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On multicast throughput scaling of hybrid wireless networks with general node density

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ABSTRACT

In this paper, we consider *hybrid wireless networks* with a general *node density* $\lambda \in [1, n]$, where n ad hoc nodes are uniformly distributed and m base stations (BSs) are regularly placed in a square region $\mathcal{A}(n,A) = \left[1, \sqrt{A}\right] \times \left[1, \sqrt{A}\right]$ with $A \in [1, n]$. We focus on multicast sessions in which each ad hoc node as a user chooses randomly d ad hoc nodes as its destinations. Specifically, when $d = 1$ (or $d = n - 1$), a multicast session is essentially a unicast (or broadcast) session. We study the asymptotic multicast throughput for such a hybrid wireless network according to different cases in terms of $m \in [1, n]$ and $d \in [1, n]$, as $n \to \infty$. To be specific, we design two types of multicast schemes, called *hybrid scheme* and *BS-based scheme*, respectively. For the hybrid scheme, there are two alternative *routing backbones*: *sparse backbones* and *dense backbones*. Particularly, according to different regimes of the node density $\lambda = \frac{n}{A}$, we derive the thresholds in terms of m and d. Depending on these thresholds, we determine which scheme is preferred for the better performance of network throughput.

1. Introduction

The last decade has witnessed the rapid development of wireless technology. While, the role of wireless technology in current communication services remains still limited. In *cellular networks* and *wireless LANs*, the wireless system involves only with the last stage of communication, from the *base stations* (in cellular networks) or *access points* (in wireless LAN) to the end users. In cellular networks, the communication between the base stations is generally taken on by wired links of high-capacity. The expensive cost and difficulty of building base stations promotes the rise of the new networking paradigm – *wireless ad hoc networks* [13]. Wireless ad hoc networks differ from the conventional infrastructure-based networks above by the fact that they rely completely on wireless communication. They are simply formed by a group of users that have transmitting and receiving capabilities. The nodes can be the mobile phones of the cellular topology, laptops like in WLANs, or sensors that measure some physical data. Whatever the application is, the common characteristic is the following: A group of nodes want to communicate with each other over the shared wireless medium but there is no additional infrastructure for assisting communication or for coordinating traffic [1,2]. In wireless ad hoc networks, when saving the investment costs of base stations, the other side of the coin is that the network throughput possibly decreases due to the aggravation of interference between the communication pairs. An interesting question is to what extent can a *given number* of base stations improve the network throughput. The aim of this paper is to contribute in this issue.

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The focus of this paper is on scaling laws, *i.e.*, scaling of the network throughput in the limit when the number of users gets large ($n \to \infty$) [1,3–12]. The scaling laws for wireless ad hoc networks have been intensively studied, especially after the milestone work done in [1]. The main advantage of studying scaling laws is to highlight qualitative and architectural properties of the system without getting involved with too many technique details [13]. The scaling laws results provide some architectural guidelines on how to design schemes that scale well, while the detailed design and performance analysis for a network with a given number of users would involve tuning of many parameters and improvements of the scheme to optimize the pre-constant in the system throughput. We consider the hybrid wireless networks that have some amount of infrastructure (*e.g.*, base stations connected by high bandwidth links) available. These base stations neither produce data nor consume data. They support the underlying ad hoc networks by relaying data packets through the infrastructure. The integrate of wireless ad hoc and cellular network architecture is often referred to as *hybrid wireless network* or *multihop cellular network* [14–17]. In such a hybrid network, data can be transported in a multi-hop fashion as in ad hoc networks or via the infrastructure as in cellular networks. The hybrid network architecture has at the same time the advantages of both types of networks. It offers the local flexibility of ad hoc networks and efficient long-distance routing of infrastructure. Then, an interesting question arises as to how much the additional infrastructure improves the capacity of pure wireless ad hoc networks. Specifically, we study a *hybrid wireless network* with a general *node density* $\lambda : [1,n]$,[1] where n ad hoc users (AUs) are uniformly distributed and m base stations (BSs) are regularly placed in a square region $\mathcal{A}(n,A) = \left[1,\sqrt{A}\right] \times \left[1,\sqrt{A}\right]$ with $A \in [1,n]$.

Multicast is an efficient method of supporting group communication, as it allows transmission and routing of packets to multiple destinations using fewer network resources. There are many important applications of wireless multicast, such as distribution of data, audio/video conference, distance education, and distributed interactive games, *etc.*, [18]. Please see the illustration in Fig. 1. An emerging typical application that has already been tested is the use of wireless ad hoc networks to broadcast replays during football games, [12]. In wireless sensor networks, multicast is an important technique for information dissemination or code updating, [19–22]. We focus on multicast sessions in which each AU as a user chooses randomly d ad hoc nodes as its destinations, and study multicast capacity scaling laws for hybrid wireless networks according to different cases in terms of $m : [1,n]$ and $d : [1,n]$, as $n \to \infty$. We design two broad types of multicast schemes: *hybrid scheme* and *BS-based scheme*. For the hybrid scheme, there are two alternative *routing backbones*, called *sparse backbones* and *dense backbones*, respectively. Hence, three schemes are produced, *i.e.*, *hybrid scheme based on sparse backbones* (H-SB scheme), *hybrid scheme based on dense backbones* (H-DB scheme), and *BS-based scheme*. According to different cases of the network parameters, *i.e.*, the node density $\lambda = \frac{n}{A}$, the number of BSs m, and the number of destinations per multicast session d, we choose the optimal one from these three schemes, and derive the optimal multicast throughput for hybrid networks. We show that under the H-SB and H-DB schemes, the bottlenecks are located at *B-O links*, *i.e.*, the links between BSs and ordinary wireless nodes. Intuitively, if the bandwidth of *B-O links* can be increased, the throughput for the network should possibly be enhanced. Hence, we designedly derive the multicast throughputs under the H-SB and H-DB schemes without taking the possible bottlenecks on the *B-O links* into account. Such results could be used when some new technical assumptions are made for the *B-O links*.

Compared to related works, this work has the following characteristics:

- **More general network scaling model**. For the scaling laws issue, in terms of scaling patterns, there are two typical network models [10,13]: *extended networks* [10,23–25,13,16,26] and *dense networks* [1,11,12,27,28]. In the former, the node density is fixed to a constant and the area of the deployment region increases to infinity; in the latter, the area is fixed to a constant and the node density increases to infinity. It is easy to see that both the extended network and dense network are indeed the special cases of our model corresponding to the cases that $A = n$ and $A = 1$, *i.e.*, $\lambda = 1$ and $\lambda = n$, respectively. The characterization of two particular scalings, *i.e.*, extended networks and dense networks, does not suffice to develop a comprehensive understanding of wireless networks, although they are representative models to some extent [13]. Hence, in this paper, we consider comprehensively the network with a general node density $\lambda : [1,n]$ rather than only the cases $\lambda = 1$ and $\lambda = n$, which can offer more insights about the scaling laws for hybrid networks.
- **More general session pattern**. Intuitively, when $d = 1$ (or $d = n - 1$), a multicast session is essentially a unicast (or broadcast) one. That is, the unicast and broadcast sessions can be regarded as the special cases of multicast. In this paper, we directly compute multicast throughput to unify the unicast and broadcast throughputs.
- **More realistic communication model**. For the scaling laws issue, there are two broad types of communication models in general. The first one is the *binary-rate model* under which if the value of a given conditional expression is beyond the threshold, the transmitter can send successfully to the receiver at a specific constant data rate; otherwise, it can not send any, *i.e.*, the transmission rate is assumed to be a binary function. The *protocol model* (ProM) and *physical model* (PhyM) defined in [1] both belong to the binary-rate model. The second one is the *continuous-rate model* that determines the transmission rate at which the transmitter can send its data to the receiver reliably, based on a continuous function of the receiver's SINR. *Gaussian channel model* [24] (also called *generalized physical model* [15,27]) is a popular continuous-rate communication model, under

[1] To simplify the expression, we let $f(n) : [f_0(n), f_1(n)]$ denote that $f(n) = \Omega(f_0(n))$ and $f(n) = O(f_1(n))$.

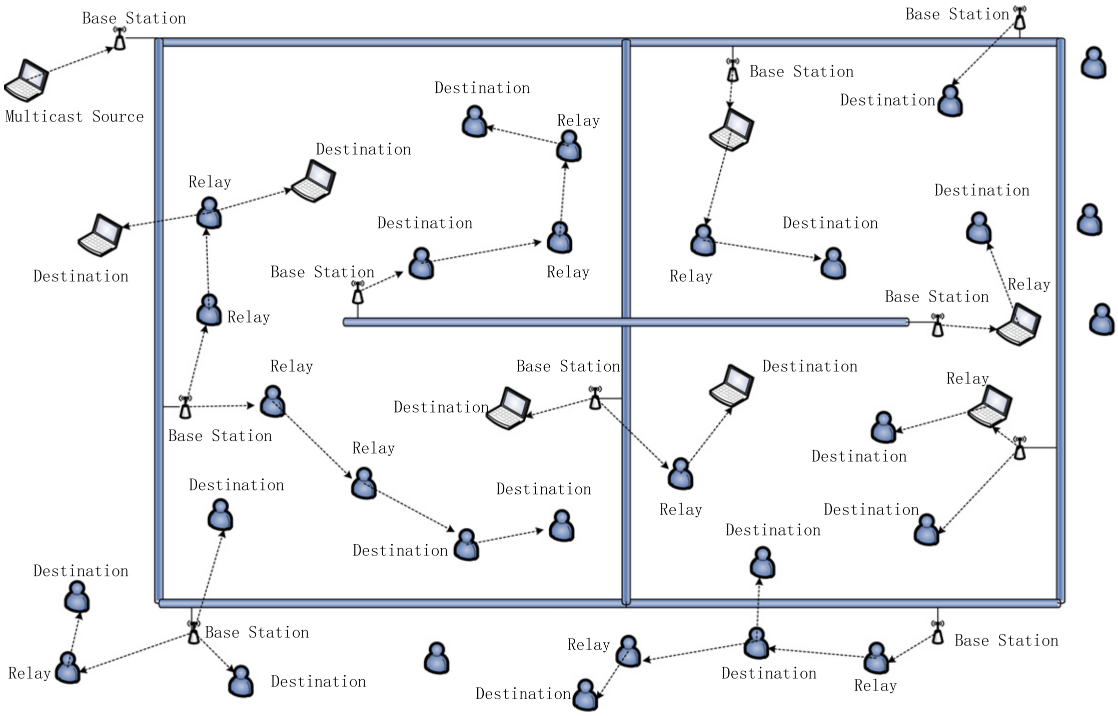


Fig. 1. An illustration of the multicast session in ad hoc networks with infrastructure support. Some of the users might be close to data sources (*e.g.*, Internet access points), and they would act as sources for the multicast traffic. Other nodes would act as relays and sinks (destinations) for the data. The base stations are connected with links of high bandwidth (*e.g.*, wired links).

which any communication pair v_i and v_j can establish a direct communication link, over a channel of bandwidth B, of rate $R(v_i, v_j) = B\log(1 + \text{SINR}(v_j))$. It has been shown that ProM and PhyM are reasonable abstraction of Gaussian channel model for dense networks, but they are over-optimistic and unrealistic for extended networks [1,24,25]. Since we consider the network model with a general node density, we adopt Gaussian channel model to *capture better* the nature of wireless channel.

The rest paper is organized as follows. In Section 2, the network model is introduced. Main results are presented in Section 3. In Section 4, we design the multicast schemes for hybrid wireless networks and derive the achievable throughput. In Section 5, we review the existing related literature. We conclude this study, and discuss the future work in Section 6.

2. System model

2.1. Network model

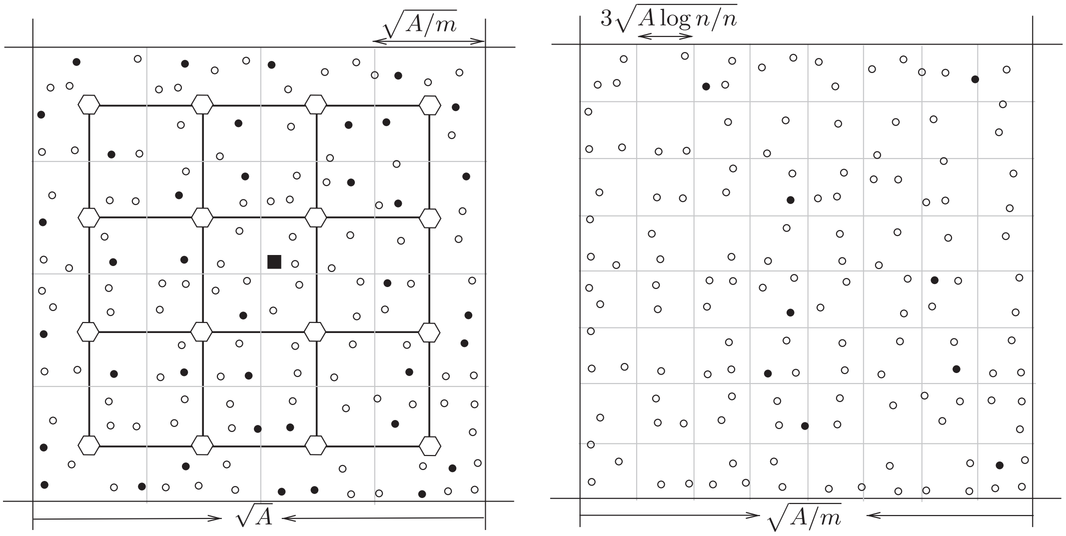
First, we build a random wireless ad hoc networks by distributing uniformly n ad hoc users (AUs) at random in a square region $\mathcal{A}(n,A) = \left[1, \sqrt{A}\right] \times \left[1, \sqrt{A}\right]$ with $A \in [1,n]$. Each AU as a source chooses randomly d ad hoc nodes as its destinations, where $d : [1,n]$. To add the support of infrastructures, we place regularly m base stations (BSs, with wireless transmitting power P) in $\mathcal{A}(n,A)$, as illustrated in Fig. 2, to construct the *hybrid network*. To be specific, divide $\mathcal{A}(n,A)$ into m *subregions* with side length $\frac{\sqrt{A}}{\sqrt{m}}$ and place one BS on the center position of each subregion. We consider the scenario where the number of BSs is no more than the number of users, *i.e.*, $m : [1,n]$.

2.2. Communication model

Assume that all nodes transmit with a constant power P, and any pairs, say v_i and v_j, can establish a direct communication link over a channel of bandwidth B, of rate $\mathbf{R}(v_i, v_j) = B\log\left(1 + \frac{P\cdot\ell(v_i,v_j)}{N_0+\sum_{v_k\in\mathcal{A}(i)/v_i} P\cdot\ell(v_k,v_j)}\right)$, where $N_0 > 0$ is the ambient noise power, $\mathcal{A}(i)$ is the set of nodes that transmit when v_i is scheduled, and $\ell(v_i, v_j)$ denotes the power attenuation function. Following the setting in [10,24,25], let $\ell(v_i, v_j) = d_{ij}^{-\alpha}$ with the power attenuation exponent $\alpha > 2$.

3. Main results

We design two alternative *routing backbones*, called *sparse backbones* and *dense backbones*, respectively. Two types of multicast schemes are proposed: *hybrid scheme* and *BS-based scheme*. The hybrid scheme can be further based on the sparse backbones or dense backbones. Consequently, three schemes are produced, *i.e.*, *hybrid scheme*



(a) Subregions Partitions (b) Cells Partitions

Fig. 2. The small hexagons are the BSs that are placed in the center positions of the subregions of area $\frac{n}{m}$. The shaded small square is the source of a given multicast session. The small circles are the ordinary ad hoc nodes, and the shaded circles are the destinations of the given multicast session.

based on sparse backbones (H-SB scheme, $\mathbb{M}_{\text{H-SB}}$), *hybrid scheme based on dense backbones* (H-DB scheme, $\mathbb{M}_{\text{H-DB}}$), and *BS-based scheme* $\mathbb{M}_{\text{B}}$.

3.1. Optimal multicast throughput based on three schemes

By adopting cooperatively three schemes, we derive the optimal multicast throughput as described in Figs. 3 and 4 that are obtained by Theorems 8, 14 and 18. Please see the details of the results in Table 1.

3.2. Bottlenecks on base stations under schemes $\mathbb{M}_{\text{H-SB}}$ and $\mathbb{M}_{\text{H-DB}}$

According to the analysis of multicast throughput under the schemes $\mathbb{M}_{\text{H-SB}}$ and $\mathbb{M}_{\text{H-DB}}$, the bottlenecks are both located on the wireless links between the ad hoc nodes and BSs, called *B-O links*. Note that this result holds under the assumption that BSs have the same capability as the ordinary ad hoc nodes when they transmit or receive data along the B-O links. Intuitively, this assumption is a bit conservative. We can improve further the network throughput by improving the capability of BSs. Hence, we purposely derive the multicast throughput under the schemes $\mathbb{M}_{\text{H-SB}}$ and $\mathbb{M}_{\text{H-DB}}$ without considering the bottlenecks on BSs, which are presented in Theorems 12 and 16, respectively. We expect that these results can be directly exploited in the future work that introduces some communication techniques for BSs. For example, when BSs are permitted to receive the data from k ad hoc nodes simultaneously, the multicast throughput can increase to k times *as long as* the improved throughput via BSs does *not* exceed the throughput derived without considering bottlenecks on BSs (Theorem 12 or Theorem 16).

4. Multicast schemes

Our multicast schemes are cell-based, then we first give a notion called *scheme lattice* for succinctness of the description.

Definition 1 (*Scheme lattice*). Divide the deployment region $\mathcal{A}(n,A) = \left[0,\sqrt{A}\right]^2$ into a lattice consisting of square cells of side length l, we call the lattice *scheme lattice* and denote it by $\mathbb{L}\left(\sqrt{A},l\right)$.[2]

4.1. Division of subregions

As illustrated in Fig. 2(b), the BS b_i, $i = 1,2,\ldots,n$, is placed in the center position of the ith cell (subregion) in $\mathbb{L}\left(\sqrt{A},\sqrt{A/m}\right)$ after giving each BS and subregion a unique index in a certain order.

4.2. Routing backbones

We partition each subregion, denoted by $\mathcal{R}_i$, into a lattice $\mathbb{L}_i\left(\sqrt{\frac{A}{m}},\min\left\{3\sqrt{\frac{A\log n}{n}},\sqrt{\frac{A}{m}}\right\}\right)$, where $i = 1,2,\ldots,m$. We give a lemma to bound the number of ad hoc nodes in each cell. By using Lemma 20, we can easily obtain that

Lemma 1. *For all cells in* $\mathbb{L}_i\left(\sqrt{A/m},\min\ \{3\sqrt{A\log n/n},\sqrt{A/m}\}\right)$, $i = 1,2,\ldots,m$, *the number of nodes is w.h.p. of order* $\Theta(\mathbf{N})$, *where*

[2] In the following content, we assume that $\frac{\sqrt{A}}{l}$ is always an integer, which has no impact on our final results due to the characteristics of scaling laws issue.

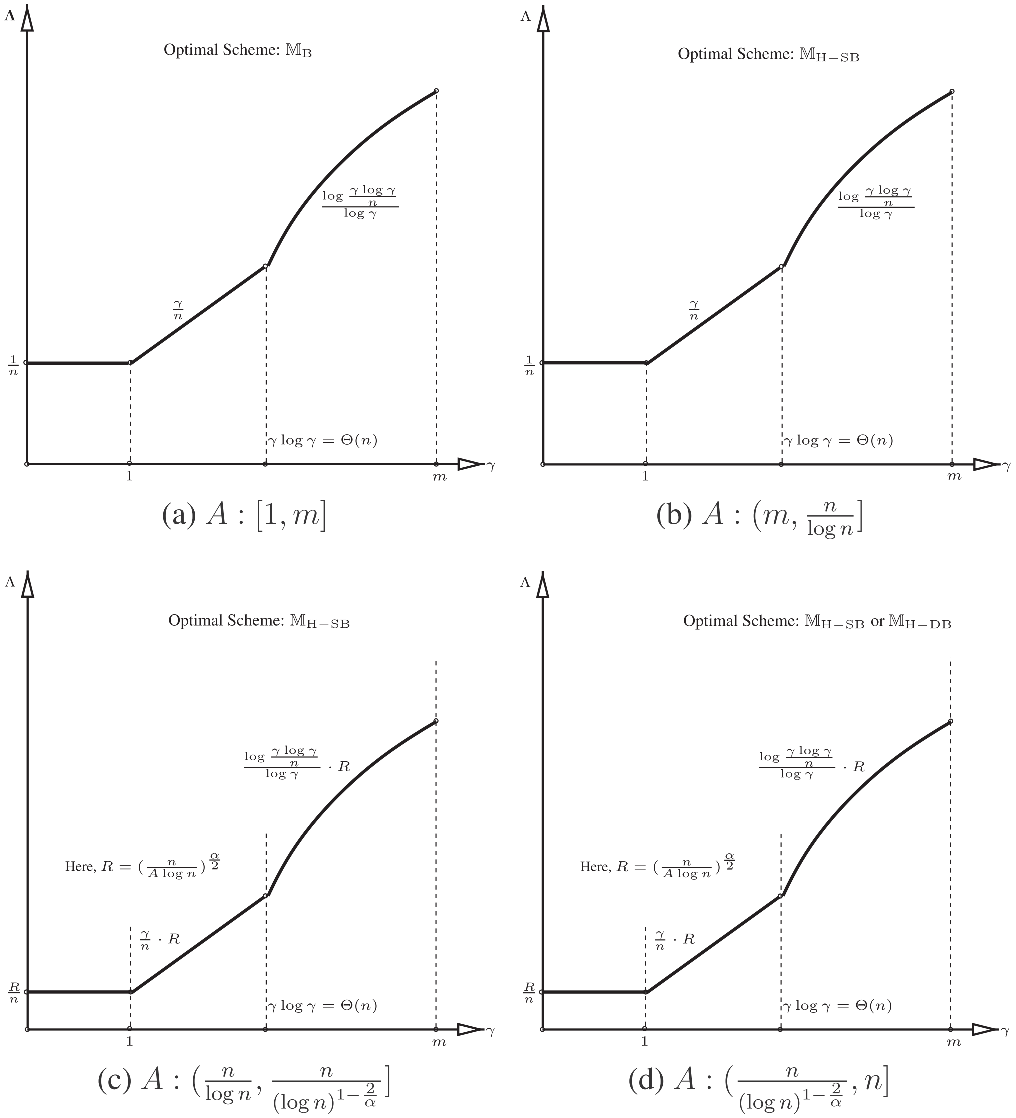


Fig. 3. Illustrations of results for the case that $m : \left[1, \frac{n}{\log n}\right]$. Here, $\gamma = \frac{m}{d}$. The relations between the optimal multicast throughput (y-axis) and γ (x-axis) are provided.

- $\mathbf{N} \in \left[\frac{9}{2}\log n, 18\log n\right]$, *when* $3\sqrt{A\log n/n} \geqslant \sqrt{A/m}$, *i.e.,* $m \leqslant \frac{n}{9\log n}$;
- **N** = *O(logn), when* $3\sqrt{A\log n/n} \leqslant \sqrt{A/m}$, *i.e.,* $m \geqslant \frac{n}{9\log n}$.

When $m \leqslant \frac{n}{9\log n}$, Based on the scheme lattice $\mathbb{L}_i\left(\sqrt{A/m}, 3\sqrt{A\log n/n}\right)$, we build the *sparse backbones* and *dense backbones* in each subregion $\mathcal{R}_i$, for $i = 1,2,\ldots,m$.

4.2.1. Sparse backbones

We build the sparse backbones by the following operations: choose randomly one ad hoc node from each cell in $\mathbb{L}_i\left(\sqrt{A/m}, 3\sqrt{A\log n/n}\right)$, called *station*; connect those stations in a pattern as illustrated in Fig. 5. Then, we get the *sparse backbone system*. The whole sparse backbone system can be scheduled by a 9-TDMA scheme, as illustrated in Fig. 5.

Lemma 2. *Each sparse backbone can sustain a rate of order*

$$\mathbf{R}_{\mathrm{SB}}(n,A) = \begin{cases} \Omega\left(\left(\frac{n}{\log n}\right)^{\frac{\alpha}{2}} \cdot A^{-\frac{\alpha}{2}}\right) & \text{when } A : \left[\frac{n}{\log n}, n\right], \\ \Omega(1) & \text{when } A : \left[1, \frac{n}{\log n}\right]. \end{cases} \quad (1)$$

Please see the proof in Appendix B.1.

4.2.2. Dense backbones

In the center of each cell of $\mathbb{L}_i\left(\sqrt{A/m}, 3\sqrt{A\log n/n}\right)$, we set a smaller square of side length $2\sqrt{A\log n/n}$, as illustrated in Fig. 6, we call it *station-cell*. Then, by Eq. (A.2), we can prove that.

Lemma 3. *For all station-cells, the number of ad hoc users (AUs) inside is w.h.p. at least of* 2 log*n.*

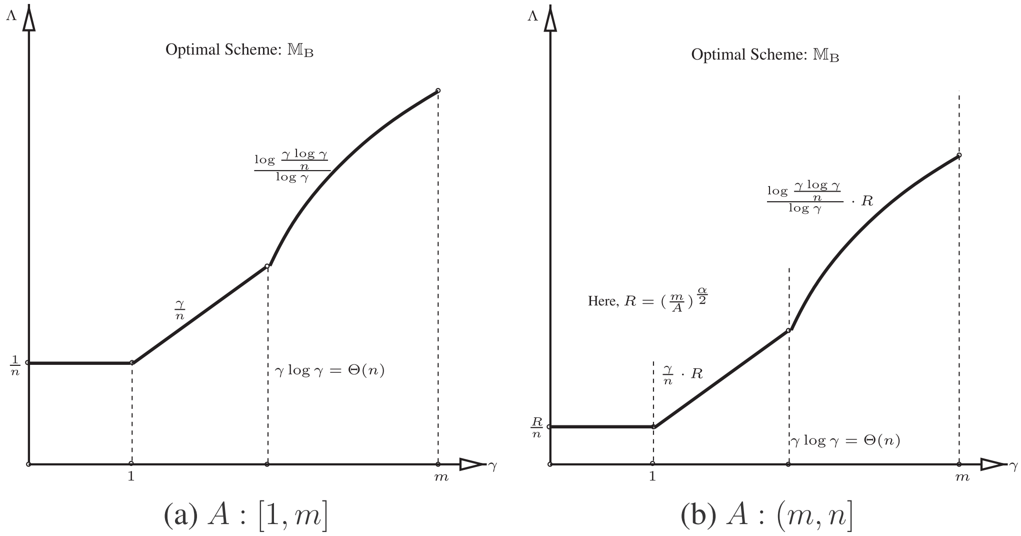


Fig. 4. Illustrations of results for the case that $m : \left(\frac{n}{\log n}, n\right]$. Here, $\gamma = \frac{m}{d}$.

Table 1
Optimal multicast scheme among the *hybrid scheme based on sparse backbones* (H-SB scheme, $\mathbb{M}_{H-SB}$), *hybrid scheme based on dense backbones* (H-DB scheme, $\mathbb{M}_{H-DB}$), and *BS-based scheme* $\mathbb{M}_B$. The corresponding throughputs are described in Figs. 3 and 4.

Number of base stations	Area of deployment region	Optimal scheme
$m : \left[1, \frac{n}{\log n}\right]$	$A : [1, m]$	$\mathbb{M}_B$
	$A : \left[m, \frac{n}{(\log n)^{1-\frac{2}{\alpha}}}\right]$	$\mathbb{M}_{H-SB}$
	$A : \left[\frac{n}{(\log n)^{1-\frac{2}{\alpha}}}, n\right]$	$\mathbb{M}_{H-SB}$ or $\mathbb{M}_{H-DB}$
$m : \left(\frac{n}{\log n}, n\right]$	$A : [1, n]$	$\mathbb{M}_B$

Thus, we can build the dense backbones by the following operations: choose randomly 2 log$n$ nodes from each station-cell, and connect them with each other by a point-to-point pattern. As illustrated in Fig. 6, we will adopt a 4-TDMA scheme to schedule the dense backbone system. Note that there are $2\log n$ links, instead of only one link, initiating from each station-cell to be scheduled simultaneously. Then, it holds that.

Lemma 4. *The rate of each dense backbone can be sustained of order*

$$\mathbf{R}_{DB}(n, A) = \begin{cases} \Omega\left(\left(\frac{n}{\log n}\right)^{\frac{\alpha}{2}} \cdot A^{-\frac{\alpha}{2}}\right) & \text{when } A : \left[\frac{n}{(\log n)^{1-\frac{2}{\alpha}}}, n\right], \\ \Omega\left(\frac{1}{\log n}\right) & \text{when } A : \left[1, \frac{n}{(\log n)^{1-\frac{2}{\alpha}}}\right]. \end{cases} \quad (2)$$

Please see the proof in Appendix B.2.

4.3. Multicast schemes

In general, there are two types of multicast routing schemes: *shortest path trees* and *minimum cost trees*, [18]. Our schemes belong to the latter type. For the multicast session $\mathcal{M}_k$, $k = 1, 2, \ldots n$, denote the set of nodes by $\mathcal{U}_k = \{v_k\} \cup \{v_{k_1}, v_{k_2}, \cdots v_{k_d}\}$, where v_k is the source node and the nodes in the latter set are the destinations of v_k. Let $\mathcal{U}_k^\imath = \{v_{k_1}^\imath, v_{k_2}^\imath, \ldots, v_{k_t}^\imath\}$ denote the set of nodes that belong to $\mathcal{U}_k$ and are located in the subregion $\mathcal{R}_\imath$, where $\mathcal{U}_k = \bigcup \mathcal{U}_k^\imath$ and $\mathcal{U}_k^{\imath_1} \cap \mathcal{U}_k^{\imath_2} = \emptyset$ for any $\imath_1 \neq \imath_2$. Define $\widetilde{\mathcal{U}}_k^\imath := \mathcal{U}_k^\imath \cup \{b_\imath\}$, where $b_\imath$ denotes the base station placed in subregion $\mathcal{R}_\imath$. Then, we can construct the Euclidean spanning tree (EST) based on every set $\widetilde{\mathcal{U}}_k^\imath$ by using the method in [29], described in Algorithm 1. Denote those ESTs as $\text{EST}(\widetilde{\mathcal{U}}_k^\imath)$, $1 \leqslant \imath \leqslant v_k$, where v_k is a random variable that represents the number of subregions containing at least one ad hoc node in $\mathcal{U}_k$.

Algorithm 1: Construction of $\text{EST}\left(\widetilde{\mathcal{U}}_k^\imath\right)$

Input : The set of nodes $\widetilde{\mathcal{U}}_k^\imath$
Output : An Euclidean spanning tree $\text{EST}(\widetilde{\mathcal{U}}_k^\imath)$.
1: In the initial state, all nodes of $\widetilde{\mathcal{U}}_k^\imath$ are isolated, then there are $d + 1$ connected components.
2: **for** $i = 1 : d$ **do**
3: Partition the deployment region $\mathcal{A}(n, A)$ into at most $d + 1 - i$ square cells, each with side length $\frac{\sqrt{A}}{\lfloor\sqrt{d+1-i}\rfloor}$;
4: Find a cell that contains two nodes of $\widetilde{\mathcal{U}}_k^\imath$ that belong to two different connected components. By connecting the pair of nodes, we merge the two connected components.
5: **end for**

Note that for all $\widetilde{\mathcal{U}}_k^\imath$ except for that one including v_k denoted as $\widetilde{\mathcal{U}}_k^{\imath_0}$, we set $b_\imath$ as the source; for $\widetilde{\mathcal{U}}_k^{\imath_0}$, we set v_k as the source. According to Lemma 3 and Lemma 10 in [17], we have the following lemma.

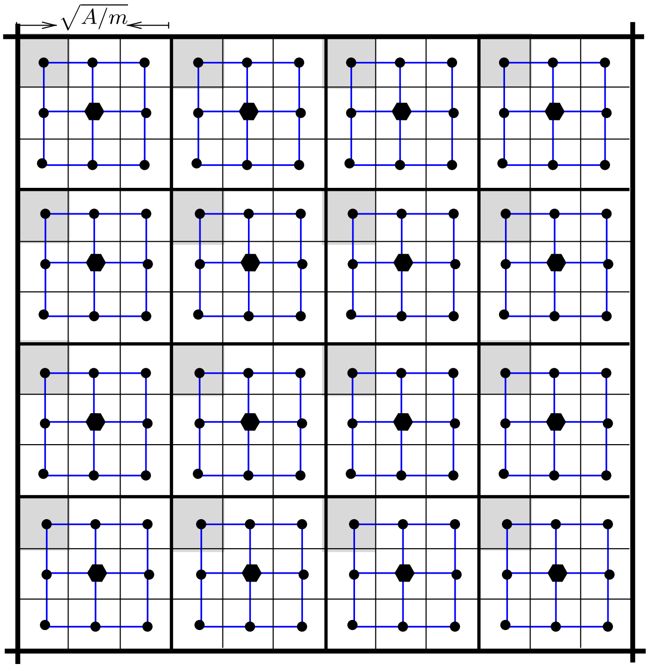


Fig. 5. Sparse backbones. The shaded cells can be scheduled simultaneously. In any time slot, there are exactly one link initiated from every activated station-cell.

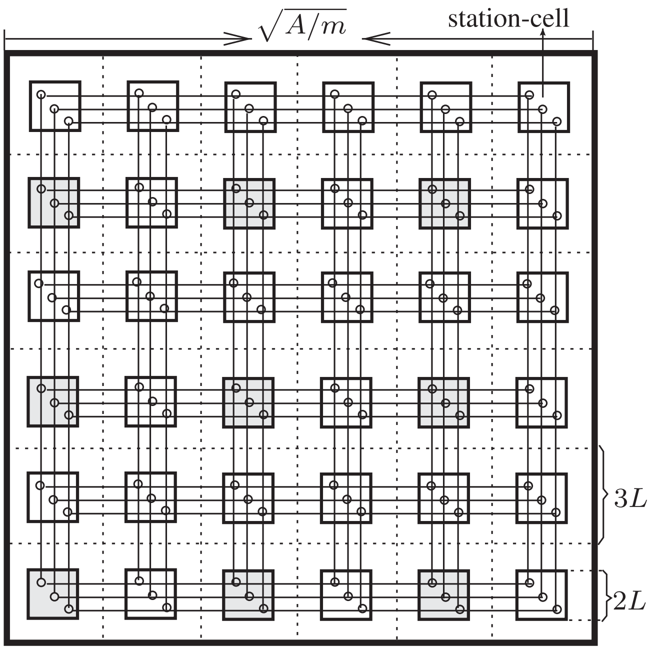


Fig. 6. Dense backbones. There is one *station-cell* centered at each cell of $\mathbb{L}_i\left(\sqrt{A/m}, 3L\right)$, where $L = \sqrt{A \log n/n}$. The shaded station-cells can be scheduled simultaneously. In any time slot, there are 2 log$n$ concurrent links initiated from every activated station-cell.

Lemma 5. *With high probability,* $v_k = \Theta(\min\{d, m\})$ *for* k, $1 \leqslant k \leqslant n$.

4.3.1. BS-based scheme $\mathbb{M}_B$

We adopt a classical BS-based scheme, in which sources deliver data to BSs during the uplink phase and BSs deliver received data to destinations during the downlink phase, as illustrated in Fig. 7(a). We present the multicast scheme $\mathbb{M}_B$ as follows:

(1) During the uplink phase, for $\iota = 1, 2, \ldots, m$, the source node in the subregion $\mathcal{R}_\iota$ transmits the packets to the BS b_ι.

(2) For $k = 1,2,\ldots,n$, the BS receiving the packets from source v_k delivers packets to the BSs that are placed in the subregions containing the destinations of v_k via BS-to-BS links.
(3) During the downlink phase, for $k = 1,2,\ldots,n$ and $\imath = 1,2,\ldots,m$, the BS $b_\imath$ delivers the packets to the destinations of source v_k that are located in the subregion $\mathcal{R}_\imath$.

Due to the regular position of BSs, we can simultaneously schedule all subregions in both uplink and downlink phases. We have

Lemma 6. *Under the scheme $\mathbb{M}_{\mathrm{B}}$, each subregion can sustain a rate of order*

$$\mathbf{R}_{\mathrm{B}}(n,A) = \begin{cases} \Omega\left(\left(\frac{m}{A}\right)^{\frac{\alpha}{2}}\right) & \text{when } A : [m,n], \\ \Omega(1) & \text{when } A : [1,m], \end{cases} \tag{3}$$

during both downlink and uplink phases.

Proof. We consider the uplink phase. All subregions are simultaneously scheduled; in each subregion, there is exactly one link from a transmitter to the BS to be permitted. For any link in any time slot, the transmitters in the eight closest cells are located at Euclidean distance at least $\frac{1}{2}\cdot\sqrt{\frac{A}{m}}$ from the receiver (BS); the 16 next closest cells are at Euclidean distance at least $\frac{3}{2}\cdot\sqrt{\frac{A}{m}}$, and so on. By extending the sum of the interferences to the whole region, this can then be bounded as follows:

$$\mathrm{I}(n,A) \leqslant \sum_{i=1}^{n} 8iP\cdot\ell\left(\left(i-\frac{1}{2}\right)\cdot\sqrt{\frac{A}{m}}\right) \leqslant 8\cdot P\cdot\left(\frac{A}{m}\right)^{\frac{\alpha}{2}}\cdot\sum_{i=1}^{\infty}\frac{i}{\left(i-\frac{1}{2}\right)^{\alpha}},$$

since $\alpha > 2$, we get that $\mathrm{I}(n,A) = O\left(\left(\frac{m}{A}\right)^{\frac{\alpha}{2}}\right)$. Because the distance of every hop is at most of $\frac{\sqrt{2}}{2}\cdot\sqrt{A/m}$, the signal strength at the receiver can be bounded as $\mathrm{S}(n,A) = \Omega\left(\left(\frac{m}{A}\right)^{\frac{\alpha}{2}}\right)$. Thus,

$$\mathbf{R}(n,A) = B\log\left(1+\frac{\mathrm{S}(n,A)}{N_0+\mathrm{I}(n,A)}\right) = \begin{cases} \Omega\left(\left(\frac{m}{A}\right)^{\frac{\alpha}{2}}\right) & \text{when } A : [m,n], \\ \Omega(1) & \text{when } A : [1,m]. \end{cases}$$

Hence, the lemma holds. □

Next, we consider the load of each subregion during the downlink phase and uplink phase. We have

Lemma 7. *Under the scheme $\mathbb{M}_{\mathrm{B}}$, the load of each subregion is of order*

$$\mathbf{L}_{\mathrm{B}} = \begin{cases} O\left(\frac{\log\gamma}{\log\frac{\gamma\log\gamma}{n}}\right) & \text{when } \gamma = \Omega(1) \text{ and } \gamma\cdot\log\gamma = \Omega(n), \\ O\left(\frac{n}{\gamma}\right) & \text{when } \gamma = \Omega(1) \text{ and } \gamma\cdot\log\gamma = O(n), \\ O(n) & \text{when } \gamma = O(1), \end{cases} \tag{4}$$

where $\gamma = \gamma(m,d) := \frac{m}{d}$.

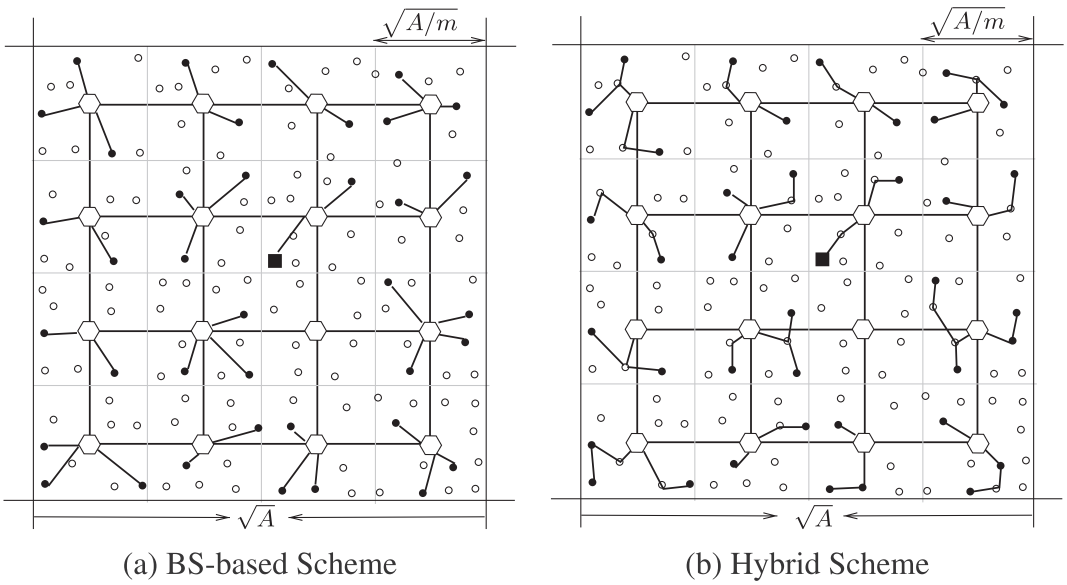


Fig. 7. Multicast schemes. (a) BS-based Scheme. The sources directly transmit data to BSs during the uplink phase and BSs also directly deliver received data to destinations during the downlink phase. (b) Hybrid Scheme. In each subregion, the data are transported to the BS by a multihop pattern. The communications between the ad hoc nodes in different subregions are relayed by the corresponding BSs.

Proof. Define an event $E_{\rm B}(k,t)$: The subregion $\mathcal{R}_t$ contains a node belonging to $\mathcal{U}_k$. Then, $\Pr(E_{\rm B}(k,t)) \leqslant \frac{d}{m} = \frac{1}{\gamma}$, for any $t = 1,2,\ldots,m$. Then, the load of each subregion is no more than $\mathbf{L}(n,\ \gamma)$, where $\gamma = \gamma(m,d) := \frac{m}{d}$ and $\mathbf{L}(n,\gamma)$ denotes the maximum number of balls in any bin when n balls are independently and uniformly at random thrown into γ bins. Note that we assume that γ is an integer, which has no impact on the result in order sense. According to Lemma 19, we can obtain Eq. (4), which completes the proof. □

Combining Lemmas 6 and 7, we finally get the following theorem.

Theorem 8. *Under the scheme $\mathbb{M}_{\rm B}$, the multicast throughput can be achieved of*

$$\Lambda_{\rm B} = \mathbf{R}_{\rm B}/\mathbf{L}_{\rm B}, \tag{5}$$

where $\mathbf{R}_B$ and $\mathbf{L}_B$ are defined in Eqs. (3) and (4), respectively.

4.3.2. Hybrid schemes based on sparse backbones $\mathbb{M}_{\rm H-SB}$

We first design Algorithm 2 to construct the multicast routing tree $\mathcal{T}_{\rm H-SB}(\mathcal{U}_k)$ for a given multicast session $\mathcal{M}_k$.

Algorithm 2: Multicast Routing Scheme of $\mathbb{M}_{\rm H-SB}$

Input: $\mathrm{EST}\left(\widetilde{\mathcal{U}}_k^{\imath}\right), 1 \leqslant \imath \leqslant v_k$.
Output: A multicast routing tree $\mathcal{T}_{\rm H-SB}(\mathcal{U}_k)$.
for each $\mathrm{EST}(\widetilde{\mathcal{U}}_k^{\imath})$ **do**
  for each link u_iu_j in $\mathrm{EST}(\widetilde{\mathcal{U}}_k^{\imath})$ **do**
    Connect u_i and u_j by using the following *Manhattan routing*:
      Denote the intersection point of the horizontal line through u_i and the vertical line through u_j as $p_{i,j}$, and denote the nearest node to $p_{i,j}$ by $u_{i,j}$; connect u_i and $u_{i,j}$ by the corresponding horizontal sparse backbone, and connect $u_{i,j}$ and u_j by the corresponding vertical sparse backbone.
  end for
  Merge the same edges (hops) and remove the circles that have no impact on the connectivity of $\mathrm{EST}(\widetilde{\mathcal{U}}_k^{\imath})$, we obtain the multicast tree $\mathcal{T}_{\rm H-SB}(\mathcal{U}_k^{\imath})$.
end for
Based on the forest consisting of the constructed trees, *i.e.*, $\mathcal{T}_{\rm H-SB}(\mathcal{U}_k^{\imath})(1 \leqslant \imath \leqslant v_k)$, we obtain the final multicast tree $\mathcal{T}_{\rm H-SB}(\mathcal{U}_k)$ by building an EST spanning the set of base stations $b_\imath(1 \leqslant \imath \leqslant v_k)$.

From Lemma 2, we can get the rate that can be sustained by every sparse backbone. Next, we aim to derive the maximum burden of each sparse backbone under the scheme $\mathbb{M}_{\rm H-SB}$.

Above all, we recall a useful result in [29].

Lemma 9. *Using Algorithm 1 with the input of $\mathcal{U}$ (a set of nodes) to built the Euclidean spanning tree spanning $\mathcal{U}$, denoted by $\mathrm{EST}(\mathcal{U})$, it holds that*

$$\|\mathrm{EST}(\mathcal{U})\| \leqslant 2\sqrt{2}\sqrt{|\mathcal{U}|}\cdot\sqrt{A},$$

where A is the area of the deployment square region and $|\mathcal{U}|$ denotes the cardinally of the set $\mathcal{U}$.

Denote the forest consisting of all $\mathrm{EST}(\widetilde{\mathcal{U}}_k^{\imath})(1 \leqslant \imath \leqslant v_k)$, by $\mathcal{F}_k$. Then, we have

Lemma 10. *With high probability, the total Euclidean edge length of $\|\mathcal{F}_k\|$ is of order $O\left(\sqrt{\frac{A\cdot d\cdot \min\{d,m\}}{m}}\right)$, for any k, $1 \leqslant k \leqslant n$.*

Proof. Denote the number of vertexes of $\mathrm{EST}(\widetilde{\mathcal{U}}_k^{\imath})$ as $x_k^{\imath}$, where $1 \leqslant \imath \leqslant v_k$ and $1 \leqslant k \leqslant n$. According to Lemma 9, $\left\|\mathrm{EST}\left(\widetilde{\mathcal{U}}_k^{\imath}\right)\right\| = O\left(\sqrt{x_k^{\imath}}\cdot\frac{\sqrt{A}}{\sqrt{m}}\right)$. Hence, there exists a constant κ_1 such that

$$\|\mathcal{F}_k\| = \sum_{\imath=1}^{v_k}\|\mathrm{EST}(\widetilde{\mathcal{U}}_k^{\imath})\| \leqslant \frac{\kappa_1\sqrt{A}}{\sqrt{m}}\cdot\sum_{\imath=1}^{v_k}\sqrt{x_k^{\imath}}.$$

By Cauchy–Schwartz Inequality, we have

$$\sum_{\imath=1}^{v_k}\sqrt{x_k^{\imath}} \leqslant \sqrt{v_k\cdot\sum_{\imath=1}^{v_k}x_k^{\imath}} \leqslant \sqrt{v_k\cdot(v_k+d)} \leqslant \sqrt{2d\cdot v_k}.$$

Then, $\|\mathcal{F}_k\| = O\left(\frac{\sqrt{A}}{\sqrt{m}}\cdot\sqrt{d\cdot v_k}\right)$, which completes the proof. □

Lemma 11. *Under the scheme $\mathbb{M}_{\rm H-SB}$, the burden of each sparse backbone is of*

$$\mathbf{L}_{\rm H-SB,1} = \begin{cases} O(d\cdot\sqrt{\frac{n\log n}{m}}) & \text{when } d = O(m), \\ O(\sqrt{n\cdot d\cdot\log n}) & \text{when } d = \Omega(m) \text{ and } d = O(\frac{n}{\log n}), \\ O(n) & \text{when } d = \Omega\left(\frac{n}{\log n}\right). \end{cases} \tag{6}$$

Proof. Given a station on a sparse backbone, say s_t, define an event $E_{\rm H\text{-}SB,1}(k,t)$: The multicast session $\mathcal{M}_k$ passes through s_t. Obviously, $E_{\rm H\text{-}SB,1}(k,t)$ happens if there exists an edge $u_iu_j \in \mathcal{F}_k$ to be routed through s_t, *i.e.*, $u_iu_{i,j}$ or $u_{i,j}u_j$ passes through s_t. Hence, by $|u_iu_{i,j}| + |u_{i,j}u_j| \leqslant \sqrt{2}|u_iu_j|$,

$$\begin{aligned}
&\Pr(E_{\rm H-SB,1}(k,t)) \\
&\leqslant \frac{3\sqrt{A\log n/n}}{\sqrt{A}}\cdot\frac{\sum_{u_iu_j\in\mathcal{F}_k}\left(|u_iu_{i,j}|+|u_{i,j}u_j|+2\cdot 3\sqrt{A\log n/n}\right)}{\sqrt{A}} \\
&\leqslant 18\log n/n\cdot(d+v_k)+3\sqrt{\frac{2\log n}{An}}\cdot\sum_{u_iu_j\in\mathcal{F}_k}|u_iu_j| \\
&\leqslant 36d\log n/n+3\sqrt{\frac{2\log n}{An}}\cdot\|\mathcal{F}_k\| \\
&\leqslant 36d\log n/n+3\kappa_2\sqrt{2\log n/n}\cdot\sqrt{\frac{d\cdot\min\{d,m\}}{m}},
\end{aligned}$$

where κ_2 is a constant and the last inequality supported by Lemma 10. That is, $\Pr(E_{\rm H\text{-}SB,1}(k,t)) = O\left(\frac{d\log n}{n} + \right.$

$\sqrt{\frac{\log n}{n}} \cdot \sqrt{\frac{d \cdot \min\{d,m\}}{m}}$). According to Lemma 11, we get Eq. (6), and complete the proof. □

Combining Lemmas 2 and 11, we can obtain Theorem 12.

Theorem 12. *Under the scheme $\mathbb{M}_{\text{H-SB}}$, taking no account of the possible bottleneck on BSs, the multicast throughput can be achieved of*

$$\Lambda_{\text{H-SB},1} = \mathbf{R}_{\text{H-SB}} / \mathbf{L}_{\text{H-SB},1}, \tag{7}$$

where $\mathbf{R}_{H\text{-}SB}$ and $\mathbf{L}_{H\text{-}SB,1}$ are defined in Eqs. (1) and (6), respectively.

Next, we consider the throughput via BSs. Based on Lemmas 2 and 7, it is easy to obtain that,

Lemma 13. *Under the scheme $\mathbb{M}_{\text{H-SB}}$, the throughput via BSs can be achieved of*

$$\Lambda_{\text{H-SB},2} = \mathbf{R}_{\text{H-SB}} / \mathbf{L}_{\text{B}}, \tag{8}$$

where $\mathbf{R}_{H\text{-}SB}$ and $\mathbf{L}_{B}$ are defined in Eqs. (1) and (4), respectively.

Combining Theorem 12 and Lemma 13, we can prove Theorem 14.

Theorem 14. *Under the scheme $\mathbb{M}_{\text{H-SB}}$, the multicast throughput is achieved of*

$$\Lambda_{\text{H-SB}} = \min\{\Lambda_{\text{H-SB},1}, \Lambda_{\text{H-SB},2}\} = \Lambda_{\text{H-SB},2}, \tag{9}$$

where $\Lambda_{H\text{-}SB,1}$ and $\Lambda_{H\text{-}SB,2}$ are defined in Eqs. (7) and (8), respectively.

4.3.3. Hybrid schemes based on dense backbones $\mathbb{M}_{\text{H-DB}}$

We first design the multicast routing in Algorithm 3.

Algorithm 3: Multicast Routing scheme of $\mathbb{M}_{\text{H-DB}}$

Input: $\text{EST}(\widetilde{\mathcal{U}}_k^\iota),\ 1 \leqslant \iota \leqslant v_k$.
Output: A multicast routing tree $\mathcal{T}_{\text{H-DB}}(\mathcal{U}_k)$.
1: **for** each $\text{EST}(\widetilde{\mathcal{U}}_k^\iota)$ **do**
2: **for** each link $u_i u_j$ in $\text{EST}\left(\widetilde{\mathcal{U}}_k^\iota\right)$ **do**
3: Connect u_i to u_j by using Manhattan routing via the dense backbones.
4: **end for**
5: Merge the same edges (hops) and remove the circles that have no impact on the connectivity of $\text{EST}(\widetilde{\mathcal{U}}_k^\iota)$, we obtain the multicast tree $\mathcal{T}_{\text{H-DB}}(\mathcal{U}_k^\iota)$.
6: **end for**
7: Based on the forest consisting of the constructed trees, *i.e.*, $\mathcal{T}_{\text{H-DB}}(\mathcal{U}_k^\iota)(1 \leqslant \iota \leqslant v_k)$, we obtain the final multicast tree $\mathcal{T}_{\text{H-DB}}(\mathcal{U}_k)$ by building an EST spanning the set of base stations $b_\iota(1 \leqslant \iota \leqslant v_k)$.

Next, we consider the burden of each dense backbone under the scheme $\mathbb{M}_{\text{H-DB}}$. By a similar procedure to the proof of Lemma 11, we can get that

Lemma 15. *Under the scheme $\mathbb{M}_{\text{H-DB}}$, the burden of each dense backbone is of*

$$\mathbf{L}_{\text{H-DB},1} = \begin{cases} O\left(\frac{d\sqrt{n}}{\sqrt{m\log n}}\right) & \text{when } d : [1, m] \\ O\left(\frac{\sqrt{nd}}{\sqrt{\log n}}\right) & \text{when } d : \left[m, \frac{n}{\log n}\right], \\ O(d) & \text{when } d : \left[\frac{n}{\log n}, n\right]. \end{cases} \tag{10}$$

Combining Lemmas 4 and 15, we can obtain Theorem 16.

Theorem 16. *Under the scheme $\mathbb{M}_{\text{H-DB}}$, taking no account of the possible bottleneck on BSs, the multicast throughput can be achieved of*

$$\Lambda_{\text{H-DB},1} = \mathbf{R}_{\text{H-DB}} / \mathbf{L}_{\text{H-DB},1}, \tag{11}$$

where $\mathbf{R}_{H\text{-}DB}$ and $\mathbf{L}_{H\text{-}DB,1}$ are defined in Eqs. (2) and (10), respectively.

Next, we consider the throughput via BSs. It is easy to obtain that,

Lemma 17. *Under the scheme $\mathbb{M}_{\text{H-DB}}$, the throughput via BSs can be achieved of*

$$\Lambda_{\text{H-DB},2} = \mathbf{R}_{\text{H-DB}} / \mathbf{L}_{\text{B}}, \tag{12}$$

where $\mathbf{R}_{H\text{-}DB}$ and $\mathbf{L}_{B}$ are defined in Eqs. (2) and (4), respectively.

Combining Theorem 16 and Lemma 17, we can prove Theorem 18.

Theorem 18. *Under the scheme $\mathbb{M}_{\text{H-DB}}$, the multicast throughput is achieved of*

$$\Lambda_{\text{H-DB}} = \min\left\{\Lambda_{\text{H-DB},1}, \Lambda_{\text{H-DB},2}\right\} = \Lambda_{\text{H-DB},2}, \tag{13}$$

where $\Lambda_{H\text{-}DB,1}$ and $\Lambda_{H\text{-}DB,2}$ are defined in Eqs. (11) and (12), respectively.

5. Literature review

We limit the scope of this paper to the multicast at network layer [12,30,24,25] that is different from that at link layer, [31–35]. We review the related work on capacity scaling laws of static wireless networks, including *wireless ad hoc networks* and *hybrid wireless networks*.

5.1. Wireless ad hoc networks

Gupta and Kumar [1] studied the *unicast* capacity for *dense networks* under the protocol model (ProM) and physical model (PhyM). They showed that direct communication between source and destination pairs is not preferable, as the interference generated would preclude most of the other nodes from communicating. On the contrary, the optimal scheme is to confine to nearest neighbor communication and maximize the number of simultaneous transmissions (spatial reuse). However, this means that each packet has to be retransmitted many times before getting to the final destination, leading to a sublinear scaling of system throughput. Specifically, they obtained that the unicast throughput under ProM and PhyM for *random dense networks* is of order $\Omega\left(\frac{1}{\sqrt{n\log n}}\right)$. Keshavarz-Haddad et al. [11] studied the broadcast capacity under ProM for an arbitrary network, and showed that the per session broadcast capacity is only of order $\Theta(1/n)$. Li [29] proved that by using the multicast scheme based on Euclidean spanning tree (EST) and Manhattan routing, the multicast capacity for *random networks* under ProM can be achieved of order $\Omega\left(\frac{1}{\sqrt{dn\log n}}\right)$ when $d = O\left(\frac{n}{\log n}\right)$, and is of order $\Omega(\frac{1}{n})$ when $d = \Omega\left(\frac{n}{\log n}\right)$. Here, d denotes the number of destinations per multicast session. Shakkottai et al. [12] designed a novel routing scheme, called *multicast comb*, by which the achievable multicast throughput is of order $\Omega(\frac{1}{\sqrt{n^{\epsilon}\log n}})$ when the number of multicast sources is n^{ϵ}, for some $\epsilon > 0$, and the number of destinations per multicast session is $n^{1-\epsilon}$.

By introducing the *percolation-based routing*, Franceschetti et al. [10] proved that the unicast throughput under the *Gaussian channel model* (GCM) for both *random dense networks* and *random extended networks*, can be achieved of order $\Omega(1/\sqrt{n})$. Also based on the percolation theory [36], Zheng [37] proved that the broadcast capacity for random extended networks is of order $\Theta\left(\frac{1}{n}\cdot(\log n)^{-\frac{\alpha}{2}}\right)$, where α is the power attenuation exponent of GCM. Later, Li et al. [24] showed that, when $d = O\left(\frac{n}{(\log n)^{2\alpha+6}}\right)$ and $n = \Omega\left(n^{\frac{1}{2}+\theta}\right)$, the multicast throughput for random networks can be achieved of $\Omega\left(\frac{\sqrt{n}}{n\sqrt{d}}\right)$. Wang et al. [25] improved the threshold of d above to $d = O\left(\frac{n}{(\log n)^{\alpha+1}}\right)$ by designing a technique called *parallel scheduling scheme*. Keshavarz-Haddad and Riedi [27,28] proposed a useful technical tool called *arena* to study upper bounds of capacity, and designed a scheme to derive the achievable multicast throughput for random dense networks.

5.2. Hybrid wireless networks

Earlier, Liu et al. [5] introduced a network model where m base stations (BSs) are regularly placed and n ad hoc nodes are randomly distributed in a deployment region of fixed area. The results of [5] showed that if m grows asymptotically slower than $\sqrt{n}$, the benefit of adding base stations on *unicast capacity* is insignificant. However, if m grows faster than $\sqrt{n}$, the unicast capacity increases linearly with the number of base stations. The scaling model of [5] is indeed the *hybrid dense network* that can be regarded as a special case of the model of this paper by letting $A = 1$. Note that the results in [5] are derived under the *protocol model* (ProM). Also adopting the ProM, Mao et al. examined the *multicast capacity* of hybrid wireless networks for the case of $m = O\left(\frac{n}{\log n}\right)$. A characteristic of the work in [17] is that the network is with a general node density as in the model of this paper. However, they assumed that the communication range under the ProM can increase linearly with the side-length of deployment region, *i.e.*, $\sqrt{A}$. That implies that the transmitting power of each ad hoc nodes will enhance to infinity when the area of deployment region goes to infinity. It is obviously unrealistic for the practical wireless networks where all users (ad hoc nodes) are power-limited. Thus, the results of [17] are only applicable to the *hybrid dense networks* indeed for which the ProM is reasonable, [13,25]. There are also some other works to study the unicast capacity under ProM for hybrid dense networks, such as [15,38,39]. Later, as another representative scaling model, the *hybrid extended network* was studied by [16,26]. Taking the limitation of ProM and PhyM into account, both works properly adopted the *Gaussian channel model* (GCM), *i.e.*, the *generalize physical model*. Liu et al. [16] focused on the *unicast capacity* for hybrid extended networks, and showed that the condition $m = \Omega(\sqrt{n})$ is also necessary to obtain a linear gain of unicast capacity under GCM for hybrid extended networks. While, Wang et al. [26] investigated the multicast capacity under GCM only for *hybrid extended networks*. Compared to [26], our work in this paper is more general in terms of scaling models, which can offer more insights about the scaling behaviors for hybrid wireless networks. Besides this, by introducing the *sparse backbone* system, instead of the *parallel connectivity paths* system in [26], we can further improve the multicast throughput for the case that $m = \Omega(d)$ and $\frac{m}{d}\log\frac{m}{d} = \Omega(n)$.

6. Conclusion

We study the achievable multicast throughput for the hybrid wireless network with a general node density under Gaussian Channel model. As in most existing works for the capacity of hybrid wireless networks, we also assume that the links between base stations and ordinary ad hoc nodes (we call such links *B-O links*) have the same bandwidth as links between ordinary ad hoc nodes. While, we prove that under the hybrid schemes the bottlenecks are located on *B-O links*. Therefore, if the bandwidth of *B-O links* can be increased, the throughput of the network can be enhanced. We designedly derive the multicast throughput without considering the possible bottlenecks on the *B-O links*. These results could be used when some new assumptions are made for the *B-O links*.

Since our schemes do not use the method based on percolation theory [36], it is a future work to improve the throughput by exploiting the connection between percolation theory and the way to scale the transmission ranges of nodes [10]. On the other hand, due to not using percolation theory, our schemes have no bottleneck on the accessing paths into the *highways* [10,28,24,25]. Hence, for some cases of n and d, our schemes can act as the complements of those schemes based on percolation routing [10,24,25].

Finally, to the best of our knowledge, even for pure wireless ad hoc networks, there are still no matching upper bounds and lower bounds for multicast capacity under Gaussian Channel model. The same question holds for *hybrid networks*. Then, it is also an interesting issue to be studied.

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Appendix A. Useful known results

A.1. Useful results of occupancy theory

We use the results on the *maximum occupancy* to derive the lower bounds of the multicast throughput. We recall the following result from [40–42].

Lemma 19. *Let* $\mathbf{L}(m,n)$ *be the random variable that counts the maximum number of balls in any bin, if we throw m balls independently and uniformly at random into n bins. Then, it holds w.h.p. that,*

$$\mathbf{L}(m,n)=\begin{cases}\Theta\left(\frac{\log n}{\log\frac{n}{m}}\right) & \text{when } m:\left[1,\frac{n}{\text{polylog}(n)}\right)\\ \Theta\left(\frac{\log n}{\log\frac{n\log n}{m}}\right) & \text{when } m:\left[\frac{n}{\text{polylog}(n)},n\log n\right)\\ \Theta\left(\frac{m}{n}\right) & \text{when } m=\Omega(n\log n)\end{cases} \tag{A.1}$$

A.2. The tail of binomial distribution

Lemma 20 [43]. *Consider n independent random variables* $X_i\in\{0,1\}$ *with* $p=Pr(X_i=1)$. *Let* $X=\sum_{i=1}^{n}X_i$. *Then,*

$$\Pr(X\leqslant\xi)\leqslant e^{\frac{-2\cdot(n\cdot p-\xi)^2}{n}}\quad \text{when } 0<\xi\leqslant n\cdot p, \tag{A.2}$$

$$\Pr(X>\xi)\leqslant\frac{\xi\cdot(1-p)}{(\xi-n\cdot p)^2}\quad \text{when } \xi>n\cdot p. \tag{A.3}$$

Appendix B. Proofs of some lemmas

B.1. Proof of Lemma 2

Proof. For any link on the sparse backbone in any time slot, the transmitters in the eight closest cells are located at Euclidean distance at least $3\sqrt{A\log n/n}$ from the receiver; the 16 next closest cells are at Euclidean distance at least $4\times\left(3\sqrt{A\log n/n}\right)$, and so on. By extending the sum of the interferences to the whole region, this can then be bounded as follows:

$$\mathrm{I}(n,A)\leqslant\sum_{i=1}^{n}8iP\cdot\ell\left((3i-2)\cdot3\sqrt{\frac{A\log n}{n}}\right)$$
$$\leqslant 9^{1-\frac{\alpha}{2}}\cdot P\cdot\left(\frac{n}{A\log n}\right)^{\frac{\alpha}{2}}\cdot\sum_{i=1}^{\infty}\frac{i}{(3i-2)^{\alpha}}$$

since $\alpha>2$, we get that $\mathrm{I}(n,A)=O\left(\left(\frac{n}{A\log n}\right)^{\frac{\alpha}{2}}\right)$. Because the distance of every hop is at most of $\sqrt{5}\cdot3\sqrt{A\log n/n}$, the signal strength at the receiver can be bounded as $\mathrm{S}(n,A)\geqslant45^{-\frac{\alpha}{2}}P\cdot(n/A\log n)^{\frac{\alpha}{2}}$.Then, $\mathrm{S}(n,A)=\Omega\left(\left(\frac{n}{A\log n}\right)^{\frac{\alpha}{2}}\right)$. Thus,

$$\mathbf{R}(n,A)=\frac{1}{9}\cdot B\log\left(1+\frac{\mathrm{S}(n,A)}{N_0+\mathrm{I}(n,A)}\right)$$
$$=\begin{cases}\Omega\left(\left(\frac{n}{\log n}\right)^{\frac{\alpha}{2}}\cdot A^{-\frac{\alpha}{2}}\right) & \text{when } A:\left[\frac{n}{\log n},n\right],\\ \Omega(1) & \text{when } A:\left[1,\frac{n}{\log n}\right].\end{cases}$$

Hence, the lemma holds. □

B.2. Proof of Lemma 4

Proof. Let $\lambda:=\frac{n}{A}$. For any link on the dense backbone in any time slot, since the length of the link is at least of $\sqrt{A\log n/n}$, we can bound the sum of interferences to the receivers as:

$$\mathrm{I}(n,A)\leqslant P\cdot(2\log n-1)\cdot\ell\left(\sqrt{\frac{\log n}{\lambda}}\right)+\sum_{i=1}^{n}8i\cdot P\cdot(2\log n)$$
$$\times\ell\left(((2i-2)\times3+1)\cdot\sqrt{\frac{\log n}{\lambda}}\right)$$
$$\leqslant 2^{1-\frac{3}{2}\alpha}\cdot(\log n)^{1-\frac{\alpha}{2}}\cdot\lambda^{\frac{\alpha}{2}}\times\left(1+\lim_{n\to\infty}\sum_{i=1}^{n}\frac{8i}{(6i-5)^{\alpha}}\right).$$

The latest limitation obviously converges when $\alpha>2$. Then,

$$\mathrm{I}(n,A)=O\left((\log n)^{1-\frac{\alpha}{2}}\cdot\lambda^{\frac{\alpha}{2}}\right). \tag{B.1}$$

Since the distance of every hop is at most $\sqrt{2^2+5^2}\cdot\left(\sqrt{\log n/\lambda}\right)$, we have the signal $\mathrm{S}(n,A)$ at the receiver can be bounded as

$$\mathrm{S}(n,A)\geqslant P\cdot29^{-\frac{\alpha}{2}}\cdot(\log n)^{-\frac{\alpha}{2}}\cdot\lambda^{\frac{\alpha}{2}}.$$

Then, we get that

$$\mathrm{S}(n,A)=\Omega\left((\log n)^{-\frac{\alpha}{2}}\cdot\lambda^{\frac{\alpha}{2}}\right). \tag{B.2}$$

From Eqs. (B.1) and (B.2), we have:

Case 1: When $\lambda:\left[1,(\log n)^{1-\frac{2}{\alpha}}\right]$, it holds that $\frac{\mathrm{S}(n,A)}{N_0+\mathrm{I}(n,A)}:\left[\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}},1\right)$, then,

$$\mathbf{R}(n,A)=\frac{1}{4}\cdot B\log\left(1+\frac{\mathrm{S}(n,A)}{N_0+\mathrm{I}(n,A)}\right)=\Omega\left(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}\right).$$

Case 2: When $\lambda : \left[(\log n)^{1-\frac{2}{\alpha}}, n\right]$, it holds that $\frac{S(n,A)}{N_0+I(n,A)} = \Omega\left(\frac{1}{\log n}\right)$, then, $\mathbf{R}(n,A) = \Omega\left(\frac{1}{\log n}\right)$.

Combining two cases, we complete the proof. □

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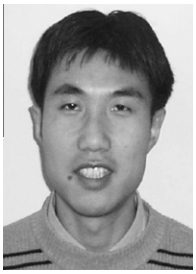

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