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Brief paper

Robust decentralized stabilization of Markovian jump large-scale systems: A neighboring mode dependent control approach☆

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ABSTRACT

This paper is concerned with the decentralized stabilization problem for a class of uncertain large-scale systems with Markovian jump parameters. The controllers use local subsystem states and neighboring mode information to generate local control inputs. A sufficient condition involving rank constrained linear matrix inequalities is proposed for the design of such controllers. A numerical example is given to illustrate the developed theory.

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1. Introduction

Many physical systems, such as power systems and economic systems, often suffer from random changes in their parameters. These parameter changes may result from abrupt environmental disturbances, component failures or repairs, etc. In many cases, a Markov chain provides a suitable model to describe the system parameter changes. A Markovian jump system is a hybrid system with different operation modes. Each operation mode corresponds to a deterministic system and the jumping transition from one mode to another is governed by a Markov chain. Recently, Markovian jump systems have received a lot of attention and many control issues have been studied, such as stability and stabilization (de Souza, 2006; Feng, Lam, & Shu, 2010), time delay (Fei, Gao, & Shi, 2009; Sun, Lam, Xu, & Zou, 2007), filtering (Shi, Boukas, & Agarwal, 1999; Wu, Shi, Gao, & Wang, 2008), H_2 control (Dong & Yang, 2008), H_∞ control (Li & Ugrinovskii, 2007; Xu & Chen, 2002), and model reduction (Zhang, Huang, & Lam, 2003). For more information on Markovian jump systems, we refer the reader to Mao and Yuan (2006).

In this paper, we consider the decentralized stabilization problem for a class of uncertain Markovian jump large-scale systems. The aim is to design a set of appropriate local feedback control laws, such that the resulting closed-loop large-scale system is stable even in the presence of uncertainties. Recently, the decentralized stabilization problem for Markovian jump large-scale systems has been investigated in the literature; see e.g., Li, Ugrinovskii, and Orsi (2007), Ugrinovskii and Pota (2005) and the references therein. It is important to point out that the stabilizing techniques developed in Li et al. (2007), Ugrinovskii and Pota (2005) and many other papers are built upon an implicit assumption that the mode information of the large-scale system must be known to all of the local controllers. In other words, the mode information of all the subsystems must be measured and then broadcast to every local controller. Such an assumption, however, may be unrealistic either because the broadcast of mode information among the subsystems is impossible in practice or because the implementation is expensive.

To eliminate the need for broadcasting mode information, a local mode dependent control approach has been developed in Xiong, Ugrinovskii, and Petersen (2009, 2010). This control approach is fully decentralized. The local controllers use only local subsystem states or outputs and local subsystem mode information to generate local control inputs. To emphasize this feature, this type of controller is referred to as a *local mode dependent controller* in Xiong et al. (2009, 2010). As pointed out

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in Xiong et al. (2009, 2010), the local mode dependent control approach offers many advantages in practice. First, it eliminates the need for broadcasting mode information among the subsystems and hence is more suitable for practical applications. Second, it significantly reduces the number of control gains and hence results in cost reduction, easier installation and maintenance.

In this paper, we focus on the state feedback case of Markovian jump large-scale systems and aim to build a bridge between the results in Ugrinovskii and Pota (2005) and Xiong et al. (2009). We assume that each local controller is able to access and utilize mode information of its neighboring subsystems including the subsystem it controls. This assumption is motivated by the fact that some subsystems may be close to each other in practice and hence exchange of mode information may be possible among these subsystems. Under this assumption, we develop an approach, which we call a *neighboring mode dependent control approach*, to stabilize Markovian jump large-scale systems. Compared to the local mode dependent control approach, our approach can stabilize a wider range of large-scale systems in practice. It is demonstrated in the numerical section that the system performance will improve as more detailed mode information is available to the local controllers. Hence the system performance achieved by our approach is better than that achieved by the local mode dependent control approach. Furthermore, both the global and the local mode dependent control approaches proposed in Ugrinovskii and Pota (2005) and Xiong et al. (2009) can be regarded as special cases of the neighboring mode dependent control approach.

Notation: $\mathbb{R}^+$ denotes the set of positive real numbers; $\mathbb{S}^+$ denotes the set of positive definite matrices; $\mathbb{R}^m$ denotes the set of real $m \times 1$ vectors; $\mathbb{R}^{m\times n}$ denotes the set of real $m \times n$ matrices. $\mathrm{diag}[F_1, \ldots, F_N]$ denotes a block diagonal matrix with $F_1, \ldots, F_N$ on the main diagonal. I is the identity matrix. For real symmetric matrices X and Y, $X \geq Y$ (respectively, $X > Y$) means that $X - Y$ is positive semi-definite (respectively, positive definite). $\|\cdot\|$ denotes either the Euclidean norm for vectors or the induced 2-norm for matrices. The superscript "T" denotes transpose of a vector or a matrix. $\mathrm{E}(\cdot)$ denotes the expectation operator with respect to the underlying complete probability space $(\Omega, \mathcal{F}, \mathrm{Pr})$.

2. Problem formulation

Consider a Markovian jump large-scale system $\mathcal{S}$ comprising N subsystems $\mathcal{S}_i$, $i \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$. The ith subsystem $\mathcal{S}_i$ is of the following form (Xiong et al., 2009):

$$\mathcal{S}_i : \begin{cases} \dot{x}_i(t) = A_i(\eta_i(t))x_i(t) + B_i(\eta_i(t))u_i(t) \\ \qquad\quad + E_i(\eta_i(t))\xi_i(t) + L_i(\eta_i(t))r_i(t), \\ \zeta_i(t) = H_i(\eta_i(t))x_i(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state, $u_i(t) \in \mathbb{R}^{m_i}$ is the input, $\xi_i(t) \in \mathbb{R}^{g_i}$ is the local uncertainty input, $r_i(t) \in \mathbb{R}^{s_i}$ is the interconnection input, which describes the effect of the other subsystems $\mathcal{S}_j, j \neq i$, on $\mathcal{S}_i$. $\zeta_i(t) \in \mathbb{R}^{h_i}$ is the uncertainty output. The initial state $x_i(0)$ is denoted by x_{i0}. The random process $\eta_i(t)$ denotes the mode switching of the subsystem $\mathcal{S}_i$; it takes values in a finite set $\mathcal{M}_i \triangleq \{1, 2, \ldots, M_i\}$. The structure of $\mathcal{S}_i$ is shown in Fig. 1.

Because the mode process $\eta_i(t)$ describes the mode switching of the ith subsystem $\mathcal{S}_i$, the vector process $[\eta_1(t), \ldots, \eta_N(t)]^T$ naturally describes the mode switching of the entire large-scale system $\mathcal{S}$. We assume that $[\eta_1(t), \ldots, \eta_N(t)]^T$ takes values in a set (denoted by $\mathcal{M}_V$) consisting of M distinct vectors. If any $\eta_i(t), i \in \mathcal{N}$, changes its value, the vector $[\eta_1(t), \ldots, \eta_N(t)]^T$ will take a different value. Hence $M_i \leq M$. In addition, we have $M \leq \prod_{i=1}^N M_i$ (not necessarily "=", because the mode processes $\eta_i(t), i \in \mathcal{N}$, may depend on each other Xiong et al., 2009). Let $\mathcal{M}_S \triangleq \{1, 2, \ldots, M\}$; then a bijective mapping $\psi : \mathcal{M}_V \to \mathcal{M}_S$

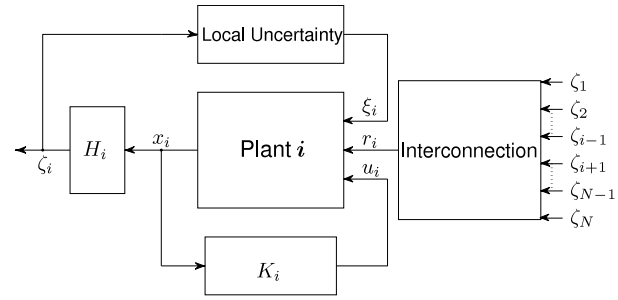

Fig. 1. The subsystem $\mathcal{S}_i$.

exists, because $\mathcal{M}_V$ and $\mathcal{M}_S$ have the same number of elements. Let $\eta(t) \triangleq \psi([\eta_1(t), \ldots, \eta_N(t)]^T)$. Thus the random vector process $[\eta_1(t), \ldots, \eta_N(t)]^T$ is transformed into the random scalar process $\eta(t)$, which carries the same mode information of the large-scale system $\mathcal{S}$. For this reason, $\eta(t)$ is referred to as the global mode process in the sequel. The inverse function $\psi^{-1} : \mathcal{M}_S \to \mathcal{M}_V$ is given by $\psi^{-1}(\mu) = [\mu_1, \ldots, \mu_N]^T, \mu \in \mathcal{M}_S, \mu_i \in \mathcal{M}_i, i \in \mathcal{N}$. Then the ith element μ_i can be determined uniquely from the global mode μ. Therefore μ_i is also a function of μ and we write: $\mu_i = \psi_i^{-1}(\mu), i \in \mathcal{N}$.

We assume here that $\eta(t)$ is a stationary Markov process. The infinitesimal generator matrix of $\eta(t)$ is $\mathbf{Q} = [q_{\mu\nu}] \in \mathbb{R}^{M\times M}$, where $q_{\mu\nu} \geq 0$ if $\nu \neq \mu$, and $q_{\mu\mu} = -\sum_{\nu=1,\nu\neq\mu}^M q_{\mu\nu}$. The initial distribution of the process $\eta(t)$ is $\pi = [\pi_1, \ldots, \pi_M]^T$ with $\pi_\mu \geq 0$, $\forall \mu \in \mathcal{M}_S$.

Assumption 1 (*Ugrinovskii & Pota, 2005*)**.** Given any locally square integrable signals $u_i(t), \xi_i(t), r_i(t)$, for any initial conditions $x_i(0) = x_{i0}, \eta_i(0) = \eta_{i0}$, the solution $x_i(t)$ to each subsystem (1) exists and is locally square integrable.

Remark 2. Recall that a signal $s(t)$ is said to be locally square integrable if it satisfies the condition $\mathrm{E}\left(\int_0^{\mathcal{T}} \|s(t)\|^2\, dt\right) < \infty$ for any finite time $\mathcal{T}$. The term "locally" here means that square integrability is only required on bounded time intervals.

The local uncertainty inputs and the interconnection inputs of the large-scale system (1) are assumed to satisfy the following integral quadratic constraints (IQCs).

Definition 3 (*Li et al., 2007*)**.** Given a set of positive definite matrices $\bar{S}_i, i \in \mathcal{N}$. A locally square integrable signal $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T$ represents an admissible local uncertainty input for the large-scale system (1) if, given any locally square integrable signals $[u_1^T(t), \ldots, u_N^T(t)]^T, [r_1^T(t), \ldots, r_N^T(t)]^T$, there exists a time sequence $\{t_l\}_{l=1}^\infty$, $t_l \to \infty$, such that for all l and for all $i \in \mathcal{N}$,

$$\mathrm{E}\left(\int_0^{t_l} \left[\|\zeta_i(t)\|^2 - \|\xi_i(t)\|^2\right] dt\right) \geq -x_{i0}^T \bar{S}_i x_{i0}. \tag{2}$$

The set of all such admissible local uncertainty inputs is denoted by Ξ.

Definition 4 (*Li et al., 2007*)**.** Given a set of positive definite matrices $\tilde{S}_i, i \in \mathcal{N}$. A locally square integrable signal $[r_1^T(t), \ldots, r_N^T(t)]^T$ represents an admissible interconnection input for the large-scale system (1) if, given any locally square integrable signals $[u_1^T(t), \ldots, u_N^T(t)]^T, [\xi_1^T(t), \ldots, \xi_N^T(t)]^T$, there exists a time sequence $\{t_l\}_{l=1}^\infty$, $t_l \to \infty$, such that for all l and for all $i \in \mathcal{N}$,

$$\mathrm{E}\left(\int_0^{t_l} \left[\sum_{j=1, j\neq i}^N \left\|\zeta_j(t)\right\|^2 - \|r_i(t)\|^2\right] dt\right) \geq -x_{i0}^T \tilde{S}_i x_{i0}. \tag{3}$$

The set of all such admissible interconnections is denoted by Π. We assume that the same time sequences $\{t_l\}_{l=1}^{\infty}$ are chosen in Definitions 3 and 4 whenever they correspond to the same signals $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T$, $[r_1^T(t), \ldots, r_N^T(t)]^T$, $[u_1^T(t), \ldots, u_N^T(t)]^T$.

Remark 5. The IQCs are used to describe relations between the input and output signals in the uncertainty blocks in Fig. 1. The constant terms on the right-hand sides of the inequalities (2) and (3) allow for nonzero initial conditions in the uncertainty dynamics. These definitions can capture a broad class of uncertainties such as nonlinear, time-varying, dynamic uncertainties; see Petersen, Ugrinovskii, and Savkin (2000, Chapter 2.3) for details.

Let $\mathcal{C} = [c_{ij}] \in \mathbb{R}^{N\times N}$ be a given binary matrix, where $c_{ij} = 1$ if the mode of the subsystem $\mathcal{S}_j$ is available to the ith local controller and $c_{ij} = 0$, otherwise. Then the total mode information accessed by the ith local controller can be written as $[c_{i1}\eta_1(t), \ldots, c_{iN}\eta_N(t)]^T$. A zero entry in this vector means that the mode information of the corresponding subsystem is not available. We assume that the random vector process $[c_{i1}\eta_1(t), \ldots, c_{iN}\eta_N(t)]^T$ takes values in a set (denoted by $\mathcal{M}_{Vi}$) consisting of M_{ci} distinct vectors. Obviously, $M_{ci} \le M$, $i \in \mathcal{N}$. Let $\mathcal{M}_{Si} \triangleq \{1, \ldots, M_{ci}\}$. Also, there exists a bijective mapping $\varphi_i : \mathcal{M}_{Vi} \to \mathcal{M}_{Si}$ with $\varphi_i([c_{i1}\mu_1, \ldots, c_{iN}\mu_N]^T) = \sigma_i$, $\mu_i \in \mathcal{M}_i$, $\sigma_i \in \mathcal{M}_{Si}$, $i \in \mathcal{N}$. Let $\aleph_i(t) \triangleq \varphi_i([c_{i1}\eta_1(t), \ldots, c_{iN}\eta_N(t)]^T)$, $i \in \mathcal{N}$. It can be seen that $\aleph_i(t)$ contains essentially the same mode information as $[c_{i1}\eta_1(t), \ldots, c_{iN}\eta_N(t)]^T$. Hence $\aleph_i(t)$ is referred to as a neighboring mode process in the sequel.

Remark 6. Both the global and the local mode dependent control problems studied in Ugrinovskii and Pota (2005) and Xiong et al. (2009) can be regarded as special cases of the neighboring mode dependent control problem with $\mathcal{C} = \mathbf{1}_{N\times N}$ (a matrix with all the elements being ones) and I, respectively.

For the large-scale system (1) with the uncertainty constraints (2), (3), our objective is to design a neighboring mode dependent decentralized control law

$$u_i(t) = K_i(\aleph_i(t))x_i(t), \quad i \in \mathcal{N}, \tag{4}$$

such that the resulting closed-loop large-scale system is robustly stochastically stable in the following sense.

Definition 7 (*Xiong et al., 2009*)**.** The closed-loop large-scale system corresponding to the uncertain large-scale system (1)–(3) and the controller (4) is said to be robustly stochastically stable if there exists a finite constant $\lambda \in \mathbb{R}^+$ such that

$$\mathrm{E}\left(\int_0^\infty \sum_{i=1}^N \|x_i(t)\|^2\, dt\right) \le \lambda\, \|x_0\|^2 \tag{5}$$

for any $x_0 = [x_{10}^T, \ldots, x_{N0}^T]^T$, and any uncertainties $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T \in \Xi$, $[r_1^T(t), \ldots, r_N^T(t)]^T \in \Pi$.

For convenience, a set of many-to-one mappings ϕ_i: $\mathcal{M}_S \to \mathcal{M}_{Si}$, $i \in \mathcal{N}$, is introduced below:

$$\phi_i(\mu) = \varphi_i(\mathrm{diag}[c_{i1}, \ldots, c_{iN}] \cdot \psi^{-1}(\mu)). \tag{6}$$

Note that ϕ_i, $i \in \mathcal{N}$, are also surjective mappings.

Example 8. Suppose $N = 3$, $M_1 = M_2 = M_3 = 2$. When the mode processes $\eta_i(t)$, $i \in \mathcal{N}$, are independent of each other, the vector set $\mathcal{M}_V$ contains 8 elements, i.e., $\mathcal{M}_V = \{[\mu_1, \mu_2, \mu_3]^T : \mu_i = 1, 2, i = 1, 2, 3\}$. Now we assume that the mode processes $\eta_i(t)$, $i \in \mathcal{N}$, are subject to the constraints below:

$$\eta_1(t) = \eta_2(t) \quad \text{if } \eta_3(t) = 1,$$
$$\eta_2(t) = 1 \quad \text{if } \eta_1(t) = 2.$$

Then $\mathcal{M}_V$ contains only four elements, i.e., $\mathcal{M}_V = \{[1, 1, 1]^T, [1, 1, 2]^T, [1, 2, 2]^T, [2, 1, 2]^T\}$. Thus $\mathcal{M}_S = \{1, 2, 3, 4\}$. The mappings ψ, ψ^{-1} between $\mathcal{M}_V$ and $\mathcal{M}_S$ can be defined as follows:

$$[1, 1, 1]^T \underset{\psi^{-1}}{\overset{\psi}{\leftrightarrows}} 1, \qquad [1, 1, 2]^T \underset{\psi^{-1}}{\overset{\psi}{\leftrightarrows}} 2,$$
$$[1, 2, 2]^T \underset{\psi^{-1}}{\overset{\psi}{\leftrightarrows}} 3, \qquad [2, 1, 2]^T \underset{\psi^{-1}}{\overset{\psi}{\leftrightarrows}} 4.$$

Suppose, for example, that $\mathcal{C} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Then we have $\mathcal{M}_{V1} = \{[1, 1, 0]^T, [1, 2, 0]^T, [2, 1, 0]^T\}$. Thus $\mathcal{M}_{S1} = \{1, 2, 3\}$. The mapping $\varphi_1 : \mathcal{M}_{V1} \to \mathcal{M}_{S1}$ can be defined as follows:

$$[1, 1, 0]^T \xrightarrow{\varphi_1} 1, \qquad [1, 2, 0]^T \xrightarrow{\varphi_1} 2, \qquad [2, 1, 0]^T \xrightarrow{\varphi_1} 3.$$

In this case, by (6), the many-to-one mapping $\phi_1 : \mathcal{M}_S \to \mathcal{M}_{S1}$ is given by:

$$1 \xrightarrow{\phi_1} 1, \qquad 2 \xrightarrow{\phi_1} 1, \qquad 3 \xrightarrow{\phi_1} 2, \qquad 4 \xrightarrow{\phi_1} 3.$$

3. Controller design

In this section, we first turn to a new uncertain Markovian jump large-scale system which is similar to the large-scale system (1). Global mode dependent stabilizing controllers are designed for this new large-scale system using the results of Ugrinovskii and Pota (2005). Then we will show how to derive neighboring mode dependent stabilizing controllers for the large-scale system (1) from these obtained global mode dependent controllers. Finally, all of the conditions for the existence of such neighboring mode dependent controllers are combined as a feasible LMI problem with rank constraints.

Consider a new large-scale system $\tilde{\mathcal{S}}$ comprising N subsystems $\tilde{\mathcal{S}}_i$, $i \in \mathcal{N}$. The ith subsystem $\tilde{\mathcal{S}}_i$ is as follows (Xiong et al., 2009):

$$\tilde{\mathcal{S}}_i : \begin{cases} \dot{\tilde{x}}_i(t) = \tilde{A}_i(\eta(t))\tilde{x}_i(t) + \tilde{B}_i(\eta(t))\left[\tilde{u}_i(t) + \tilde{\xi}_i^u(t)\right] \\ \qquad\qquad + \tilde{E}_i(\eta(t))\tilde{\xi}_i(t) + \tilde{L}_i(\eta(t))\tilde{r}_i(t), \\ \tilde{\zeta}_i(t) = \tilde{H}_i(\eta(t))\tilde{x}_i(t), \end{cases} \tag{7}$$

where $\tilde{A}_i(\mu) = A_i(\mu_i)$, $\tilde{B}_i(\mu) = B_i(\mu_i)$, $\tilde{E}_i(\mu) = E_i(\mu_i)$, $\tilde{L}_i(\mu) = L_i(\mu_i)$, $\tilde{H}_i(\mu) = H_i(\mu_i)$ for all $\mu \in \mathcal{M}_S$ and $\mu_i = \psi_i^{-1}(\mu) \in \mathcal{M}_i$, $i \in \mathcal{N}$. The initial state $\tilde{x}_{i0} = x_{i0}$, $i \in \mathcal{N}$. The uncertainties $\tilde{\xi}_i(t)$, $\tilde{r}_i(t)$, $\tilde{\xi}_i^u(t)$, $i \in \mathcal{N}$, satisfy the following constraints, respectively.

Definition 9. A locally square integrable signal $[\tilde{\xi}_1^T(t), \ldots, \tilde{\xi}_N^T(t)]^T$ represents an admissible local uncertainty input for the large-scale system (7) if, given any locally square integrable signals $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T$, $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T$, $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T$, there exists a time sequence $\{t_l\}_{l=1}^{\infty}$, $t_l \to \infty$, such that for all l and for all $i \in \mathcal{N}$,

$$\mathrm{E}\left(\int_0^{t_l} \left[\left\|\tilde{\zeta}_i(t)\right\|^2 - \left\|\tilde{\xi}_i(t)\right\|^2\right] dt\right) \ge -\tilde{x}_{i0}^T \bar{S}_i \tilde{x}_{i0}. \tag{8}$$

The set of all such admissible local uncertainty inputs is denoted by $\tilde{\Xi}$.

Definition 10. A locally square integrable signal $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T$ represents an admissible interconnection input for the large-scale system (7) if, given any locally square integrable signals $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T$, $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T$, $[\tilde{\xi}_1^T(t), \ldots, \tilde{\xi}_N^T(t)]^T$,

there exists a time sequence $\{t_l\}_{l=1}^{\infty}$, $t_l \to \infty$, such that for all l and for all $i \in \mathcal{N}$,

$$\mathrm{E}\left(\int_0^{t_l}\left[\sum_{j=1,j\neq i}^{N}\left\|\tilde{\zeta}_j(t)\right\|^2-\left\|\tilde{r}_i(t)\right\|^2\right]dt\right)\geq -\tilde{x}_{i0}^T\tilde{S}_i\tilde{x}_{i0}. \tag{9}$$

The set of all such admissible interconnection inputs is denoted by $\tilde{\Pi}$.

Definition 11 (*Xiong et al., 2009*)**.** Suppose $\beta_i^u(\mu) \in \mathbb{R}^+, i \in \mathcal{N}$, $\mu \in \mathcal{M}_S$. A locally square integrable signal $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T$ represents an admissible input uncertainty for the large-scale system (7) if, for all locally square integrable signals $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T$, $[\tilde{\xi}_1^T(t), \ldots, \tilde{\xi}_N^T(t)]^T$, $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T$ and for all $i \in \mathcal{N}$,

$$\mathrm{E}\left(\beta_i^u(\eta(t))\left\|\tilde{x}_i(t)\right\|^2-\left\|\tilde{\xi}_i^u(t)\right\|^2\right)\geq 0. \tag{10}$$

The set of all such admissible input uncertainties is denoted by $\tilde{\Xi}^u$.

We assume that the same sequences $\{t_l\}_{l=1}^{\infty}$ are chosen in Definitions 9, 10 whenever they correspond to the same signals $[\tilde{\xi}_1^T(t), \ldots, \tilde{\xi}_N^T(t)]^T$, $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T$, $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T$, $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T$. Furthermore, one can verify that the system (7) has the same system matrices as the system (1), i.e., $\tilde{A}_i(\cdot) = A_i(\cdot)$, $\tilde{B}_i(\cdot) = B_i(\cdot)$, $\tilde{E}_i(\cdot) = E_i(\cdot)$, $\tilde{L}_i(\cdot) = L_i(\cdot)$, $\tilde{H}_i(\cdot) = H_i(\cdot)$ at any time t. Using this fact, we will show that $\tilde{\Xi} = \Xi$, $\tilde{\Pi} = \Pi$.

For convenience, let $\mathcal{L}^m(t)$ denote the set of all locally square integrable signals of dimension $m = \sum_{i=1}^N m_i$, and let $\mathcal{L}^s(t)$ denote the set of all locally square integrable signals of dimension $s = \sum_{i=1}^N s_i$. Given $[\tilde{\xi}_1^T(t), \ldots, \tilde{\xi}_N^T(t)]^T \in \tilde{\Xi}$. By Definition 9, the inequality (8) holds for any signals $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T \in \mathcal{L}^m(t)$, $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T \in \mathcal{L}^m(t)$, $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T \in \mathcal{L}^s(t)$. This implies that the inequality (8) holds for any $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T \in \mathcal{L}^m(t)$, $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T \in \mathcal{L}^s(t)$ and $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T \equiv 0$. This is indeed the case defined by Definition 3. Thus we have $[\tilde{\xi}_1^T(t), \ldots, \tilde{\xi}_N^T(t)]^T \in \Xi$, i.e., $\tilde{\Xi} \subset \Xi$.

To show $\Xi \subset \tilde{\Xi}$, suppose $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T \in \Xi$. Then we shall prove $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T \in \tilde{\Xi}$. By Definition 9, we need to prove that the inequality (8) holds when we apply this signal $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T$ and any other signals $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T \in \mathcal{L}^m(t)$, $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T \in \mathcal{L}^m(t)$, $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T \in \mathcal{L}^s(t)$ to the large-scale system (7). Note that the two inputs $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T$, $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T$ in the large-scale system (7) can be considered as an equivalent input $[\hat{u}_1^T(t), \ldots, \hat{u}_N^T(t)]^T$. For any $[\tilde{u}_1^T(t), \ldots, \tilde{u}_N^T(t)]^T \in \mathcal{L}^m(t)$, $[\tilde{\xi}_1^{uT}(t), \ldots, \tilde{\xi}_N^{uT}(t)]^T \in \mathcal{L}^m(t)$, we have $[\hat{u}_1^T(t), \ldots, \hat{u}_N^T(t)]^T \in \mathcal{L}^m(t)$. Thus, it suffices to prove that the inequality (8) holds when we apply $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T$ and any other signals $[\hat{u}_1^T(t), \ldots, \hat{u}_N^T(t)]^T \in \mathcal{L}^m(t)$, $[\tilde{r}_1^T(t), \ldots, \tilde{r}_N^T(t)]^T \in \mathcal{L}^s(t)$. But this follows directly from Definition 3 and the fact that $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T \in \Xi$. Hence $[\xi_1^T(t), \ldots, \xi_N^T(t)]^T \in \tilde{\Xi}$ and $\Xi \subset \tilde{\Xi}$. Therefore $\tilde{\Xi} = \Xi$. In a similar way, we can prove that $\tilde{\Pi} = \Pi$.

We also mention that the values of $\beta_i^u(\mu)$, $\mu \in \mathcal{M}_S$, $i \in \mathcal{N}$, in Definition 11 can either be given appropriately in advance, or be solved from numerical computation as illustrated in Theorem 15.

Associated with the large-scale system (7) the quadratic cost functional is as follows (Xiong et al., 2009):

$$J \triangleq \mathrm{E}\left(\int_0^{\infty}\sum_{i=1}^{N}[\tilde{x}_i^T(t)\tilde{R}_i(\eta(t))\tilde{x}_i(t)+\tilde{u}_i^T(t)\tilde{G}_i(\eta(t))\tilde{u}_i(t)]dt\right), \tag{11}$$

where $\tilde{R}_i(\mu) \in \mathbb{S}^+$, $\tilde{G}_i(\mu) \in \mathbb{S}^+$, $\mu \in \mathcal{M}_S$, $i \in \mathcal{N}$, are given weighting matrices.

For the large-scale system (7) with the uncertainty constraints (8)–(10), global mode dependent stabilizing controllers can be designed using the technique developed in Ugrinovskii and Pota (2005). Furthermore, applying these controllers to the large-scale system (7) will yield a cost upper bound, i.e., $\sup_{\tilde{\Xi},\tilde{\Pi},\tilde{\Xi}^u} J < c_1$, $c_1 \in \mathbb{R}^+$. This result is stated in the following theorem.

Theorem 12 (*Xiong et al., 2009*)**.** *If there exist matrices $X_i(\mu) \in \mathbb{S}^+$ and scalars $\tau_i \in \mathbb{R}^+$, $\theta_i \in \mathbb{R}^+$, $\tau_i^u \in \mathbb{R}^+$, $\mu \in \mathcal{M}_S$, $i \in \mathcal{N}$, such that*

$$\begin{aligned}&\tilde{A}_i^T(\mu)X_i(\mu)+X_i(\mu)\tilde{A}_i(\mu)+\sum_{\nu=1}^{M}q_{\mu\nu}X_i(\nu)+\tilde{R}_i(\mu)\\&\quad+X_i(\mu)\left(\bar{B}_{2i}(\mu)\bar{B}_{2i}^T(\mu)-\tilde{B}_i(\mu)\tilde{G}_i^{-1}(\mu)\tilde{B}_i^T(\mu)\right)X_i(\mu)\\&\quad+\tau_i^u\beta_i^u(\mu)I+(\tau_i+\bar{\theta}_i)\tilde{H}_i^T(\mu)\tilde{H}_i(\mu)<0,\end{aligned} \tag{12}$$

where $\bar{B}_{2i}(\mu) = \left[(\tau_i^u)^{-1/2}\tilde{B}_i(\mu)\ \tau_i^{-1/2}\tilde{E}_i(\mu)\ \theta_i^{-1/2}\tilde{L}_i(\mu)\right]$ and $\bar{\theta}_i = \sum_{j=1,j\neq i}^N \theta_j$, then the global mode dependent controllers given by

$$\begin{cases}\tilde{u}_i(t)=\tilde{K}_i(\eta(t))\tilde{x}_i(t),\\ \tilde{K}_i(\mu)=-\tilde{G}_i^{-1}(\mu)\tilde{B}_i^T(\mu)X_i(\mu),\end{cases} \tag{13}$$

$\mu \in \mathcal{M}_S$, $i \in \mathcal{N}$, robustly stabilize the uncertain large-scale system (7) with the uncertainty constraints (8)–(10), and achieve a bounded system cost $J \leq \sum_{i=1}^N \tilde{x}_{i0}^T\left[\sum_{\mu=1}^M \pi_\mu X_i(\mu)+\tau_i\bar{S}_i+\theta_i\tilde{S}_i\right]\tilde{x}_{i0}$.

After obtaining the global mode dependent stabilizing controllers (13) for the large-scale system (7), the next step is to derive neighboring mode dependent stabilizing controllers for the large-scale system (1). The following result is an extension of Theorem 1 in Xiong et al. (2009) to the neighboring mode dependent control case. The proof is similar to that of Theorem 1 in Xiong et al. (2009) and hence is omitted.

Theorem 13. *Given the global mode dependent controllers (13) which stabilize the large-scale system (7) with the uncertainty constraints (8)–(10). If the gains $K_i(\cdot)$ in the controllers (4) are chosen to satisfy*

$$\left\|K_i(\sigma_i)-\tilde{K}_i(\mu)\right\|^2 \leq \beta_i^u(\mu) \tag{14}$$

for all $\mu \in \mathcal{M}_S$, $\sigma_i = \phi_i(\mu) \in \mathcal{M}_{Si}$, $i \in \mathcal{N}$, then the neighboring mode dependent controllers (4) stabilize the large-scale system (1) with the uncertainty constraints (2), (3).

In the following remark, we use an example to illustrate the fact that Theorem 13 is less conservative than Theorem 1 in Xiong et al. (2009).

Remark 14. Consider the Markovian jump large-scale system in Example 8. Three control gains need to be scheduled for the first local controller if using the neighboring mode dependent control approach, while two control gains are needed if using the local mode dependent control approach. We denote the three neighboring mode dependent control gains as $K_1(\sigma_1)$, $\sigma_1 \in \mathcal{M}_{S1} = \{1, 2, 3\}$ and the two local mode dependent control gains as $\mathcal{K}_1(\mu_1)$, $\mu_1 \in \mathcal{M}_1 = \{1, 2\}$. For comparison, given the global mode dependent control gains $\tilde{K}_1(\mu)$ and the scalars $\beta_1^u(\mu)$, $\mu = 1, 2, 3$, the constraints imposed on $\mathcal{K}_1(1)$, $K_1(1)$ are specified as follows

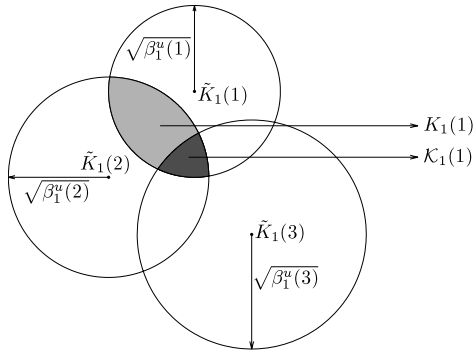

Fig. 2. Illustration of the constraints.

based on Theorem 1 in Xiong et al. (2009) and our Theorem 13, respectively:

$$\begin{cases} \left\| \mathcal{K}_1(1) - \tilde{K}_1(1) \right\|^2 \le \beta_1^u(1), \\ \left\| \mathcal{K}_1(1) - \tilde{K}_1(2) \right\|^2 \le \beta_1^u(2), \\ \left\| \mathcal{K}_1(1) - \tilde{K}_1(3) \right\|^2 \le \beta_1^u(3), \end{cases} \tag{15}$$

$$\begin{cases} \left\| K_1(1) - \tilde{K}_1(1) \right\|^2 \le \beta_1^u(1), \\ \left\| K_1(1) - \tilde{K}_1(2) \right\|^2 \le \beta_1^u(2). \end{cases} \tag{16}$$

These inequalities are illustrated in Fig. 2 where each circle denotes a Euclidean ball. $\tilde{K}_1(\mu)$ is the center and $\sqrt{\beta_1^u(\mu)}$ the radius of the ball for $\mu = 1, 2, 3$. As shown in Fig. 2, the set where $\mathcal{K}_1(1)$ takes values is only a subset of the set where $K_1(1)$ takes values. Hence the proposed framework provides greater flexibility in choosing control gains. Potentially, this will allow one to achieve better system performance than obtained using local mode dependent controllers. We also mention that if the Euclidean ball centered at $\tilde{K}_1(3)$ does not intersect the Euclidean ball centered at $\tilde{K}_1(1)$ (or $\tilde{K}_1(2)$), then no local mode dependent controllers exist. However, the existence of the neighboring mode dependent controllers is not affected. Therefore our technique potentially produces less conservative results than that in Xiong et al. (2009).

Next, the conditions in Theorems 12 and 13 are combined and recast as a rank constrained LMI problem. Although rank constrained LMI problems are non-convex in general, numerical methods such as the LMIRank toolbox (Orsi, 2005) often yield good results in solving these problems.

Theorem 15. *Suppose there exist matrices $X_i(\mu) \in \mathbb{S}^+$, $Y_i(\mu) \in \mathbb{S}^+$, $K_i(\sigma_i) \in \mathbb{R}^{m_i \times n_i}$ and scalars $\bar{\beta}_i(\mu) \in \mathbb{R}^+$, $\tilde{\beta}_i(\mu) \in \mathbb{R}^+$, $\tilde{\tau}_i^u \in \mathbb{R}^+$, $\tilde{\tau}_i \in \mathbb{R}^+$, $\tilde{\theta}_i \in \mathbb{R}^+$, $\mu \in \mathcal{M}_S$, $i \in \mathcal{N}$, such that the following inequalities hold:*

$$\begin{bmatrix} \mathcal{G}_{i11}(\mu) & \mathcal{G}_{i12}(\mu) & \mathcal{G}_{i13}(\mu) \\ \mathcal{G}_{i12}^T(\mu) & \mathcal{G}_{i22}(\mu) & 0 \\ \mathcal{G}_{i13}^T(\mu) & 0 & \mathcal{G}_{i33}(\mu) \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} -\tilde{\tau}_i^u I & \Upsilon_i^T(\mu) \\ \Upsilon_i(\mu) & -\tilde{\beta}_i(\mu) I \end{bmatrix} \le 0, \tag{18}$$

$$\operatorname{rank}\left(\begin{bmatrix} \bar{\beta}_i(\mu) & 1 \\ 1 & \tilde{\beta}_i(\mu) \end{bmatrix} \right) \le 1, \tag{19}$$

$$\operatorname{rank}\left(\begin{bmatrix} Y_i(\mu) & I \\ I & X_i(\mu) \end{bmatrix} \right) \le n_i, \tag{20}$$

where

$$\begin{aligned} \mathcal{G}_{i11}(\mu) &= Y_i(\mu)\tilde{A}_i^T(\mu) + \tilde{A}_i(\mu)Y_i(\mu) + q_{\mu\mu}Y_i(\mu) \\ &\quad - \tilde{B}_i(\mu)\tilde{G}_i^{-1}(\mu)\tilde{B}_i^T(\mu) + \tilde{\tau}_i^u\tilde{B}_i(\mu)\tilde{B}_i^T(\mu) \\ &\quad + \tilde{\tau}_i\tilde{E}_i(\mu)\tilde{E}_i^T(\mu) + \tilde{\theta}_i\tilde{L}_i(\mu)\tilde{L}_i^T(\mu), \\ \mathcal{G}_{i12}(\mu) &= Y_i(\mu)[\sqrt{q_{\mu 1}}I \ \cdots \ \sqrt{q_{\mu(\mu-1)}}I \sqrt{q_{\mu(\mu+1)}}I \ \cdots \ \sqrt{q_{\mu M}}I], \\ \mathcal{G}_{i13}(\mu) &= Y_i(\mu)\left[I \ I \ \tilde{H}_i^T(\mu) \ \cdots \ \tilde{H}_i^T(\mu) \right], \\ \mathcal{G}_{i22}(\mu) &= -\operatorname{diag}[Y_i(1), \ldots, Y_i(\mu-1), Y_i(\mu+1), \ldots, Y_i(M)], \\ \mathcal{G}_{i33}(\mu) &= -\operatorname{diag}[\tilde{R}_i^{-1}(\mu), \bar{\beta}_i(\mu)I, \tilde{\tau}_i I, \\ &\quad \tilde{\theta}_1 I, \ldots, \tilde{\theta}_{i-1}I, \tilde{\theta}_{i+1}I, \ldots, \tilde{\theta}_N I], \\ \Upsilon_i(\mu) &= K_i(\phi_i(\mu)) + \tilde{G}_i^{-1}(\mu)\tilde{B}_i^T(\mu)X_i(\mu). \end{aligned}$$

Then a stabilizing controller (4) is given by: $u_i(t) = K_i(\sigma_i)x_i(t)$, for $\aleph_i(t) = \sigma_i \in \mathcal{M}_{Si}$, $i \in \mathcal{N}$.

Proof. From $X_i(\mu) \in \mathbb{S}^+$, $Y_i(\mu) \in \mathbb{S}^+$ and (20), we have $Y_i(\mu) = (X_i(\mu))^{-1}$. Similarly, $\tilde{\beta}_i(\mu) = (\bar{\beta}_i(\mu))^{-1}$. On the other hand, if (17) is satisfied, by setting $\tau_i^u = (\tilde{\tau}_i^u)^{-1}$, $\tau_i = (\tilde{\tau}_i)^{-1}$, $\theta_i = (\tilde{\theta}_i)^{-1}$, $\beta_i^u(\mu) = (\bar{\beta}_i(\mu))^{-1}\tilde{\tau}_i^u = \tilde{\beta}_i(\mu)\tilde{\tau}_i^u$, and applying the Schur complement equivalence, the inequality (12) is satisfied. Then, by Theorem 12, the global mode dependent controllers (13) can be designed to stabilize the large-scale system (7) with the uncertainty constraints (8)–(10).

Also, the LMI (18) and the equation (13) imply that $\left\| \tilde{K}_i(\mu) - K_i(\sigma_i) \right\|^2 \le \beta_i^u(\mu)$ for all $\mu \in \mathcal{M}_S$, $\sigma_i = \phi_i(\mu)$, $i \in \mathcal{N}$. That is, the inequality (14) holds. Then, by Theorem 13, the constructed controllers (4) stabilize the large-scale system (1) with the uncertainty constraints (2), (3). □

Remark 16. In Xiong et al. (2009), a control gain form has been proposed for the design of local mode dependent controllers. That is, each local mode dependent control gain is chosen to be a weighted average of the related global mode dependent control gains. This particular gain form is then incorporated into the coupled LMIs from which the local mode dependent control gains are computed; see Theorems 3 and 4 in Xiong et al. (2009) for details. Unfortunately, choosing such a gain form is not helpful in terms of an improvement in system performance, and sometimes may even result in infeasibility of the corresponding LMIs. A demonstration of this fact is given in Section 4. Indeed, such a gain form imposes an additional constraint and hence is not used in this paper.

4. Numerical example

Consider the Markovian jump large-scale system given in Xiong et al. (2009). The mode information is $\mathcal{M}_V = \left\{ [1, 1, 1]^T, [1, 2, 2]^T, [2, 1, 2]^T, [2, 2, 1]^T \right\}$. The initial distribution of $\eta(t)$ is assumed to be the same as its stationary distribution $\pi_\infty = [\pi_{\infty 1}, \ldots, \pi_{\infty M}]^T$, which can be computed from the infinitesimal generator matrix $\mathbf{Q}$. Given a neighboring mode information pattern $\mathcal{C}$, our objective is to find the corresponding neighboring mode dependent stabilizing controllers for this large-scale system. An upper bound on the quadratic cost (11) is also evaluated for the resulting closed-loop large-scale system. The main software we use is the LMIRank toolbox (Orsi, 2005). The procedure is summarized as follows:

1. Solve the optimization problem

   $\min \gamma$ subject to

   $$\sum_{i=1}^{N} x_{i0}^T \left[\sum_{\mu=1}^{M} \pi_{\infty\mu} X_i(\mu) + \tau_i \bar{S}_i + \theta_i \tilde{S}_i \right] x_{i0} < \gamma,$$

   and (17)–(20).

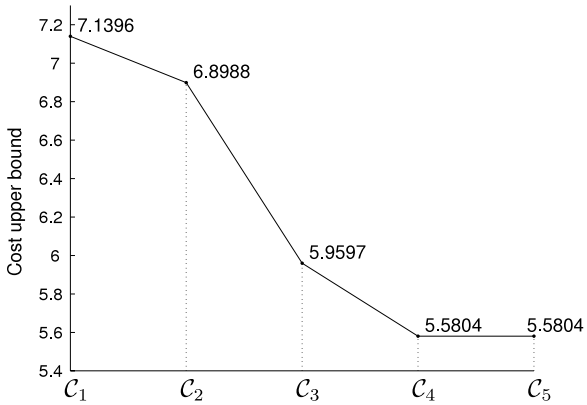

Fig. 3. Cost upper bounds for the closed-loop systems.

If an optimal value γ is found, feasible neighboring mode dependent control gains (4) are obtained.

2. Apply the obtained controllers to the large-scale system (1) and compute the cost upper bound for the resulting closed-loop large-scale system. The method for computing this upper bound is taken from Ugrinovskii and Pota (2005). It involves solving a worst-case performance analysis problem.

Five cases are considered, i.e.,

$$\mathcal{C}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{C}_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathcal{C}_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathcal{C}_4 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \qquad \mathcal{C}_5 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

It can be seen that each neighboring mode information pattern contains more mode information than the preceding one. $\mathcal{C}_1$ corresponds to the local mode dependent control case, while $\mathcal{C}_5$ corresponds to the global mode dependent control case. By using the preceding procedure, neighboring mode dependent stabilizing controllers are found for each of these cases. Furthermore, if we apply the obtained controllers to the large-scale system, the cost upper bounds for the resulting closed-loop large-scale systems are shown in Fig. 3.

Note that the cost upper bound found here in the local mode dependent control case is different from (in fact, less than) that in Xiong et al. (2009). This is because the gain form proposed by Theorem 3 in Xiong et al. (2009) is not used in our computation. One may also notice that the cost upper bound found in the case of $\mathcal{C}_4$ is the same as the one in the case of $\mathcal{C}_5$. We now explain why this happens. In the case of $\mathcal{C}_4$, each local controller obtains two subsystem modes directly. In fact, the third subsystem mode can be derived from these two modes based on possible mode combinations in $\mathcal{M}_V$. Hence $\mathcal{C}_4$ and $\mathcal{C}_5$ are equivalent, in the sense that they yield the same performance. This example demonstrates that the system achieves better (or at least equal) performance if more information about the subsystem modes is available to the local controllers. It also shows that sometimes complete information about the global mode of the large-scale system may be redundant.

5. Conclusions

This paper has presented a decentralized control scheme for uncertain Markovian jump large-scale systems. The proposed controllers use local subsystem states and neighboring mode information to generate local control inputs. A computational algorithm involving rank constrained LMIs has been developed for the design of such controllers. The developed theory is illustrated by a numerical example.

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