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Brief paper

Output feedback negative imaginary synthesis under structural constraints[☆]

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ABSTRACT

The negative imaginary property is a property that many practical systems exhibit. This paper is concerned with the negative imaginary synthesis problem for linear time-invariant systems by output feedback control. Sufficient conditions are developed for the design of static output feedback controllers, dynamic output feedback controllers and observer-based feedback controllers. Based on the design conditions, a numerical algorithm is suggested to find the desired controllers. Structural constraints can be imposed on the controllers to reflect the practical system constraints. Also, the separation principle is shown to be valid for the observer-based design. Finally, three numerical examples are presented to illustrate the efficiency of the developed theory.

1. Introduction

The study of negative imaginary systems has attracted much attention in recent years (Cai & Hagen, 2010; Ferrante & Ntogramatzidis, 2013; Lanzon & Petersen, 2008; Mabrok, Kallapur, Petersen, & Lanzon, 2015, 2014; Wang, Lanzon, & Petersen, 2015a,b; Xiong, Petersen, & Lanzon, 2012). By appropriately choosing the system input and output, many practical dynamic systems can be modelled as negative imaginary systems. Examples could be found in active vibration control systems (Das, Pota, & Petersen, 2015; Fanson & Caughey, 1990; Moheimani, Vautier, & Bhikkaji, 2006; Petersen & Lanzon, 2010) and circuit systems (Petersen & Lanzon, 2010). An important class of results for negative imaginary systems is the stability results developed in Lanzon and Petersen (2008), Xiong, Petersen, and Lanzon (2010), Mabrok et al. (2014), Liu and Xiong (2015). For positive-feedback interconnected negative imaginary systems, necessary and sufficient conditions are established to test the system stability. These results can be considered as a generalization of the positive position control results in Moheimani et al. (2006), Fanson and Caughey (1990), and depend only on the system gains at zero and infinite frequencies. In other words, the interconnected systems might have large control gains over other frequencies. In contrast, the small gain theorem requires that the control gains be small over all frequencies. An important application of the results is to robust control problems, where system uncertainty can be modelled by a negative imaginary system. Then, the closed-loop system will be stable as long as the controller is negative imaginary and satisfies the gain conditions. An illustrative example can be found in Xiong et al. (2010), where the uncertainty parameter is allowed to be arbitrarily large. Therefore, the results in Lanzon and Petersen (2008), Xiong et al. (2010), Mabrok et al. (2014), Liu and Xiong (2015) provide an attractive tool for robust control.

The underlying motivation of this study is to extend the application areas of negative imaginary systems theory. Consider the case that the uncertainty part in a system is negative imaginary while the remaining part of the system is not. The stability results in Lanzon and Petersen (2008), Xiong et al. (2010), Mabrok et al. (2014), Liu and Xiong (2015) will not be applicable. To make them applicable, one has to design controllers such that the remaining part of the system is negative imaginary; see examples in Petersen and Lanzon (2010), Song, Lanzon, Patra, and Petersen (2012), Mabrok et al. (2015). The problem of designing controllers for

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non-negative imaginary systems such that the resulting closed-loop systems become negative imaginary is called the negative imaginary synthesis problem, and the designed controllers are called negative imaginary controllers. When the full system state is available, state feedback negative imaginary controllers can be designed, and the corresponding design conditions have been established in Petersen and Lanzon (2010), Song et al. (2012), Mabrok et al. (2015) for both minimal and nonminimal state-space realizations. However, in practice, the system state is often not available and only measurement output can be used when designing controllers. Also in many cases, the desired controllers have to meet structural constraints in the system design (Lin, Fardad, & Jovanović, 2011; Rubió-Massegú, Rossell, Karimi, & Palacios-Quiñonero, 2013; Siljak, 1991; Zečević & Šiljak, 2008). For example, the controllers have to be of a block diagonal structure in the decentralized control of large-scale systems. Therefore, the design of output feedback negative imaginary controllers with structural constraints is an appealing practice use of the stability results in negative imaginary systems theory. The design of output feedback controllers has been recognized as a hard problem in general (Abbaszadeh & Marquez, 2009; Dinh, Gumussoy, Michiels, & Diehl, 2012; Shu, Lam, & Xiong, 2010; Syrmos, Abdallah, Dorato, & Grigoriadis, 1997).

This paper studies the negative imaginary synthesis problem when designing output feedback controllers. Firstly, the design of static output feedback controllers is considered. A sufficient condition is established in terms of a linear matrix inequality and a linear matrix equality. For the established design condition, an arbitrarily structural constraint can be readily imposed on the controller to meet practical requirements. It deserves mentioning that the solvability of the condition depends on the choice of the right inverse of the measurement matrix. An iterative algorithm is proposed to search for an approximate right inverse. When the measurement output equals to the system state, our result recovers the available ones in Petersen and Lanzon (2010), Song et al. (2012). Then, the design condition is extended to design dynamic output feedback controllers and observer-based state feedback controllers. In particular, for observer-based control, the separation principle is shown to hold. Finally, three numerical examples are used to demonstrate the developed design theory. The first example demonstrates the application of the results to a robust stabilization problem where the uncertainty is modelled by a strictly negative imaginary system. The conservatism of the developed design condition is studied via the first example. The second example validates the applicability of the results to MIMO systems. The third example illustrates how structural constraints are imposed on the designed controllers. The designed controller is a decentralized reduced-order dynamic output feedback controller. In all, the contribution of this paper is that a systematic design theory for output feedback negative imaginary controllers is developed and controller structural constraints can be enforced.

Notation: Let $\mathbb{R}^{m\times n}$ and $\mathcal{R}^{m\times n}$ denote the set of $m\times n$ real matrices and real-rational proper transfer function matrices, respectively. A^{T} and A^* denotes the transpose and the complex conjugate transpose of a complex matrix A, respectively. $\Re[\cdot]$ is the real part of a complex number. The notation $X>0$ or $X\geq 0$, where X is a real symmetric matrix, means that the matrix X is positive definite or positive semidefinite, respectively.

2. Problem formulation

Consider a linear time-invariant system

$$\begin{cases}\dot{x}(t) = Ax(t)+B_1w(t)+B_2u(t),\\ z(t) = C_1x(t),\\ y(t) = C_2x(t),\end{cases} \tag{1}$$

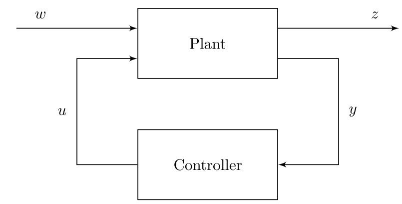

Fig. 1. Negative imaginary synthesis using output feedback control.

where $x(t)\in\mathbb{R}^n$ is the system state, $u(t)\in\mathbb{R}^p$ is the control input, $y(t)\in\mathbb{R}^q$ is the measurement output, $w(t)\in\mathbb{R}^m$ is the system input, $z(t)\in\mathbb{R}^m$ is the system output. The matrices $A\in\mathbb{R}^{n\times n}$, $B_1\in\mathbb{R}^{n\times m}$, $B_2\in\mathbb{R}^{n\times p}$, $C_1\in\mathbb{R}^{m\times n}$ and $C_2\in\mathbb{R}^{q\times n}$ are known constant matrices. The measurement output matrix C_2 is assumed to be of full row rank without loss of generality. The system is chosen to be strictly proper to keep the results simple and tractable.

The objective of the paper is to design output feedback controllers, as shown in Fig. 1, such that the resulting closed-loop system is negative imaginary. The closed-loop system is negative imaginary if its transfer function is a negative imaginary transfer function. The following is the definition for negative imaginary transfer functions.

Definition 1 (*Xiong et al., 2010*)**.** A transfer function matrix $R(s)\in\mathcal{R}^{m\times m}$ is *negative imaginary* if

(1) $R(s)$ has no poles at the origin and in $\Re[s]>0$;
(2) $j[R(j\omega)-R^*(j\omega)]\geq 0$ for all $\omega\in(0,\infty)$ except values of ω where $j\omega$ is a pole of $R(s)$;
(3) If $j\omega_0$, $\omega_0\in(0,\infty)$, is a pole of $R(s)$, it is at most a simple pole, and the residue matrix $K_0\triangleq\lim_{s\to j\omega_0}(s-j\omega_0)jR(s)$ is positive semidefinite Hermitian.

To determine whether a transfer function is negative imaginary or not, the negative imaginary lemma provides a necessary and sufficient condition in terms of the minimal state-space realization of the transfer function.

Lemma 1 (*Negative Imaginary Lemma (Xiong et al., 2010)*). *Let (A,B,C,D) be a minimal state-space realization of a transfer function matrix $R(s)\in\mathcal{R}^{m\times m}$, where $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{m\times n}$, $D\in\mathbb{R}^{m\times m}$. Then $R(s)$ is negative imaginary if and only if*

(1) $\det(A)\neq 0$, $D=D^{\mathsf{T}}$;
(2) *there exists a matrix $Y\in\mathbb{R}^{n\times n}$, $Y=Y^{\mathsf{T}}>0$, such that*

$$AY+YA^{\mathsf{T}}\leq 0 \quad \text{and} \quad B+AYC^{\mathsf{T}}=0.$$

When the realization (A,B,C,D) is not minimal, the conditions in Lemma 1 are only sufficient to test the negative imaginariness of $R(s)$; see Corollary 1 of Song et al. (2012).

Remark 1. The notation of negative imaginary systems has been extended to the cases where zero or infinite poles are allowed in the system (Ferrante, Lanzon, & Ntogramatzidis, 2016; Ferrante & Ntogramatzidis, 2013; Liu, J, & Xiong, 2016; Mabrok et al., 2014). New versions of negative imaginary lemmas have been reported in Mabrok et al. (2015). However, the results in Mabrok et al. (2015) cannot be considered as a generalization of Lemma 1. Lemma 1 is used in this paper to help the controller design, and the requirement of the closed-loop system having no poles at the origin is not a strong condition.

Before presenting the main results, some notation is defined to simplify the presentation. Let $C_2^{\perp}\in\mathbb{R}^{(n-q)\times n}$ such that $C_2C_2^{\perp\mathsf{T}}=0$ and $C_2^{\perp}C_2^{\perp\mathsf{T}}=I$; in other words, the rows of $C_2^{\perp}$ consist of the basis of the orthogonal complement subspace of the subspace spanned by the rows of C_2. One has that $[C_2^{\mathsf{T}}\ C_2^{\perp\mathsf{T}}]$ is invertible. Let $C_2^{+}\in\mathbb{R}^{n\times q}$

be the Moore–Penrose inverse of C_2; that is, $C_2^+ = C_2^\top(C_2C_2^\top)^{-1}$. Let $C_2^R \in \mathbb{R}^{n\times q}$ be a right inverse of C_2; in other words, C_2^R satisfies $C_2C_2^R = I$ and is of full column rank. All the right inverses of C_2 are given by $C_2^R = C_2^+ + (QC_2^\perp)^\top$ where $Q \in \mathbb{R}^{q\times(n-q)}$ is an arbitrary matrix.

3. Static output feedback control design

A static output feedback controller is of the form

$$u(t) = Fy(t), \tag{2}$$

where $F \in \mathbb{R}^{p\times q}$ is the control gain to be determined. The resulting closed-loop system is

$$\begin{cases} \dot{x}(t) = (A + B_2FC_2)x(t) + B_1w(t), \\ z(t) = C_1x(t). \end{cases} \tag{3}$$

The transfer function of the system (3) is given by

$$R(s) = C_1(sI - A - B_2FC_2)^{-1}B_1. \tag{4}$$

According to Lemma 1, $R(s)$ is negative imaginary if and only if the matrix $A + B_2FC_2$ is invertible, and there exists a matrix $Y \in \mathbb{R}^{n\times n}$, $Y = Y^\top > 0$, such that

$$(A + B_2FC_2)Y + Y(A + B_2FC_2)^\top \le 0 \tag{5}$$

$$B_1 + (A + B_2FC_2)YC_1^\top = 0. \tag{6}$$

The following result gives a necessary and sufficient condition for the solvability of the conditions in (5)–(6). This result will be used to derive our main results.

Lemma 2. *There exist matrices $F \in \mathbb{R}^{p\times q}$, $Y \in \mathbb{R}^{n\times n}$, $Y = Y^\top > 0$, such that (5) and (6) hold, if and only if, there exist matrices $Q \in \mathbb{R}^{q\times(n-q)}$, $Y_1 \in \mathbb{R}^{q\times q}$, $Y_1 = Y_1^\top > 0$, $Y_2 \in \mathbb{R}^{(n-q)\times(n-q)}$, $Y_2 = Y_2^\top > 0$, $M \in \mathbb{R}^{p\times q}$ such that*

$$A\overline{Y} + \overline{Y}A^\top + B_2\overline{M} + \overline{M}^\top B_2^\top \le 0 \tag{7}$$

$$B_1 + (A\overline{Y} + B_2\overline{M})C_1^\top = 0 \tag{8}$$

hold, where $\overline{Y} = C_2^RY_1(C_2^R)^\top + C_2^{\perp\top}Y_2C_2^\perp$, $\overline{M} = M(C_2^R)^\top$, and $C_2^R = C_2^+ + (QC_2^\perp)^\top$.

Proof. Define $\overline{C}_2 = [C_2^+ \quad C_2^{\perp\top}] \in \mathbb{R}^{n\times n}$, which is an invertible matrix.

($\Rightarrow$) Firstly, $Y > 0$ implies that

$$\overline{C}_2^{-1}Y\overline{C}_2^{-\top} = \begin{bmatrix} \widetilde{Y}_1 & \widetilde{Y}_3 \\ \widetilde{Y}_3^\top & \widetilde{Y}_2 \end{bmatrix} > 0, \tag{9}$$

where $\widetilde{Y}_1 \in \mathbb{R}^{q\times q}$, $\widetilde{Y}_2 \in \mathbb{R}^{(n-q)\times(n-q)}$ and $\widetilde{Y}_3 \in \mathbb{R}^{q\times(n-q)}$.

Next, we will verify that $Q = \widetilde{Y}_1^{-1}\widetilde{Y}_3$, $Y_1 = \widetilde{Y}_1 > 0$, $Y_2 = \widetilde{Y}_2 - \widetilde{Y}_3^\top\widetilde{Y}_1^{-1}\widetilde{Y}_3 > 0$, $M = F\widetilde{Y}_1$ are a set of solutions to (7) and (8).

Note that $\overline{Y}$ can be rewritten as

$$\begin{aligned}\overline{Y} &= \overline{C}_2\begin{bmatrix} I & 0 \\ Q^\top & I \end{bmatrix}\begin{bmatrix} Y_1 & 0 \\ 0 & Y_2 \end{bmatrix}\begin{bmatrix} I & Q \\ 0 & I \end{bmatrix}\overline{C}_2^\top \\ &= \overline{C}_2\begin{bmatrix} Y_1 & Y_1Q \\ Q^\top Y_1 & Q^\top Y_1Q + Y_2 \end{bmatrix}\overline{C}_2^\top \\ &= \overline{C}_2\begin{bmatrix} \widetilde{Y}_1 & \widetilde{Y}_3 \\ \widetilde{Y}_3^\top & \widetilde{Y}_2 \end{bmatrix}\overline{C}_2^\top = Y,\end{aligned}$$

and that

$$\begin{aligned}FC_2Y &= FC_2\overline{Y} \\ &= FC_2\overline{C}_2\begin{bmatrix} Y_1 & Y_1Q \\ Q^\top Y_1 & Q^\top Y_1Q + Y_2 \end{bmatrix}\overline{C}_2^\top \\ &= F\begin{bmatrix} I & 0 \end{bmatrix}\begin{bmatrix} Y_1 & Y_1Q \\ Q^\top Y_1 & Q^\top Y_1Q + Y_2 \end{bmatrix}\overline{C}_2^\top \\ &= FY_1(C_2^R)^\top = M(C_2^R)^\top = \overline{M}.\end{aligned}$$

Therefore, the Eqs. in (5) and (6) can be written as the Eqs. in (7) and (8), respectively.

($\Leftarrow$) We will verify that $Y = \overline{Y} > 0$ and $F = MY_1^{-1}$ are a set of solutions to (5) and (6). Due to the particular structure of $\overline{Y}$, it can be verified that $FC_2Y = \overline{M}$, as shown in the necessity part, still holds. Therefore, (7) and (8) can be written as (5) and (6), respectively. □

The conditions in (7) and (8) are still hard to solve. Fortunately, by fixing Q, (7) reduces to a linear matrix inequality and (8) a linear matrix equality, and hence can be solved efficiently. The proof of Lemma 2 also shows that once a set of solutions to (7) and (8) is found, the static output feedback controller gain can be designed as $F = MY_1^{-1}$. Hence the following result is obtained.

Theorem 1. *Consider plant (1) and static output feedback controller (2). For any given matrix $Q \in \mathbb{R}^{q\times(n-q)}$, if there exist matrices $Y_1 \in \mathbb{R}^{q\times q}$, $Y_1 = Y_1^\top > 0$, $Y_2 \in \mathbb{R}^{(n-q)\times(n-q)}$, $Y_2 = Y_2^\top > 0$, $M \in \mathbb{R}^{p\times q}$ such that linear matrix inequality (7) and the linear matrix equality (8) hold, then the controller gain in (2) is given by $F = MY_1^{-1}$, and the closed-loop system (3) is negative imaginary provided that $\det(A + B_2FC_2) \neq 0$.*

According to Lemma 2, the existence of the negative imaginary controller (2) guarantees the existence of Q in Theorem 1 such that (7) and (8) are feasible. On the other hand, it is observed from the proof of Lemma 2 that fixing the value of Q in Theorem 1 is essentially equivalent to fixing the value of $\widetilde{Y}_3$ in (9). Based on this observation, the following algorithm is suggested to search for an appropriate Q.

Algorithm 1. (1) Choose any starting value $Q^{(0)}$, two relatively large feasibility multipliers $\gamma^{(0)} > 0$ and $\epsilon^{(0)} > 0$. Pick up two reduction factors $1 > \beta_\gamma > 0$, $1 > \beta_\epsilon > 0$ and stopping tolerance $\delta > 0$. Initialize iteration counter $k \leftarrow 0$.

(2) Check the feasibility of (7) and (8) in Theorem 1. If a feasible solution to (7) and (8) is found, then a controller gain is given by $F = MY_1^{-1}$. STOP.

(3) If either $\gamma^{(k)} < \delta$ or $\epsilon^{(k)} < \delta$ is true, then the controller (2) cannot be designed. STOP.

(4) Solve the following feasible problem

$$Y^{(k)} = Y^{(k)\top} > 0$$

$$\Gamma_1^{(k)} + \Gamma_1^{(k)\top} \le \gamma^{(k)}I$$

$$\begin{bmatrix} -\epsilon^{(k)}I & \Gamma_2^{(k)} \\ \Gamma_2^{(k)\top} & -I \end{bmatrix} \le 0$$

where

$$\begin{aligned}\Gamma_1^{(k)} &= A\overline{Y}^{(k)} + B_2\overline{M}^{(k)} \\ \Gamma_2^{(k)} &= B_1 + \Gamma_1^{(k)}C_1^\top \\ Y^{(k)} &= \begin{bmatrix} Y_1 & Y_3 \\ Y_3^\top & Q^{(k)\top}Y_1Q^{(k)} + Y_2 \end{bmatrix} \\ \overline{Y}^{(k)} &= \overline{C}_2Y^{(k)}\overline{C}_2^\top \\ \overline{M}^{(k)} &= M\begin{bmatrix} I & Q^{(k)} \end{bmatrix}\overline{C}_2^\top \\ \overline{C}_2 &= \begin{bmatrix} C_2^+ & C_2^{\perp\top} \end{bmatrix}\end{aligned}$$

for matrix variables Y_1, Y_2, Y_3 and M.

(5) Set $Q^{(k+1)} \leftarrow Y_1^{-1}Y_3$ and decrease the feasibility multipliers as $\gamma^{(k+1)} \leftarrow \beta_\gamma \gamma^{(k)}$ and $\epsilon^{(k+1)} \leftarrow \beta_\epsilon \epsilon^{(k)}$. Then advance $k \leftarrow k+1$ and return to (2).

Algorithm 1 is designed by considering Q in Theorem 1 as a matrix variable again. Let $Y_3 = Y_1 Q$, then $\overline{Y}$ is rewritten as

$$\overline{Y} = \overline{C}_2 \begin{bmatrix} Y_1 & Y_3 \\ Y_3^\mathsf{T} & Q^\mathsf{T} Y_1 Q + Y_2 \end{bmatrix} \overline{C}_2^\mathsf{T}.$$

In each iteration of Algorithm 1, Q is firstly fixed so that the feasible problem in Step (4) is convex and then updated according to $Q = Y_1^{-1}Y_3$ in Step (5).

The multipliers $\gamma^{(k)}$ and $\epsilon^{(k)}$ are initially chosen relatively large so that the feasible problem in Step (4) is feasible and then reduced in Step (5) to help find solutions to (7) and (8).

Remark 2. The equality condition (8) in Theorem 1 is inherited from Lemma 1. In contrast, the equality conditions in Corollary 2 of Abbaszadeh and Marquez (2009) stems from the need to tackle the static output feedback controller design. It is worth emphasizing that the solution set of (7) and (8) in Theorem 1 is a convex set. Free toolboxes such as YALMIP can be used to solve such conditions directly.

Remark 3. When $C_2 = I$, the static output feedback controller in (2) reduces to the state feedback controller $u(t) = Fx(t)$. In this case, the matrices $\overline{Y}$ and $\overline{M}$ in Lemma 2 and Theorem 1 become $\overline{Y} = Y_1$ and $\overline{M} = M$, respectively; the state feedback control gain is given by $F = \overline{M}\overline{Y}^{-1}$. Then, Theorem 1 coincides with the results in Petersen and Lanzon (2010); Song et al. (2012).

Remark 4. Due to the simple form for the design of the static output feedback control gain $F = MY_1^{-1}$ in Theorem 1, an arbitrarily structural constraint (Rubió-Massegú et al., 2013; Zečević & Šiljak, 2008) can be imposed on the gain by specifying the same structure on M and a block diagonal structure on Y_1. For example, suppose the output feedback control gain F in distributed control be of the form

$$F = \begin{bmatrix} \bullet & \bullet & 0 & 0 \\ 0 & \bullet & \bullet & 0 \\ 0 & 0 & \bullet & \bullet \end{bmatrix},$$

where the zero blocks indicate the places where the entries of F must be zeros (representing the structural constraints), the bullet blocks represent the entries of F that are to be designed. To force the designed controller gain having the above form, the matrix variables M and Y_1 in Theorem 1 need to have the following form

$$M = \begin{bmatrix} \bullet & \bullet & 0 & 0 \\ 0 & \bullet & \bullet & 0 \\ 0 & 0 & \bullet & \bullet \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} \bullet & 0 & 0 & 0 \\ 0 & \bullet & 0 & 0 \\ 0 & 0 & \bullet & 0 \\ 0 & 0 & 0 & \bullet \end{bmatrix}.$$

In addition, a decentralized controller design example is given in Section 6.

Remark 5. If the system input $w(t)$ is available for the controller design, then a static output feedback controller of the form

$$u(t) = Fy(t) + F_w w(t)$$

can be constructed such that the resulting closed-loop system is negative imaginary. In this case, the result in Theorem 1 remains true after the matrix B_1 in (8) is replaced with $B_1 + B_2 F_w$. Here, F_w is an additional matrix variable to be constructed. The introduction of F_w provides us more freedom to search for negative imaginary controllers.

4. Dynamic output feedback control design

In this section, instead of designing the static feedback controller (2), we are interested in the design of dynamic output feedback controllers. A dynamic output feedback controller is of the form

$$\begin{cases} \dot{x}_F(t) = A_F x_F(t) + B_F y(t), \\ u(t) = C_F x_F(t) + D_F y(t), \end{cases} \tag{10}$$

where $x_F(t) \in \mathbb{R}^{n_F}$ is the state of the controller. The matrices $A_F \in \mathbb{R}^{n_F \times n_F}$, $B_F \in \mathbb{R}^{n_F \times q}$, $C_F \in \mathbb{R}^{p \times n_F}$ and $D_F \in \mathbb{R}^{p \times q}$ are to be designed.

The resulting closed-loop system of plant (1) controlled by (10) is given by

$$\begin{cases} \dot{\overline{x}}(t) = \overline{A}\overline{x}(t) + \widetilde{B}_1 w(t), \\ z(t) = \widetilde{C}_1 \overline{x}(t), \end{cases} \tag{11}$$

where $\overline{x}(t) \triangleq \begin{bmatrix} x(t) \\ x_F(t) \end{bmatrix}$ is the state, and the system matrices are given by

$$\overline{A} = \begin{bmatrix} A + B_2 D_F C_2 & B_2 C_F \\ B_F C_2 & A_F \end{bmatrix}, \qquad \widetilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \qquad \widetilde{C}_1 = \begin{bmatrix} C_1 & 0 \end{bmatrix}.$$

To design the dynamic output feedback controller (10) in a unified way as in designing (2), we rewrite the system matrix $\overline{A}$ as

$$\overline{A} = \widetilde{A} + \widetilde{B}_2 \widetilde{F} \widetilde{C}_2,$$

where

$$\widetilde{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \qquad \widetilde{B}_2 = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix},$$

$$\widetilde{F} = \begin{bmatrix} A_F & B_F \\ C_F & D_F \end{bmatrix}, \qquad \widetilde{C}_2 = \begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix}.$$

As a result, the closed-loop system (11) can be rewritten as

$$\begin{cases} \dot{\overline{x}}(t) = (\widetilde{A} + \widetilde{B}_2 \widetilde{F} \widetilde{C}_2)\overline{x}(t) + \widetilde{B}_1 w(t), \\ z(t) = \widetilde{C}_1 \overline{x}(t), \end{cases} \tag{12}$$

which is of the same form as in (3). Therefore, Theorem 1 is applicable to the dynamic output feedback control design, and the following result is readily obtained.

Corollary 1. *Consider plant (1) and dynamic output feedback controller (10). For any given matrix $Q \in \mathbb{R}^{(n_F+q)\times(n-q)}$, if there exist matrices $Y_1 \in \mathbb{R}^{(n_F+q)\times(n_F+q)}$, $Y_1 = Y_1^\mathsf{T} > 0$, $Y_2 \in \mathbb{R}^{(n-q)\times(n-q)}$, $Y_2 = Y_2^\mathsf{T} > 0$ and $M \in \mathbb{R}^{(n_F+p)\times(n_F+q)}$ such that*

$$\widetilde{A}\overline{Y} + \overline{Y}\widetilde{A}^\mathsf{T} + \widetilde{B}_2\overline{M} + \overline{M}^\mathsf{T}\widetilde{B}_2^\mathsf{T} \le 0 \tag{13}$$

$$\widetilde{B}_1 + (\widetilde{A}\overline{Y} + \widetilde{B}_2\overline{M})\widetilde{C}_1^\mathsf{T} = 0 \tag{14}$$

hold, where $\overline{Y} = \widetilde{C}_2^\mathrm{R} Y_1 (\widetilde{C}_2^\mathrm{R})^\mathsf{T} + \widetilde{C}_2^{\perp\mathsf{T}} Y_2 \widetilde{C}_2^{\perp}$, $\overline{M} = M(\widetilde{C}_2^\mathrm{R})^\mathsf{T}$, and $\widetilde{C}_2^\mathrm{R} = \widetilde{C}_2^{+} + (Q\widetilde{C}_2^{\perp})^\mathsf{T}$. Then the system matrices of the dynamic controller (10) are given by $\widetilde{F} = MY_1^{-1}$, and the closed-loop system (12) is negative imaginary provided that $\det(\widetilde{A} + \widetilde{B}_2\widetilde{F}\widetilde{C}_2) \neq 0$.

Remark 6. When the system input $w(t)$ is available for the controller, it can be used to help design negative imaginary controllers. In this case, the dynamic output feedback controller can be of the form

$$\begin{cases} \dot{x}_F(t) = A_F x_F(t) + B_F y(t) + B_w w(t), \\ u(t) = C_F x_F(t) + D_F y(t) + D_w w(t), \end{cases}$$

and the resulting closed-loop system is the same as the one in (12) except that the matrix $\widetilde{B}_1$ is given by $\begin{bmatrix} B_1 + B_2 D_w \\ B_w \end{bmatrix}$. Fortunately, the introduction of B_w and D_w into $\widetilde{B}_1$ does not increase computation complexity of the solvability of the conditions in (13) and (14), but gives more freedom to find a negative imaginary controller.

Remark 7. By the virtue of the similarity of the design methods for the static output feedback control gain F in Theorem 1 and the dynamic output feedback control matrices $\widetilde{F}$ in Corollary 1, an arbitrarily structural constraint can be imposed on the dynamic controller (10) as well (see the example section for an illustration).

5. Observer-based state feedback control design

In this section, observer-based state feedback controllers are to be designed for the output feedback negative imaginary synthesis problem. Consider a typical observer-based state feedback controller of the form

$$\begin{cases}\dot{\widehat{x}}(t) = A\widehat{x}(t) + B_2u(t) + L(y(t) - C_2\widehat{x}(t)),\\ u(t) = K\widehat{x}(t),\end{cases} \tag{15}$$

where $L \in \mathbb{R}^{n\times q}$ and $K \in \mathbb{R}^{p\times n}$ are the observer gain matrix and the state feedback gain matrix, respectively. The matrices L and K are to be determined.

Let $\bar{x}(t) = \begin{bmatrix} x(t) \\ \widehat{x}(t)\end{bmatrix}$. The closed-loop system of plant (1) under control of (15) is given by

$$\begin{cases}\dot{\bar{x}}(t) = (\widetilde{A} + \widetilde{B}_2\widetilde{F}\widetilde{C}_2)\bar{x}(t) + \widetilde{B}_1w(t),\\ z(t) = \widetilde{C}_1\bar{x}(t),\end{cases} \tag{16}$$

which has the same form as in (12), except that the component matrices are different. Here,

$$\widetilde{A} = \begin{bmatrix} A & 0 \\ 0 & A\end{bmatrix}, \quad \widetilde{B}_2 = \begin{bmatrix} B_2 & 0 \\ B_2 & I\end{bmatrix}, \quad \widetilde{F} = \begin{bmatrix} K & 0 \\ 0 & L\end{bmatrix},$$
$$\widetilde{C}_2 = \begin{bmatrix} 0 & I \\ C_2 & -C_2\end{bmatrix}, \quad \widetilde{B}_1 = \begin{bmatrix} B_1 \\ 0\end{bmatrix}, \quad \widetilde{C}_1 = \begin{bmatrix} C_1 & 0\end{bmatrix}.$$

By Theorem 1, the following result can be obtained for the design of the matrices L and K.

Corollary 2. *Consider plant (1) and observer-based controller (15). For any given matrix $Q \in \mathbb{R}^{(n+q)\times(n-q)}$, if there exist matrices $Y_1 \triangleq \begin{bmatrix} Y_{11} & 0 \\ 0 & Y_{12}\end{bmatrix} \in \mathbb{R}^{(n+q)\times(n+q)}$, $Y_{11} \in \mathbb{R}^{n\times n}$, $Y_{12} \in \mathbb{R}^{q\times q}$, $Y_1 = Y_1^{\mathsf{T}} > 0$, $Y_2 \in \mathbb{R}^{(n-q)\times(n-q)}$, $Y_2 = Y_2^{\mathsf{T}} > 0$ and $M \triangleq \begin{bmatrix} M_1 & 0 \\ 0 & M_2\end{bmatrix} \in \mathbb{R}^{(p+n)\times(n+q)}$, $M_1 \in \mathbb{R}^{p\times n}$, $M_2 \in \mathbb{R}^{n\times q}$, such that*

$$\widetilde{A}\overline{Y} + \overline{Y}\widetilde{A}^{\mathsf{T}} + \widetilde{B}_2\overline{M} + \overline{M}^{\mathsf{T}}\widetilde{B}_2^{\mathsf{T}} \leq 0 \tag{17}$$
$$\widetilde{B}_1 + (\widetilde{A}\overline{Y} + \widetilde{B}_2\overline{M})\widetilde{C}_1^{\mathsf{T}} = 0 \tag{18}$$

hold, where $\overline{Y} = \widetilde{C}_2^{\mathrm{R}}Y_1(\widetilde{C}_2^{\mathrm{R}})^{\mathsf{T}} + \widetilde{C}_2^{\perp\mathsf{T}}Y_2\widetilde{C}_2^{\perp}$, $\overline{M} = M(\widetilde{C}_2^{\mathrm{R}})^{\mathsf{T}}$, and $\widetilde{C}_2^{\mathrm{R}} = \widetilde{C}_2^{+} + (Q\widetilde{C}_2^{\perp})^{\mathsf{T}}$. Then with the observer gain matrix given by $L = M_2Y_{12}^{-1}$ and the state feedback gain given by $K = M_1Y_{11}^{-1}$, the closed-loop system (16) is negative imaginary provided that $\det(\widetilde{A} + \widetilde{B}_2\widetilde{F}\widetilde{C}_2) \neq 0$.

When the system input $w(t)$ is available for the observer design, it can significantly simplify the negative imaginary synthesis problem. Now consider an observer-based state feedback controller of the form

$$\begin{cases}\dot{\widehat{x}}(t) = A\widehat{x}(t) + B_1w(t) + B_2u(t) + L(y(t) - C_2\widehat{x}(t)),\\ u(t) = K\widehat{x}(t).\end{cases} \tag{19}$$

Let $e(t) = x(t) - \widehat{x}(t)$. The resulting closed-loop system is given by

$$\begin{cases}\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \widetilde{B}_1w(t),\\ z(t) = \widetilde{C}_1\bar{x}(t),\end{cases} \tag{20}$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ e(t)\end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A + B_2K & -B_2K \\ 0 & A - LC_2\end{bmatrix},$$
$$\widetilde{B}_1 = \begin{bmatrix} B_1 \\ 0\end{bmatrix}, \quad \widetilde{C}_1 = \begin{bmatrix} C_1 & 0\end{bmatrix}.$$

The transfer function of the closed-loop system (20) is given by

$$R(s) = \widetilde{C}_1(sI - \bar{A})^{-1}\widetilde{B}_1 = C_1(sI - A - B_2K)^{-1}B_1.$$

As a result, a version of separation principle holds for the negative imaginary synthesis problem, and the result is summarized in the following theorem.

Theorem 2. *Consider plant (1) and observer-based controller (19). If the observer gain is designed such that $A - LC_2$ is stable, the state feedback gain is designed such that $(A + B_2K, B_1, C_1)$ is negative imaginary, then the closed-loop system (20) is negative imaginary.*

6. Illustrative examples

Three examples are provided in this section. The first example demonstrates the application of the developed theory to a robust stabilization problem. The conservatism of the results in the paper is studied in the first example. The second example validates the applicability of the results to MIMO systems. The third example illustrates the design of structured controllers.

Example 1 (*Robust Stabilization*)**.** Consider the uncertain system in Petersen and Lanzon (2010); Song et al. (2012). A state-space realization of the uncertain system is given by

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1\end{bmatrix}x(t) + \begin{bmatrix} 0 \\ 0 \\ 1\end{bmatrix}w(t) + \begin{bmatrix} -2 \\ 1 \\ 0\end{bmatrix}u(t),$$
$$z(t) = \begin{bmatrix} 0 & 1 & 0\end{bmatrix}x(t),$$
$$\hat{w}(s) = \Delta(s)\hat{z}(s).$$

The system uncertainty is modelled by $\Delta(s)$ as an unknown transfer function, which satisfies the strictly negative imaginary property (Xiong et al., 2010). Furthermore, the uncertainty is assumed to satisfy $\Delta(0) \leq 1$ and $\Delta(\infty) \geq 0$. The readers are referred to Petersen and Lanzon (2010); Song et al. (2012) for further details about the example.

According to the stability result in negative imaginary systems theory (that is, Theorem 1 in Xiong et al. (2010)), if the transfer function (4) is negative imaginary and satisfies $R(0) < 1$ and $R(\infty) = 0$, then the uncertain closed-loop system in this example is internally stable. In Petersen and Lanzon (2010); Song et al. (2012), static state feedback controllers have been successfully found to achieve robust stability of the uncertain system. Here, we are interested in the design of static output feedback controllers to achieve the same goal.

Suppose the measurement output of the system is given by

$$y(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}x(t).$$

Algorithm 1 is used to find the desired controller. To start Algorithm 1, choose $Q^{(0)} = 0, \gamma^{(0)} = 100, \epsilon^{(0)} = 100, \beta_\gamma = 0.618$, $\beta_\epsilon = 0.618$ and $\delta = 0.0001$. After fifteen iterations, the algorithm successfully found a set of feasible solutions to (7) and (8), which was given by

$$Q = \begin{bmatrix} -0.3909 \\ -0.8451\end{bmatrix}, \quad Y_1 = \begin{bmatrix} 0.9345 & 1.9345 \\ 1.9345 & 6.0240\end{bmatrix},$$
$$Y_2 = 0.8451, \quad M = \begin{bmatrix} 1.0000 & 2.7572\end{bmatrix}.$$

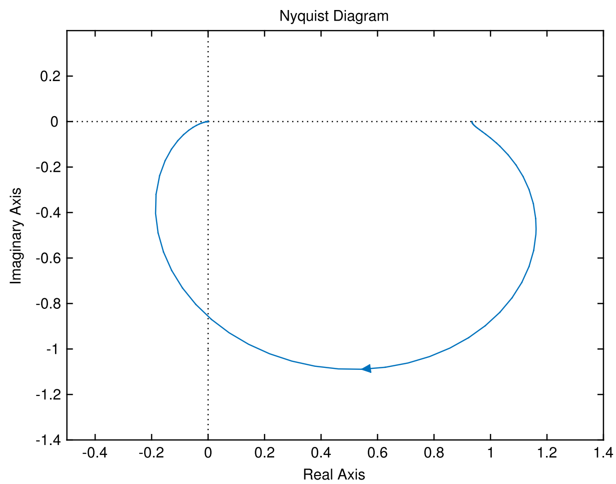

Fig. 2. Nyquist plot of closed-loop system $R(s)$.

The designed controller was given by

$$u(t) = MY_1^{-1}y(t) = \begin{bmatrix}0.3658 & 0.3402\end{bmatrix} y(t).$$

With this controller, it can be verified that the transfer function $R(s)$ in (4) is negative imaginary, $R(0) = 0.9345$, and $R(\infty) = 0$. Therefore, the designed static output feedback controller robustly stabilizes the uncertain plant. The Nyquist plot of $R(s)$ is shown in Fig. 2.

To gain insight into the conservatism of the design conditions in Theorem 1, we firstly calculated out the exact region for the controller parameters such that the transfer function $R(s)$ in (4) is both negative imaginary and $R(0) < 1$. Then we searched the region around the Q given by Algorithm 1 to find as many control gains as possible that satisfy the conditions in (7) and (8) and $R(0) < 1$. The result is depicted in Fig. 3. The grey area is the exact region found in theory, the dotted points are the control gains found using Theorem 1. We might conclude from the figure that the conservatism in Theorem 1 is not significant, at least for this particular example.

Example 2 (*MIMO Systems*)**.** Consider an MIMO system given by

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t),$$

$$z(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(t),$$

$$y(t) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(t).$$

Starting Algorithm 1 with the same setup as in Example 1, a set of solutions was successfully found after one iteration. The designed static output feedback controller is given by

$$u(t) = \begin{bmatrix}-2.0000 & -1.0809\end{bmatrix} y(t).$$

The resulting closed-loop system is negative imaginary, which can be verified by Lemma 1.

Example 3 (*Decentralized Control*)**.** Consider a system of the form (1). The system data are borrowed from Rubió-Massegú et al. (2013) and given by

$$A = \begin{bmatrix} -4 & 0 & -2 & 0 & 0 \\ 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 & -1 \\ 0 & -2 & 0 & -1 & 0 \\ 3 & 0 & -2 & 0 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

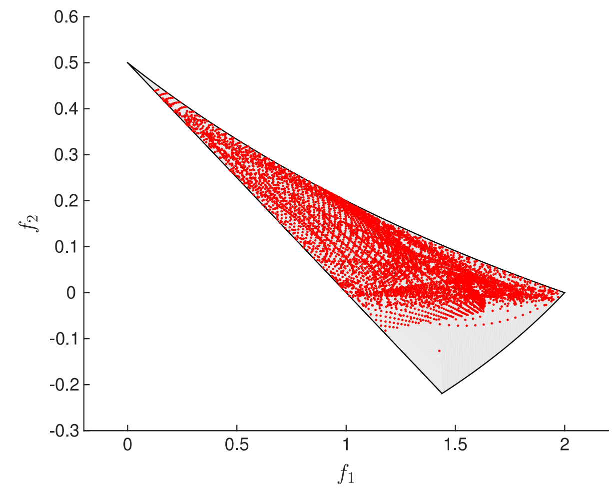

Fig. 3. Exact region of robust controller (grey area) and feasible points based on Theorem 1.

$$B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Here, the system output equation (that is, C_1) has been changed to suit our model (1). In Rubió-Massegú et al. (2013), a decentralized static output feedback controller was designed such that the closed-loop system is stable and has a specified H_∞ performance. Here, we want to design a decentralized reduced-order dynamic output feedback controller such that the closed-loop system is negative imaginary. Applying Algorithm 1 with the same setup as in Example 1, a desired controller of the form (10) was found after two iterations. The controller matrices are given by

$$A_F = \begin{bmatrix} -0.5007 & 0 & 0 \\ 0 & -0.9975 & 0 \\ 0 & 0 & -0.5001 \end{bmatrix},$$

$$B_F = \begin{bmatrix} 0.1062 & 0 & 0 \\ 0 & -2.2445 & 0 \\ 0 & 0 & 0.0506 \end{bmatrix},$$

$$C_F = \begin{bmatrix} -0.0263 & 0 & 0 \\ 0 & -0.3837 & 0 \\ 0 & 0 & -0.0288 \end{bmatrix},$$

$$D_F = \begin{bmatrix} 0.3570 & 0 & 0 \\ 0 & 2.0114 & 0 \\ 0 & 0 & -2.9114 \end{bmatrix}.$$

7. Conclusions

This paper studied the output feedback negative imaginary synthesis problem. Sufficient conditions have been established for static output feedback control design, dynamic output feedback control design and observer-based control design, respectively. For the design conditions, arbitrarily structural constraints can be readily imposed on the designed controllers. The efficiency of these conditions has been illustrated by three numerical examples. The conservatism of the conditions has also been discussed. The questions of how to extend the results of this paper to deal with either systems with zero or infinite poles or systems with parameter uncertainties are interesting areas for future research.

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