Localized Construction of Low Weighted Structure and Its Applications in Wireless Ad Hoc Networks

Xiang-Yang Li

***Abstract*— We consider a wireless network composed of a set of n wireless nodes distributed in a two dimensional plane. The signal sent by every node can be received by all nodes within its transmission range, which is uniform and normalized to one unit. We present the first distributed method to construct a bounded degree planar connected structure LRNG, whose total link length is within a constant factor of the minimum spanning tree [1] using total $O(n)$ messages under the broadcast communication model. Moreover, in our method, every node only uses its two-hop information to construct such structure, i.e., it is localized method. We show that some two-hop information is necessary to construct any low-weighted structure. We also study the application of this structure in efficient broadcasting in wireless ad hoc networks. We prove that, for broadcasting, the relative neighborhood graph (RNG), which is the previously best-known sparse structure that can be constructed locally, could use energy $O(n)$ times the total energy used by our structure LRNG. Our simulations show that the broadcasting based on LRNG consumes energy about 36% more than that by MST, and broadcasting based on RNG consumes energy about 64% more than that by MST. We also show that no localized method can construct a structure for broadcasting with total power consumption asymptotically better than LRNG.**

***Index Terms*— Broadcasting, energy conservation, low weight, minimum spanning tree, ad hoc networks.**

I. Introduction

Recent years saw a great amount of research in wireless networks, especially ad hoc wireless networks due to its potential applications in various situations such as battlefield, emergency relief, and so on. We assume that each wireless node has an omni-directional antenna and a single transmission of a node can be received by *any* node within its vicinity which, we assume, is a disk centered at this node. A wireless node can receive the signal from another node if it is within the transmission range of the sender. Otherwise, they communicate through multi-hop wireless links by using intermediate nodes to relay the message. Consequently, each node in the wireless network also acts as a router, forwarding data packets for other nodes.

We consider a wireless ad hoc network (or sensor network) with all nodes distributed in a two-dimensional plane. Assume that all wireless nodes have distinctive identities, and the identity of a node u is denoted by $ID(u)$. We also assume that each static wireless node knows its position information either through a low-power Global Position System (GPS) receiver or through some other way. More specifically, it is enough for our protocol when each node knows the relative position of its one-hop neighbors. The relative position of neighbors can be estimated by the *direction of arrival* and the *strength of signal*. For simplicity, we also assume that all wireless nodes have the same maximum transmission range and we normalize it to one unit. By one-hop broadcasting, each node u can gather the location information of all nodes within the transmission range of u. Consequently, all wireless nodes V together define a unit-disk graph (UDG), which has an link uv iff the Euclidean distance $\|uv\|$ is less than one unit.

Energy conservation is a critical issue in *ad hoc* wireless network for the node and network life, as the nodes are powered by batteries only. In the most common power-attenuation model, the power needed to support a link uv is $\|uv\|^\beta$, where $\|uv\|$ is the Euclidean distance between u and v, β is a real constant between 2 and 5 dependent on the wireless transmission environment. This power consumption is typically called *path loss*. We assume that the path loss is the major part of power consumption to transmit signals. Notice that, practically, there is some other overhead cost for each device to receive and then to process the signal. For simplicity, this overhead cost can be integrated into one cost, denoted by c, which is almost the same for all nodes. However, we will assume that $c = 0$ for the rest of this paper.

Wireless ad hoc network needs some special treatment as it intrinsically has its own special characteristics and some unavoidable limitations compared with wired networks. For example, wireless nodes are often powered by batteries only and they often have limited memories. A transmission by a wireless device is often received by many nodes within its vicinity. This causes the signal interference if there are at least two nodes wanting to send a signal to a node. On the other hand, we can also utilize this broadcasting property to save the communications needed to send some information. Throughout this paper, a *local broadcast* by a node means it sends the message to all nodes within its transmission range; a *global broadcast* by a node means it tries to send the message to all nodes in the network by the possible relaying of other nodes. Since the main communication cost in wireless networks is to send out the signal while the receiving cost of a message is neglected here, a protocol's message complexity is only measured by how many messages are sent out by all nodes.

Wireless ad hoc networks require efficient distributed algorithms and especially efficient localized algorithms for fast updating due to node's mobility. Here a distributed algorithm is called a *localized algorithm* if every node only uses the information of nodes within a constant number of hops (plus some additional information, if necessary, provided that it can be represented in a constant number of bits). A structure can be ef-

Department of Computer Science, Illinois Institute of Technology, 10 W. 31st Street, Chicago, IL 60616, USA. Email: xli@cs.iit.edu,

[1] The structure whose total link length is within a constant factor of the minimum spanning tree is called low-weighted hereafter.

ficiently updated if it is constructed by a localized algorithm since when a node moves, it only affects the structure within a constant number of hops.

In recent years, there has been a substantial amount of research on topology control for wireless ad hoc networks [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. Topology control is to select a subset of links among all possible wireless links for communication. These algorithms are designed for different objectives. Some of the algorithms [1], [14], [4], [9] try to minimize the maximum link length while maintaining the network connectivity. Some algorithms [3], [6], [11] bound the number of neighboring nodes each node has to connect to. The method proposed in [6] also guarantees that the constructed structure is a length spanner. Here a structure H is a spanner if, for any two nodes, the length of the shortest-path connecting them in H is no more than a constant factor of the length of the shortest-path connecting them in the original communication graph. A spanner structure can guarantee that the total power consumption needed by the least cost path to connect any two nodes is within a small constant factor of the optimum if all possible links are kept. In [5], we proposed the first algorithm that can construct a planar spanner in a localized manner. Planar structures are used by several localized routing algorithms [15], [16]. Recently, we [17] proposed the first algorithm that can construct a bounded degree planar spanner in a localized manner. However, no localized algorithm is known on how to construct a structure whose total link length is within a constant factor of that of the minimum spanning tree. We call such structure as *low weight* structure.

It was recently shown [18] that a broadcasting based on the minimum spanning tree consumes energy within a constant factor of the optimum. The best distributed algorithm [19], [20], [21] can compute the minimum spanning tree in $O(n)$ rounds using $O(m+n\log n)$ communications for a general graph with m edges and n nodes. The relative neighborhood graph, the Gabriel graph and the Yao graph all have $O(n)$ edges and contain the Euclidean minimum spanning tree for wireless ad hoc networks modeled by UDG. This implies that we can construct the minimum spanning tree in a distributed manner using $O(n\log n)$ messages. Unfortunately, even for wireless network modeled by a ring, the $O(n\log n)$ number of messages is still necessary for constructing the minimum spanning tree. This is expensive especially when we have to update the MST frequently due to the frequent nodes' movement. Thus, it is nature to ask whether we can approximate the MST efficiently instead of constructing it exactly. Relative neighborhood graph (RNG) has been used for broadcasting in wireless ad hoc networks [22], [23]. The ratio of the weight (total link length here) of RNG over the weight of MST could be $O(n)$ for n points set [24]. The same example also shows that, the total power used for broadcasting based on RNG is as large as $O(n^2)$ of that based on MST.

We present the first localized method to construct a bounded degree planar connected structure, namely LRNG, whose total edge length is within a constant factor of that of the minimum spanning tree. The total communication cost of our method is $O(n)$ under a local broadcast communication model. In addition, every node only uses its two-hop information to construct such structure. We also show that some two-hop information is necessary to construct any low-weighted structure. We also show the application of this structure in efficient broadcasting in wireless ad hoc networks. Notice that a structure with low-weight cannot guarantee that the broadcasting based on structure LRNG consumes energy within a constant factor of the optimum. We show that the energy consumption using the structure LRNG is within $O(n^{\beta-1})$ of the optimum. Given a geometry graph G, let $\omega_b(G) = \sum_{uv\in G}\|uv\|^b$. Equivalently, we prove that $\omega_\beta(LRNG) = O(n^{\beta-1})\cdot\omega_\beta(MST)$ for any $\beta \geq 1$. This improves the previously known "lightest" structure RNG by $O(n)$ factor since in the worst case $\omega_\beta(RNG) = \Theta(n^\beta)\cdot\omega_\beta(MST)$ for any $\beta \geq 1$. Notice that, the optimum broadcasting structure consumes total node power at least $O(\omega_\beta(MST))$. At last, we show that there is no localized algorithm that can construct a structure for broadcasting whose total energy consumption is $O(\omega_\beta(MST))$.

The remainder of the paper is organized as follows. We give some geometry notations and present our efficient localized method constructing a bounded degree planar structure with low weight in Section II. The proof of the correctness of the algorithm is also given. In Section III, we discuss its applications in broadcasting, and find that it saves considerable energy consumption compared with that based on RNG. In Section IV, we conducted extensive simulations to compare the performances of the structure LRNG with previously best known structures. We conclude our paper in Section V with the discussion of possible future works.

II. Low Weight Topology

Before we present our structure LRNG, we first give some notations and review the definitions of some known structures. Let $\|xy\|$ denote the Euclidean distance between two points x and y. A disk centered at a point x with radius r, denoted by $disk(x,r)$, is the set of points whose distance to x is at most r, i.e., $disk(x,r) = \{y \mid \|xy\| \leq r\}$. Let $lune(u,v)$ defined by two points u and v be the intersection of two disks with radius $\|uv\|$ and centered at u and v respectively, i.e., $lune(u,v) = disk(u,\|uv\|)\cap disk(v,\|uv\|)$. The ***relative neighborhood graph*** [25], denoted by RNG(V), consists of all edges uv such that the *interior* of $lune(u,v)$ contains no point $w\in V$. Notice here if only the boundary of $lune(u,v)$ contains a point from V, edge uv is still included in RNG. Given a weighted geometry graph G over a set of points, let $\omega(G)$ be the total weight of the edges in G. More specifically, if the weight of an edge uv is defined as $\|uv\|^b$, then let $\omega_b(G)$ be the total weight of the weighted edges in G, i.e., $\omega_b(G) = \sum_{uv\in G}\|uv\|^b$. When $b=1$, b is often omitted from the notation. A minimum spanning tree of a set of points V is a connected graph whose weight is the minimum among all connected graphs spanning V. It is known that, given a UDG or a point set, the relative neighborhood graph always contains the minimum spanning tree as a subgraph.

A. Modified RNG

Our low-weight structure is based on a modified relative neighborhood graph. Notice that, traditionally, the relative neighborhood graph will always select an edge uv even if there is some node on the boundary of $lune(u, v)$. Thus, RNG may have unbounded node degree, e.g., considering $n-1$ points equally distributed on the circle centered at the nth point v, the degree of v is $n-1$. Notice that for the sake of lowering the weight of a structure, the structure should contain as less edges as possible without breaking the connectivity. We then naturally extend the traditional definition of RNG as follows.

The modified *relative neighborhood graph* consists of all edges uv such that (1) the *interior* of $lune(u, v)$ contains no point $w \in V$ and, (2) there is no point $w \in V$ with $ID(w) < ID(v)$ on the boundary of $lune(u, v)$ and $\|wv\| < \|uv\|$, and (3) there is no point $w \in V$ with $ID(w) < ID(u)$ on the boundary of $lune(u, v)$ and $\|wu\| < \|uv\|$, and (4) there is no point $w \in V$ on the boundary of $lune(u, v)$ with $ID(w) < ID(u)$, $ID(w) < ID(v)$, and $\|wu\| = \|uv\|$. See Figure 1 for an illustration when an edge uv is not included in the modified relative neighborhood graph. We denote such structure by RNG' hereafter. Obviously, RNG' is a subgraph of traditional RNG. We prove that RNG' has a maximum node degree 6 and still contains a minimum spanning tree as a subgraph.

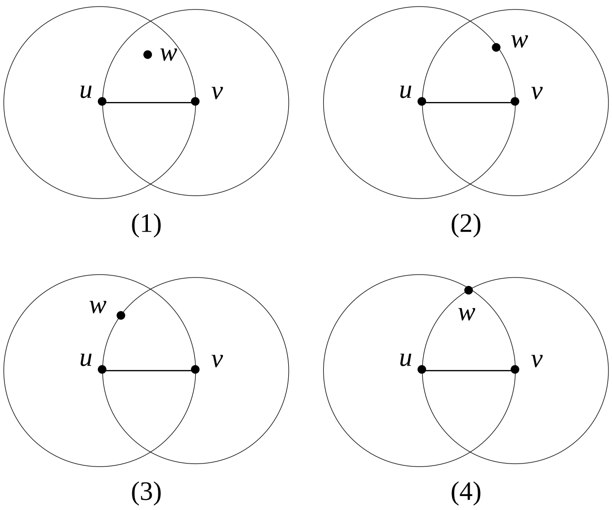

Fig. 1. Which edges are not in the modified RNG.

Lemma 1: The maximum node degree in modified relative neighborhood graph RNG' is at most 6.

PROOF. Consider any node u. For the sake of contradiction, assume that u has degree larger than 6, i.e., it has at least 7 neighbors in RNG'. By the pigeonhole principle, obviously, two of its neighbors in RNG', say v_1 and v_2, form an angle $\angle v_1uv_2$ less than $\pi/3$. Assume that $\|uv_1\| \leq \|uv_2\|$ and if $\|uv_1\| = \|uv_2\|$ assume that $ID(v_1) < ID(v_2)$. Obviously, node v_1 is then inside the interior of $lune(u, v_2)$ or if it is on the boundary then $\|v_1v_2\| < \|uv_2\|$ and $ID(v_1) < ID(v_2)$. In both cases, it is a contradiction to the existence of edge uv_2 in RNG'. This finishes the proof. ◻

Similar to the above proof, it is not difficult to show that the maximum node degree in graph RNG' is at most 5 actually.

Lemma 2: The graph RNG' contains an Euclidean minimum spanning tree as a subgraph.

PROOF. One way to construct MST is to add edges in the order of their lengths to the MST if it does not create a cycle with previously added edges. If there are two edges with the same length, we break the tie by comparing the larger ID of the two end-points then comparing smaller ID of the two-end points. In other words, we label an edge uv by $(\|uv\|, \max(ID(u), ID(v)), \min(ID(u), ID(v)))$, and an edge uv is ordered before an edge xy if the lexicographic order of the label of uv is less than that of xy. Let T be the minimum spanning tree constructed using the above edge ordering. We will show that $T \subseteq RNG'$.

For the sake of contradiction, assume that there is an edge in T that is not in RNG'. Consider such edge uv with the smallest ordering. By definition of RNG', there are only four cases for $uv \notin RNG'$.

The first case is that there is a $w \in V$ in the interior of $lune(u, v)$. Obviously, edges uw and wv are shorter than uv. No matter whether the edge uw is in T, we know that there is a path in T (could be edge uw) connecting u and w using edges with length at most uw. Similarly, the same property holds for points w and v. Thus, when we add edge uv to T, it will create a cycle with edges already in T since the edges in path from u to w to v definitely are shorter than uv, i.e., added before edge uv. This is a contradiction to the existence of uv in T.

The second case is that there is a point $w \in V$ on the boundary of $lune(u, v)$ with $ID(w) < ID(v)$ and $\|wv\| < \|uv\|$. We only need to show that edge uw and edge wv are ordered before edge uv; the remaining proof is similar to the first case. Since $ID(w) < ID(v)$, it is easy to show that the label of uw is lexicographically less than that of edge uv. Edge wv is ordered before uv since $\|wv\| < \|uv\|$.

The similar proof carries over to the third and the fourth cases. This finishes the proof. ◻

Obviously, graph RNG' still can be constructed using n messages. Each node first locally broadcasts its location and ID to its one-hop neighbors. Then every node decides which edge to keep solely based on the one-hop neighbors' location information collected. Since the definition is still symmetric, the edges constructed by different nodes are consistent, i.e., an edge uv is kept by a node u iff it is also kept by node v. The computational cost of a node u is still $O(d \log d)$ by using Delaunay triangulation, where d is its degree in UDG. A simple edge by edge testing method has time complexity $O(d^2)$.

Although graph RNG' has possibly less edges than RNG, its total edge weight could still be arbitrarily large compared with the MST. Figure 2 (a) illustrates an example where $\omega(RNG')/\omega(MST) = O(n)$ for a set of n points. Here $n/2$ points are equally distributed with separation $\epsilon \leq 2/n$ on two parallel vertical segments with distance 1 respectively. Obviously, all edges form RNG' have total weight $n/2 + (n/2-1)\epsilon$ and the MST has total weight $1 + (n/2-1)\epsilon$. On the other hand, the following lemma bound the weight and spanning ratio from above.

Lemma 3: For any sparse graph G with $O(n)$ edges, containing MST as subgraph, $\omega_b(G) = O(n^b) \cdot \omega_b(MST)$ for $b \geq 1$, and it has length spanning ratio at most $O(n)$.

PROOF. For any edge $uv \in G$, if $uv \in MST$, then $\|uv\|^b <$

$\omega_b(MST)$. If $uv \notin MST$, then there is a path in MST with edges not longer than uv connecting u and v. Let a_j, $1 \le j \le k \le n$ be the length of these edges. Then $\|uv\| < \sum_{1 \le j \le k} a_j$. Thus,

$$\|uv\|^b < (\sum_{1\le j\le k} a_j)^b \le n^{b-1} \sum_{1\le j\le k} a_j^b \le n^{b-1} \cdot \omega_b(MST).$$

Consequently, $\omega_b(G) = O(n^b) \cdot \omega_b(MST)$ since G has only $O(n)$ edges. Similar proof can show that G has length spanning ratio at most $O(n)$. □

B. Low Weight Topology

So far RNG' is the previously best known connected structures that can be constructed locally and has a small total edge weight. As shown by Figure 2 (a), its total weight could still be as large as $O(n)$ times of $\omega(MST)$. In this section, we give a communication efficient method to construct a sparse topology from RNG' whose total edge weight is within a constant factor of $\omega(MST)$. Previously no method is known to construct a structure with weight $O(\omega(MST))$ in a localized manner.

We first show by example that it is *impossible* to construct a low-weighted structure using only one hop neighbor information. Assume that there is such algorithm. Consider a set of points illustrated by Figure 2 (a). Let's see what this hypothetical algorithm will do to this point set. Since it uses only one-hop information, at every node, the algorithm only knows that there are a sequence of nodes are evenly distributed with small separation, and another node which is one-unit away from current node. Since the algorithm has the same (or almost same) information at each node, the algorithm cannot decide whether to keep the long edge. If it keeps the long edge, then the total weight of the final structure is $O(n \cdot \omega(MST))$. If it discards the long edge, however, it may disconnect the graph since the nodes known at the algorithm at one node may be the whole network. See Figure 2 (b) for an illustration.

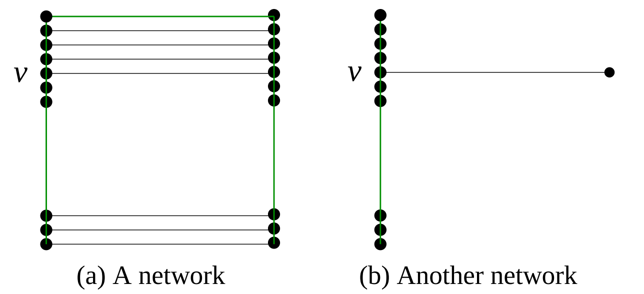

Fig. 2. The hypothetical algorithm cannot distinguish two cases here.

We now present our localized algorithm that constructs a low-weighted structure using only some two hops information.

Algorithm 1: Construct Low Weight Structure

1) All nodes together construct the modified relative neighborhood graph RNG' in a localized manner.
2) Each node u locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node u listens to the messages from its one-hop neighbors.
3) If node u received a message informing existence of edge xy from its neighbor x, for each edge uv in RNG', if uv is the longest among xy, ux, and vy, node u removes edge uv. Ties are broken by the label of the edges. Here assume that $uvyx$ is the convex hull of u, v, x, and y.

Let *LRNG* be the final structure formed by all remaining edges in RNG', and we call it low-weighted modified relative neighborhood graph (LRNG). Obviously, if an edge uv is kept by node u, then it is also kept by node v. To study the total weight of this structure, we will show that the edges in LRNG satisfies the *isolation property* [26]. Let $c > 0$ be a constant. Let E be a set of edges in d-dimensional space, and let $e \in E$ be an edge of weight l. If it is possible to place a hyper-cylinder B of radius and height $c \cdot l$ each, such that the axis of B is a subedge of e and B does not intersect with any other edge, i.e., $B \cap (E - \{e\}) = \phi$, then edge e is said to be *isolated* [26]. If all the edges in E are isolated, then E is said to satisfy the *isolation property*. The following theorem is proved by Das *et al.* [26].

Theorem 4: [26] If a set of line segments E in d-dimensional space satisfies the isolation property, then $\omega(E) = O(1) \cdot \omega(SMT)$.

Here SMT is the Steiner minimum tree over the end points of E. Obviously, total edge weight of SMT is no more than that of the minimum spanning tree. Generally, $\omega(MST) = O(\omega(SMT))$ for a set of points in Euclidean space. It is also known [26] that, in the definition of the isolation property, we can replace the hyper-cylinder by a hypersphere, a hypercube etc., without affecting the correctness of the above theorem. We will use a disk and call it *protecting disk*. Specifically, the protecting disk of a segment uv is $disk(p, \frac{\sqrt{3}}{4}\|uv\|)$, where p is the midpoint of segment uv. Obviously, we need all such disks do not intersect any edge except the one that defines it.

We first partition the edges of LRNG into at most 7 groups such that the edges in each group satisfy the isolation property. Notice, given any node u, any cone apexed at u with angle less than $\pi/3$ will contain at most one edge of LRNG incident on u since $LRNG \subseteq RNG'$. Thus, we partition the region surrounded the origin by 7 equal-sized cones, say $\mathbb{C}_1, \mathbb{C}_2, \cdots, \mathbb{C}_7$ (the cone is half-open and half-close). The cones at different nodes are just a simple shifting of cones from the origin. Let E_i be the set of edges at cone C_i (one end-point is the apex of the cone and the other end-point is inside the cone). We then show that:

Lemma 5: No two edges in E_i share an end-point.

PROOF. Assume that two edges xu and yu share a common node u. Obviously, these two edges cannot be from the cone apexed at node u; see Figure 3 for an illustration. Clearly, angle

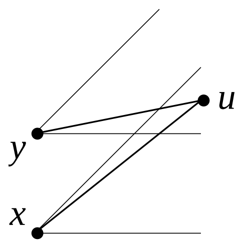

Fig. 3. Edges share a common end-point.

$\angle xuy \leq 2\pi/7$. However, we already showed that there are no two edges incident on u forming an angle less than $\pi/3$. This finishes the proof. $\square$

We then prove the main theorem of this paper.

Theorem 6: The total edge weight of LRNG is within a constant factor of that of the minimum spanning tree.

PROOF. We basically just show that the edges in E_i satisfy the isolation property, for $1 \leq i \leq 7$. For the sake of contradiction, assume that E_i does not satisfy the isolation property. Consider any edge uv from E_i and assume that it is not isolated. Thus, there is an edge, say xy, intersects the protecting disk of uv. There are four different cases: Case (a): $x \in disk(u, \|uv\|)$ and $y \in disk(v, \|uv\|)$; Case (b): $x \in disk(u, \|uv\|)$ and $y \notin disk(v, \|uv\|)$; Case (c): $x \notin disk(u, \|uv\|)$ and $y \in disk(v, \|uv\|)$; Case (d): $x \notin disk(u, \|uv\|)$ and $y \notin disk(v, \|uv\|)$. These four cases are illustrated by Figure 4. Remember that $lune(u, v)$ is empty. We will show that none of these four cases is possible.

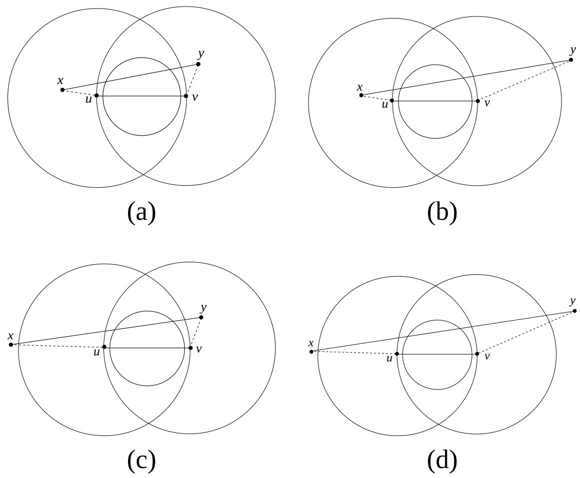

Fig. 4. Four cases that an edge uv is not isolated. Assume edge xy intersects the protecting disk.

For the first case, since x is in $disk(u, \|uv\|)$ and y is in $disk(v, \|uv\|)$, we know that xu and yv are both shorter than uv. Here, xu and yv need not be in the structure E_i. Thus, either uv or xy is the longest edge among uv, xy, xu and yv. Consequently, our algorithm will remove either uv or xy (whichever is longer).

For the remaining three cases, we will show that edge xy is the longest of these four edges. First of all, nodes x and y cannot be on the different side of the line passing through nodes u and v. Assume that they do, and x is below the line uv. We also assume that x is outside of the disk centered at u with radius $\|uv\|$ since one of the x and y is outside of the corresponding disk. See Figure 5 for an illustration. We first show that $\angle yxu < \pi/3$. Let point q be the intersection point of segment xy with line uv. Let point p be the corner point of the lune $lune(u, v)$ that is on the same side of uv as y. Obviously, $\angle yxu < \angle yqu < \angle pqu < \angle puv = \pi/3$. We then show that $\|xy\| > \|xu\|$. Let z be the intersection point of xy with the boundary of $lune(u, v)$ and closer to u than v. Obviously, $\angle xuz > \pi/2$, thus, $\|xy\| > \|xz\| > \|xu\|$. Consequently,

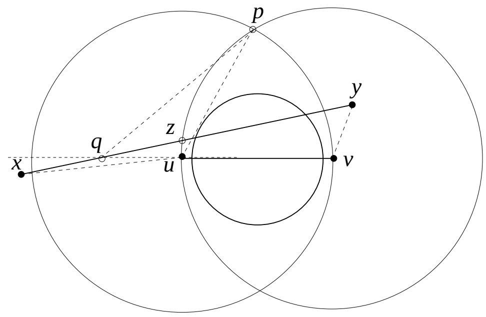

Fig. 5. Node x is below line uv and y is above.

point u is inside the lune defined by points x and y, which is a contradiction to the fact that $xy \in RNG'$.

We then prove that the Case (b) is impossible. Assume that y is outside of disk centered at v with radius $\|uv\|$. See Figure 6 for an illustration of the proof that follows. Let z be the

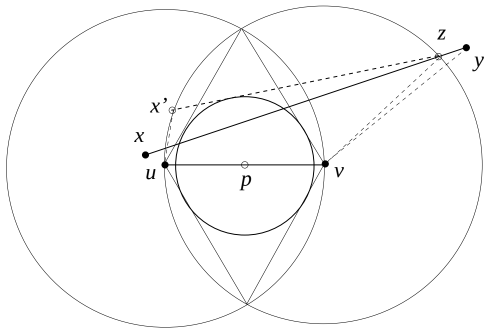

Fig. 6. Case (b) is impossible.

intersection point of xy with $disk(v, \|uv\|)$ that is closer to y. Let x' be the point on the disk $disk(v, \|uv\|)$ such that segment zx' is tangent to the protecting disk of segment uv. Obviously, $\angle ux'z > \pi/2$. Then $\|zx\| > \|zx'\|$. We can show that $\|zx'\|$ is at least $\|zv\|$ (the proof is presented in next paragraph). Then

$$\|yx\| > \|yz\| + \|zx'\| > \|yv\| - \|zv\| + \|zx'\| > \|yv\|$$

Then xy is the longest segment of the convex hull $xyvu$ since $\|xu\| \leq \|uv\| \leq \|vy\|$. This is a contradiction since our algorithm will remove edge xy. Notice here edge xy is the longest edge implies that node u is a neighbor of x and node v is a neighbor of y. Thus both node x and node y will know the existence of edge uv, and thus will remove edge xy according to our algorithm.

Figure 7 illustrates the proofs of $\|zx'\| \geq \|zv\|$ that follows. Consider any chord xy tangent on the protecting disk for uv. We will show that $\|xy\| \geq \|yv\| = \|uv\|$. Let z be the midpoint of xy, i.e., vz is perpendicular to xy. To make xy shorter, segment vz must be as long as possible. Let p be the midpoint of uv and s be the point on xy such that segment ps is perpendicular to segment xy. Then clearly, $\|vz\| = \|ps\| + \|pv\| \cdot \cos(\angle ups)$. Thus, xy is minimized when angle $\angle ups$ is minimized. However, $\angle ups > \angle upw$ since x and y are all above the line uv. Here w is the only intersecting point of chord ut with the protecting disk $disk(p, \frac{\sqrt{3}}{4} \cdot \|uv\|$. It is easy to show that $\|ut\| = lengthtv = \|uv\|$. Thus, the minimum length of segment xy is $\|uv\|$ when $\angle ups = \angle upw$.

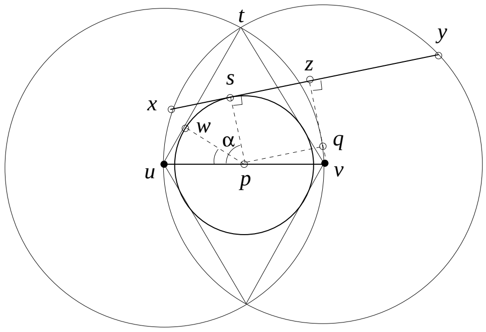

Fig. 7. Edge xy is the longest edge.

The proof of Case (c) is exactly the same as that for Case (b). For Case (d), same to the proof of Case (b), we know that $\|xu\| < \|xy\|$ and $\|vy\| < \|xy\|$. Then edge xy is also the longest edge of the convex hull $xyvu$. This is a contradiction since our algorithm will remove edge xy (nodes x and y will be informed by u and v respectively of the existence of edge uv since $\|xu\| < 1$ and $\|vy\| < 1$). This finishes the proof. ◻

Notice that, from the above proof, we generally proved the following corollary.

Corollary 7: A subgraph $G \subseteq RNG'$ is low-weighted if for any two edges $uv \in G$ and $xy \in G$, neither uv nor xy is the longest edge of the quadrilateral $uvyx$.

Based on this corollary, several new low weighted structures had been proposed recently [27], [28]. We then show that the topology LRNG does contain an Euclidean minimum spanning tree as a subgraph, thus it is still a connected graph.

Lemma 8: The constructed topology LRNG still contains a minimum spanning tree as a subgraph.

PROOF. Consider the minimum spanning tree T constructed in the proof of Lemma 2. We will prove that $T \subseteq LRNG$ by induction on the order of the edges added to the minimum spanning tree T. For the edge with the smallest order, it is clearly still in LRNG. Assume that the first $k-1$ edges added to T are still in LRNG. Consider the kth edge, say uv, added to T. If uv is not in LRNG, there must have two points x and y such that edge uv has the largest lexicographical label among edges on the convex hull $uvyx$.

Notice that for RNG', it is easy to show by induction that, for any two points p and q, there is a path in RNG' connecting p and q, whose edges have label less than that of pq. For any edge in this path, if it is not in T, then by definition of T, we know that there is another path with edges in T to connect the two endpoints of this edge. Thus, for any two points p and q, there is a path in T connecting p and q, whose edges have label less than that of pq.

Consequently, for points u and v, there is a path in T connecting them using edges with label lexicographically less than uv. This is a contradiction to the fact that uv is also in the minimum spanning tree T. This finishes the proof. ◻

III. APPLICATION IN BROADCASTING

Minimum-energy broadcast/multicast routing in a simple ad hoc networking environment has been addressed in [29], [30], [31], [32]. To assess the complexities one at a time, the nodes in the network are assumed to be static. Nevertheless, as argued in [32], the impact of mobility can be incorporated into this static model because the transmission power can be adjusted to accommodate the new locations of the nodes as necessary. In other words, the capability to adjust the transmission power provides considerable “elasticity” to the topological connectivity, and hence may reduce the need for hand-offs and tracking. In addition, as assumed in [32], there are sufficient bandwidth and transceiver resources. Under these assumptions, centralized (as opposed to distributed) algorithms were presented by [32] for minimum-energy broadcast/multicast routing. These centralized algorithms, in this simple networking environment, are expected to serve as the basis for further studies on distributed algorithms in a more practical network environment, with limited bandwidth and transceiver resources, as well as the node mobility.

Any broadcast routing is viewed as an arborescence (a directed tree) T, rooted at the source node of the broadcasting, that spans all nodes. Let $f_T(p)$ denote the transmission power of the node p required by T. For any leaf node p of T, $f_T(p) = 0$. For any internal node p of T,

$$f_T(p) = \max_{pq \in T} \|pq\|^{\beta},$$

in other words, the β-th power of the longest distance between p and its children in T. The total energy required by T is $\sum_{p \in V} f_T(p)$. Thus the minimum-energy broadcast routing problem is different from the conventional link-based minimum spanning tree problem. Indeed, while the MST can be solved in polynomial time by algorithms such as Prim's algorithm and Kruskal's algorithm [33], it is known [29] that the minimum-energy broadcast routing problem cannot be solved in polynomial time if $P \neq NP$.

Three greedy heuristics were proposed in [32] for the minimum-energy broadcast routing problem: MST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). The MST heuristic first applies the Prim's algorithm to obtain a MST, and then orient it as an arborescence rooted at the source node. The SPT heuristic applies the Dijkstra's algorithm to obtain a SPT rooted at the source node. The BIP heuristic is the node version of Dijkstra's algorithm for SPT. It maintains, throughout its execution, a single arborescence rooted at the source node. The arborescence starts from the source node, and new nodes are added to the arborescence one at a time on the minimum incremental cost basis until all nodes are included in the arborescence. The incremental cost of adding a new node to the arborescence is the minimum *additional* power increased by some node in the current arborescence to reach this new node. The implementation of BIP is based on the standard Dijkstra's algorithm, with one fundamental difference on the operation whenever a new node q is added. Whereas the Dijkstra's algorithm updates the node weights (representing the current knowing distances to the source node), BIP updates the cost of each link (representing the incremental power to reach the head node of the directed link). This update is performed by subtracting the cost of the added link pq from the cost of every link qr that starts from q

to a node r not in the new arborescence.

For a pure illustration purpose, another slight variation of BIP was discussed in detail in [34]. This greedy heuristic is similar to the Chvatal's algorithm [35] for the set cover problem and is a variation of BIP. Like BIP, an arborescence, which starts with the source node, is maintained throughout the execution of the algorithm. However, unlike BIP, many new nodes can be added one at a time. Similar to the Chvatal's algorithm [35], the new nodes added are chosen to have the minimal *average* incremental cost, which is defined as the ratio of the minimum additional power increased by some node in the current arborescence to reach these new nodes to the number of these new nodes. They called this heuristic as the Broadcast Average Incremental Power (BAIP). In contrast to the $1 + \log m$ approximation ratio of the Chvatal's algorithm [35], where m is the largest set size in the Set Cover Problem, they showed that the approximation ratio of BAIP is at least $\frac{4n}{\ln n} - o(1)$, where n is the number of receiving nodes.

The heuristics based broadcasting methods BIP, MST, and SPT have been evaluated through simulations in [32], but little is known about their analytical performances in terms of the approximation ratio. Wan *et al.* [34], [18] showed that the approximation ratios of MST and BIP are between 6 and 12 and between $\frac{13}{3}$ and 12 respectively; on the other hand, the approximation ratios of SPT and BAIP are at least $\frac{n}{2}$ and $\frac{4n}{\ln n} - o(1)$ respectively, where n is the number of nodes. The following lemma was proved in [18].

Lemma 9: For any point set V in the plane, the total energy required by any broadcasting among V is at least $\omega_\beta(MST)/C_{mst}$, where $6 \le C_{mst} \le 12$ is a constant related to the geometry minimum spanning tree.

Unfortunately, none of these underlying structures can be constructed in a localized manner, i.e., each node cannot determine which edge is in the defined structure by purely using the information of the nodes within some constant hops. RNG has been used for broadcasting in wireless ad hoc networks [22], [23]. Obviously, the ratio of the total link lengths in RNG over the total link lengths of MST could be $O(n)$ for a UDG of n points set. Figure 8 illustrates an example that the total energy used by broadcasting on RNG could be about $O(n^\beta)$ times of the minimum-energy used by an optimum method. Here the n nodes are evenly distributed on the arc xu_1, segment u_1u_k, arc u_ky, arc yv_k, segment v_kv_1, and arc v_1x. Here four nodes u_1, u_k, v_1, and v_k form a unit square. It is not difficult to show that $\omega(MST) = \Theta(1)$ and $\omega_\beta(MST) = \Theta(1/n^{\beta-1})$, while $\omega(RNG) = \Theta(n)$ and $\omega_\beta(RNG) = \Theta(n)$.

Together with Lemma 3, we know that in the worst case, $\omega_\beta(RNG') = \Theta(n^\beta) \cdot \omega_\beta(MST)$.

Lemma 10: In the worst case, $\omega_\beta(RNG) = \Theta(n^\beta) \cdot \omega_\beta(MST)$.

Notice that even the structure LRNG has total edge length $\omega(LRNG) \le c \cdot \omega(MST)$ for some constant c, it does not guarantee that $\omega_\beta(LRNG)$ is within a constant factor of $\omega_\beta(MST)$ for $\beta > 1$. Figure 9 illustrates such an example. Here the segment uv has length 1. The other $n - 1$ nodes are evenly distributed along the three segments of a square (with side length $1 + \epsilon$) such that the lines drawn in Figure 9 is indeed the graph RNG'. It is not difficult to show that all edges in RNG' are

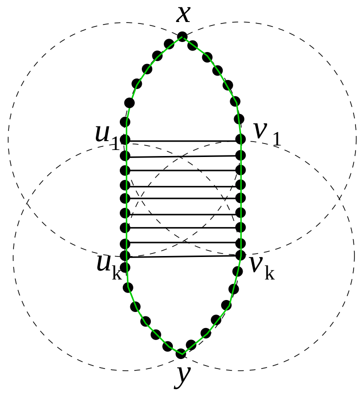

Fig. 8. Broadcasting based on RNG could be $\Theta(n^\beta)$ times the optimum.

still kept by our algorithm, i.e., LRNG = RNG'. Obviously, $\omega_\beta(LRNG) = O(1)$ and $\omega_\beta(MST) = O(1/n^{\beta-1})$ for any $\beta > 1$.

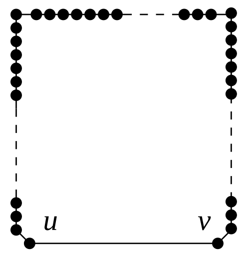

Fig. 9. $\omega_\beta(LRNG) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$.

On the other hand, we can show that the worst-case performance of LRNG on broadcasting is better than that based on RNG. Actually, the total energy consumption of broadcasting based on LRNG is no more than $O(n^{\beta-1})$ times of the optimum.

Lemma 11: $\omega_\beta(LRNG) \le O(n^{\beta-1}) \cdot \omega_\beta(MST)$.

PROOF. Assume that $\omega(LRNG) \le c \cdot \omega(MST)$ for a constant c. Let a_i, $1 \le i \le k$ be the edge lengths of LRNG, and b_i, $1 \le i \le n-1$ be the edge lengths of MST. Here $k = O(n)$ is the number of edges in LRNG. Then

$$\begin{aligned} \sum_{1 \le i \le k} a_i^\beta &\le (\sum_{1 \le i \le k} a_i)^\beta \\ &\le c^\beta \cdot (\sum_{1 \le i \le n-1} b_i)^\beta \\ &\le c^\beta \cdot n^{\beta-1} \cdot \sum_{1 \le i \le n-1} b_i^\beta. \end{aligned}$$

This finishes the proof. □

Consequently, we know that in the worst case, $\omega_\beta(LRNG) = \Theta(n^{\beta-1}) \cdot \omega_\beta(MST)$. Figure 9 shows that, to get a structure with weight $O(\omega_\beta(MST))$, we have to construct a minimum spanning tree for that example. Notice that the worst case communication cost to build a minimum spanning tree is $O(n \log n)$ under the wireless communication model. It seems that the we may have to spend $O(n \log n)$ communications to build a structure with weight

$O(\omega_\beta(MST))$. However, this worst case may not happen at all: for the configurations of nodes that need $O(n \log n)$ communications to build the minimum spanning tree, the structure built by our method may be good enough; on the other hand, for the example that our algorithm does not perform well, we may find an efficient way to build a minimum spanning tree using $o(n \log n)$ messages. We leave it as an open problem whether we can construct a structure G whose weight $\omega_\beta(G)$ is $O(\omega_\beta(MST))$ using only $o(n \log n)$ messages, or even $O(n)$ messages. Here each message has $O(\log n)$ bits always.

Here we show that no *localized* algorithm can construct a structure for broadcasting, whose total power consumption is at most $o(n^{\beta-1})$ times the optimum. Assume that there is such a deterministic localized algorithm A that uses k-hop information. Figure 10 illustrates such an example that algorithm A cannot approximate the optimum broadcasting structure within factor $o(n^{\beta-1})$. In the example, the shortest hop distance between nodes u and x is more than k hops. Then algorithm A will have the same information at node u for both configurations (a) and (b). If A decides to keep edge uv, then for configuration (a), the broadcasting based on the final structure (with all edges shown in Figure 10 (a)) will consume power about $1 + (\frac{2k+1}{n})^\beta \cdot n = 1 + \frac{(2k+1)^\beta}{n^{\beta-1}}$. Notice that, the optimum broadcasting structure will not use link uv, and has total power consumption about $(\frac{2k+1}{n})^\beta \cdot n = \frac{(2k+1)^\beta}{n^{\beta-1}}$. Consequently, the constructed structure uses power about $1 + \frac{n^{\beta-1}}{(2k+1)^\beta} = \Theta(n^{\beta-1})$ times of the optimum, since k is a constant here. If A decides not to keep edge uv, then the structure constructed by A is not connected for configuration illustrated by Figure 10 (b). Thus, we have the following theorem.

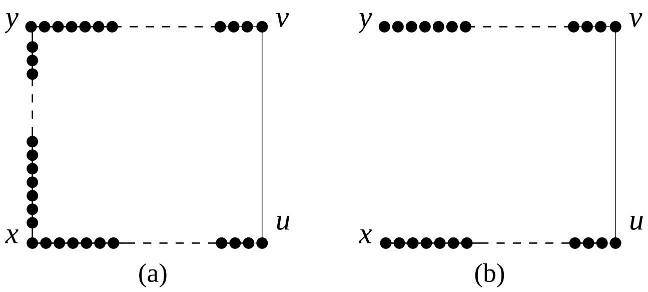

Fig. 10. No localized algorithm that approximates the optimum broadcasting within factor $o(n^{\beta-1}$.

Theorem 12: No localized method can construct a structure for broadcasting with total power consumption asymptotically better than LRNG.

IV. Experiments

We conducted extensive simulations to study the performance of our structure in terms of the longest edge length, the total edge length, maximum node power, total node power and so on. Although network throughput is an important performance metric, it is influenced by many other factors such as the MAC protocol, routing protocol and so on. Therefore, most related work does not test the throughput performance. We will use the following metrics to compare the performance:

1) **Total Messages**: In wireless networks, less messages to construct a topology will save energy consumption. We already showed that the total messages of constructing LRNG is at most $3n$.
2) **Max Messages**: We also test what is the maximum number of messages a node will send in building this structure. A large number of messages at some node will delay the topology updating and drain out its battery power quickly.
3) **Average Node Degree**: A smaller average node degree often implies less contention and interference for signal and thus a better frequency spatial reuse, which in turn will improve the throughput of the network.
4) **Max Node Degree**: We also test the maximum node degree. A larger node degree at some node will cause more contention and interference for signal, and also may drain out its battery power quickly.
5) **Max Node Power**: Notice that each user u will set its transmission range equal to the length of the longest edge incident on u, called *node power* in this paper. Thus, a smaller node power will always save the power consumption. The max-node-power captures the maximum power used by all nodes. Here, in all our simulations, we set the constant $\beta = 2$, so that the power needed to support a link uv is $\|uv\|^2$.
6) **Total Node Power**: The total node power approximates the total power used by all nodes to keep the connectivity.
7) **Total Edge Length**: We proved that all structures proposed in this paper have the total edge length within a constant factor of MST, while no previously known structures having this property.
8) **Total Link Power**: It was also proved in [18] that a broadcasting based on MST consumes energy within a constant factor of the optimum. We thus compare the total link power used by our structure with previously known structures.

In the simulations, we will only test the performances of structure LRNG and compare it with previously known structure G_0^- in [36], RNG in terms of the above metrics. The reason for only selecting G_0^- and RNG is that in [36], their simulations already show that G_0^- out-performs other previously known structures in terms of the node degree, max node power, and the total node power. Hereafter, we use $LMST$ instead of G_0^- in the experiments, if it is clear.

In the first simulation, we randomly generate 100 nodes uniformly in a $1000m \times 1000m$ region. The maximum transmission range of each node is set as $250m$ for all the nodes. The topology derived using the maximum transmission power (UDG), MST, RNG, LMST, and LRNG are shown in Figure 11 respectively. To make the performance testing precise, we generate 100 sets of 100 node sets and compute the performance metrics accordingly. The corresponding performances are illustrated in the following Table IV. Here for max node degree, max message and max node power, we show both the maximum and average values over the 100 sets.

We then vary the number of nodes in the region from 50 to 500. The transmission range of each node is still set as $250m$. We plotted the performances of all structures in Figure 12.

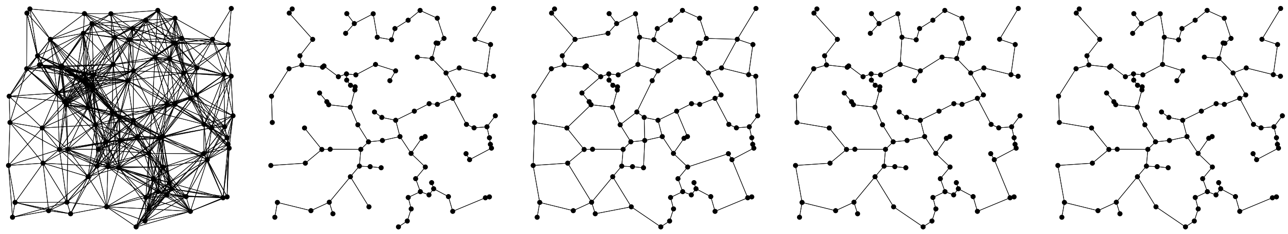

Fig. 11. Different structures from a UDG.

TABLE I

THE PERFORMANCES COMPARISON OF SEVERAL STRUCTURES.

	MST	RNG	LMST	LRNG
MaxMaxMsg	-	1.00	5.00	4.00
AvgMaxMsg	-	1.00	4.68	4.00
TotMsg	-	100.00	305.96	238.98
MaxMaxDeg	4.00	4.00	4.00	4.00
AvgMaxDeg	3.68	4.00	3.68	3.68
AvgDeg	1.98	2.38	2.06	2.03
MaxMaxNPow	3.59	5.90	5.90	4.06
AvgMaxNPow	2.27	4.02	3.85	3.16
TotNPow	208.91	368.40	343.27	285.35
TotLength	135.41	189.26	148.18	143.84
TotLPow	114.86	192.71	136.16	128.53

All the results show that LRNG has better performance than LMST and RNG. In other words, LRNG has less length cost and power cost for broadcasting; it has smaller node power to keep the connectivity. The messages used for constructing LRNG are also less than the one of LMST. The simulation results confirm all of our theoretical analysis. Remember that LRNG maybe spend $O(n^{\beta-1})$ times of power of the optimum for broadcasting in the worst case. However, our simulations show that the energy consumption of broadcasting based on LRNG is within a small factor of that based on the MST and is much better than the energy consumed based on RNG and LMST. In summary, the LRNG is the best among all these known local structures; additionally, it can approximate MST theoretically and be used for energy efficient broadcasting.

V. CONCLUSION

We consider a wireless network composed of n a set of wireless nodes distributed in a two dimensional plane. We presented the first localized method to construct a bounded degree planar connected structure LRNG whose total edge length is within a constant factor of that of the minimum spanning tree, i.e., $\omega(LRNG) = O(1) \cdot \omega(MST)$. The total communication cost of our method is $O(n)$, and every node only uses its two-hop information to construct such structure. We showed that some two-hop information is necessary to construct any low-weighted structure. We also studied the application of this structure in efficient broadcasting in wireless ad hoc networks. We showed that the energy consumption using this structure is within $O(n^{\beta-1})$ of the optimum, i.e., $\omega_\beta(LRNG) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$ for any $\beta \geq 1$. This improves the previously known "lightest" structure RNG by $O(n)$ factor since $\omega(RNG) = \Theta(n) \cdot \omega(MST)$ and $\omega_\beta(RNG) = O(n^\beta) \cdot \omega_\beta(MST)$. We also showed that *no* localized method can construct a structure such that the broadcasting based on this structure consumes power within factor $o(n^{\beta-1})$ of the optimum.

On one aspect, a structure with low-weight does not guarantee that it approximates the optimum broadcasting structure in terms of the total energy consumption. On the other hand, a structure for broadcasting whose total energy consumption is within a constant factor of optimum does not guarantee that it is low-weight. We can show that its total edge length is within $O(\sqrt{n})$ of $\omega(MST)$ for a n-nodes network. Considering this "non-relevance" of the low-weight structure and the optimum broadcasting structure, it remains open how to construct a topology that approximates the optimum broadcasting structure using messages $o(n \log n)$.

The constructed structure is bounded degree, planar, and low-weight. We [37] recently gave an $O(n \log n)$-time centralized algorithm constructing a bounded degree, planar, and low-weighted spanner. However, we cannot make that a distributed algorithm without sacrificing the spanner property [17]. It remains open how to construct a bounded degree, planar, and low-weighted *spanner* in a distributed manner using only $O(n)$ communications under the local broadcasting communication model.

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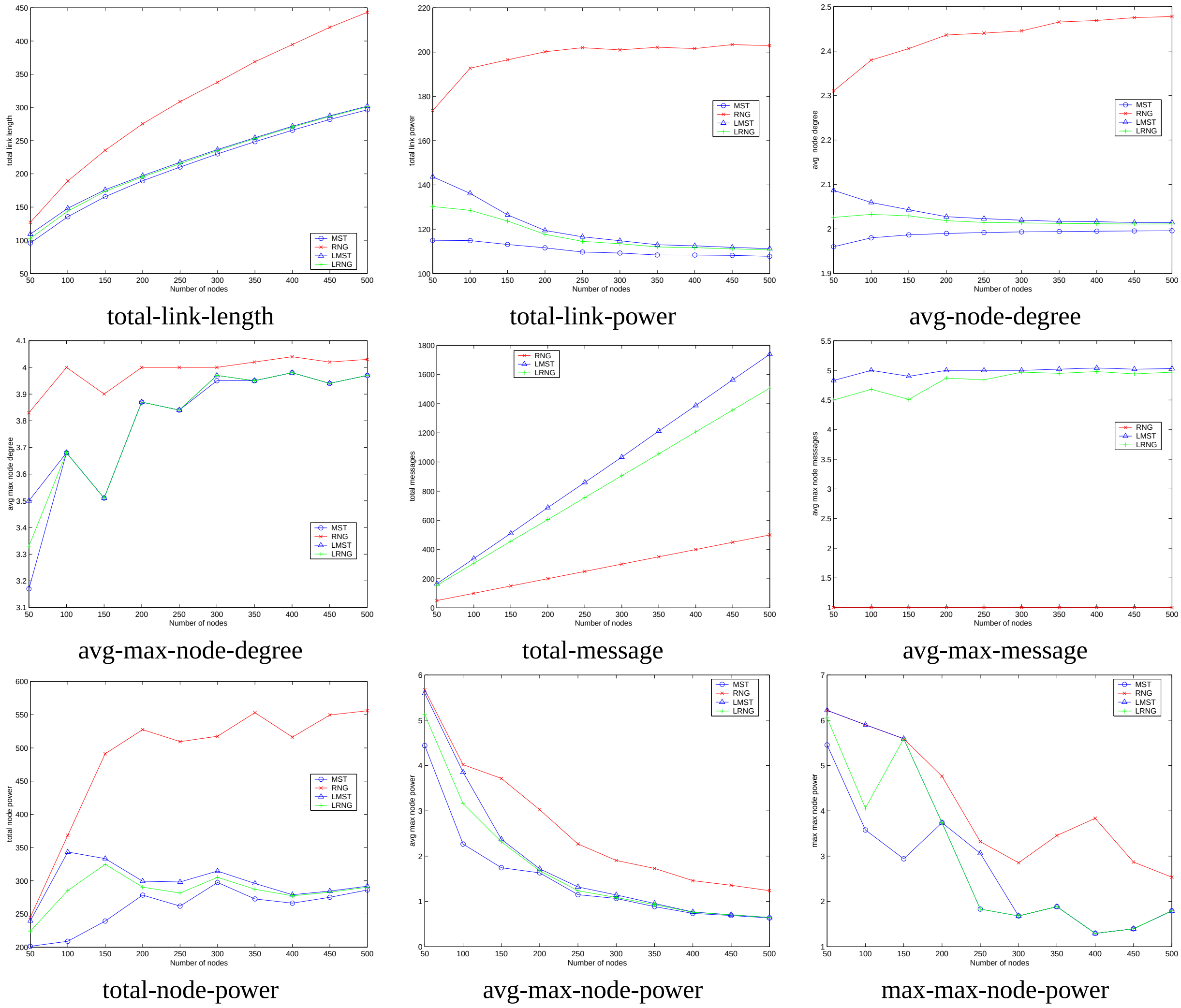

Fig. 12. Results when the number of nodes in the networks are different (from 50 to 500). Here the transmission range is set as $250m$.

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