$CS06201a01$: Network Computing and Efficient Algorithms

Lecture 13: Game

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Review of related concepts

Game Theory is the formal study of strategic interaction

- In a strategic setting the actions of several agents are interdependent.
- Each agent's outcome depends not only on his auctions, but also on the actions of other agents.
- How to predict opponents' play and respond optimally?

Review of related concepts

- Everything is a game $\cdots$
  - poker,chess,soccer,driving,dating,stock market
  - advertising, setting prices, entering new markets, building a reputation
  - bargaining, partnerships, job market search and screening
  - designing contracts, auctions, insurance, environmental regulations
  - international relations, trade agreements, electoral campaigns

Everything is a game ⋯

- Most modern economic research includes game theoretical elements.
- Eleven game theorists have won the economics Nobel Prize so far.

Key Elements of Game

Games are thought experiments to help us learn how to predict rational behavior in situations of conflict.

- Situation of conflict: Everybody's actions affect others. This is captured by the tabular game formalism.
- Rational Behavior: The players want to maximize their own expected utility. No altruism, envy, masochism, or externalities (if my neighbor gets the money, he will buy louder stereo, so I will hurt a little myself...).
- Intelligence The players are able to calculate their optimal strategies.
- Predict: We want to know what happens in a game. Such predictions are called solution concepts (e.g., Nash equilibrium).

Key Elements of Game

- Players: Who is interacting?
- Strategies: What are the options of each player? In what order do players act?
- Payoffs: How do strategies translate into outcomes? What are players' preferences over possible outcomes?
- Information/Beliefs: What do players know/believe about the situation and about one another? What actions do they observe before making decisions?
- Rationality: How do players think?

Concepts

- Decision-makers choice(s) in any given situation
- Fully known to the decision-maker
- Examples
  - Price set by a firm
  - Bids in an auction
  - Routing decision by a routing algorithm
- Strategy space: set of all possible actions
  - Finite vs infinite strategy space
- Pure vs mixed strategies
  - Pure: deterministic actions
  - Mixed: randomized actions

Concepts (Cont'd)

- Preference Transitive ordering among strategies if

$$a >> b, b >> c, \text{ then } a >> c$$

- Payoff
  - An order-preserving mapping from preference to R+
  - Example: in flow control,

$$U(x) = log(1+x)px$$

Two axiomatic assumptions on games

- In any given situation a decision-maker always chooses the action which is the best according to his/her preferences (a.k.a. rational play).
- Rational play is common knowledge among all players in the game.

Game Variants

- Static vs multi-stage
- Stackelberg Games
- Complete and incomplete information
- Non-cooperative game and Cooperative game

Static vs Multi-stage

- Static: game is played only once
  - Prisoners dilemma
- Multi-stage: game is played in multiple rounds
  - Multi-round auctions, chess games

Stackelberg Games

- Model
  - One player (leader) has dominate influence over another
  - Typically there are two stages
  - One player moves first
  - Then the other follows in the second stage
  - Can be generalized to have
    - multiple groups of players
    - Static games in both stages
- Main Theme
  - Leader plays by backwards induction, based on the anticipated behavior of his/her follower.

Complete and Incomplete information

- Complete information
  - The utility functions (including risk aversion), payoffs, strategies and types of players are common knowledge.
- Incomplete information
  - Players do not posses full information about their opponents.
  - Examples: Auctions: sellers, buyers unsure of other buyers valuations

Cooperative and Non-cooperative game

- Cooperative game
  - Players can coordinate their strategies and share the payoff.
  - Focus on predicting which coalitions will form, the joint actions that groups take and the resulting collective payoffs.
- Non-cooperative game
  - Game with competition between individual players
  - Focus on predicting players individual strategies and payoffs and to find Nash equilibria.

Example 1: Prisoners Dilemma

- The prisoner's dilemma is a paradox in decision analysis in which two individuals acting in their own self-interests do not produce the optimal outcome.
- The typical prisoner's dilemma is set up in such a way that both parties choose to protect themselves at the expense of the other participant.
- As a result, both participants find themselves in a worse state than if they had cooperated with each other in the decision-making process.
- The prisoner's dilemma is one of the most well-known concepts in modern game theory.

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