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Brief Paper

On robust stabilization of Markovian jump systems with uncertain switching probabilities☆

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Abstract

This brief paper is concerned with the robust stabilization problem for a class of Markovian jump linear systems with uncertain switching probabilities. The uncertain Markovian jump system under consideration involves parameter uncertainties both in the system matrices and in the mode transition rate matrix. First, a new criterion for testing the robust stability of such systems is established in terms of linear matrix inequalities. Then, a sufficient condition is proposed for the design of robust state-feedback controllers. A globally convergent algorithm involving convex optimization is also presented to help construct such controllers effectively. Finally, a numerical simulation is used to illustrate the developed theory.

Keywords: Linear matrix inequalities; Markovian parameters; Robust stability; Uncertain systems

1. Introduction

Markovian jump linear systems (MJLSs) have been intensively studied over the past decade. The reason is mainly that MJLS is a suitable mathematical model to represent a class of dynamic systems subject to random abrupt variations in their structures, and has many applications such as target tracking problems, manufactory processes, and fault-tolerant systems (Mariton, 1990). From a mathematical point of view, MJLSs can be regarded as a special class of stochastic systems with system matrices changed randomly at discrete time instances governed by a Markov process, and remain linear time invariant between the random jumps. MJLSs also belong to the category of hybrid systems with finite operation modes. Each operation mode corresponds to some dynamic system, and the mode transitions from one to another are governed by a Markov process as well. The analysis and synthesis problems have been extensively studied, such as the controllability and observability (Ji & Chizeck, 1990), stability and stabilization (Ji & Chizeck, 1990; Feng, Loparo, Ji, & Chizeck, 1992; El Ghaoui & Rami, 1996; Boukas, Shi, & Benjelloun, 1999; Mao, 2002), H_2 control (Costa, Val, & Geromel, 1999; do Val, Geromel, & Goncalves, 2002), H_∞ control (de Farias, Geromel, do Val, & Costa, 2000), filtering (Xu, Chen, & Lam, 2003) and model reduction (Zhang, Huang, & Lam, 2003). In particular, for linear continuous-time MJLSs with uncertainties only in system matrices, the robust stability property can be tested by checking the existence of the solution to a set of coupled linear matrix inequalities (LMIs) (Shi, Boukas, & Agarwal, 1999).

Unfortunately, almost all of the work done on robust control of MJLSs is built upon the assumption that switching probabilities are known precisely a priori. However, in

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practice, only estimated values of mode transition rates are available, and estimation errors, referred to as switching probability uncertainties, may also lead to instability or at least degraded performance of a system as the uncertainties in system matrices do. This point is demonstrated by the numerical simulation presented in this paper. Therefore, further work is needed to tackle the more realistic situation with uncertain switching probabilities. In the literature, two different types of descriptions about uncertain switching probabilities have been considered in the context of robust stabilization. The first one is the polytopic description where the mode transition rate matrix is assumed to be in a convex hull with known vertices (El Ghaoui & Rami, 1996; Costa et al., 1999). The other type is described in an element-wise way. In this case, the elements of the mode transition rate matrix are measured in practice and error bounds are given at the same time (Benjelloun, Boukas, & Shi, 1997; Boukas et al., 1999). In many situations, the element-wise uncertainty description can be more convenient as well as natural. On the other hand, the element-wise description can be formulated into an equivalent polytopic description, but the total number of vertex matrices of the convex hull will be extremely large when the total number of system modes is greater than *three*. In this paper, we consider the element-wise uncertainties in the mode transition rate matrix and propose a new criterion for robust stability and a new approach for the robust stabilization of the uncertain MJLSs.

It is important to point out that the feasible solution set to the robust stabilization problem in this paper is not convex. Similar conditions also appear in the H_∞ control problem for linear time-invariant systems using reduced-order output-feedback controllers (El Ghaoui, Oustry, & Rami, 1997; Leibfritz, 2001), and the robust H_∞ control problem for linear time-delay systems (Lee, Moon, Kwon, & Park, 2004). Numerically, it is difficult to solve such non-convex problem directly. However, the cone complementarity linearization algorithm (El Ghaoui et al., 1997) and the sequential linear programming matrix method (SLPMM) (Leibfritz, 2001) have been developed for this purpose based on the LMI machinery. Consequently, such problems can be solved systematically and effectively.

This paper considers the robust stabilization problem for MJLSs with uncertain switching probabilities. The aim is to design a robust state-feedback controller such that the closed-loop system is quadratically mean square stable over all admissible uncertainties both in system matrices and in the mode transition rate matrix. The analysis problem can be tackled in terms of the solvability of a set of coupled LMIs, and the associated synthesis problem can be dealt with using the SLPMM algorithm. A numerical example is used to demonstrate the usefulness and applicability of the developed theory.

Notation: The notations in this paper are standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $m \times n$ real matrices. $\mathbb{R}^+$ and $\mathbb{S}^{n\times n}$ denote the set of all positive real numbers and the set of all $n \times n$ real symmetric positive definite matrices, respectively. The notation $X \geqslant Y$ (respectively, $X > Y$) where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive-definite). I is the identity matrix with compatible dimensions. The superscript "T" denotes the transpose for vectors or matrices, and trace$(\cdot)$ stands for the trace of a square matrix. $\|\cdot\|_2$ refers to the Euclidean norm for vectors. Moreover, let $(\Omega, \mathscr{F}, \mathrm{P})$ be a complete probability space. $\mathrm{E}(\cdot)$ stands for the mathematical expectation operator.

2. Problem statement

Consider the following MJLSs, defined on a complete probability space $(\Omega, \mathscr{F}, \mathrm{P})$:

$$\dot{x}(t) = \hat{A}(\hat{r}(t))x(t) + \hat{B}(\hat{r}(t))u(t), \quad t \geqslant 0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input. The mode jumping process $\{\hat{r}(t), t \geqslant 0\}$ is a continuous-time, discrete-state homogeneous Markov process on the probability space, takes values in a finite state space $\mathscr{S} \triangleq \{1, 2, \ldots, s\}$, and has the mode transition probabilities

$$\Pr(\hat{r}(t+\delta t) = j|\hat{r}(t) = i) = \begin{cases} \hat{\pi}_{ij}\delta t + o(\delta t) & \text{if } j \neq i, \\ 1 + \hat{\pi}_{ii}\delta t + o(\delta t) & \text{if } j = i, \end{cases}$$

where $\delta t > 0$, $\lim_{\delta t \to 0} o(\delta t)/\delta t = 0$, and $\hat{\pi}_{ij} \geqslant 0$ $(i, j \in \mathscr{S}, j \neq i)$ denotes the switching rate from mode i to mode j and $\hat{\pi}_{ii} \triangleq -\sum_{j=1, j\neq i}^{s} \hat{\pi}_{ij}$ for all $i \in \mathscr{S}$.

In this paper, both system matrices $\hat{A}_i \triangleq \hat{A}(\hat{r}(t) = i)$, $\hat{B}_i \triangleq \hat{B}(\hat{r}(t) = i)$, $i \in \mathscr{S}$, and the mode transition rate matrix $\hat{\Pi} \triangleq (\hat{\pi}_{ij})$ are not precisely known a priori, but belong to the following admissible uncertainty domains, respectively:

$$\mathscr{D}_a \triangleq \{A_i + E_i F_i H_{ai} : F_i^{\mathrm{T}} F_i \leqslant I \text{ for all } i \in \mathscr{S}\}, \tag{2a}$$

$$\mathscr{D}_b \triangleq \{B_i + E_i F_i H_{bi} : F_i^{\mathrm{T}} F_i \leqslant I \text{ for all } i \in \mathscr{S}\}, \tag{2b}$$

$$\mathscr{D}_\pi \triangleq \{\Pi + \Delta\Pi : |\Delta\pi_{ij}| \leqslant \varepsilon_{ij}, \ \varepsilon_{ij} \geqslant 0 \text{ for all } i, j \in \mathscr{S}, j \neq i\}, \tag{2c}$$

where matrices $A_i, B_i, E_i, H_{ai}, H_{bi}$ and $\Pi \triangleq (\pi_{ij})$ are known constant real matrices of appropriate dimensions, while F_i and $\Delta\Pi \triangleq (\Delta\pi_{ij})$ denote the uncertainties in the system matrices and the mode transition rate matrix, respectively. For all $i, j \in \mathscr{S}, j \neq i$, $\pi_{ij} (\geqslant 0)$ denotes the estimated value of $\hat{\pi}_{ij}$, and the error between them is referred as to $\Delta\pi_{ij}$ which can take any value in $[-\varepsilon_{ij}, \varepsilon_{ij}]$; For all $i \in \mathscr{S}$, $\pi_{ii} \triangleq -\sum_{j=1, j\neq i}^{s} \pi_{ij}$ and $\Delta\pi_{ii} \triangleq -\sum_{j=1, j\neq i}^{s} \Delta\pi_{ij}$.

Let $x(t; x_0, \hat{r}_0)$ be the trajectory of the system state of (1) from initial system state $x_0 \triangleq x(0) \in \mathbb{R}^n$ and initial operation mode $\hat{r}_0 \triangleq \hat{r}(0) \in \mathscr{S}$. We have the following definitions and proposition throughout the paper.

Definition 1 (de Farias et al., 2000). The nominal Markovian jump system of (1) (with $u(t) \equiv 0$) is said to be mean square stable if

$$\lim_{t\to\infty} \mathrm{E}\left(\|x(t; x_0, \hat{r}_0)\|_2^2\right) = 0$$

for any initial conditions $x_0 \in \mathbb{R}^n$ and $\hat{r}_0 \in \mathscr{S}$.

Proposition 2 (de Farias et al., 2000). *The nominal Markovian jump system of* (1) (*with* $u(t) \equiv 0$) *is mean square stable if and only if the coupled LMIs*

$$A_i^{\mathrm{T}} P_i + P_i A_i + \sum_{j=1}^{s} \pi_{ij} P_j < 0 \quad \text{for all } i \in \mathscr{S} \tag{3}$$

are feasible for a set of matrices $\{P_i : P_i \in \mathbb{S}^{n\times n}, i \in \mathscr{S}\}$.

Definition 3. The uncertain Markovian jump system (1) (with $u(t) \equiv 0$) is said to be quadratically mean square stable if there exists a set of matrices $\{P_i : P_i \in \mathbb{S}^{n\times n}, i \in \mathscr{S}\}$ such that

$$\hat{A}_i^{\mathrm{T}} P_i + P_i \hat{A}_i + \sum_{j=1}^{s} \hat{\pi}_{ij} P_j < 0 \quad \text{for all } i \in \mathscr{S} \tag{4}$$

hold over all admissible uncertainty domains in (2).

3. Robust stability analysis

The goal of this section is to develop a new analysis result of robust stability for system (1) with uncertainty domains (2). Here, we establish a new criterion for testing the robust stability property in terms of LMIs.

Theorem 4. *Uncertain Markovian jump system* (1) (*with* $u(t) \equiv 0$) *is quadratically mean square stable if there exist sets*: $\{P_i : P_i \in \mathbb{S}^{n\times n}, i \in \mathscr{S}\}$, $\{\lambda_i : \lambda_i \in \mathbb{R}^+, i \in \mathscr{S}\}$ *and* $\{\lambda_{ij} : \lambda_{ij} \in \mathbb{R}^+, i, j \in \mathscr{S}, j \neq i\}$ *such that*

$$\begin{bmatrix} Q_i & P_i E_i & M_i \\ E_i^{\mathrm{T}} P_i & -\lambda_i I & 0 \\ M_i^{\mathrm{T}} & 0 & -\Lambda_i \end{bmatrix} < 0 \quad \text{for all } i \in \mathscr{S}, \tag{5}$$

where

$$Q_i = A_i^{\mathrm{T}} P_i + P_i A_i + \sum_{j=1}^{s} \pi_{ij} P_j + \frac{1}{4} \sum_{j=1, j\neq i}^{s} \lambda_{ij} \varepsilon_{ij}^2 I + \lambda_i H_{ai}^{\mathrm{T}} H_{ai},$$

$$M_i = \begin{bmatrix} P_i - P_1 & \cdots & P_i - P_{i-1} & P_i - P_{i+1} & \cdots & P_i - P_s \end{bmatrix},$$

$$\Lambda_i = \mathrm{diag}(\lambda_{i1} I, \ldots, \lambda_{i(i-1)} I, \lambda_{i(i+1)} I, \ldots, \lambda_{is} I).$$

Proof. Consider the uncertain system (1) with $u(t) \equiv 0$ and the admissible uncertainty domains (2), we have $\hat{A}_i = A_i + E_i F_i H_{ai}$ and $\hat{\pi}_{ij} = \pi_{ij} + \Delta\pi_{ij}$ for all $i, j \in \mathscr{S}$. According to Definition 3, the uncertain Markovian jump system (1) is quadratically mean square stable if there exists a set of matrices $\{P_i : P_i \in \mathbb{S}^{n\times n}, i \in \mathscr{S}\}$ such that

$$(A_i + E_i F_i H_{ai})^{\mathrm{T}} P_i + P_i (A_i + E_i F_i H_{ai}) + \sum_{j=1}^{s} (\pi_{ij} + \Delta\pi_{ij}) P_j < 0$$

for all $i \in \mathscr{S}$. This inequality can be further written as

$$A_i^{\mathrm{T}} P_i + P_i A_i + \sum_{j=1}^{s} \pi_{ij} P_j + H_{ai}^{\mathrm{T}} F_i^{\mathrm{T}} E_i^{\mathrm{T}} P_i + P_i E_i F_i H_{ai} + \sum_{j=1, j\neq i}^{s} \left[\frac{1}{2}\Delta\pi_{ij}(P_j - P_i) + \frac{1}{2}\Delta\pi_{ij}(P_j - P_i)\right] < 0.$$

The above inequality holds for all $F_i^{\mathrm{T}} F_i \leqslant I$ and $|\Delta\pi_{ij}| \leqslant \varepsilon_{ij}$ if there exist real numbers $\lambda_i \in \mathbb{R}^+$, $\lambda_{ij} \in \mathbb{R}^+ (i, j \in \mathscr{S}, j \neq i)$ such that

$$A_i^{\mathrm{T}} P_i + P_i A_i + \sum_{j=1}^{s} \pi_{ij} P_j + \lambda_i H_{ai}^{\mathrm{T}} H_{ai} + \frac{1}{\lambda_i} P_i E_i E_i^{\mathrm{T}} P_i + \sum_{j=1, j\neq i}^{s} \left[\frac{\lambda_{ij}}{4} \varepsilon_{ij}^2 I + \frac{1}{\lambda_{ij}} (P_j - P_i)^2\right] < 0, \tag{6}$$

which is equivalent to inequality (5) in view of Schur complement equivalence. □

Remark 5. Although the uncertainty domain (2c) can be formulated into a fix polytope (El Ghaoui & Rami, 1996; Costa et al., 1999) by introducing $L \triangleq 2^{s(s-1)}$ vertex matrices, the test for the robust stability of the uncertain system (1) needs to check the solvability of a $(L+1)sn \times (L+1)sn$ (compared to $(s+1)sn \times (s+1)sn$ here) linear matrix inequality system with respect to $(n(n+1)/2)s$ (compared to $(n(n+1)/2)s + s(s-1)$ here) scalar variables based on Theorem 3.3 of El Ghaoui and Rami (1996). For example, consider the case $n = 2$, $s = 4$, to test the robust stability, we can translate the uncertainty domain (2c) into a fix polytopic description by introducing 4096 vertex matrices, then we need to test the solvability of a 32776×32776 (compared to 40×40 here) linear matrix inequality system with respect to 12 (compared to 24 here) scalar variables. On the other hand, the literature (Benjelloun et al., 1997; Boukas et al., 1999) have considered the robust stability problem with a similar uncertainty domain to (2c), where they bounded $\Delta\pi_{ij}$ by the upper bound ε_{ij} for all $i, j \in \mathscr{S}$ in every LMI (Theorem 2.2 of Benjelloun et al. (1997), Theorem 3 of Boukas et al. (1999)). This approach may be more conservative than ours in many cases based on numerical experiences.

4. Robust stability synthesis

This section deals with the robust stabilization problem for MJLSs with uncertain switching probabilities. We aim to design a state-feedback controller such that the resulting closed-loop system is quadratically mean square stable over all admissible uncertainty domains in (2).

Consider the state-feedback control law

$$u(t)=K(\hat{r}(t))x(t), \tag{7}$$

where $K_i \triangleq K(\hat{r}(t)=i)\in\mathbb{R}^{m\times n}$ $(i\in\mathscr{S})$ is the controller to be determined. The closed-loop system is

$$\begin{aligned}\dot{x}(t)=\{&A(\hat{r}(t))+B(\hat{r}(t))K(\hat{r}(t))+E(\hat{r}(t))F(\hat{r}(t))\\ &\times[H_a(\hat{r}(t))+H_b(\hat{r}(t))K(\hat{r}(t))]\}x(t).\end{aligned} \tag{8}$$

The following result solves the *robust stabilization problem* (RSP) for system (1) with uncertain switching probabilities.

Theorem 6. *Consider uncertain Markovian jump system* (1), *there exists a state-feedback control law* (7) *such that the closed-loop system* (8) *is quadratically mean square stable if there exist sets of matrices* $P\triangleq\{P_i : P_i\in\mathbb{S}^{n\times n}, i\in\mathscr{S}\}$, $X\triangleq\{X_i : X_i\in\mathbb{S}^{n\times n}, i\in\mathscr{S}\}$, $V\triangleq\{V_i : V_i\in\mathbb{S}^{n\times n}, i\in\mathscr{S}\}$, $Z\triangleq\{Z_i : Z_i\in\mathbb{S}^{n\times n}, i\in\mathscr{S}\}$, $Y\triangleq\{Y_i : Y_i\in\mathbb{R}^{m\times n}, i\in\mathscr{S}\}$, $\Xi\triangleq\{\alpha_i : \alpha_i\in\mathbb{R}^+, i\in\mathscr{S}\}$, $\Lambda\triangleq\{\lambda_{ij} : \lambda_{ij}\in\mathbb{R}^+, i,j\in\mathscr{S}, j\neq i\}$ *satisfying the coupled LMIs*

$$\begin{bmatrix} Q_{1i} & (H_{ai}X_i+H_{bi}Y_i)^{\mathrm{T}} & X_i\\ H_{ai}X_i+H_{bi}Y_i & -\alpha_i I & 0\\ X_i & 0 & -Z_i\end{bmatrix}<0, \tag{9}$$

$$\begin{bmatrix} Q_{2i} & M_i\\ M_i^{\mathrm{T}} & -\Lambda_i\end{bmatrix}\leqslant 0 \tag{10}$$

with equality constraints

$$P_iX_i=I,\quad V_iZ_i=I \tag{11}$$

for all $i\in\mathscr{S}$, *where*

$$\begin{aligned}Q_{1i}&=(A_iX_i+B_iY_i)^{\mathrm{T}}+(A_iX_i+B_iY_i)+\alpha_iE_iE_i^{\mathrm{T}},\\ Q_{2i}&=-V_i+\sum_{j=1}^{s}\pi_{ij}P_j+\frac{1}{4}\sum_{j=1,j\neq i}^{s}\lambda_{ij}\varepsilon_{ij}^2I,\\ M_i&=\begin{bmatrix}P_i-P_1 & \cdots & P_i-P_{i-1} & P_i-P_{i+1} & \cdots & P_i-P_s\end{bmatrix},\\ \Lambda_i&=\operatorname{diag}(\lambda_{i1}I,\ldots,\lambda_{i(i-1)}I,\lambda_{i(i+1)}I,\ldots,\lambda_{is}I).\end{aligned}$$

In this case, controller (7) *is given by* $K_i=Y_iP_i$, $i\in\mathscr{S}$.

Proof. Consider inequality (6), let $V_i\triangleq Z_i^{-1}\in\mathbb{S}^{n\times n}$ for all $i\in\mathscr{S}$ such that

$$\sum_{j=1}^{s}\pi_{ij}P_j+\sum_{j=1,j\neq i}^{s}\left[\frac{\lambda_{ij}}{4}\varepsilon_{ij}^2I+\frac{1}{\lambda_{ij}}(P_i-P_j)^2\right]\leqslant V_i$$

which is equivalent to (10) in view of Schur complement equivalence. Replacing A_i and H_{ai} in (6) with $A_i+B_iK_i$ and $H_{ai}+H_{bi}K_i$, respectively, yields

$$\begin{aligned}&(A_i+B_iK_i)^{\mathrm{T}}P_i+P_i(A_i+B_iK_i)+\frac{1}{\lambda_i}P_iE_iE_i^{\mathrm{T}}P_i\\ &\quad+\lambda_i(H_{ai}+H_{bi}K_i)^{\mathrm{T}}(H_{ai}+H_{bi}K_i)+V_i<0.\end{aligned}$$

Therefore, the closed-loop system (8) is quadratically mean square stable if the above inequality holds for all $i\in\mathscr{S}$. Now, pre- and post-multiply both sides of the above inequality by P_i^{-1} and apply the changes of variables $X_i\triangleq P_i^{-1}$, $Y_i\triangleq K_iX_i$, $\alpha_i\triangleq\lambda_i^{-1}$, we have

$$\begin{aligned}&(A_iX_i+B_iY_i)^{\mathrm{T}}+(A_iX_i+B_iY_i)+\alpha_iE_iE_i^{\mathrm{T}}\\ &\quad+\alpha_i^{-1}(H_{ai}X_i+H_{bi}Y_i)^{\mathrm{T}}(H_{ai}X_i+H_{bi}Y_i)+X_iV_iX_i<0\end{aligned}$$

which is equivalent to (9) by Schur complement equivalence again. □

4.1. Computational method

Define the solution set of Theorem 6 as

$$\mathscr{X}\triangleq\{(P,X,V,Z,Y,\Xi,\Lambda) : (9),(10),(11)\text{ are satisfied}\}.$$

Although the set $\mathscr{X}$ is not convex due to equality constraints (11), SLPMM (Leibfritz, 2001) can be used to solve such non-convex problems.

Firstly, for computational purpose, we introduce a sufficiently small scalar $\beta\in\mathbb{R}^+$ and replace Q_{1i} by $Q_{1i}+\beta I$ for all $i\in\mathscr{S}$, then inequality (9) becomes

$$\begin{bmatrix} Q_{1i}+\beta I & (H_{ai}X_i+H_{bi}Y_i)^{\mathrm{T}} & X_i\\ H_{ai}X_i+H_{bi}Y_i & -\alpha_i I & 0\\ X_i & 0 & -Z_i\end{bmatrix}\leqslant 0. \tag{12}$$

Secondly, to find a solution in $\mathscr{X}$, the equality constraints (11) can be weakened to the semi-definite programming conditions

$$\begin{bmatrix} P_i & I\\ I & X_i\end{bmatrix}\geqslant 0,\quad \begin{bmatrix} V_i & I\\ I & Z_i\end{bmatrix}\geqslant 0\quad\text{for all } i\in\mathscr{S} \tag{13}$$

Notice that the equality constraints in (11) correspond to the boundaries of the convex sets in (13). Finally, let

$$\mathscr{X}_\beta\triangleq\{(P,X,V,Z,Y,\Xi,\Lambda) : (10),(12),(13),\ \text{are satisfied}\}.$$

Now, $\mathscr{X}_\beta$ is a closed and convex set. The SLPMM algorithm can be employed to find a solution of Theorem 6. The solution of RSP is summarized below.

Algorithm RSP. For a given precision $\delta\in\mathbb{R}^+$, let N be the maximum number of iterations, a sufficiently small number $\beta\in\mathbb{R}^+$ be given and $\mathscr{X}_\beta\neq\varnothing$.

(1) Determine $(P^0,X^0,V^0,Z^0,Y^0,\Xi^0,\Lambda^0)\in\mathscr{X}_\beta$, let $k:=0$.

(2) Solve the following convex optimization problem for the variables $(P,X,V,Z,Y,\Xi,\Lambda)\in\mathscr{X}_\beta$:

$$\min_{\mathscr{X}_\beta}\sum_{i=1}^{s}\operatorname{trace}(P_iX_i^k+P_i^kX_i+V_iZ_i^k+V_i^kZ_i),$$

where $P_i^k\in P^k$, $X_i^k\in X^k$, $V_i^k\in V^k$, $Z_i^k\in Z^k$, $i\in\mathscr{S}$.

(3) Let $T_i^k:=P_i$, $L_i^k:=X_i$, $U_i^k:=V_i$, $R_i^k:=Z_i$ for all $i\in\mathscr{S}$.

(4) If

$$\left| \sum_{i=1}^{s} \mathrm{trace}(T_i^k X_i^k + P_i^k L_i^k + U_i^k Z_i^k + V_i^k R_i^k) - 2 \sum_{i=1}^{s} \mathrm{trace}(P_i^k X_i^k + V_i^k Z_i^k) \right| < \delta$$

then go to step (7), else go to step (5).

(5) Compute $\theta^* \in [0, 1]$ by solving

$$\min_{\theta \in [0,1]} g(\theta)$$

where

$$g(\theta) \triangleq \sum_{i=1}^{s} \mathrm{trace}([P_i^k + \theta(T_i^k - P_i^k)][X_i^k + \theta(L_i^k - X_i^k)] + [V_i^k + \theta(U_i^k - V_i^k)][Z_i^k + \theta(R_i^k - Z_i^k)])$$

(6) Let $P_i^{k+1} := P_i^k + \theta^*(T_i^k - P_i^k)$, $X_i^{k+1} := X_i^k + \theta^*(L_i^k - X_i^k)$, $V_i^{k+1} := V_i^k + \theta^*(U_i^k - V_i^k)$, $Z_i^{k+1} := Z_i^k + \theta^*(R_i^k - Z_i^k)$ for all $i \in \mathscr{S}$, and $k := k + 1$. If $k < N$, then go to step (2), else go to step (7).

(7) If $\sum_{i=1}^{s} \mathrm{trace}(P_i^k X_i^k + V_i^k Z_i^k) = 2sn$, then a solution is found successfully, else a solution cannot be found.

5. Numerical example

To illustrate the usefulness and flexibility of the theory developed in the paper, we present a simulation example. Attention is focused on the design of a robust stabilizing controller for a Markovian jump system with uncertain switching probabilities.

Consider a uncertain Markovian jump system (1) with two operation modes. The system data and the initial conditions of (1) are as follows:

$$A_1 = \begin{bmatrix} 0.1769 & 0.7843 \\ 0.9266 & 0.1363 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.5478 & 0.1279 \\ 0.6160 & 0.9657 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2995 \\ 0.4471 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.7417 \\ 0.7957 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} -6.7000 & 6.7000 \\ 6.9180 & -6.9180 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \hat{r}_0 = 1.$$

The uncertainties in the mode transition rate matrix Π are such that $|\Delta\pi_{12}| \leqslant \varepsilon_{12}$ with $\varepsilon_{12} \triangleq \pi_{12}/2$, $|\Delta\pi_{21}| \leqslant \varepsilon_{21}$ with $\varepsilon_{21} \triangleq \pi_{21}/2$. The open-loop system is unstable.

If no uncertainties exist, a stabilizing controller for the nominal system can be obtained by Theorem 6 of (Shi et al., 1999) as

$$K_1 = [-5.8205 - 7.1201], \quad K_2 = [-2.7969 - 3.7968].$$

Applying this controller makes the resulting closed-loop system become mean square stable (see Fig. 1). However, the closed-loop system turns out to be unstable (see Fig. 2) if there exist switching probability uncertainties, say, $\Delta\pi_{12} = \varepsilon_{12}$ and $\Delta\pi_{21} = -\varepsilon_{21}$. Hence, it is necessary to consider the uncertainties in Π when designing controller (7).

Using Algorithm RSP, a robust controller (7) could be achieved such that the closed-loop system is robustly mean square stable over all admissible uncertainties $|\Delta\pi_{12}| \leqslant \varepsilon_{12}$ and $|\Delta\pi_{21}| \leqslant \varepsilon_{21}$ (see Fig. 3). To compute with Algorithm RSP for this example, it is chosen that $\delta = 10^{-10}$, $N = 100$ and $\beta = 0.01$. The controller obtained is

$$K_1 = [-1.6077 - 1.4896], \quad K_2 = [-0.3103 - 2.6795].$$

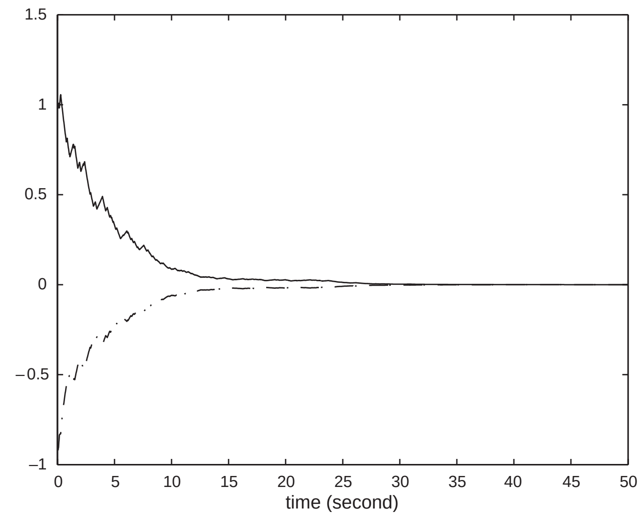

Fig. 1. Closed-loop system without uncertainties.

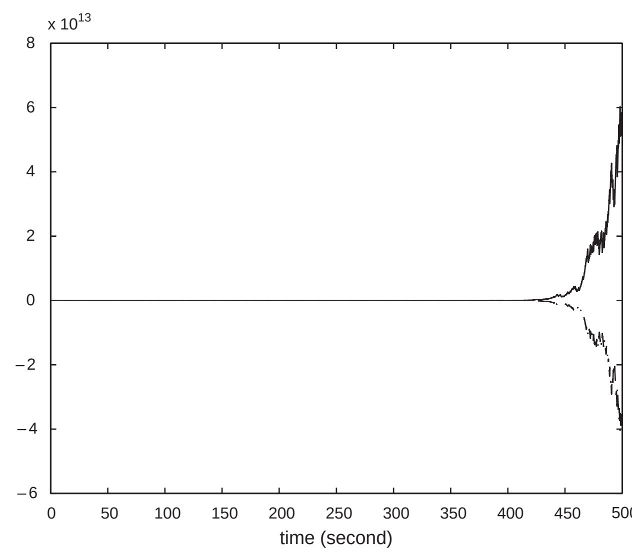

Fig. 2. Closed-loop system with uncertainties.

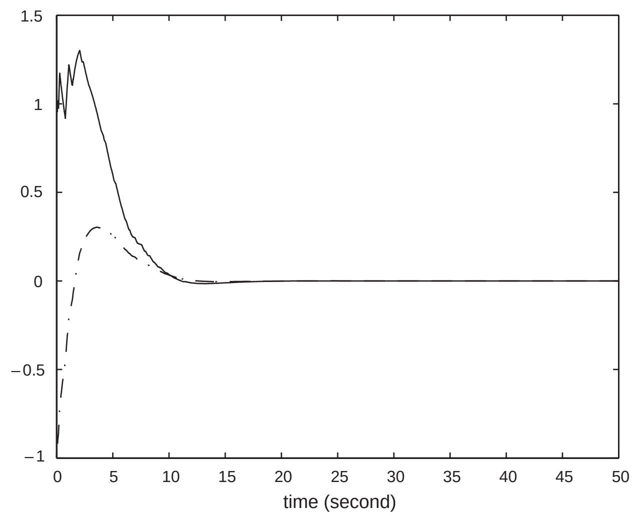

Fig. 3. Closed-loop system with uncertainties.

6. Conclusion

This paper has proposed a robust stabilization method for a class of Markovian jump linear systems with uncertain switching probabilities. The robust stability of such systems can be tested based on the feasibility of a set of coupled linear matrix inequalities. An algorithm involving convex optimization was also proposed to construct a controller such that the uncertain system can be stabilized over all admissible uncertainties in the system matrices as well as in the mode transition rate matrix. A numerical example illustrated that the constructed controller could tolerate the uncertainties in the switching probabilities.

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References

Benjelloun, K., Boukas, E. K., & Shi, P., (1997). Robust stabilizability of uncertain linear systems with Markovian jumping parameters. In *Proceedings of the American control conference*. New Mexico USA, (pp. 866–867).

Boukas, E. K., Shi, P., & Benjelloun, K. (1999). On stabilization of uncertain linear systems with jump parameters. *International Journal of Control*, *72*(9), 842–850.

Costa, O. L. V., Val, J. B. R., & Geromel, J. C. (1999). Continuous-time state-feedback H_2-control of Markovian jump linear system via convex analysis. *Automatica*, *35*, 259–268.

de Farias, D. P., Geromel, J. C., do Val, J. B. R., & Costa, O. L. V. (2000). Output feedback control of Markov jump linear systems in continuous-time. *IEEE Transactions on Automatic Control*, *45*(5), 944–949.

do Val, J. B. R., Geromel, J. C., & Goncalves, A. P. C. (2002). The H_2-control for jump linear systems: cluster observations of the Markov state. *Automatica*, *38*, 343–349.

El Ghaoui, L., Oustry, F., & Rami, M. A. (1997). A cone complementarity linearization algorithm for static output-feedback and related problems. *IEEE Transactions on Automatic Control*, *42*(8), 1171–1176.

El Ghaoui, L., & Rami, M. A. (1996). Robust state-feedback stabilization of jump linear systems via LMIs. *International Journal of Robust and Nonlinear Control*, *6*(9–10), 1015–1022.

Feng, X., Loparo, K. A., Ji, Y., & Chizeck, H. J. (1992). Stochastic stability properties of jump linear systems. *IEEE Transactions on Automatic Control*, *37*(1), 38–53.

Ji, Y., & Chizeck, H. J. (1990). Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control. *IEEE Transactions on Automatic Control*, *35*(7), 777–788.

Leibfritz, F. (2001). An LMI-based algorithm for designing suboptimal static H_2/H_∞ output feedback controllers. *SIAM Journal on Control and Optimization*, *39*(6), 1711–1735.

Lee, Y. S., Moon, Y. S., Kwon, W. H., & Park, P. G. (2004). Delay-dependent robust H_∞ control for uncertain systems with a state-delay. *Automatica*, *40*, 65–72.

Mao, X. (2002). Exponential stability of stochastic delay interval systems with Markovian switching. *IEEE Transactions on Automatic Control*, *47*(10), 1604–1612.

Mariton, M. (1990). *Jump linear systems in automatic control*. New York: Marcel Dekker.

Shi, P., Boukas, E. K., & Agarwal, R. K. (1999). Kalman filtering for continuous-time uncertain systems with Markovian jumping parameters. *IEEE Transactions on Automatic Control*, *44*(8), 1592–1597.

Xu, S., Chen, T., & Lam, J. (2003). Robust H_∞ filtering for uncertain Markovian jump systems with mode-dependent time delays. *IEEE Transactions on Automatic Control*, *48*(5), 900–907.

Zhang, L., Huang, B., & Lam, J. (2003). H_∞ model reduction of Markovian jump linear systems. *Systems & Control Letters*, *50*, 103–118.

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