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Decentralized control scheme for large-scale systems defined over a graph in presence of communication delays and random missing measurements[☆]

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ABSTRACT

This paper studies the decentralized output-feedback control of large-scale systems defined over a directed connected graph with communication delay and random missing measurements. The nodes in the graph represent the subsystems, and the edges represent the communication connection. The information travels across an edge in the graph and suffers from one step communication delay. For saving the storage space, the information delayed more than D step times is discarded. In addition, to model the system in a more practical case, we assume that the observation for the subsystem output suffers random missing. Under this new information pattern, the optimal output-feedback control problem is non-convex, what is worse, the separation principle fails. This implies that the optimal control problem with the information pattern introduced above is difficult to solve. In this paper, a new decentralized control scheme is proposed. In particular, a new estimator structure and a new controller structure are constructed, and the gains of the estimator and the controller are designed simultaneously. An optimality condition with respect to the gains is established. Based on the optimality condition, an iterative algorithm is exploited to design the gains numerically. It is shown that the exploited algorithm converges to Nash optimum. Finally, the proposed theoretical results are illustrated by a physical system which is a heavy duty vehicles platoon.

1. Introduction

Recently, much research attention has been paid to large-scale systems in which subsystems exchange information through a communication network, usually wireless. Such systems can be found in engineering fields, such as smart grids (Aldeen, Saha, Alpcan, & Evans, 2015), smart vehicle formations (Fax & Murray, 2004), and sensor network (Sivakumar, Sadagopan, & Baskaran, 2016). One feature of such systems is that the system performance is severely affected by the imperfections of the communication network (Heemels, Teel, Van de Wouw, & Nesic, 2010), such as packet losses, network delay, and communication constraint. To understand and counteract the effects of the communication network imperfections is becoming increasingly important. Especially, how to counteract the effects induced by the network delay for large-scale systems is a hot research topic.

Decentralized state-feedback linear quadratic Gaussian (LQG) control for large-scale systems defined over a directed connected graph with communication delay has been studied in Feyzmahdavian, Alam, and Gattami (2012), Lamperski and Doyle (2012), Lamperski and Lessard (2015), Matni and Doyle (2013). The authors of Lamperski and Doyle (2012) and Lamperski and Lessard (2015) designed an explicit optimal state-feedback LQG controller based on the independence decomposition of the process noise history. The varying communication delay case was investigated in Matni and Doyle (2013). The result of Matni and Doyle (2013) is only suitable for two-player systems. The design methods of Lamperski and Doyle (2012), Lamperski and Lessard (2015) and Matni and Doyle (2013) cannot be extended to the output-feedback case. The reason is that the independence decomposition for the measurements is not valid. In addition, the results of Lamperski and Doyle (2012), Lamperski and Lessard (2015) and Matni and Doyle (2013) were established under the assumption that the process noises of different subsystems are independent of each other. Removing this assumption, the explicit optimal state-feedback controller was

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found by the vectorization technique in Feyzmahdavian, Alam et al. (2012). The result of Feyzmahdavian, Alam et al. (2012) was extended to the output-feedback case in Feyzmahdavian, Gattami, and Johansson (2012). However, the result of Feyzmahdavian, Gattami et al. (2012) is only for the three-player systems with chain structure, and is unlikely to be extended to the large-scale systems composed of N subsystems. For large-scale systems composed of N subsystems, decentralized output feedback control with delay sharing pattern was investigated in Kurtaran and Sivan (1974) and Nayyar, Mahajan, and Teneketzis (2011). The authors of Kurtaran and Sivan (1974) designed an optimal output feedback LQG controller under one step delay sharing pattern by dynamic programming. Two structural results for multiple step delays sharing pattern were established in Nayyar et al. (2011). In addition, the decentralized output feedback controller with asymmetric one step delay sharing pattern was designed in Nayyar, Kalathil, and Jain. (2018). However, for the delay model defined over a directed connected graph, the decentralized output feedback control of large-scale systems composed of N subsystems is not fully studied.

On the other hand, in Feyzmahdavian, Gattami et al. (2012), Kurtaran and Sivan (1974) and Nayyar et al. (2018, 2011), it is assumed that the observation for the subsystem output is always valid. However, the observation may be affected by uncertain factors in engineering practice, and thus may suffer from random missing. For systems with random missing measurements (uncertain observation), the linear filtering problems have been studied, see (Ma & Sun, 2011; Moayedi, Foo, & Soh, 2010); however, the decentralized controller design considering random missing measurements is still an open problem. To design the optimal decentralized controller under random missing measurement is a challenge task, because the separation principle (Yoshikawa & Kobayashi, 1978) may fail.

In this paper, we focus on the decentralized output feedback LQG control for large-scale systems with communication delays and random missing measurements. The large-scale system is composed of N subsystems, and is defined over a directed connected graph. The nodes in the graph represent the subsystems. The measurement output in each subsystem contains valid measurement or noise only (random missing measurements). The edges in the graph represent the communication network. The information travels across an edge with one step delay. Such a delay model was introduced in Lamperski and Doyle (2012) and Lamperski and Lessard (2015), and was applied to vehicle formations control in Feyzmahdavian, Alam et al. (2012). In this paper, it is assumed that each subsystem maintains a buffer of length $D+1$ such that the information delayed more than D step times is discarded. Under this setup, the corresponding optimal LQG control problem is non-convex. To solve this optimal control problem, we propose a new decentralized control scheme. Firstly, a new estimator structure and a new controller structure are constructed. It is shown that the separation principle (Yoshikawa & Kobayashi, 1978) fails. Secondly, an optimality condition with respect to the gains of the estimator and the controller is established. Thirdly, we give an iterative algorithm to find the gains of the estimator and the controller simultaneously, and we show that the algorithm converges to Nash optimum. Lastly, we use a heavy duty vehicles platoon to illustrate the theoretical results proposed in this paper.

Notation. For a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, $\mathcal{V}=\{1,\dots,N\}$ is the node set and $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ is the edge set; define $\mathcal{N}_i=\{j:(j,i)\in\mathcal{E}\}$, where (j,i) is an arrow from j to i. The sequence $\{x_0,\dots,x_t\}$ is denoted by $x_{0:t}$. Let $\mathrm{tr}(X)$ denote the trace of the square matrix X. $\mathbb{E}(x)$ is the expectation of the random variable x, and $\mathbb{E}(x|y)$ is the conditional expectation of x given y. Let A^{T} denote the transpose of the matrix A. The notations $X\succ 0$ and $X\succeq 0$ mean that X is a positive definite matrix and a positive semi-definite matrix, respectively. The $m\times n$ zero matrix is denoted by $0_{m\times n}$, and the $n\times n$ zero matrix is denoted by 0_n. For a matrix A, $(A)^n$ denotes the nth power of A, and $(A)_{ij}$ denotes the ith row, jth column element of A. $\Pr(\cdot)$ is the probability measure. For two sets $\mathbb{X}_1$, $\mathbb{X}_2$, define $\mathbb{X}_1\setminus\mathbb{X}_2\triangleq\{x:x\in\mathbb{X}_1 \text{ and } x\notin\mathbb{X}_2\}$.

2. Problem statement

Consider a large-scale system defined over a connected directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ with $\mathcal{V}=\{1,\dots,N\}$, where the nodes represent the subsystems. The ith subsystem is of the form

$$x_{t+1}^i = A^{ii}x_t^i + \sum_{j\in\mathcal{N}_i} A^{ij}x_t^j + B^i u_t^i + \omega_t^i, \tag{1}$$

$$y_t^i = \delta_t^i C^i x_t^i + \upsilon_t^i. \tag{2}$$

For the ith subsystem, $x^i\in\mathbb{R}^{n_i}$ is the state; $u^i\in\mathbb{R}^{l_i}$ is the control input; $y^i\in\mathbb{R}^{m_i}$ is the measurement output; $\omega^i\in\mathbb{R}^{n_i}$ is the process noise; $\upsilon^i\in\mathbb{R}^{m_i}$ is the measurement noise. The matrices A^{ij}, B^i and C^i are of proper dimensions for all $i,j\in\mathcal{V}$; δ_t^i is a random binary variable, and satisfies

$$\Pr(\delta_t^i=1)=\lambda,\quad \Pr(\delta_t^i=0)=1-\lambda,$$

where $\delta_t^i=0$ means that the measurement of the ith subsystem is missing (the output contains noise only); $\delta_t^i=1$ implies that the measurement of the ith subsystem is valid. It is assumed that $\delta_{t_1}^i$ is independent of $\delta_{t_2}^j$ for any $t_1\neq t_2$ or $i\neq j$.

Define the following matrices

$$A=\begin{bmatrix} A^{11} & \cdots & A^{1N}\\ \vdots & \ddots & \vdots\\ A^{N1} & \cdots & A^{NN}\end{bmatrix},\ A^{ij}=0 \text{ for } (i,j)\notin\mathcal{E},$$

$$B=\begin{bmatrix} B^1 & & \\ & \ddots & \\ & & B^N\end{bmatrix},\ C=\mathrm{diag}\{C^1,\dots,C^N\},$$

$$\delta_t=\mathrm{diag}\{\delta_t^1 I_{m_1},\dots,\delta_t^N I_{m_N}\}.$$

Stacking x^i, u^i, ω^i, υ^i and y^i into augmented vectors

$$x=\begin{bmatrix} x^1\\ \vdots\\ x^N\end{bmatrix},\quad u=\begin{bmatrix} u^1\\ \vdots\\ u^N\end{bmatrix},\quad y=\begin{bmatrix} y^1\\ \vdots\\ y^N\end{bmatrix},$$

$$\omega=\begin{bmatrix} \omega^1\\ \vdots\\ \omega^N\end{bmatrix},\quad \upsilon=\begin{bmatrix} \upsilon^1\\ \vdots\\ \upsilon^N\end{bmatrix}.$$

The large-scale system (1)–(2) can be written as

$$x_{t+1} = Ax_t + Bu_t + \omega_t, \tag{3}$$

$$y_t = \delta_t C x_t + \upsilon_t, \tag{4}$$

where the initial state x_0 is a Gaussian variable with $x_0\sim\mathcal{N}(\bar{x},\Theta_0)$, $\Theta_0\succ 0$. The noises ω_t and υ_t are the independent Gaussian processes with $\omega_t\sim\mathcal{N}(0,W_t)$, $W_t\succ 0$. and $\upsilon_t\sim\mathcal{N}(0,V_t)$, $V_t\succ 0$, respectively. Assume that x_0, ω_{t_1} and υ_{t_2} are pairwise independent for all t_1, t_2. In addition, the system parameters A, B, C, Θ_0, W_t and V_t are known to all subsystems.

The information pattern in this paper is described as follows. The information travels across an edge in the graph and requires one step time. Define τ_{ij} as the length of the shortest path from the ith subsystem to the jth subsystem, where $\tau_{ii}=0$. Hence,

τ_{ij} step times are needed for the information to flow from the ith subsystem to the jth subsystem. For example, consider the system defined over the graph in Fig. 1, the information to flow between the 1st subsystem and the 2nd subsystem needs one step time. Three step times are needed for the information to go across from the 3rd subsystem to the 1st subsystem, and so on. In addition, for saving the storage space, the information delayed more than D step times will be discarded. As a result, the measurement output available to the ith subsystem is

$$\mathcal{L}_t^i = \{y^j_{t-D:t-\tau_{ij}} : j \in \bar{\mathcal{V}}_i\},$$

where $\bar{\mathcal{V}}_i = \{j \in \mathcal{V} : \tau_{ij} \le D\}$. Assume that $D < \max_{i,j\in\mathcal{V}} \tau_{ij}$. In this paper, the control input of the ith subsystem is restricted to the form

$$u_t^i = \gamma_t^i(\mathcal{L}_t^i), \tag{5}$$

where γ_t^i is a linear function.

Define the following cost function

$$J \triangleq \mathbb{E}\left\{\sum_{t=0}^{T-1} x_t^{\mathrm{T}} Q_t x_t + u_t^{\mathrm{T}} R_t u_t + x_T^{\mathrm{T}} Q_T x_T\right\}, \tag{6}$$

where $Q_t \succeq 0$ $(t = 0, \ldots, T)$ and $R_t \succ 0$ $(t = 0, \ldots, T-1)$ are known to the subsystems. In this paper, we assume that $T \gg \max_{i,j\in\mathcal{V}} \tau_{ij} > D$, and T is finite. In general, the parameter T in (6) is chosen to be the system running time. Our objective is to find the control input of the form (5) to minimize the cost function (6).

Remark 1. The communication delay and dropped messages (the information delayed more than D step times will be discarded) lead to that the available measurement output to the ith subsystem $\mathcal{L}_t^i$ is incomplete information. Thus, the controller performance that is evaluated by the cost function (6) is deteriorated by the communication delay and dropped messages, because in theory, the optimal value of the cost function gets larger by the controller using less information. Note that $\mathcal{L}_t^i$ depends on D. If the choice of D leads to that the information contained in $\mathcal{L}_t^i$ is reduced dramatically, then the optimal value of the cost function would increase dramatically.

Remark 2. The assumption that the graph $\mathcal{G}$ is connected and directed is without loss of generality. The reason is given as follows:

- If the graph $\mathcal{G}$ is unconnected and contains ξ connected subgraphs, the corresponding LQG problem can be decomposed into ξ independent subproblems, and each subproblem is defined over a connected subgraph. We focus on solving the subproblems which are defined over connected graph.
- The undirected graph set is a subset of the directed graph set.

Remark 3. For all $i, j \in \mathcal{V}$, $y^j_{t-D-1} \in \mathcal{L}^j_{t-1}$, but $y^j_{t-D-1} \notin \mathcal{L}^i_t$. It follows that $\mathcal{L}_t^i \not\supseteq \mathcal{L}^j_{t-1}$ for all $i, j \in \mathcal{V}$. According to Definition 3 in Ho and Chu (1972), the information pattern considered in this paper is not partially nested. In addition, Theorem 2 in Ho and Chu (1972) shows that the optimal control law under partially nested information pattern is linear for the quadratic cost function. As a result, the optimal control law γ_t^i for the cost function (6) may be nonlinear. Nevertheless, for implementation simplicity, we focus on designing the linear γ_t^i in this paper.

Remark 4. For the graphs satisfying $\{j : \tau_{ij} > D\} \neq \emptyset$ for any $i \in \mathcal{V}$, the set $\mathcal{L}_t^i$ does not contain the information of the jth subsystem for any j satisfies $\tau_{ij} > D$. This means that $\mathcal{L}_t^i$ is not a sufficient statistic for optimal decentralized decision-making. Hence, for the cost function (6), the optimal current control law γ_t^i depends on all the control law at $t \in \{0, \ldots, T-1\}$, that is $\gamma^j_{0:T-1}$ for $i, j \in \mathcal{V}$. This implies that the optimization problem (to minimize (6) subjecting to (1), (2), (5)) is non-convex. Consequently, the techniques of independence decomposition (Feyzmahdavian, Alam et al., 2012), dynamic programming (Lamperski & Doyle, 2012), Behrman equation (Åström, 2012) are not suitable in this paper.

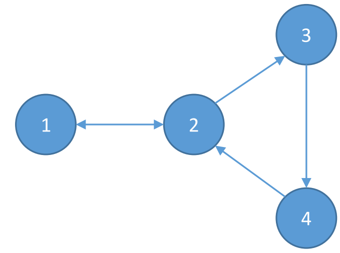

Fig. 1. A large-scale system defined over a directed graph with four nodes.

3. Decentralized control scheme

A new decentralized control scheme is proposed in this section. In Section 3.1, a new estimator structure and a new controller structure are constructed. An optimality condition with respect to the gains of the estimator and the controller are established in Section 3.2. In Section 3.3, an algorithm is exploited to compute the gains of the estimator and the controller numerically, and the realization of the control scheme is presented.

3.1. Framework of the control scheme

In this subsection, the estimator structure and the controller structure are constructed. In addition, for facilitating the gain matrices design in the following subsection, we use the augmented vector approach to rewrite the estimator and the controller in a compact form.

To provide a tradeoff between the online computational burden in the estimator and the controller, we choose a scalar M $(0 < M \le D)$, and decompose the set $\mathcal{L}_t^i$ into two parts: $\mathcal{L}_t^i = \mathcal{L}_t^{i1} \cup \mathcal{L}_t^{i2}$, where $\mathcal{L}_t^{i1} = \{y^j_{t-D:t-M} : j \in \bar{\mathcal{V}}_i\}$, $\mathcal{L}_t^{i2} = \{y^j_{t-M+1:t-\tau_{ij}} : j \in \tilde{\mathcal{V}}_i\}$, $\tilde{\mathcal{V}}_i = \{j \in \bar{\mathcal{V}}_i,\ \tau_{ij} < M\}$. Then, $\mathcal{L}_t^{i1}$ is used to estimate the subsystem state x_t^i in the ith estimator, and $\mathcal{L}_t^{i2}$ is used to generate the control input directly.

The estimated subsystem state denoted by $\hat{x}_t^i$ is computed in the ith subsystem, and is transmitted to the other subsystems via the communication network. The information travelling across an edge suffers from one step delay, and the information delayed by more than D step times is discarded. It follows that the estimated subsystem state available to the ith subsystem is

$$\mathcal{X}_t^i = \{\hat{x}^j_{t-D:t-\tau_{ij}} : j \in \bar{\mathcal{V}}_i\}.$$

To obtain the estimated subsystem state $\hat{x}_t^i$ based on the information set $\mathcal{L}_t^{i1}$ and $\mathcal{X}_t^i$, we propose the following local estimator

$$\begin{cases} \hat{x}_t^{i-} = A^{ii}\hat{x}^i_{t-1} + \sum\limits_{j\in\mathcal{N}_i} A^{ij}\hat{x}^j_{t-1} + B^i u^i_{t-1}, \\ \hat{x}_t^i = \hat{x}_t^{i-} + \sum\limits_{t_0=M}^{D} K_t^{i,t_0}(y_{t-t_0} - C_s\hat{x}_{t-t_0}), \end{cases} \tag{7}$$

where $\hat{x}_0^i = \bar{x}^i$; C_s is a matrix with proper dimensions; $K_t^{i,t_0} = \begin{bmatrix} K_t^{i,t_0,1} & \cdots & K_t^{i,t_0,N} \end{bmatrix}$, $K_t^{i,t_0,j} \in R^{n_i\times m_j}$. Moreover, $K_t^{i,t_0,j} = 0$ if

$S_{t_0}^{ij}=0$, where $S_{t_0}^{ij}$ is defined as follows

$$\left[\begin{array}{c} \mathcal{A}\in\mathbb{R}^{n\times n},\quad n=\sum_{i=1}^{N}n_i,\\ (\mathcal{A})_{ij}=\begin{cases}1, & (A)_{ij}\neq 0;\\ 0, & (A)_{ij}=0;\end{cases}\\ \begin{bmatrix} S_{t_0}^{11} & \cdots & S_{t_0}^{1N}\\ & \ddots & \\ S_{t_0}^{N1} & \cdots & S_{t_0}^{NN}\end{bmatrix}=(\mathcal{A})^{t_0},\ S_{t_0}^{ij}\in\mathbb{R}^{n_i\times n_j}, \end{array}\right]$$

where A is the system matrix defined in Eq. (3).

Remark 5. Note that $S_{t_0}^{ij}=0_{n_i\times n_j}$ means that the information from the jth subsystem cannot arrive to the ith subsystem within t_0 time step. As a result, $K_t^{i,t-t_0,\alpha}=0$, for any $\alpha\in\{j\in\mathcal{V}:y_{t-t_0}^j\notin\mathcal{L}_t^i$ or $\hat{x}_{t-t_0}^j\notin\mathcal{X}_t^i\}$. This implies that the estimator (7) is designed using the information in $\mathcal{L}_t^{i1}\cup\mathcal{X}_t^i$ only. The information set $\mathcal{L}_t^{i1}\cup\mathcal{X}_t^i$ is induced by the new information pattern described in Section 2, and is different from the information sets used to design estimator in related literature, such as (Das & Moura, 2015; Khan & Moura, 2008). The information set $\mathcal{L}_t^{i1}\cup\mathcal{X}_t^i$ has two features: (1) $\mathcal{L}_t^{i1}\cup\mathcal{X}_t^i$ contains the information of different times. (2) The information of subsystem j for $\tau_{ij}>D$, is not contained in $\mathcal{L}_t^{i1}\cup\mathcal{X}_t^i$ at any time (incomplete information). The information set with features (1)–(2) complicate the estimator design.

To minimize the cost function (6) by the control input of the form (5), we propose the following local controller based on $\mathcal{L}_t^{i2}$ and $\mathcal{X}_t^i$, that is,

$$u_t^i=\gamma_t^{i1}(\mathcal{L}_t^{i2})+\gamma_t^{i2}(\mathcal{X}_t^i),\tag{8}$$

where γ_t^{i1} and γ_t^{i2} are linear functions.

Remark 6. $\mathcal{L}_t^{i2}$ is not used to estimate the subsystem state. However, $\mathcal{L}_t^{i2}$ is used to design the local control input directly. To minimize the cost function (6), $\mathcal{L}_t^{i2}$ is not discarded. The first term of (8) can be viewed as a correction.

Remark 7. The local controller (8) involves the parameters M and D. Under the constraints of the computing power of estimator and controller, we should choose a larger D and a smaller M to achieve better system performance. The reason is stated as follows. Firstly, to analyse how the system performance varies with M, we take M as a variable. Note that $\mathcal{L}_t^{i1}$, $\mathcal{L}_t^{i2}$, and $\mathcal{X}_t^i$ depend on M, thus, are denoted by $\mathcal{L}_t^{i1}(M)$, $\mathcal{L}_t^{i2}(M)$, and $\mathcal{X}_t^i(M)$, respectively. In theory, the optimal controller using more information should achieve better performance. Now, we want to show that $\mathcal{L}_t^{i2}(M-1)\cup\mathcal{X}_t^i(M-1)$ contains more information than $\mathcal{L}_t^{i2}(M)\cup\mathcal{X}_t^i(M)$. From the definition of $\mathcal{L}_t^{i2}(M)$, we have that $\mathcal{L}_t^{i2}(M)\setminus\mathcal{L}_t^{i2}(M-1)=\{y_{t-M+1}^j:j\in\tilde{\mathcal{V}}_i\}$. From (7), we have that $\sum_{t_0=M-1}^{D}K_t^{i,t_0}y_{t-t_0}\triangleq\mu_t^i(\mathcal{L}_t^{i1}(M-1))$ is a linear combination of the elements in $\mathcal{X}_t^i(M-1)$. It is easy to see that $\{y_{t-M+1}^j:j\in\tilde{\mathcal{V}}_i\}\subseteq\mathcal{L}_t^{i1}(M-1)$. This implies that $\{y_{t-M+1}^j:j\in\tilde{\mathcal{V}}_i\}$ can be replaced by $\mathcal{X}_t^i(M-1)$ without losing optimality, because K_t^{i,t_0}, $t_0\in\{M-1,\ldots,D\}$ are to be chosen through optimization procedure. On the other hand, according to (7), one has that $\hat{x}_{t-\tau_{ij}}^j(M)$ and $\hat{x}_{t-\tau_{ij}}^j(M-1)$ are computed based on $\mathcal{L}_{t-\tau_{ij}}^{j1}(M)$ and $\mathcal{L}_{t-\tau_{ij}}^{j1}(M-1)$, respectively, where

$$\mathcal{L}_{t-\tau_{ij}}^{j1}(M)=\{y_{t-\tau_{ij}-D:t-\tau_{ij}-M}^{\bar{j}}:\bar{j}\in\bar{\mathcal{V}}_j\},$$
$$\mathcal{L}_{t-\tau_{ij}}^{j1}(M-1)=\{y_{t-\tau_{ij}-D:t-\tau_{ij}-M+1}^{\bar{j}}:\bar{j}\in\bar{\mathcal{V}}_j\}.$$

It follows that $\mathcal{L}_{t-\tau_{ij}}^{j1}(M-1)\setminus\mathcal{L}_{t-\tau_{ij}}^{j1}(M)=\{y_{t-\tau_{ij}-M+1}^{\bar{j}}:\bar{j}\in\bar{\mathcal{V}}_j\}$, and $\{y_{t-\tau_{ij}-M+1}^{\bar{j}}:\bar{j}\in\bar{\mathcal{V}}_j\}\nsubseteq\mathcal{L}_t^{i1}(M)\cup\mathcal{L}_t^{i2}(M)$ if $\tau_{ij}+M-1>D$. Note that, for $j\in\bar{\mathcal{V}}_i$, $\hat{x}_{t-\tau_{ij}}^j(M)\in\mathcal{X}_t^i(M)$ and $\hat{x}_{t-\tau_{ij}}^j(M-1)\in\mathcal{X}_t^i(M-1)$. Based on the above discussion, we can say that $\mathcal{L}_t^{i2}(M-1)\cup\mathcal{X}_t^i(M-1)$ contains more information than $\mathcal{L}_t^{i2}(M)\cup\mathcal{X}_t^i(M)$. As a result, to achieve better system performance, we should choose a smaller M. Similarly, one has that $\mathcal{L}_t^{i1}(D+1)\supseteq\mathcal{L}_t^{i1}(D)$ and $\mathcal{L}_t^{i2}(D+1)=\mathcal{L}_t^{i2}(D)$. This implies that we should choose a larger D to achieve better system performance. Also, the computational burden of estimator and controller depends on M and D. Hence, the choice of M and D should satisfy the constraints of the computing power of estimator and controller.

In the following, we focus on designing K_t^{i,t_0}, γ_t^{i1}, and γ_t^{i2} to minimize (6), for $i\in\mathcal{V}$, $t_0\in\{M,M+1,\ldots,D\}$.

Define the following augmented vectors:

$$X_t=\begin{bmatrix}x_t\\ \vdots\\ x_{t-D}\end{bmatrix},\ \hat{X}_t=\begin{bmatrix}\hat{x}_t\\ \vdots\\ \hat{x}_{t-D}\end{bmatrix},\ Y_t=\begin{bmatrix}y_t\\ \vdots\\ y_{t-D}\end{bmatrix},$$

and denote $n=\sum_{i=1}^N n_i$, $m=\sum_{i=1}^N m_i$, and $l=\sum_{i=1}^N l_i$. The system dynamics and the estimator can be written as

$$\begin{cases}X_{t+1} &=\bar{A}X_t+\bar{B}u_t+\bar{\omega}_t,\\ Y_t &=\bar{\delta}_t\bar{C}X_t+\bar{\upsilon}_t,\end{cases}$$

and

$$\begin{cases}\hat{X}_{t+1}^- &=\bar{A}\hat{X}_t+\bar{B}u_t,\\ \hat{X}_{t+1} &=\hat{X}_{t+1}^-+\Omega_t(Y_t-\bar{C}_s\hat{X}_t),\end{cases}\tag{9}$$

where

$$\bar{A}=\begin{bmatrix}\hat{A} & 0_{n\times n}\\ I_{nD} & 0_{nD\times n}\end{bmatrix},\ \hat{A}=\begin{bmatrix}A & 0_{n\times n(D-1)}\end{bmatrix},$$
$$\bar{C}=\mathrm{diag}\{\underbrace{C,\ldots,C}_{D+1}\},\ \bar{C}_s=\mathrm{diag}\{\underbrace{C_s,\ldots,C_s}_{D+1}\},$$
$$\bar{B}_t=\begin{bmatrix}B_t\\ 0_{nD\times l}\end{bmatrix},\ \bar{\omega}_t=\begin{bmatrix}\omega_t\\ 0_{nD\times 1}\end{bmatrix},\ \bar{\upsilon}_t=\begin{bmatrix}\upsilon_t\\ \upsilon_{t-1}\\ \vdots\\ \upsilon_{t-D}\end{bmatrix},$$
$$\bar{\delta}_t=\begin{bmatrix}\delta_t & & \\ & \ddots & \\ & & \delta_{t-D}\end{bmatrix},\ \Omega_t=\begin{bmatrix}\bar{K}_t\\ 0_{nD\times m(D+1)}\end{bmatrix},$$
$$\bar{K}_t=\begin{bmatrix}0_{n_1\times mM} & K_t^{1,M} & \cdots & K_t^{1,D}\\ \vdots & \vdots & \vdots & \\ 0_{n_N\times mM} & K_t^{N,M} & \cdots & K_t^{N,D}\end{bmatrix}.$$

In addition, (8) can be rewritten as

$$u_t=F_tY_t+G_t\hat{X}_t,\tag{10}$$

where F_t and G_t are the gain matrices of the form

$$F_t=\begin{bmatrix}F_t^{1,0} & \cdots & F_t^{1,M-1} & 0_{l_1\times m(D-M+1)}\\ \vdots & \vdots & \vdots & \vdots\\ F_t^{N,0} & \cdots & F_t^{N,M-1} & 0_{l_N\times m(D-M+1)}\end{bmatrix},$$
$$G_t=\begin{bmatrix}G_t^{1,0} & \cdots & G_t^{1,D}\\ \vdots & \vdots & \vdots\\ G_t^{N,0} & \cdots & G_t^{N,D}\end{bmatrix},$$
$$F_t^{i,t_0}=\begin{bmatrix}F_t^{i,t_0,1} & \cdots & F_t^{i,t_0,N}\end{bmatrix},\ F_t^{i,t_0,j}\in\mathbb{R}^{l_i\times m_j},$$
$$G_t^{i,t_0}=\begin{bmatrix}G_t^{i,t_0,1} & \cdots & G_t^{i,t_0,N}\end{bmatrix},\ G_t^{i,t_0,j}\in\mathbb{R}^{l_i\times n_j},$$

where for any $t_0 \in \{0, 1, \ldots, M-1\}$, if $S_{t_0}^{ij} = 0$ then $F_t^{i,t_0,j} = 0$; and for any $t_0 \in \{0, 1, \ldots, D\}$, if $S_{t_0}^{ij} = 0$, then $G_t^{i,t_0,j} = 0$. Furthermore, the cost function (6) can be rewritten as

$$J \triangleq \mathbb{E}\left\{\sum_{t=0}^{T-1} X_t^{\mathsf{T}}\bar{Q}_t X_t + u_t^{\mathsf{T}}R_t u_t + X_T^{\mathsf{T}}\bar{Q}_T X_T\right\}, \tag{11}$$

where $\bar{Q}_t = \begin{bmatrix} Q_t & 0_{n\times nD} \\ 0_{nD\times n} & 0_{nD\times nD}\end{bmatrix}$.

Now, our aim is to design F_t, G_t and Ω_t to minimize (11). Note that F_t, G_t and Ω_t can be designed off-line by each subsystem, because they depend only on the known system parameters and the statistical properties of the noises.

3.2. Optimality condition

An optimality condition with respect to the gains of the estimator and controller is found in this subsection.

Define $e_t = X_t - \hat{X}_t$. The control input can be written as

$$u_t = (F_t\bar{\delta}_t\bar{C} + G_t)\hat{X}_t + F_t\bar{\delta}_t\bar{C}e_t + F_t\bar{\upsilon}_t. \tag{12}$$

Then, $\hat{X}_t$ and e_t can be written as

$$\begin{cases}\hat{X}_{t+1} = \Xi_t^1\hat{X}_t + \Xi_t^2 e_t + \Xi_t^3\bar{\upsilon}_t,\\ e_{t+1} = \Lambda_t^1\hat{X}_t + \Lambda_t^2 e_t + \bar{\omega}_t - \Omega_t\bar{\upsilon}_t,\end{cases} \tag{13}$$

where

$$\begin{aligned}
\Xi_t^1 &= \bar{A} + \Omega_t(\bar{\delta}_t\bar{C} - \bar{C}_s) + \bar{B}(F_t\bar{\delta}_t\bar{C} + G_t),\\
\Xi_t^2 &= \Omega_t\bar{\delta}_t\bar{C} + \bar{B}F_t\bar{\delta}_t\bar{C},\\
\Xi_t^3 &= \Omega_t + \bar{B}F_t,\\
\Lambda_t^1 &= -\Omega_t(\bar{\delta}_t\bar{C} - \bar{C}_s),\\
\Lambda_t^2 &= \bar{A} - \Omega_t\bar{\delta}_t\bar{C}.
\end{aligned}$$

Note that (9) is an unbiased estimator if and only if $\mathbb{E}(e_t) = 0$, $t \in \{0, \ldots, T-1\}$. From (13), we know that $\mathbb{E}(e_t) = 0$ if and only if $\mathbb{E}(\Lambda_t^1) = 0$ for $t \in \{0, \ldots, T-1\}$. Let $\mathbb{E}(\Lambda_t^1) = 0$, we have,

$$\bar{C}_s = \sum_{\bar{\delta}_t}\Pr(\bar{\delta}_t)\bar{\delta}_t\bar{C}. \tag{14}$$

Remark 8. In this paper, we design $\bar{C}_s$ using (14). This design has been proposed in Moayedi et al. (2010) (see Eq. (25) in Moayedi et al., 2010).

Under the information pattern described in Section 2, the gain matrix Ω_t must satisfy the sparsity constraint defined in (9). This means that Ω_t cannot be chosen to be Kalman filter gain. Thus, one has that $\hat{X}_t \neq \mathbb{E}(X_t \mid Y_t)$, which implies that $\mathbb{E}(e_t\hat{X}_t^{\mathsf{T}}) \neq 0$. To minimize the cost function (11), the controller and the estimator are coupled via the term $\mathbb{E}(e_t^{\mathsf{T}}\bar{Q}\hat{X}_t)$. In addition, it follows from (13) that the estimation error dynamics depends on $\hat{X}_t$, because $\Lambda_t^1 \neq 0$ $(\bar{\delta}_t\bar{C} - \bar{C}_s \neq 0)$ if $\lambda \neq 1$. As a result, the separation principle is not valid. We need to design the gain matrices F_t, G_t and Ω_t simultaneously. Define $Z_t = \begin{bmatrix} G_t & F_t \\ 0_{n(D+1)} & \Omega_t\end{bmatrix}$, one has that

$$G_t = \mathcal{J}_1 Z_t\mathcal{J}_2, \quad F_t = \mathcal{J}_1 Z_t\mathcal{J}_3, \quad \Omega_t = \mathcal{J}_4 Z_t\mathcal{J}_3,$$

where $\mathcal{J}_1 = \begin{bmatrix} I_l & 0_{l\times\varrho}\end{bmatrix}$, $\mathcal{J}_2 = \begin{bmatrix} I_\varrho \\ 0_{\bar{\varrho}\times\varrho}\end{bmatrix}$, $\mathcal{J}_3 = \begin{bmatrix} 0_{\varrho\times\bar{\varrho}} \\ I_{\bar{\varrho}}\end{bmatrix}$, $\mathcal{J}_4 = \begin{bmatrix} 0_{\varrho\times l} & I_\varrho\end{bmatrix}$, $\varrho = n(D+1)$, $\bar{\varrho} = m(D+1)$.

Define $\Sigma_t = \mathbb{E}\left\{\begin{bmatrix}\hat{X}_t\hat{X}_t^{\mathsf{T}} & \hat{X}_t e_t^{\mathsf{T}} \\ e_t\hat{X}_t^{\mathsf{T}} & e_t e_t^{\mathsf{T}}\end{bmatrix}\middle|\delta_{0:t}\right\}$, we have,

$$\Sigma_{t+1} = \Pi_t^1\Sigma_t\Pi_t^{1^{\mathsf{T}}} + \Pi_t^2\Upsilon_t\Pi_t^{2^{\mathsf{T}}},$$

where $\Pi_t^1 = \begin{bmatrix}\Xi_t^1 & \Xi_t^2 \\ \Lambda_t^1 & \Lambda_t^2\end{bmatrix}$, $\Pi_t^2 = \begin{bmatrix} 0_\varrho & \Xi_t^3 \\ I_\varrho & -\Omega_t\end{bmatrix}$, $\Upsilon_t = \mathbb{E}\left(\begin{bmatrix}\bar{\omega}_t\\ \bar{\upsilon}_t\end{bmatrix}\begin{bmatrix}\bar{\omega}_t\\ \bar{\upsilon}_t\end{bmatrix}^{\mathsf{T}}\right)$. Then, the cost function (11) can be written as

$$\begin{aligned}
J &= \sum_{t=0}^{T}\mathbb{E}\Big((\hat{X}_t + e_t)^{\mathsf{T}}\bar{Q}_t(\hat{X}_t + e_t) + u_t^{\mathsf{T}}R_t u_t\Big)\\
&= \sum_{t=0}^{T}\mathbb{E}\Big[\Big(\begin{bmatrix} I_\varrho & I_\varrho\end{bmatrix}\begin{bmatrix}\hat{X}_t\\ e_t\end{bmatrix}\Big)^{\mathsf{T}}\bar{Q}_t\Big(\begin{bmatrix} I_\varrho & I_\varrho\end{bmatrix}\begin{bmatrix}\hat{X}_t\\ e_t\end{bmatrix}\Big)\\
&\quad + \Big(\begin{bmatrix} F_t\bar{\delta}_t\bar{C} + G_t & F_t\bar{\delta}_t\bar{C}\end{bmatrix}\begin{bmatrix}\hat{X}_t\\ e_t\end{bmatrix}\\
&\quad\quad + \begin{bmatrix} 0_{l\times\varrho} & F_t\end{bmatrix}\begin{bmatrix}\bar{\omega}_t\\ \bar{\upsilon}_t\end{bmatrix}\Big)^{\mathsf{T}}\\
&\quad \times R_t\Big(\begin{bmatrix} F_t\bar{\delta}_t\bar{C} + G_t & F_t\bar{\delta}_t\bar{C}\end{bmatrix}\begin{bmatrix}\hat{X}_t\\ e_t\end{bmatrix}\\
&\quad\quad + \begin{bmatrix} 0_{l\times\varrho} & F_t\end{bmatrix}\begin{bmatrix}\bar{\omega}_t\\ \bar{\upsilon}_t\end{bmatrix}\Big)\Big]\\
&= \sum_{\delta_{0:T-1}}\Pr(\delta_{0:T-1})\hat{J},
\end{aligned}$$

where $R_T = 0_l$, $\hat{J} = \sum_{t=0}^{T}\Big\{\mathrm{tr}\Big((\underline{Q}_t + \tilde{F}_t^{\mathsf{T}}R_t\tilde{F}_t)\Sigma_t\Big) + \mathrm{tr}\Big(\hat{F}_t^{\mathsf{T}}R_t\hat{F}_t\Upsilon_t\Big)\Big\}$, $\underline{Q}_t = \begin{bmatrix}\bar{Q}_t & \bar{Q}_t \\ \bar{Q}_t & \bar{Q}_t\end{bmatrix}$, $\tilde{F}_t = \begin{bmatrix} F_t\bar{\delta}_t\bar{C} + G_t & F_t\bar{\delta}_t\bar{C}\end{bmatrix}$, and $\hat{F}_t = \begin{bmatrix} 0_{l\times\varrho} & F_t\end{bmatrix}$; for the above equation, the first equality is obtained by plugging $X_t = \hat{X}_t + e_t$ into (11); the second equality is obtained by substituting (12) into the term $u_t^{\mathsf{T}}R_t u_t$; the third equality is derived using the formula $\mathbb{E}(x^{\mathsf{T}}Ax) = \mathrm{tr}(A\mathbb{E}(xx^{\mathsf{T}}))$, and the conditional expectation formula.

Now, the optimal F_t, G_t and Ω_t minimizing the cost function (11) can be obtained by solving the following optimization problem.

$$\begin{aligned}
&\min_{Z_t} \quad J = \sum_{\delta_{0:T-1}}\Pr(\delta_{0:T-1})\hat{J},\\
&\text{subject to} \quad \Sigma_{t+1} = \Pi_t^1\Sigma_t\Pi_t^{1^{\mathsf{T}}} + \Pi_t^2\Upsilon_t\Pi_t^{2^{\mathsf{T}}},\\
&\qquad\qquad F_t = \mathcal{J}_1 Z_t\mathcal{J}_3,\\
&\qquad\qquad G_t = \mathcal{J}_1 Z_t\mathcal{J}_2,\\
&\qquad\qquad \Omega_t = \mathcal{J}_4 Z_t\mathcal{J}_3.
\end{aligned} \tag{15}$$

To solve problem (15), we define the following Hamiltonian function based on the matrix minimum principle (Athans, 1967):

$$\begin{aligned}
\mathcal{H} &= \mathrm{tr}\Big((\underline{Q}_t + \tilde{F}_t^{\mathsf{T}}R_t\tilde{F}_t)\Sigma_t\Big) + \mathrm{tr}\Big(\hat{F}_t^{\mathsf{T}}R_t\hat{F}_t\Upsilon_t\Big)\\
&\quad + \mathrm{tr}\Big((\Pi_t^1\Sigma_t\Pi_t^{1^{\mathsf{T}}} + \Pi_t^2\Upsilon_t\Pi_t^{2^{\mathsf{T}}})\Psi_{t+1}\Big)\\
&\quad + \mathrm{tr}(Z_t\mathcal{M}_t^{\mathsf{T}}),
\end{aligned} \tag{16}$$

where $\Psi_{t+1} \in \mathbb{R}^{2n(D+1)\times 2n(D+1)}$; $\mathcal{M}_t \in \mathbb{R}^{(l+n(D+1))\times(n+m)(D+1)}$, and satisfies $(\mathcal{M}_t)_{ij} = 0$ if $(Z_t)_{ij} \neq 0$.

An optimality condition with respect to $F_{0:T-1}$, $G_{0:T-1}$ and $\Omega_{0:T-1}$ for problem (15) is presented as follows.

Theorem 1. *Consider problem (15), the optimal $F_{0:T-1}$, $G_{0:T-1}$ and $\Omega_{0:T-1}$ satisfy the matrix equations (17)–(23) given in Box I.*

Proof. Consider problem (15) and the Hamiltonian function (16), the optimal Ψ_t is computed by $\Psi_t = \frac{\partial\mathcal{H}}{\partial\Sigma_t}$, that is,

$$\Psi_t = (\underline{Q}_t + \tilde{F}_t^{\mathsf{T}}R_t\tilde{F}_t) + (\Pi_t^1)^{\mathsf{T}}\Psi_{t+1}\Pi_t^1.$$

$$\bar{\Psi}_t = \begin{cases} \underline{Q}_t, & t = T; \\ (\underline{Q}_t + \tilde{F}_t^{\mathsf{T}} R_t \tilde{F}_t) + (\Pi_t^1)^{\mathsf{T}} \underline{Q}_t \Pi_t^1, & t = T-1; \\ \sum_{\delta_t} \Pr(\delta_t)(\underline{Q}_t + \tilde{F}_t^{\mathsf{T}} R_t \tilde{F}_t) + (\Pi_t^1)^{\mathsf{T}} \bar{\Psi}_{t+1} \Pi_t^1, & t \le T-2. \end{cases} \tag{17}$$

$$\begin{cases} \sum_{\bar{\delta}_t} \Pr(\bar{\delta}_t)\Big(\begin{bmatrix} I_\kappa & 0_\kappa \end{bmatrix} \bar{T}_t^1 \begin{bmatrix} I_\varrho \\ 0_\varrho \end{bmatrix} + \begin{bmatrix} 0_\kappa & I_\kappa \end{bmatrix} \bar{T}_t^1 \begin{bmatrix} 0_\varrho \\ I_\varrho \end{bmatrix} + \bar{T}_t^2 + \mathcal{M}_t^{\mathsf{T}}\Big) = 0, \\ \tilde{F}_t = \mathcal{J}_1 Z_t \begin{bmatrix} \mathcal{J}_3 \bar{\delta}_t \bar{C} + \mathcal{J}_2 & \mathcal{J}_3 \bar{\delta}_t \bar{C} \end{bmatrix}, \\ \hat{F}_t = \mathcal{J}_1 Z_t \begin{bmatrix} 0_{(\varrho+\bar{\varrho})\times\varrho} & \mathcal{J}_3 \end{bmatrix}, \\ G_t = \mathcal{J}_1 Z_t \mathcal{J}_2, \\ F_t = \mathcal{J}_1 Z_t \mathcal{J}_3, \\ \Omega_t = \mathcal{J}_4 Z_t \mathcal{J}_3, \end{cases} \tag{18}$$

where $\kappa = n(D+1) + l$, and

$$\bar{T}_t^1 = \mathcal{W}_2 \bar{\Sigma}_t {\Pi_t^1}^{\mathsf{T}} \bar{\Psi}_{t+1} \mathcal{W}_1 + \mathcal{W}_4 \bar{\Sigma}_t {\Pi_t^1}^{\mathsf{T}} \bar{\Psi}_{t+1} \mathcal{W}_3 + \mathcal{W}_6 \bar{\Sigma}_t {\Pi_t^1}^{\mathsf{T}} \bar{\Psi}_{t+1} \mathcal{W}_5 + \mathcal{W}_8 \Upsilon_t {\Pi_t^2}^{\mathsf{T}} \bar{\Psi}_{t+1} \mathcal{W}_7, \tag{19}$$

$$\bar{T}_t^2 = \Big(\begin{bmatrix} \mathcal{J}_3 \bar{\delta}_t \bar{C} + \mathcal{J}_2 & \mathcal{J}_3 \bar{\delta}_t \bar{C} \end{bmatrix} \bar{\Sigma}_t \tilde{F}_t^{\mathsf{T}} + \begin{bmatrix} 0_{(\varrho+\bar{\varrho})\times\varrho} & \mathcal{J}_3 \end{bmatrix} \Upsilon_t \hat{F}_t^{\mathsf{T}}\Big) R_t \mathcal{J}_1, \tag{20}$$

$$\mathcal{W}_1 = \begin{bmatrix} \mathcal{J}_4 + \bar{B}\mathcal{J}_1 & \mathcal{J}_4 + \bar{B}\mathcal{J}_1 \\ -\mathcal{J}_4 & -\mathcal{J}_4 \end{bmatrix}, \ \mathcal{W}_2 = \begin{bmatrix} \mathcal{J}_3 \bar{\delta}_t \bar{C} & 0_{(\varrho+\bar{\varrho})\times\varrho} \\ 0_{(\varrho+\bar{\varrho})\times\varrho} & \mathcal{J}_3 \bar{\delta}_t \bar{C} \end{bmatrix}, \ \mathcal{W}_3 = \begin{bmatrix} \bar{B}\mathcal{J}_1 & 0_{\varrho\times\kappa} \\ 0_{\varrho\times\kappa} & 0_{\varrho\times\kappa} \end{bmatrix}, \ \mathcal{W}_4 = \begin{bmatrix} \mathcal{J}_2 & 0_{(\varrho+\bar{\varrho})\times\varrho} \\ 0_{(\varrho+\bar{\varrho})\times\varrho} & 0_{(\varrho+\bar{\varrho})\times\varrho} \end{bmatrix}, \tag{21}$$

$$\mathcal{W}_5 = \begin{bmatrix} -\mathcal{J}_4 & 0_{\varrho\times\kappa} \\ \mathcal{J}_4 & 0_{\varrho\times\kappa} \end{bmatrix}, \ \mathcal{W}_6 = \begin{bmatrix} \mathcal{J}_3 \bar{C}_s & 0_{(\varrho+\bar{\varrho})\times\varrho} \\ 0_{(\varrho+\bar{\varrho})\times\varrho} & 0_{(\varrho+\bar{\varrho})\times\varrho} \end{bmatrix}, \ \mathcal{W}_7 = \begin{bmatrix} 0_{\varrho\times\kappa} & \mathcal{J}_4 + \bar{B}\mathcal{J}_1 \\ 0_{\varrho\times\kappa} & -\mathcal{J}_4 \end{bmatrix}, \ \mathcal{W}_8 = \begin{bmatrix} 0_{(\varrho+\bar{\varrho})\times\varrho} & 0_{(\varrho+\bar{\varrho})\times\bar{\varrho}} \\ 0_{(\varrho+\bar{\varrho})\times\varrho} & \mathcal{J}_3 \end{bmatrix}, \tag{22}$$

$$\bar{\Sigma}_{t+1} = \begin{cases} \Sigma_{t+1}, & t < D \\ \sum_{\delta_{t-D}} \Pr(\delta_{t-D}) \Pi_t^1 \bar{\Sigma}_t {\Pi_t^1}^{\mathsf{T}} + \Pi_t^2 \Upsilon_t {\Pi_t^2}^{\mathsf{T}}, & t \ge D \end{cases} \tag{23}$$

Box I.

In Hamiltonian function (16), we know $R_t \succ 0$, $\Sigma_t \succeq 0$, $\Upsilon_t \succeq 0$, and $\Psi_{t+1} \succeq 0$. According to the formula $\mathrm{tr}(AXBX^{\mathsf{T}}) = vec^{\mathsf{T}}(X)(B^{\mathsf{T}} \otimes A)vec(X)$, where $vec(\cdot)$ is the vectorization operator, $\otimes$ is the Kronecker product, and the fact that $B^{\mathsf{T}} \otimes A \succeq 0$ if $B^{\mathsf{T}} \succeq 0$, $A \succeq 0$, one has that the Hamiltonian function (16) with respect to Z_t is convex. Thus, the optimal Z_t exists, and is obtained by solving $\sum_{\delta_{0:T-1}} \Pr(\delta_{0:T-1}) \frac{\partial \mathcal{H}}{\partial Z_t} = 0$. Now, we derive the partial derivatives of $\mathcal{H}$ with respect to Z_t, that is,

$$\begin{aligned} \partial\mathcal{H} =& 2\,\mathrm{tr}(\Sigma_t \tilde{F}_t^{\mathsf{T}} R_t (\partial \tilde{F}_t)) + 2\,\mathrm{tr}(\Upsilon_t \hat{F}_t^{\mathsf{T}} R_t (\partial \hat{F}_t)) \\ &+ 2\,\mathrm{tr}(\Sigma_t {\Pi_t^1}^{\mathsf{T}} \Psi_{t+1} (\partial \Pi_t^1)) \\ &+ 2\,\mathrm{tr}(\Upsilon_t {\Pi_t^2}^{\mathsf{T}} \Psi_{t+1} (\partial \Pi_t^2)) \\ &+ 2\,\mathrm{tr}((\partial Z_t)\mathcal{M}_t^{\mathsf{T}}), \end{aligned}$$

where

$$\begin{aligned} \partial \tilde{F}_t &= \mathcal{J}_1 \partial(Z_t) \begin{bmatrix} \mathcal{J}_3 \bar{\delta}_t \bar{C} + \mathcal{J}_2 & \mathcal{J}_3 \bar{\delta}_t \bar{C} \end{bmatrix}, \\ \partial \hat{F}_t &= \mathcal{J}_1 \partial(Z_t) \begin{bmatrix} 0_{(\varrho+\bar{\varrho})\times\varrho} & \mathcal{J}_3 \end{bmatrix}, \\ \partial \Pi_t^1 &= \mathcal{W}_1 \partial(\mathrm{diag}(Z_t, Z_t)) \mathcal{W}_2 + \mathcal{W}_3 \partial(\mathrm{diag}(Z_t, Z_t)) \mathcal{W}_4 \\ &\quad + \mathcal{W}_5 \partial(\mathrm{diag}(Z_t, Z_t)) \mathcal{W}_6, \\ \partial \Pi_t^2 &= \mathcal{W}_7 \partial(\mathrm{diag}(Z_t, Z_t)) \mathcal{W}_8, \end{aligned}$$

and the matrices $\mathcal{W}_1, \ldots, \mathcal{W}_8$ are defined in (21)–(22). Then, we have

$$\begin{aligned} \partial\mathcal{H} =& \mathrm{tr}\Big(T_t^1 \partial(\mathrm{diag}(Z_t, Z_t))\Big) \\ &+ \mathrm{tr}\Big(T_t^2 (\partial Z_t)\Big) + \mathrm{tr}\Big(\mathcal{M}_t^{\mathsf{T}} (\partial Z_t)\Big), \end{aligned}$$

where

$$\begin{aligned} T_t^1 =& \mathcal{W}_2 \Sigma_t {\Pi_t^1}^{\mathsf{T}} \Psi_{t+1} \mathcal{W}_1 + \mathcal{W}_4 \Sigma_t {\Pi_t^1}^{\mathsf{T}} \Psi_{t+1} \mathcal{W}_3 \\ &+ \mathcal{W}_6 \Sigma_t {\Pi_t^1}^{\mathsf{T}} \Psi_{t+1} \mathcal{W}_5 + \mathcal{W}_8 \Upsilon_t {\Pi_t^2}^{\mathsf{T}} \Psi_{t+1} \mathcal{W}_7, \\ T_t^2 =& \Big(\begin{bmatrix} \mathcal{J}_3 \bar{\delta}_t \bar{C} + \mathcal{J}_2 & \mathcal{J}_3 \bar{\delta}_t \bar{C} \end{bmatrix} \Sigma_t \tilde{F}_t^{\mathsf{T}} \\ &+ \begin{bmatrix} 0_{(\varrho+\bar{\varrho})\times\varrho} & \mathcal{J}_3 \end{bmatrix} \Upsilon_t \hat{F}_t^{\mathsf{T}}\Big) R_t \mathcal{J}_1. \end{aligned}$$

Note that

$$\begin{aligned} \partial(\mathrm{diag}(Z_t, Z_t)) =& \begin{bmatrix} I_\varrho \\ 0_\varrho \end{bmatrix} \partial(Z_t) \begin{bmatrix} I_{\varrho+\bar{\varrho}} & 0_{\varrho+\bar{\varrho}} \end{bmatrix} \\ &+ \begin{bmatrix} 0_\varrho \\ I_\varrho \end{bmatrix} \partial(Z_t) \begin{bmatrix} 0_{\varrho+\bar{\varrho}} & I_{\varrho+\bar{\varrho}} \end{bmatrix}. \end{aligned}$$

As a result, one has

$$\begin{aligned} \partial\mathcal{H} = \mathrm{tr}\Big(\Big(&\begin{bmatrix} I_{\varrho+\bar{\varrho}} & 0_{\varrho+\bar{\varrho}} \end{bmatrix} T_t^1 \begin{bmatrix} I_\varrho \\ 0_\varrho \end{bmatrix} \\ &+ \begin{bmatrix} 0_{\varrho+\bar{\varrho}} & I_{\varrho+\bar{\varrho}} \end{bmatrix} T_t^1 \begin{bmatrix} 0_\varrho \\ I_\varrho \end{bmatrix} + T_t^2 + \mathcal{M}_t^{\mathsf{T}}\Big)(\partial Z_t)\Big). \end{aligned}$$

Thus, $\sum_{\delta_{0:T-1}} \Pr(\delta_{0:T-1})\frac{\partial \mathcal{H}}{\partial Z_t} = 0$ gives that

$$\sum_{\delta_{0:T-1}} \Pr(\delta_{0:T-1})\Big\{\begin{bmatrix} I_{\varrho+\bar{\varrho}} & 0_{\varrho+\bar{\varrho}}\end{bmatrix} T_t^1 \begin{bmatrix} I_\varrho \\ 0_\varrho \end{bmatrix} + \begin{bmatrix} 0_{\varrho+\bar{\varrho}} & I_{\varrho+\bar{\varrho}}\end{bmatrix} T_t^1 \begin{bmatrix} 0_\varrho \\ I_\varrho \end{bmatrix} + T_t^2 + \mathcal{M}_t^{\mathrm{T}}\Big\} = 0.$$

In addition, one can compute

$$\begin{aligned}
&\sum_{\delta_{0:T-1}} \Pr(\delta_{0:T-1}) T_t^1 \\
&= \sum_{\bar{\delta}_t} \Pr(\bar{\delta}_t)\Big\{ \mathcal{W}_2\Big(\sum_{\delta_{0:t-D-1}} \Pr(\delta_{0:t-D-1})\Sigma_t\Big)\Pi_t^{1^{\mathrm{T}}} \\
&\quad \times \Big(\sum_{\delta_{t+1:T-1}} \Pr(\delta_{t+1:T-1})\Psi_{t+1}\Big)\mathcal{W}_1 \\
&\quad + \mathcal{W}_4\Big(\sum_{\delta_{0:t-D-1}} \Pr(\delta_{0:t-D-1})\Sigma_t\Big)\Pi_t^{1^{\mathrm{T}}} \\
&\quad \times \Big(\sum_{\delta_{t+1:T-1}} \Pr(\delta_{t+1:T-1})\Psi_{t+1}\Big)\mathcal{W}_3 \\
&\quad + \mathcal{W}_6\Big(\sum_{\delta_{0:t-D-1}} \Pr(\delta_{0:t-D-1})\Sigma_t\Big)\Pi_t^{1^{\mathrm{T}}} \\
&\quad \times \Big(\sum_{\delta_{t+1:T-1}} \Pr(\delta_{t+1:T-1})\Psi_{t+1}\Big)\mathcal{W}_5 \\
&\quad + \mathcal{W}_8 \Upsilon_t \Pi_t^{2^{\mathrm{T}}}\Big(\sum_{\delta_{t+1:T-1}} \Pr(\delta_{t+1:T-1})\Psi_{t+1}\Big)\mathcal{W}_7\Big\} \\
&= \sum_{\bar{\delta}_t} \Pr(\bar{\delta}_t)\Big(\mathcal{W}_2\bar{\Sigma}_t\Pi_t^{1^{\mathrm{T}}}\bar{\Psi}_{t+1}\mathcal{W}_1 + \mathcal{W}_4\bar{\Sigma}_t\Pi_t^{1^{\mathrm{T}}}\bar{\Psi}_{t+1}\mathcal{W}_3 \\
&\quad + \mathcal{W}_6\bar{\Sigma}_t\Pi_t^{1^{\mathrm{T}}}\bar{\Psi}_{t+1}\mathcal{W}_5 + \mathcal{W}_8\Upsilon_t\Pi_t^{2^{\mathrm{T}}}\bar{\Psi}_{t+1}\mathcal{W}_7\Big) \\
&= \sum_{\bar{\delta}_t} \Pr(\bar{\delta}_t)\bar{T}_t^1,
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{\delta_{0:T-1}} \Pr(\delta_{0:T-1}) T_t^2 \\
&= \sum_{\bar{\delta}_t} \Pr(\bar{\delta}_t)\Big\{\begin{bmatrix} \mathcal{J}_3\bar{\delta}_t\bar{C} + \mathcal{J}_2 & \mathcal{J}_3\bar{\delta}_t\bar{C}\end{bmatrix} \\
&\quad \times \Big(\sum_{\delta_{0:t-D-1}} \Pr(\delta_{0:t-D-1})\Sigma_t\Big)\tilde{F}_t^{\mathrm{T}} \\
&\quad + \begin{bmatrix} 0_{(\varrho+\bar{\varrho})\times\varrho} & \mathcal{J}_3\end{bmatrix}\Upsilon_t\hat{F}_t^{\mathrm{T}}\Big\}R_t\mathcal{J}_1 \\
&= \sum_{\bar{\delta}_t} \Pr(\bar{\delta}_t)\bar{T}_t^2,
\end{aligned}$$

where $\bar{\Psi}_{t+1}$, $\bar{T}_t^1$, $\bar{T}_t^2$, and $\bar{\Sigma}_t$ are defined in (17), (19), (20), and (23), respectively. Hence, $\sum_{\delta_{0:T-1}} \Pr(\delta_{0:T-1})\frac{\partial \mathcal{H}}{\partial Z_t} = 0$ gives (18). Thus, we have proved that the optimal $Z_{0:T-1}$ satisfies the matrix equations (17)–(23). The proof is completed. ■

3.3. Iterative algorithm and the control scheme realization

An iterative algorithm is exploited to design the gain matrices numerically in this subsection. In addition, the control scheme realization is presented.

From the optimality condition (17)–(23), we can see that if $Z_{0:\tau-1}$ and $Z_{\tau+1:T-1}$ are given, then the optimal Z_τ can be obtained by solving linear matrix equation (18), for any $\tau \in \{0, \ldots, T-1\}$. This implies that we can obtain a Nash optimal $Z_{0:T-1}$ through alternating iterative.

For ease of notation, we denote

$$\begin{aligned}
\bar{\Sigma}_0 &= \begin{bmatrix} \Theta_0 & 0_{n\times nD} \\ 0_{nD\times n} & 0_{nD}\end{bmatrix}, \\
\ell &= \sum_{t=0}^{T-1}\Big\{\|F_t^{(i)} - F_t^{(i-1)}\| + \|G_t^{(i)} - G_t^{(i-1)}\| \\
&\qquad + \|\Omega_t^{(i)} - \Omega_t^{(i-1)}\|\Big\}.
\end{aligned}$$

Then, the gain matrices $F_{0:T-1}$, $G_{0:T-1}$ and $\Omega_{0:T-1}$ can be computed by the following iterative algorithm off-line.

Algorithm 1.

1: Given $\bar{\Sigma}_0$, and $\bar{\Psi}_t^{(-1)}$, $t = 1, \cdots, T$.
2: Set $i = 0$.
3: **for** $t = 0 : T-1$ **do**
4: Obtain $F_t^{(i)}$, $G_t^{(i)}$, $\Omega_t^{(i)}$ via solving the linear matrix equations (18) by letting $\bar{\Sigma}_t = \bar{\Sigma}_t^{(i)}$, $\bar{\Psi}_{t+1} = \bar{\Psi}_{t+1}^{(i-1)}$.
5: Let $F_t = F_t^{(i)}$, $G_t = G_t^{(i)}$, $\Omega_t = \Omega_t^{(i)}$, and update $\bar{\Sigma}_{t+1}^{(i)}$ according to (23).
6: **end for**
7: **if** $\ell > \varepsilon$ (ε is a small positive) **then**
8: Update $\bar{\Psi}_{1:T}^{(i)}$ by (17) recursively using $F_{0:T-1}^{(i)}$, $G_{0:T-1}^{(i)}$, $\Omega_{0:T-1}^{(i)}$.
9: Set $i = i + 1$, and return to step 3.
10: **else**
11: Obtain the suboptimal gains: $F_t^* = F_t^{(i)}$, $G_t^* = G_t^{(i)}$, $\Omega_t^* = \Omega_t^{(i)}$
12: **end if**

Remark 9. To solve the linear matrix equation (18) is the main computational burden of Algorithm 1. Assume that the computational burden of solving linear matrix equation (18) is ϖ. The computational burden of the algorithm to compute estimator and controller gains is $iT\varpi$, here i is the number of iterations. The computational burden of the algorithm increases as i increases, and is a multiple of i.

Now we show that Algorithm 1 converges to a Nash optimal solution to the optimization problem (15).

Theorem 2. *Consider problem (15). The gain matrices F_t^*, G_t^*, and Ω_t^* returned by Algorithm 1 converge to Nash optimum, that is*

$$J(F_{0:T-1}^*, G_{0:T-1}^*, \Omega_{0:T-1}^*) \le \hat{J}, \tag{24}$$

holds when $i \to \infty$, where $\hat{J}$ is given by (26) in Box II.

Proof. To analyse the iterative process of Algorithm 1, one has that we obtain $Z_t^{(i)}$ by letting $\bar{\Sigma}_t = \bar{\Sigma}_t^{(i)}$, and $\bar{\Psi}_{t+1} = \bar{\Psi}_{t+1}^{(i-1)}$, where $\bar{\Sigma}_t^{(i)}$ is computed based on $Z_{0:t-1}^{(i)}$, and $\bar{\Psi}_{t+1}^{(i-1)}$ is computed using $Z_{t+1:T-1}^{(i-1)}$. Thus, we have that the matrix $Z_t^{(i)}$ computed in Algorithm 1 satisfies

$$\begin{aligned}
&J\Big(Z_0^{(i)}, \ldots, Z_\tau^{(i)}, Z_{\tau+1}^{(i-1)}, \ldots, Z_{T-1}^{(i-1)}\Big) \\
&\quad \le J\Big(Z_0^{(i)}, \ldots, Z_{\tau-1}^{(i)}, Z_\tau^{(i-1)}, \ldots, Z_{T-1}^{(i-1)}\Big).
\end{aligned} \tag{25}$$

From (25), let $i \to \infty$, we have (24). The proof is completed. ■

$$\hat{J} = J\Big(F_0^*, G_0^*, \Omega_0^*, \ldots, F_{\tau-1}^*, G_{\tau-1}^*, \Omega_{\tau-1}^*, F_\tau, G_\tau, \Omega_\tau, F_{\tau+1}^*, G_{\tau+1}^*, \Omega_{\tau+1}^*, \ldots, F_{T-1}^*, G_{T-1}^*, \Omega_{T-1}^*\Big). \tag{26}$$

Box II.

$$A = \begin{bmatrix} \Theta_1 & \varpi_2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \xi_2 & \Theta_2 & \varpi_3 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_3 & \Theta_3 & \cdots & \varpi_4 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \Theta_{N-1} & \varpi_{N-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \xi_N & \Theta_N \end{bmatrix}, \; B = \begin{bmatrix} k_{u_1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & k_{u_2}^1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & k_{u_2}^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & k_{u_N}^1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & k_{u_N}^2 \end{bmatrix}, \; x = \begin{bmatrix} \triangle\tilde{v}^1 \\ \triangle d^{12} \\ \triangle\tilde{v}^2 \\ \triangle d^{23} \\ \triangle\tilde{v}^3 \\ \vdots \\ \triangle\tilde{v}^{N-1} \\ \triangle d^{(N-1)N} \\ \triangle\tilde{v}^N \end{bmatrix} \tag{27}$$

Box III.

After the gains of the estimator and the controller are designed off-line, the local estimator and local controller can have the realization:

$$\begin{cases} \hat{x}_t^{i-} &= A^{ii}\hat{x}_{t-1}^i + \sum\limits_{j\in\mathcal{N}_i} A^{ij}\hat{x}_{t-1}^j + B^i u_{t-1}^i, \\ \hat{x}_t^i &= \hat{x}_t^{i-} + \sum\limits_{t_0=M}^{D}\sum\limits_{j\in\daleth_{t_0}^{i1}} K_t^{i,t_0,j}(y_{t-t_0}^j - C_s^j\hat{x}_{t-t_0}^j), \end{cases}$$

$$u_t^i = \sum_{t_0=0}^{M-1}\sum_{j\in\daleth_{t_0}^{i2}} F_t^{i,t_0,j} y_{t-t_0}^j + \sum_{t_0=0}^{D}\sum_{j\in\daleth_{t_0}^{i3}} G_t^{i,t_0,j}\hat{x}_{t-t_0}^j,$$

where $\daleth_{t_0}^{i1} = \{j : y_{t-t_0}^j \in \mathcal{L}_t^{i1}\}$, $\daleth_{t_0}^{i2} = \{j : y_{t-t_0}^j \in \mathcal{L}_t^{i2}\}$, $\daleth_{t_0}^{i3} = \{j : \hat{x}_{t-t_0}^j \in \mathcal{X}_t^i\}$.

Remark 10. When $D = 0$, $\mathcal{L}_t^i$ includes the measurements of the ith subsystem only. In this case, u_t^i is a fully decentralized controller. When $D = 1$, $\mathcal{L}_t^i$ includes the measurements of the ith subsystem and its neighbours. This setup is usually used in multi-agent systems (Ji & Egerstedt, 2007). Note that, if we choose a bigger D, then the system performance is better, however, the control scheme design becomes more complex. The designer can make a tradeoff between the system performance and the complexity of the control scheme design through choosing a proper D.

4. Application to vehicle formations

In this section, we use a platoon of heavy duty vehicles (HDVs) as an example to illustrate the effectiveness of the proposed theoretical results.

Consider a platoon of N HDV as depicted in Fig. 2. The dynamics of a single HDV are given by Feyzmahdavian, Alam et al. (2012)

$$\begin{aligned} \dot{s} &= \tilde{v} \\ m\dot{\tilde{v}} &= F_{engine} - F_{brake} - F_{airdrag}(\tilde{v}) \\ &\quad - F_{roll}(\alpha) - F_{gravity}(\alpha), \\ &= k_u u - k_b F_{brake} - k_d\tilde{v}^2 \\ &\quad - k_{f_r}\cos\alpha - k_g\sin\alpha, \end{aligned}$$

where $\tilde{v}$ denotes the velocity, m is the mass, u is the net engine torque. The coefficients for the engine, brake, air drag, road friction and gravitation are denoted by k_u, k_b, k_d, k_{f_r} and k_g respectively.

Fig. 2. A platoon of heavy duty vehicle.

When we focus on maintaining a constant intermediate distance between the HDVs, using one step forward discretization, the discrete time model for the HDV platoon is of the form given by (3)–(4). According to Eq. (2) in Feyzmahdavian, Alam et al. (2012), the matrices A, B, and the state vector x are given by (27) in Box III, where $\triangle\tilde{v}^i = \tilde{v}^i - \bar{\tilde{v}}$, $\triangle d^{i-1,i} = d^{i-1,i} - \bar{d}$, which are the velocity deviation, and the intermediate distance deviation, respectively. $\bar{\tilde{v}}$ is the desired velocity; $\bar{d}$ is the desired intermediate distance; $d^{i-1,i}$ is the longitudinal relative distance between the $(i-1)$th vehicle and the ith vehicle.

The corresponding states for each subsystem are $x_t^1 = \triangle\tilde{v}_t^1, x_t^i = \begin{bmatrix} \triangle d^{i-1,i} \\ \triangle\tilde{v}^i \end{bmatrix}$ $(i = 2, \ldots, T)$. In addition, from Eq. (8) in Feyzmahdavian, Alam et al. (2012), the cost function is defined by

$$J = \sum_{t=0}^{T-1}\sum_{i=2}^{N} \begin{bmatrix} \triangle\tilde{v}_t^{i-1} \\ \triangle d_t^{i-1,i} \\ \triangle\tilde{v}_t^i \end{bmatrix}^\top Q^i \begin{bmatrix} \triangle\tilde{v}_t^{i-1} \\ \triangle d_t^{i-1,i} \\ \triangle\tilde{v}_t^i \end{bmatrix} + R^i(u_t^i)^2 + \pi^{v^1}(\triangle\tilde{v}_t^1)^2 + \pi^{u^1}(u_t^1)^2,$$

where

$$Q^i = \begin{bmatrix} \pi_i^{\Delta v} & 0 & -\pi_i^{\Delta v} \\ 0 & \pi_i^d + \pi_i^\tau & -\tau\pi_i^\tau \\ -\pi_i^{\Delta v} & -\tau\pi_i^\tau & \tau^2\pi_i^\tau + \pi_i^{\Delta v} + \pi_i^v \end{bmatrix},$$

$$Q^1 = \begin{bmatrix} \pi^{v^1} & 0 \\ 0 & \pi^{u^1} \end{bmatrix}, \quad R^i = \pi_i^{u^i},$$

where $\pi_i^{\Delta v}, \pi_i^d, \pi_i^\tau, \pi_i^v, \pi^{v^1}$ and π^{u^1} are positive scalars.

We consider a platoon composed of $N = 3$ identical vehicles. The mass of each vehicle is $m = 40000$ kg. It is expected that each vehicle travels in the steady velocity $\bar{\tilde{v}} = 19.44$ m/s (70 km/h) and the desired relative distance $\bar{d} = \tau\bar{\tilde{v}}$, where we set $\tau = 1$ s. Let the sampling time $T_s = 1$ s. According to the parameters used in Al Alam, Gattami, and Johansson (2011), we choose $\Theta_i = 0.9999$,

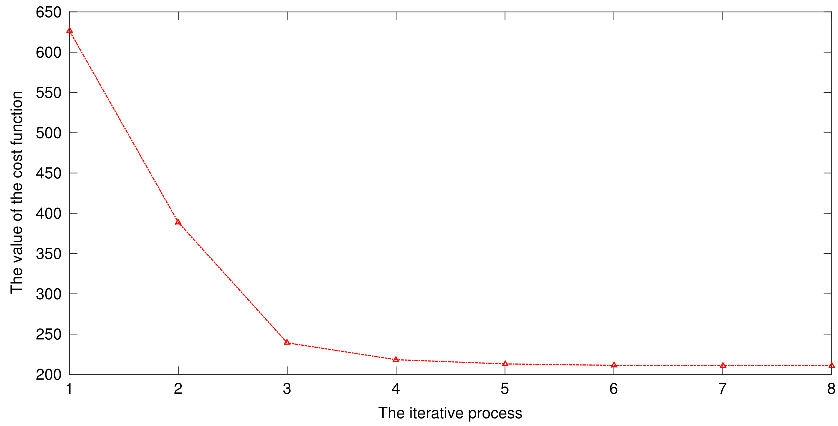

Fig. 3. The iterative process of Algorithm 1 .

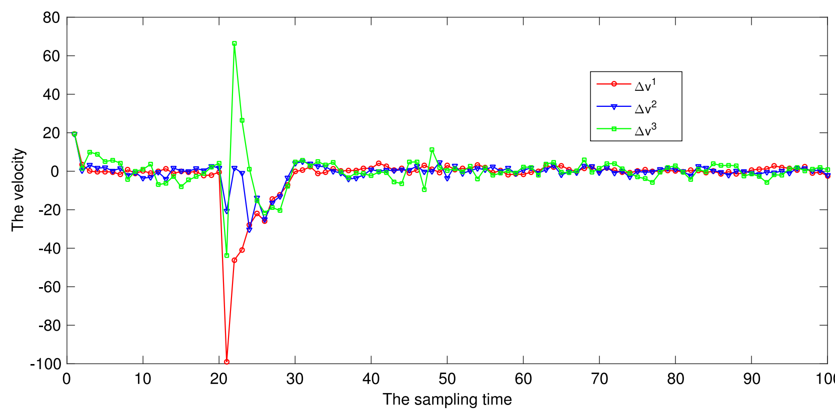

Fig. 4. The velocity of the HDV platoon.

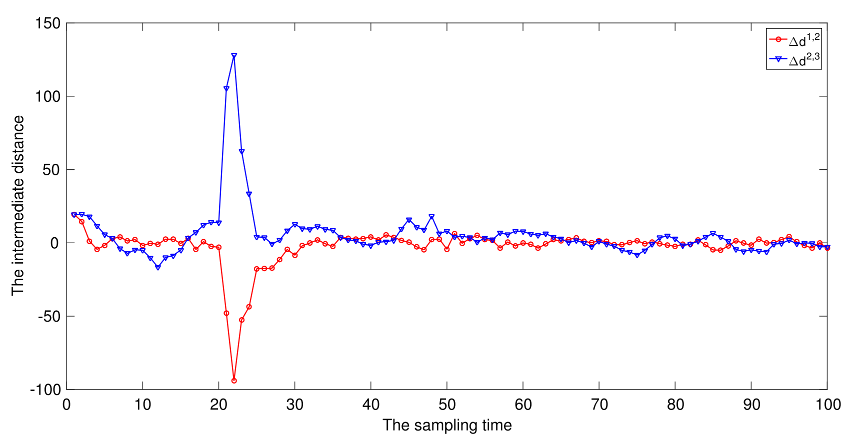

Fig. 5. The intermediate distance of the HDV platoon.

$\varpi_i = 0.1476$ in system matrix A. For control matrix B, $k_{u_1} = k^1_{u_2} = k^2_{u_2} = k^2_{u_3} = 1.48$, and $k^1_{u_3} = 0$. For the weight matrices Q and R in the cost function, $\pi_i^{\tau} = \pi_i^{\Delta v} = 0.1$, $\pi_i^{u^i} = 0.02$, $\pi_i^{d} = \pi_i^{v} = 0.0001$. In Eqs. (3) and (4), $C^1 = 1$, $C^2 = I_2$, $C^3 = \begin{bmatrix} 1 & 0 \end{bmatrix}$; the noises ω_t and υ_t are zero-mean Gaussian noises with identity covariance matrices. Assume that the information delayed more than $D = 1$ step times is discarded. Let $M = 1$, $T = 100$, and $\lambda = 0.93$. For simplicity, in this example, we assume $\delta_t^i = \delta_t^j$ for all $i, j \in \mathcal{V}$. The gains of the estimator and the controller are computed by Algorithm 1 off-line. To run Algorithm 1, we choose $\varepsilon = 0.05$, and $\Psi_t^{(0)} = I_{2\varrho}$ for $t = 1, \ldots, T$. The iterative process is shown by Fig. 3.

The estimator and the controller are designed successfully by (7) and (8). The trajectories of the velocity deviation and the intermediate distance deviation are shown in Figs. 4 and 5. All the data between $t = 0$ to $t = 100$ is simulated. The velocity and intermediate distance are retained around the desired values all the times except for $t = 20$ to 25. During $t = 20$ to 21, we simulate that the platoon suffers from a serious disturbance from external environment. That is, $\Pr(|\omega_t^i| \gg 0) = 1$ for $t = 20,21$ and any $i \in \mathcal{V}$. We assume that the serious disturbance is due to the road condition, sensor (network) temporary failure, and human factors, and so on. Then the velocity and the intermediate distance deviate from the desired values a lot. The HDV platoon regulates the velocity and the intermediate distance to the neighbourhood

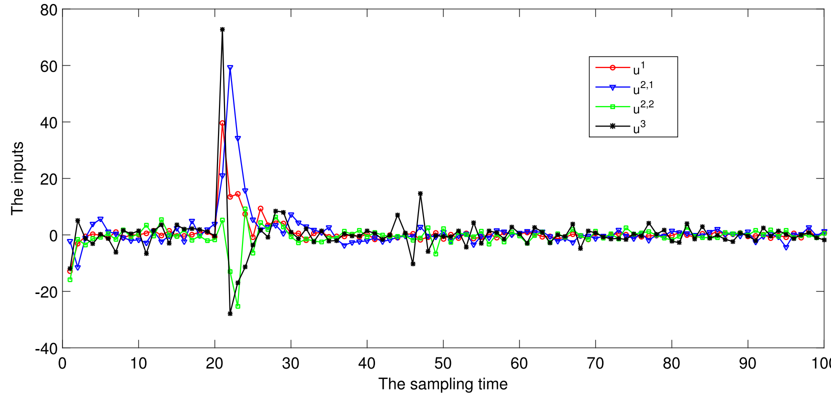

Fig. 6. The inputs of the HDV platoon.

of the desired values within few times (from $t = 22$ to $t = 25$) by our controller. In addition, the control input is presented by Fig. 6. The values of the input are maintained around the origin all the time except for $t = 20, 21, \ldots, 25$. The value of the input is proportional to the input energy. From $t = 20$ to 25, it requires more energy for the HDV to deal with the serious disturbance. However, at other times, the HDV only needs less energy to deal with the process noise.

Note that the spectral radius of the system matrix A is 1.36. Hence, the HDV platoon system without control is unstable. However, the system is stable and is maintained at a desired state by the proposed controller. This implies that the proposed control scheme in this paper is effective.

5. Conclusion

In this paper, we focused on the decentralized output-feedback LQG control for a large-scale system in presence of communication delay and random missing measurement. The large-scale system was defined over a directed connected graph. One step time was required for the information to travel across an edge in the graph. The information delayed more than D step times was discarded. In addition, the observation for the subsystem output was uncertain. To solve the optimal LQG problem under above setup, a new control scheme was proposed. A new estimator structure and a new controller structure were constructed. An optimality condition with respect to the gains of the estimator and the controller was established, and an algorithm was given to compute the gains numerically. Once the gain matrices were obtained, the estimator and the controller of each subsystem could operate on-line based on local available information only. Finally, our proposed methods were applied to control a platoon of HDV. The HDV platoon could be maintained at the desired state by the proposed control scheme.

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