omega_{\beta}(G) = \sum_{uv \in G} \|uv\|^{\beta}$. It is easy to show that the total power consumption of broadcasting based on any connected structure H is at most $2\omega_{\beta}(H)$. Let $T \subseteq H$ be the tree used for broadcasting. The power consumption of each node u is at most $||uv||^{\beta}$, where uv is the longest link incident on u in T. The claim follows from that any such link uv will be used at most twice to define the power for a node. It is also known the minimum power consumption of broadcasting is at least $\omega_{\beta}(MST)/12$. Consequently, we have the following theorem.

Theorem 16: If H is a low-weighted structure, then the power consumption of broadcasting based on H is at most $O(n^{\beta-1})$ times of the optimum.

We then show that there is a configuration of nodes such that the broadcastings based on the low-weighted structures $LMST_k$ and IMRG do consume power $\Theta(n^{\beta-1})$ times of the optimum. Consider the example illustrated by Figure 3 (a). Clearly, our structures will keep the link uv. Thus, the total power consumptions based on our structures are $O(1)$, while the optimum structure (without link uv) has power consumption only $1/n^{\beta-1}$. Notice that, this example shows that the broadcasting based on *any* locally constructed structure has power consumption at least $\Theta(n^{\beta-1})$ times of the optimum in the worst case.

casting based on this structure is within $o(n^{\beta-1})$ of the optimum. Here assume that the power needed to support a link uv is $\|uv\|^{\beta}$.

It has been shown in [14] that the broadcastings based on RNG could consume power $\Theta(n^\beta)$ times of the optimum. The same example can also show that the broadcastings based on one hop local minimum spanning tree G_0 [8] could consume power $\Theta(n^\beta)$ times of the optimum. Thus, our low-weighted structures improve the performances for broadcasting of previously proposed structures by $\Theta(n)$ factor in the worst case.

We then consider the scenario when the receiver node does consume a power to receive the signal, and we assume that this power is no more than the power consumed by the sender always. Notice that in all our structures, there are at most 6 receivers. Thus, the total power consumed by both senders and receivers in this new energy model is no more than 7 times of the total power consumption of all senders in the previous energy model. We also show that the broadcasting based on MST is still a good approximation. Let $E_0(G)$ be the energy consumption of the broadcasting based on a structure G when assume that the power needed to support the communication between a link uv is $\|uv\|^\beta$ and the receiver does not consume power. Let OPT_0 be the optimum structure for broadcasting in this model. Let $E_1(G)$ be the energy consumption of the broadcasting based on a structure G when the power consumed by each receiver is considered and this power is assumed to be no more than the power used by the sender. Let OPT_1 be the optimum structure for broadcasting in this model. Wan *et al.* [11] essentially proved that $E_0(MST) \leq 12E_0(OPT_0)$. We argue that $E_1(MST) \leq cE_1(OPT_1)$ for some constant c as follows. Since MST has a node degree bounded by 6, $E_1(MST) \leq 7E_0(MST)$. Notice that $E_1(OPT_1) \geq E_0(OPT_1) \geq E_0(OPT_0)$, which implies our statement. Consequently, our structures consume powers no more than $O(n^{\beta-1})$ times of the optimum.

We summarize the worst case performances of out structures LMST_k and IMRG.

Theorem 17: The power consumption of broadcasting, and the total node power needed to achieve network connectivity, based on the structure LMST_k or IMRG is at most $O(n^{\beta-1})$ times of the optimum. Our structures are asymptotically the optimum among all locally constructed structures.

B. Performances for Random Wireless Ad Hoc Networks

We then conduct extensive simulations to study the performances of our structures in terms of the maximum transmission power used by all nodes, the total transmission power used by all nodes, and the total length of links. Although network throughput is an important performance metric, it is influenced by many other factors such as the MAC protocol, routing protocol and so on. Therefore, most related works do not test the throughput performance. To study various aspects of our structures, we will use the following metrics to compare the performances:

1. **Total Messages**: In wireless networks, less messages to construct the topology will save energy consumption. We showed that the total messages of constructing IMRG is at most $13n$.

2. **Max Messages**: We also test what is the maximum number of messages a node will send in building the structure. A large number of messages sent by a node will delay the topology updating and drain out its battery power quickly.

3. **Average Node Degree**: A smaller average node degree often implies less contention and interference for signal and thus a better frequency spatial reuse, which in turn will improve the throughput of the network.

4. **Max Node Degree**: We also test the maximum node degree. A larger node degree will cause more contention and interference for signal, and also may drain out its battery power quickly.

5. **Max Node Power**: Each node u will set its transmission range equal to the length of the longest edge incident on u. A smaller node power will always save the power consumption. The max-node-power captures the maximum power used by all nodes. Here, in all our simulations, we set the constant $\beta = 2$, so that the power needed to support a link uv is $\|uv\|^2$.

6. **Total Node Power**: The total node power approximates the total power used by all nodes to keep the connectivity of the network.

7. **Total Node Power for Broadcasting**: This measures the total node power of all nodes that have a degree at least 2, i.e., internal nodes. This approximates the total power used by a broadcasting based on this structure. Notice that the nodes with degree 1 (except the possible source node) do not relay the message in a broadcasting.

8. **Total Edge Length**: We proved that all our structures have a total edge length within a constant factor of that of MST. We want to see the actual approximation performances.

9. **Total Link Power**: It was proved in [11] that the minimum total power needed for broadcasting is within a constant factor of the total link power in MST. We thus compare the total link power used by our structures with previously known structures and especially that of MST.

In the simulations, we will only test the performances of structures LRNG, LMST_2^- and IMRG^-, and compare them with previously known structures LMST_1 (called G_0 in [8]), and

RNG in terms of the above metrics. The reason for only selecting G_0^- and RNG is that in [8], their simulations already showed that G_0^- out-performs other previously known structures in terms of the node degree, max node power, and the total node power. Hereafter, we use the term LMST, LMST$_2$ and IMRG instead of G_0^-, LMST$_2^-$ and IMRG$^-$ in the experiments, if it is clear.

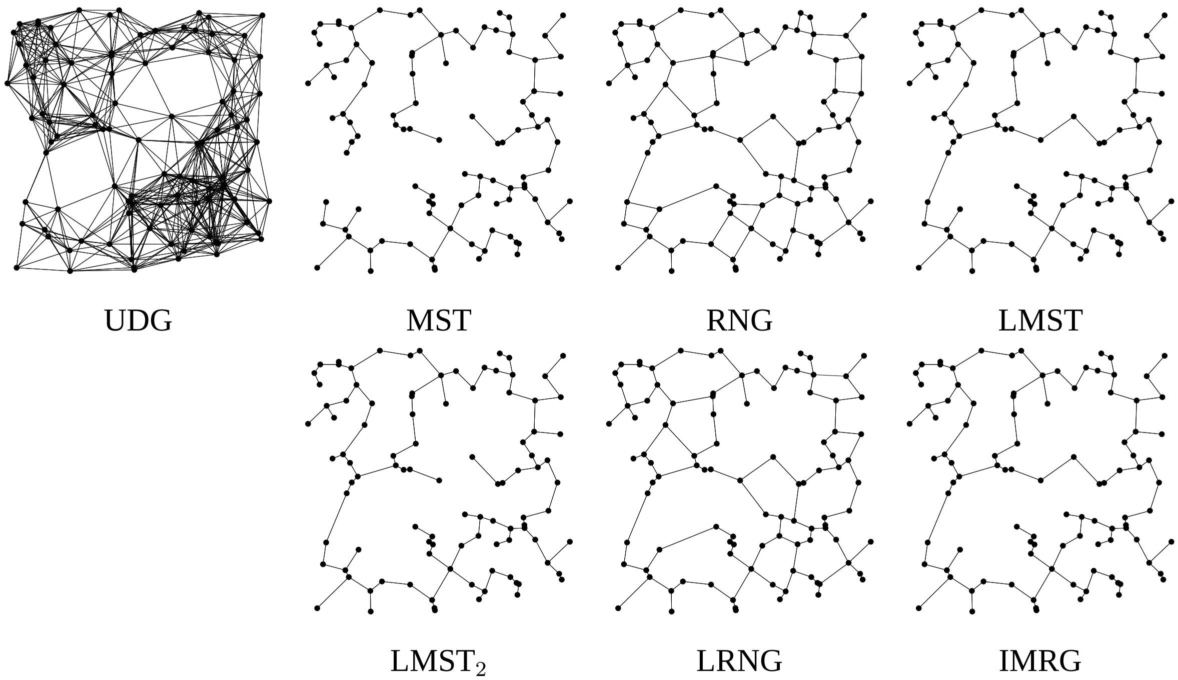


Fig. 4. Different structures from a UDG.

In the first simulation, we randomly generate 100 nodes uniformly in a $1000m \times 1000m$ region. The maximum transmission range of each node is set as $250m$. The topology derived using the maximum transmission power (UDG), MST, RNG, LMST$_1$ (or called G_0^-), LMST$_2$, LRNG, and IMRG (actually IMRG$^-$) are shown in Figure 4 respectively. To make the performance testing precise, we generate 100 sets of nodes, each of which has 100 nodes, and compute the performance metrics accordingly. The average degree of UDG is 15.37 and the maximum degree is 26. The corresponding performances are illustrated in the following Table V-B. Here for max node degree, max message and max node power, we show both the maximum and average values over the 100 sets. We found that structure LMST$_2$ outperforms all other structures in all metrics significantly (except the number of messages used). In addition, structure IMRG performs better than LMST with slightly high communication cost to construct it. For example, structure LMST uses about 5% percent more total node power than the structure IMRG for broadcasting, while

RNG consumes about 50% percent more total node power for broadcasting than the structure IMRG. We did not count the messages used to find the two hops neighbors for all nodes when computing the total messages used to construct LMST_2 (such messages number is marked by a star in our results).

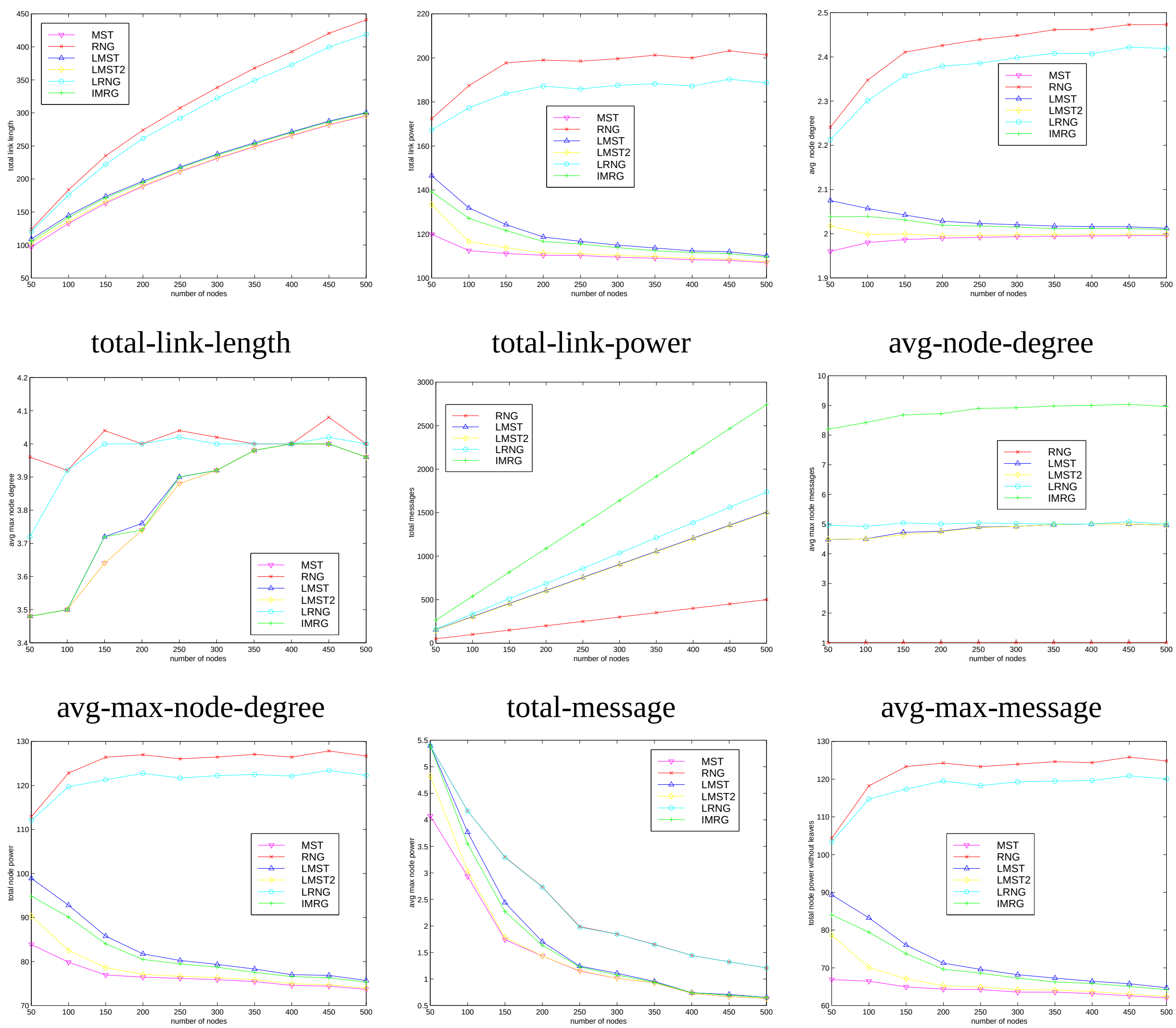
TABLE I

THE PERFORMANCES COMPARISON OF SEVERAL STRUCTURES.

	MST	RNG	LMST	LMST_2	LRNG	IMRG
MaxMaxMsg	-	1.00	5.00	5.00*	5.00	9.00
AvgMaxMsg	-	1.00	4.50	4.50*	4.92	8.42
TotMsg	-	100.00	305.72	299.88*	334.76	538.68
MaxMaxDeg	4.00	4.00	4.00	4.00	4.00	4.00
AvgMaxDeg	3.50	3.92	3.50	3.50	3.92	3.50
AvgDeg	1.98	2.35	2.06	2.00	2.30	2.04
MaxMaxNPow	4.13	5.40	4.69	4.13	5.40	4.69
AvgMaxNPow	2.93	4.17	3.77	3.03	4.17	3.55
TotNPow	79.85	122.80	92.79	82.56	119.69	90.10
TotNPowBrdcst	66.48	118.21	83.26	70.08	114.74	79.43
TotLength	132.79	183.59	144.86	135.55	175.52	141.99
TotLPow	112.47	187.37	131.85	116.56	177.29	127.13

We then vary the number of nodes in the region from 50 to 500. The transmission range of each node is still set as $250m$. We plotted the performances of all structures in Figure 5. Finally, we fix the number of nodes in the region as 500 and grow the transmission range of each node from $100m$ to $300m$. We plotted the performances of all structures in Figure 6.

All the results show that IMRG has better performances than LMST and RNG: IMRG has the least total link length and least total node power for broadcasting; it has the least node power to keep the connectivity. The number of messages used for constructing IMRG is slightly more than the number of messages used to construct LMST. The simulation results confirm all our theoretical analysis. Remember that, in the worst case, IMRG may spend $O(n^{\beta-1})$ times the total power used by the optimum broadcasting. However, our simulations show that the energy consumption of broadcasting based on IMRG is within a small constant factor (about 15% more) of that based on the MST and is much better than that based on RNG. In summary, IMRG is the best among all these known local structures; additionally, it can approximate MST theoretically and be used for energy efficient broadcasting.



total-link-length | total-link-power | avg-node-degree

avg-max-node-degree | total-message | avg-max-message

total-node-power | avg-max-node-power | total-node-power-brdcst

Fig. 5. Results when the number of nodes in the networks are different (from 50 to 500). Here the transmission range is set as $250m$.

VI. Conclusion

We defined a sequence of low-weighted sparse structures LMST$_k$, and presented an efficient method to construct them locally using only $O(n)$ messages. Here a structure is called low-

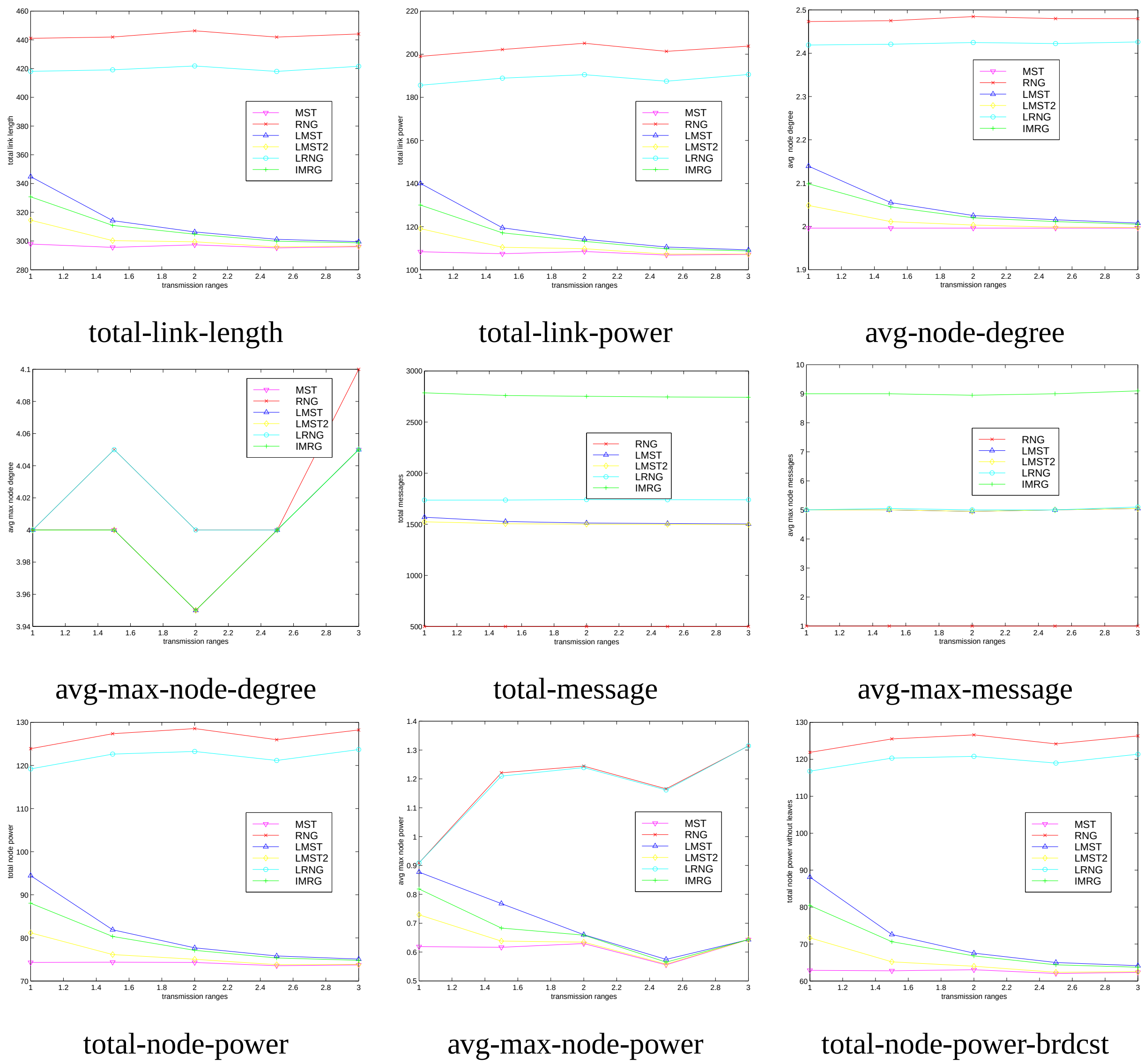


Fig. 6. Results when the transmission range are different (from $100m$ to $300m$). Here the number of nodes is 500.

weighted if its total link length is within a constant factor of that of the Euclidean minimum spanning tree. We further defined a bounded degree planar low weighted connected structure IMRG that can be constructed more efficiently. The total communication cost of our localized method is at most $13n$. We showed that both structures are asymptotically the best structures that can be constructed locally for broadcasting. We conducted extensive simulations to study the performances of our structures and compared them with previously known localized structures. Our structures out-perform all previously known structures and structure IMRG only incurs a small message overhead.

The constructed structures are planar, bounded degree, and low-weighted. Li *et al.* [35] recently gave an $O(n \log n)$-time centralized algorithm to construct a bounded degree, planar, and low-weighted *spanner*. However, it is still unknown how to make that a distributed algorithm using $O(n)$ communications without sacrificing the spanner property. On the other hand, Li *et al.* [7] showed how to construct a planar spanner with bounded degree in a localized manner (using $O(n)$ messages) for unit disk graph. However, the constructed structure does not seem to be low-weighted. It remains open how to construct a bounded degree, planar, and *low-weighted spanner* in a distributed manner using only $O(n)$ communications under the local broadcasting communication model.

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