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Synchronization analysis of network systems applying sampled-data controller with time-delay via the Bessel–Legendre inequality

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ABSTRACT

In this paper, taking advantage of aperiodic sampled data control technique, the synchronization issue of network systems is brought into focus. During the transmission of sampled data, time-varying delays are concerned. Applying input delay method, the sampled data network systems can be rebuilt via continuous systems with another new delay term in the distributed controller. Unlike the utilization of Jensen and Wirtinger-based inequalities in most literature, the Bessel–Legendre integral inequality is introduced, which can relax conservative further. The characteristics of this integral inequality are adequately merged with the establishment of augmented Lyapunov functional. Two sufficient conditions for synchronizability of network systems are established. In the end, a simulation example is illustrated to verify the efficacy and advantage of the designed approach.

1. Introduction

For the past few years, a large amount of problems of network systems have been researched extensively [1–8]. For example, in [9], based on time variant measurement topology, the issue of distributed estimation for cars formation was addressed. In [10], the synchronization problem of network systems with general linear dynamics was solved by a distributed event-triggered strategy. In particular, the synchronization problem becomes a hotspot topic in network systems investigation owing to its widespread practical applications. For instance, formation control, cooperative control of unmanned aircrafts and underwater vehicles, communication among wireless sensor networks and flocking of mobile vehicles are some typical issues in military and civilian area which the synchronization problems are involved. In [11], using a complex Laplacian-based technique, the fundamental formation control problem was investigated which required the network systems to achieve an explicit but arbitrary formation shape. In [12], for the synchronization tracking, an iterative learning control scheme was proposed, which can be utilized in the network systems with a fixed communication topology. The results were not only suitable for the homogeneous network systems, but extended to heterogeneous ones also. Afterwards, with regard to the synchronization issue of high-order nonlinear network systems, a novel distributed adaptive iterative learning control strategy was further designed in [13]. In addition, the distributed synchronization protocol design issue for network systems with oriented communication graphs and ordinary linear dynamics was discussed in [14].

For network systems, researchers poured attention into synchronization problem utilizing continuous-time control strategies. However, continuous-time control strategy sometimes is not appropriate due to some inherent characteristics of the networked scenarios. In order to address such issue, sampled-data control method should be a better choice rather than continuous-time one. The plant of sampled-data system is continuous-time, while the update mode of control law is discrete-time [15–20]. Although periodic sampling approach was widely used under some circumstances, it is necessary to employ aperiodic sampling pattern in order to saving energy further. Some crucial problems were studied in literature with the aim of analysing aperiodic sampled-data systems. In [21], stability analysis of the acylic sampled data systems was concerned for application to embedded and networked control. In [22], the synchronization of network systems with nonlinear dynamics applying aperiodic sampled data controllers was studied. In [23], the continuous time systems with time-varying sampling intervals were rebuilt as discrete time-varying systems. In [24], the initial sampled data systems were reconstructed by time delay systems employing the input delay approaches. Adopting

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sampled data control laws, a refresh method to evaluate the stability of continuous linear systems was proposed in [25]. Derived from the discrete time Lyapunov theorem, the above approach affords simpler stability conditions of the continuous time system models. In [26], utilizing Lyapunov functionals with discontinuity at the impulse moments, exponential stability for time-varying impulsive systems including nonlinear dynamics was established.

Owing to the fact that control signals are transmitted from one system to another in a wireless communication network in most situations, nonlinear dynamics and transmission delays usually arise inevitably during control process [27–33]. The problems of time delay for network systems are widely studied by researchers. In [34], the cooperative output regulation issues for discrete time linear network systems suffering time delay were studied. The network systems were subject to jointly connected switching networks. The distributed control law was composed of a distributed switched observer and a purely decentralized control law. In [35], the synchronization problems of time-delay network systems with continuous-time single-integrator dynamics were studied and the smart leader was introduced. In [36], the synchronization issues of a kind of second order continuous network systems with jointly-connected topologies and time delay were investigated. Moreover, a linear neighbor-based protocol suffering time delay was introduced. The imperfect network environment may influence the stability of network systems remarkably. As a consequence, the sampled-data control approach is popular and effective for analyzing network systems with time delay. Among the existing literature, the input delay method is a compelling approach to model the sampled-data control inputs via continuous-time functions. By applying this approach, the closed-loop control systems can be presented as switched systems with ordered and multiple time-varying delays [37,38].

More recently, by Lyapunov functional approach, the synchronization analysis of network systems with time variant delays has drew a lot of attentions. The essential technical steps concerning this approach are correlated with the design of the functional, and the choice of proper bounding approaches as well. As is known to all that the Jensen inequality is one of the bounding methods, which has been employed diffusely, despite an unavoidable conservatism. Afterwards, to relax this conservatism, extended Jensen inequalities were studied, which involved Jensen inequality via the adoption of attached quadratic terms. The alleged Wirtinger-based inequality and its further extensions were investigated in [39,40]. In addition, generalized integral inequalities were investigated in [41], which were derived from Legendre polynomials and Bessel inequality. Particularly, if the degree of Legendre polynomials is chosen as two, it can be called the second-order Bessel–Legendre inequality.

There are many progress and achievements on synchronization problems for network systems in the existing literature. However, the synchronization issue of network systems has not been fully studied and some problems are required to be probed. Based on the discussion mentioned above, this paper considers the issue of synchronization for network systems with time-varying delays utilizing aperiodic sampled-data control approach. The main contributions are as follows. First, time-varying delays with no restriction on their derivatives are considered in this paper. Furthermore, the characteristics of the Bessel-Legendre inequality are adequately adapted to the establishment of augmented Lyapunov-Krasovskii functionals. In addition, unlike the utilization of Wirtinger-based and Jensen inequalities in most literature, the Bessel-Legendre inequality is adopted to address the synchronization issue of network systems, which is less conservative.

The structure of the paper is arranged as follows. First, system model, the corresponding problem description and some useful lemmas are explained by Section 2. Then in Section 3, derived from the input delay approach and Lyapunov functional, two novel sufficient conditions for synchronizability issue of network systems with time variant delays are provided. Meanwhile, two relevant sampled-data synchronization control laws are designed. In the end, the validity of the proposed synchronization control law is testified by numerical simulation in Section IV.

Notations: Throughout the entire paper, $\mathbb{R}^n$ represents the real Euclidean space with dimension n. $\mathbb{R}^{a\times b}$ is utilized to denote the collection of all $a\times b$ real matrices. M^T is the matrix transpose of M, and M^{-1} is the inverse matrix. For any square matrix A, $\mathrm{He}(A) = A + A^T$. The notation $\|\cdot\|$ means the Euclidean vector norm or the induced matrix 2-norm. $Q > 0$ ($Q \geq 0$) represents that Q is positive definite (positive semi-definite). The set $\mathbb{S}^n_+$ stands for the collection of symmetric positive definite matrices. 0 expresses a zero matrix, and I_N stands for the N-dimensional identity matrix. $\mathrm{diag}(\cdots)$ represents a block diagonal matrix or diagonal matrix, and $\mathrm{diag}^p(\bar{J})$ denotes a block diagonal matrix with p blocks of $\bar{J}$. $col\{\cdots\}$ stands for a column vector. $P\otimes Q$ represents the Kronecker product.

2. Problem formulation and preliminaries

2.1. Network systems

So as to introduce the concept of network systems, simple knowledge of graph theory is presented first. $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{A})$ is utilized to represent an oriented graph containing the collection of vertices $\mathbf{V} = \{\nu_1, \nu_2, \dots, \nu_N\}$, the collection of oriented edges $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$, and the adjacency matrix $\mathbf{A} = [a_{ij}]_{N\times N}$. For convenience, we represent $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{A})$ as $\mathbf{G}(\mathbf{A})$ if there is no ambiguity. When a vertex j can take over signals from vertex i, then an oriented edge $e_{ij} \in \mathbf{E}$ exists in $\mathbf{G}(\mathbf{A})$, which can be represented via vertices pair of vertices (ν_i, ν_j). The element of adjacency matrix $\mathbf{A}$ is set as $a_{ij} = 1$ if and only if an oriented edge (ν_i, ν_j) consists in $\mathbf{G}(\mathbf{A})$, or else, $a_{ij} = 0$.

From vertex ν_i to ν_j, an oriented path is a series of ordered edges in $\mathbf{E}$, indicated by $(\nu_i, \nu_{c1}), (\nu_{c1}, \nu_{c2}), \dots, (\nu_{ck}, \nu_j)$ with intermediate vertices ν_{cp}, $p = 1, \dots, k$. An oriented graph is strong or strongly connected if there exists an oriented path between any two distinct vertices ν_i and ν_j. For graph $\mathbf{G}(\mathbf{A})$, the corresponding Laplacian matrix $L = [l_{ij}]_{N\times N}$ is set by

$$\begin{cases} l_{ij} = -a_{ij}, & i \neq j, \\ l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}. \end{cases} \tag{1}$$

With regard to a strong oriented graph $\mathbf{G}(\mathbf{A})$, L is subject to $\sum_{j=1}^{N} l_{ij} = 0$, which is known as the diffusion property, and is irreducible as well.

Consider the network systems with N isolated linear systems, abbreviated as nodes, described by the following dynamic characteristics:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \dots, N. \tag{2}$$

For the ith node, $x_i(t) = [x_{i1}(t), \dots, x_{in}(t)]^T \in \mathbb{R}^n$ stands for the system state and $u_i(t) \in \mathbb{R}^p$ denotes the designed control law. The constant real matrices A and B have proper dimensions.

The objective of the paper is, for network systems (2), to provide a distributed controller u_i for each node i such that the synchronization can be realized. However, to reduce the energy consumption of communication, such design task for node i only depends on the signals from its neighbors and its own at separate sampling instants instead of successive signals.

Before deriving the main results, it is essential to retrospect one definition and two lemmas as follows.

Definition 1 [42]**.** If the following equations hold,

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j = 1, \ldots, N, \tag{3}$$

the network systems (2) can be considered to achieve the synchronization for any initial conditions.

Lemma 1 *[43]*. *With regard to an oriented strong graph* **G***, its Laplacian matrix L meets the diffusive coupling condition (1) with* $\sum_{j=1}^{N} l_{ij} = 0$*, and is irreducible as well. Moreover, L has an eigenvalue 0 with multiplicity 1, and the relevant normalized left eigenvector* $\varrho = [\varrho_1, \varrho_2, \ldots, \varrho_N]^T \in \mathbb{R}^N$ *satisfying* $\sum_{i=1}^{N} \varrho_i = 1$*, namely,* $\varrho^T \cdot L = 0$*. In addition, for all* $i = 1, 2, \ldots, N$*,* $\varrho_i > 0$ *is satisfied.*

Lemma 2 *[44]*. *With regard to an oriented and strong graph* **G***, let* $\varrho = [\varrho_1, \varrho_2, \ldots, \varrho_N]^T$ *be the unique left normalized eigenvector of corresponding Laplace matrix L with regard to the 0 eigenvalue,* $\Xi = \mathrm{diag}\{\varrho_1, \varrho_2, \ldots, \varrho_N\} > 0$*, and* $W_{N\times N} = \Xi - \varrho\varrho^T$*. Accordingly,* $\forall P > 0$*, the following two equations hold:*

$$\begin{aligned}
&x^T(t)(WL \otimes PC)g(x(t)) \\
&= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \varrho_i l_{ij}\big(x_i(t) - x_j(t)\big)^T PC\big[g(x_i(t)) - g(x_j(t))\big], \\
&x^T(t)(W \otimes PC)g(x(t)) \\
&= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \varrho_i \varrho_j\big(x_i(t) - x_j(t)\big)^T PC\big[g(x_i(t)) - g(x_j(t))\big].
\end{aligned}$$

Remark 1. It can be noticed that $g(x(t))$ stands for any function of $x(t)$. For example, $\dot{x}(t)$ and $x(t)$ are the most common two alternatives of $g(x(t))$. When nonlinear dynamics exist in network systems, then $g(x(t))$ can be substituted by a nonlinear function of $x(t)$.

2.2. Bessel-Legendre integral inequality

In the first place, let us introduce an inequality which is the kernel of the theoretical derivation of this paper. It is consistent with the inequality shown in [45] recently, which is a special circumstance of the Bessel-Legendre inequality in [41] as well. The corresponding proving process can be sought out in [45] or in [41].

Lemma 3. *For a differentiable function z in* $[a, b] \to \mathbb{R}^n$*, and a matrix* $M \in \mathbb{S}_+^n$*, the inequality*

$$\int_a^b \dot{z}^T(v) M \dot{z}(v) dv \geq \frac{1}{b-a} \mathfrak{G}^T \mathrm{diag}(M, 3M, 5M) \mathfrak{G} \tag{4}$$

holds, where

$$\mathfrak{G} = \begin{bmatrix} z(b) - z(a) \\ z(b) - z(a) - \dfrac{2}{b-a}\displaystyle\int_a^b z(v)dv \\ z(b) - z(a) - \dfrac{6}{b-a}\displaystyle\int_a^b \delta_{a,b}(v) z(v) dv \end{bmatrix},$$

$$\delta_{a,b}(v) = 2\left(\frac{v-a}{b-a}\right) - 1.$$

Remark 2. The Wirtinger-based inequality of [39] is included in the inequality (4) with the aid of the third element of $\mathfrak{G}$. Except as signals $\int_a^b z(v)dv$, $z(a)$ and $z(b)$, an additional signal $\int_a^b \delta_{a,b}(v) z(v) dv$ is introduced to accomplish this improvement. Compared with the Jensen and Wirtinger-based inequalities, the utilize of the Bessel–Legendre integral inequality can relax the conservatism. In the next section, the characteristics of this integral inequality are adapted to the establishment of extended Lyapunov functional adequately. Pursuing the approach developed in [46], more precise conditions are derived in this paper.

2.3. Parameter-dependent matrix inequalities

Applying another construction method, the reciprocally convex combination inequality in [47] is shown via the following lemma.

Lemma 4. *With regard to any* $\mathfrak{R} \in \mathbb{S}_+^n$*, it is assumed that there is a matrix* $X \in \mathbb{R}^{n\times n}$ *so that* $\begin{bmatrix} \mathfrak{R} & X \\ X^T & \mathfrak{R} \end{bmatrix} \leq 0$*. Accordingly, the following inequality*

$$\begin{bmatrix} \dfrac{1}{\beta}\mathfrak{R} & 0 \\ 0 & \dfrac{1}{1-\beta}\mathfrak{R} \end{bmatrix} \leq \begin{bmatrix} \mathfrak{R} & X \\ X^T & \mathfrak{R} \end{bmatrix}, \quad \forall \beta \in (0, 1),$$

holds.

Next, a variational form of Lemma 4 is presented alternatively, which is derived from the classical bounding technique in [48].

Lemma 5. *With regard to any given* $\mathfrak{M}_1 \in \mathbb{S}_+^n$*,* $\mathfrak{M}_2 \in \mathbb{S}_+^n$*,* $Y_1 \in \mathbb{R}^{2n\times n}$ *and* $Y_2 \in \mathbb{R}^{2n\times n}$*, the inequality*

$$\begin{bmatrix} \dfrac{1}{\beta}\mathfrak{M}_1 & 0 \\ 0 & \dfrac{1}{1-\beta}\mathfrak{M}_2 \end{bmatrix} \leq \Theta_M(\beta), \quad \forall \beta \in (0, 1),$$

holds, where

$$\begin{aligned}
\Theta_M(\beta) = {} & \mathrm{He}(Y_1[I_n 0_{n\times n}] + Y_2[0_{n\times n} I_n]) \\
& - \beta Y_1 \mathfrak{M}_1^{-1} Y_1^T - (1-\beta) Y_2 \mathfrak{M}_2^{-1} Y_2^T.
\end{aligned}$$

Remark 3. It can be noticed that the key distinction between Lemmas 4 and 5 is that, the lower bound depends specifically upon the uncertain argument β in Lemma 5. Eventually, at the cost of additional decision variables, this reliance on β results in a decrease of conservatism.

3. Sampled data control law

In the paper, the synchronization control protocal with sampled data signals for system i is given as follows, $i = 1, \ldots, N$,

$$u_i(t) = \alpha K \sum_{j=1, j\neq i}^{N} a_{ij}\big[x_j(t_k - \eta(t_k)) - x_i(t_k - \eta(t_k))\big],$$
$$t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \tag{5}$$

where $\alpha > 0$ stands for a coupling strength parameter, K is the gain matrix to be designed. t_k represents the updating instant of the ZOH. Moreover, it is supposed that the sampled signals have experienced a time-varying transmission delay $\eta(t)$ from sampler to controller which satisfies $0 \leq \eta_1 \leq \eta(t) \leq \eta_2$. Hence, data information at discrete sampling instant $t_k - \eta(t_k)$ is applied to the controller design task at t_k. Then, by employing ZOH, the control signal holds between two adjacent updating moments t_k and t_{k+1}. The updating instants of ZOH are subject to $\eta(t_0) = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{t\to\infty} t_k = +\infty$. Suppose that the sampling intervals are alterable with an upper bound h_b as

$$(t_{k+1} - \eta(t_k)) - (t_k - \eta(t_k)) = t_{k+1} - t_k \leq h_b, \quad k \in \mathbb{N}.$$

By defining $h_2 \triangleq h_b + \eta_2$, one has

$$t_{k+1} - (t_k - \eta(t_k)) = t_{k+1} - t_k + \eta(t_k) \leq h_2, \quad k \in \mathbb{N}, \tag{6}$$

where h_2 represents the maximal time lag between the instant $t_k - \eta(t_k)$ at which the state information is updated, and the instant t_{k+1} at which the next sampling is triggered.

Define

$$x(t)=\begin{bmatrix}x_1(t)\\x_2(t)\\\vdots\\x_N(t)\end{bmatrix},\quad x(t_k-\eta(t_k))=\begin{bmatrix}x_1(t_k-\eta(t_k))\\x_2(t_k-\eta(t_k))\\\vdots\\x_N(t_k-\eta(t_k))\end{bmatrix}.$$

Substituting (5) into (2) leads to

$$\dot{x}(t)=(I_N\otimes A)x(t)-\alpha(L\otimes BK)x(t_k-\eta(t_k)),\tag{7}$$

where $t\in[t_k,t_{k+1})$, $k\in\mathbb{N}$. Inspired by Fridman et al. [24,37,38], the input delay approach is utilized. Defining $h(t)=t-t_k+\eta(t_k)$, $t\in[t_k,t_{k+1})$, it can be found from (6) that $h_1\le\eta(t_k)\le h(t)<t_{k+1}-t_k+\eta(t_k)\le h_2$, where $h_1=\eta_1$. Then, we rewrite (7) as

$$\begin{cases}\dot{x}(t)=(I_N\otimes A)x(t)-\alpha(L\otimes BK)x(t-h(t)),\\ x(t)=\phi(t),\quad -h_2\le t\le 0,\end{cases}\tag{8}$$

where $h_1\le h(t)\le h_2$ and $h_{12}\triangleq h_2-h_1$.

Remark 4. It can be observed that $\dot{h}(t)=1$ for $t\neq t_k$. However, no constraints are made on the derivative of the time-varying delay functions $\dot{\eta}(t)$. Therefore, $\eta(t)$ are fast variant delays which the stability conditions do not rely on the derivative of the time delay functions $\dot{\eta}(t)$ [49].

3.1. Some useful notations

On the basis of Lemma 3 together with Lemma 4 or 5, two theorems are given for the synchronization issue of system (8) with time variant delays. For the simplicity of expression, in this section we utilize the following notations.

$$\begin{aligned}
e_i&=\begin{bmatrix}0_{nN\times(i-1)nN} & I_{nN} & 0_{nN\times(14-i)nN}\end{bmatrix},\quad i=1,\dots,14,\\
w_i&=\begin{bmatrix}0_{n\times(i-1)n} & I_n & 0_{n\times(14-i)n}\end{bmatrix},\quad i=1,\dots,14,\\
\vec{\Sigma}&=(I_N\otimes A)e_1-\alpha(L\otimes BK)e_3,\\
\Sigma_{ij}&=Aw_1+\bar{\alpha}_{ij}BKw_3,\quad \bar{\alpha}_{ij}=\alpha(l_{ij}/\varrho_j),
\end{aligned}$$

$$G_2(\chi)=\begin{bmatrix}\chi_1-\chi_2\\ \chi_1+\chi_2-2\chi_5\\ \chi_1-\chi_2-6\chi_6\end{bmatrix},\quad G_3(\chi)=\begin{bmatrix}\chi_2-\chi_3\\ \chi_2+\chi_3-2\chi_7\\ \chi_2-\chi_3-6\chi_8\end{bmatrix},$$

$$G_4(\chi)=\begin{bmatrix}\chi_3-\chi_4\\ \chi_3+\chi_4-2\chi_9\\ \chi_3-\chi_4-6\chi_{10}\end{bmatrix},\quad \Gamma(\chi)=\begin{bmatrix}G_3(\chi)\\G_4(\chi)\end{bmatrix},$$

and

$$\xi_0(t)=\begin{bmatrix}x(t)\\x(t-h_1)\\x(t-h(t))\\x(t-h_2)\end{bmatrix},\quad \xi_1(t)=\frac{1}{h_1}\begin{bmatrix}\int_{-h_1}^{0}x(t+s)ds\\ \int_{-h_1}^{0}\delta_1(s)x(t+s)ds\end{bmatrix},$$

$$\xi_2(t)=\frac{1}{h(t)-h_1}\begin{bmatrix}\int_{-h(t)}^{-h_1}x(t+s)ds\\ \int_{-h(t)}^{-h_1}\delta_2(s)x(t+s)ds\end{bmatrix},$$

$$\xi_3(t)=\frac{1}{h_2-h(t)}\begin{bmatrix}\int_{-h_2}^{-h(t)}x(t+s)ds\\ \int_{-h_2}^{-h(t)}\delta_3(s)x(t+s)ds\end{bmatrix},$$

$$\xi_4(t)=(h(t)-h_1)\xi_2(t),\quad \xi_5(t)=(h_2-h(t))\xi_3(t),$$

$$\xi_6(t)=\begin{bmatrix}\int_{-h_2}^{-h_1}x(t+s)ds\\ h_{12}\int_{-h_2}^{-h_1}\delta_4(s)x(t+s)ds\end{bmatrix},$$

and looking up the functions $\delta_{a,\,b}$ given in Lemma 3, where the functions δ_i are shown as follows

$$\delta_1(s)=2\frac{s+h_1}{h_1}-1,\qquad \delta_2(s)=2\frac{s+h(t)}{h(t)-h_1}-1,$$
$$\delta_3(s)=2\frac{s+h_2}{h_2-h(t)}-1,\quad \delta_4(s)=2\frac{s+h_2}{h_{12}}-1.$$

3.2. "Reciprocally convex" – based result

Theorem 1. *It is assumed that the oriented graph* **G** *is strong. Given controller gain matrix K, time-varing delay $0\le\eta_1\le\eta(t)\le\eta_2$, and upper bound of sampling intervals $h_b>0$, if there exist $P\in\mathbb{S}_+^{5n}$, $S_1,S_2,R_1,R_2\in\mathbb{S}_+^n$, $\mathcal{M},\mathcal{N}\in\mathbb{R}^{14n\times 2n}$ and a matrix $X\in\mathbb{R}^{3n\times 3n}$ so that for any $i,j=1,\dots,N$, and any $\kappa=1,2$, the following inequalities hold:*

$$\Psi=\begin{bmatrix}\tilde{R}_2 & X\\ X^T & \tilde{R}_2\end{bmatrix}\le 0,\quad \Phi_{ij}(h_\kappa)=\Phi_0(h_\kappa)-\Gamma^T(w)\Psi\Gamma(w)<0,$$

where $h_1=\eta_1$, $h_2=h_b+\eta_2$, and for any $\theta\in\mathbb{R}$,

$$\begin{aligned}
\Phi_0(\theta)&=\mathrm{He}\Big(G_1^T(\theta)PG_0+\mathcal{M}g_1(\theta)+\mathcal{N}g_2(\theta)\Big)+\hat{S}\\
&\quad+\Sigma_{ij}^T\Big(h_1^2R_1+h_{12}^2R_2\Big)\Sigma_{ij}-G_2(w)^T\tilde{R}_1G_2(w),\\
G_1(\theta)&=\begin{bmatrix}w_1^T & h_1w_5^T & h_1w_6^T & w_{11}^T+w_{13}^T & \hat{G}_1^T(\theta)\end{bmatrix}^T,\\
\hat{G}_1(\theta)&=(h_2-\theta)(w_{11}+w_{14})+(\theta-h_1)(w_{12}-w_{13}),\\
G_0&=\begin{bmatrix}\Sigma_{ij}^T & w_1^T-w_2^T & w_1^T+w_2^T-2w_5^T & w_2^T-w_4^T & \hat{G}_0^T\end{bmatrix}^T,\\
\hat{G}_0&=h_{12}(w_2+w_4)-2(w_{11}+w_{13}),\\
\hat{S}&=\mathrm{diag}(S_1,\ -S_1+S_2,\ 0_{n\times n},\ -S_2,\ 0_{10n\times 10n}),\\
\tilde{R}_i&=\mathrm{diag}(R_i,\ 3R_i,\ 5R_i),
\end{aligned}$$

and

$$g_1(\theta)=(\theta-h_1)\begin{bmatrix}w_7\\w_8\end{bmatrix}-\begin{bmatrix}w_{11}\\w_{12}\end{bmatrix},$$

$$g_2(\theta)=(h_2-\theta)\begin{bmatrix}w_9\\w_{10}\end{bmatrix}-\begin{bmatrix}w_{13}\\w_{14}\end{bmatrix}.$$

Afterwards, the network systems (2) can realize the synchronization.

Proof. For system (8), choose the Lyapunov-Krasovskii functional as

$$V\big(x(t),\dot{x}(t)\big)=V_P\big(x(t)\big)+V_S\big(x(t)\big)+V_R\big(x(t),\dot{x}(t)\big),\tag{9}$$

where

$$\begin{aligned}
V_P\big(x(t)\big)&=\vec{x}^T(t)(W\otimes P)\vec{x}(t),\\
V_S\big(x(t)\big)&=\int_{t-h_1}^{t}x^T(s)(W\otimes S_1)x(s)ds\\
&\quad+\int_{t-h_2}^{t-h_1}x^T(s)(W\otimes S_2)x(s)ds,\\
V_R\big(x(t),\dot{x}(t)\big)&=h_1\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)(W\otimes R_1)\dot{x}(s)dsd\theta\\
&\quad+h_{12}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}^T(s)(W\otimes R_2)\dot{x}(s)dsd\theta,
\end{aligned}$$

and $\vec{x}(t)=col\{x(t),\ h_1\xi_1(t),\ \xi_6(t)\}$. □

Remark 5. The Lyapunov functionals V_S and V_R have been utilized in [47] and [39] already. About the choice of V_P, for example in [47], the usual approach is to select a quadratic term relying on the momentary system state $x(t)$ merely. While it was demonstrated in [39] that it is necessary to augment V_P for purpose of fully taking advantage of the Wirtinger-based inequality. The augmented form is comprised of $\int_{-h_1}^{0} x(t+s)ds$ and $\int_{-h_2}^{-h_1} x(t+s)ds$, which can be found apparently in the Wirtinger-based inequality. Furthermore, considering the Bessel-Legendre inequality shown in Lemma 3, which is an augmented form of the Wirtinger-based inequality, the term V_P ought to contain two extra signals $\int_{-h_1}^{0} \delta_1(s)x(t+s)ds$ and $\int_{-h_2}^{-h_1} \delta_4(s)x(t+s)ds$ as well. Such state extension in V_P was first put forward in [50].

Along the trajectories of (8), the target of the following derivation is to obtain an upper bound of the derivative of Lyapunov functional $V\big(x(t), \dot{x}(t)\big)$. Therefore, an extended state vector is utilized as

$$\varphi(t) = col\Big\{\xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t), \xi_5(t)\Big\}.$$

To avoid complexity, the time variable is left out without any possible ambiguity. Specifically, it means that, for example, h and φ represent $h(t)$ and $\varphi(t)$ in the sequel, respectively.

For $t \in [t_k, t_{k+1})$, by finding the time derivative of functional $V_P(x(t))$ along the trajectory of (8), we can see that

$$\frac{d}{dt}V_P\big(x(t)\big) = 2\vec{x}^T(t)(W \otimes P)\dot{\vec{x}}(t).$$

Accordingly, it is needed to represent $\dot{\vec{x}}(t)$ and $\vec{x}(t)$ by the extended vector φ. For one thing, it can be noticed that

$$\dot{x}(t) = \vec{\Sigma}\varphi.$$

Then, it can be found that

$$\frac{d}{dt}\int_{-h_1}^{0} x(t+s)ds = \frac{d}{dt}\int_{t-h_1}^{t} x(u)du = x(t) - x(t-h_1), \tag{10}$$

is the first component of $h_1\dot{\xi}_1(t)$. Afterwards, adopting integration by parts leads to

$$\begin{aligned}\frac{d}{dt}\int_{-h_1}^{0} s\cdot x(t+s)ds &= s\cdot x(t+s)|_{-h_1}^{0} - \int_{-h_1}^{0} x(t+s)ds \\ &= h_1x(t-h_1) - \int_{-h_1}^{0} x(t+s)ds.\end{aligned}$$

Therefore, the second component of $h_1\dot{\xi}_1(t)$ is given by

$$\begin{aligned}&\frac{d}{dt}\int_{-h_1}^{0} \delta_1(s)x(t+s)ds \\ &= \frac{d}{dt}\int_{-h_1}^{0}\left(\frac{2s}{h_1}+1\right)x(t+s)ds \\ &= \frac{2}{h_1}\cdot\frac{d}{dt}\int_{-h_1}^{0} s\cdot x(t+s)ds + \frac{d}{dt}\int_{-h_1}^{0} x(t+s)ds \\ &= 2x(t-h_1) - \frac{2}{h_1}\int_{-h_1}^{0} x(t+s)ds + x(t) - x(t-h_1) \\ &= x(t) + x(t-h_1) - \frac{2}{h_1}\int_{-h_1}^{0} x(t+s)ds.\end{aligned} \tag{11}$$

Consequently, from (10) and (11), it can be concluded that

$$h_1\dot{\xi}_1(t) = \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_5\end{bmatrix}\varphi.$$

By using similar methods, it can be seen that

$$\dot{\xi}_6(t) = \begin{bmatrix} e_2 - e_4 \\ \hat{G}_0(e)\end{bmatrix}\varphi,$$

where $\hat{G}_0(\chi) = h_{12}(\chi_2 + \chi_4) - 2(\chi_{11} + \chi_{13})$. Hence, it can be summarized that

$$\dot{\vec{x}}(t) = \vec{G}_0\varphi,$$

where

$$\vec{G}_0 = \begin{bmatrix}\vec{\Sigma}^T & e_1^T - e_2^T & e_1^T + e_2^T - 2e_5^T & e_2^T - e_4^T & \hat{G}_0^T(e)\end{bmatrix}^T.$$

For another, let us derive an expression of $\vec{x}(t)$ relying on φ. To begin with, it can be seen that

$$x(t) = e_1\varphi, \quad h_1\xi_1(t) = h_1\begin{bmatrix} e_5 \\ e_6\end{bmatrix}\varphi.$$

Then, to express $\xi_6(t)$ depending on the augmented state φ, it can be noticed that

$$\xi_6(t) = \begin{bmatrix} \left(\int_{-h}^{-h_1} + \int_{-h_2}^{-h}\right)x(t+s)ds \\ h_{12}\left(\int_{-h}^{-h_1} + \int_{-h_2}^{-h}\right)\delta_4(s)x(t+s)ds\end{bmatrix}. \tag{12}$$

On one hand, from (12), it can be seen that the first nN elements can be represented as $(e_{11} + e_{13})\varphi$. On the other hand, for the last nN elements of $\xi_6(t)$, two dissimilar expressions of $\delta_4(t)$ are needed, which are based on $\delta_2(t)$ and $\delta_3(t)$, respectively. Then, it can be found that

$$\begin{cases} h_{12}\delta_4(s) = (h-h_1)\delta_2(s) + (h_2 - h), \\ h_{12}\delta_4(s) = (h_2-h)\delta_3(s) - (h - h_1).\end{cases}$$

Therefore, it can be seen that

$$\begin{aligned}&h_{12}\left(\int_{-h}^{-h_1} + \int_{-h_2}^{-h}\right)\delta_4(s)x(t+s)ds \\ &= (h-h_1)\left(\int_{-h}^{-h_1}\delta_2(s)x(t+s)ds - \int_{-h_2}^{-h} x(t+s)ds\right) \\ &\quad + (h_2-h)\left(\int_{-h}^{-h_1} x(t+s)ds + \int_{-h_2}^{-h}\delta_3(s)x(t+s)ds\right) \\ &= \hat{G}_1(h,e)\varphi,\end{aligned}$$

where $\hat{G}_1(\theta, \chi) = (h_2 - \theta)(\chi_{11} + \chi_{14}) + (\theta - h_1)(\chi_{12} - \chi_{13})$. Accordingly, it can be concluded that

$$\xi_6(t) = \begin{bmatrix} e_{11} + e_{13} \\ \hat{G}_1(h,e)\end{bmatrix}\varphi,$$

and $\vec{x}(t) = G_1(h,e)\varphi$, where

$$G_1(\theta,\chi) = \begin{bmatrix}\chi_1^T & h_1\chi_5^T & h_1\chi_6^T & \chi_{11}^T + \chi_{13}^T & \hat{G}_1^T(\theta,\chi)\end{bmatrix}^T.$$

Furthermore, recalling the structure of the extended state φ, it can be noticed that the last four elements of φ can be regarded as linear combination of the other elements. That is, $\xi_4(t) = (h - h_1)\xi_2(t)$ and $\xi_5(t) = (h_2 - h(t))\xi_3(t)$ directly lead to

$$\varphi^T\mathrm{He}\Big((W\otimes\mathcal{M})g_1(h,e) + (W\otimes\mathcal{N})g_2(h,e)\Big)\varphi = 0,$$

where matrices $\mathcal{M}, \mathcal{N} \in \mathbb{R}^{14n\times 2n}$ and functions $g_1(\cdot)$, $g_2(\cdot)$ are defined as

$$g_1(\theta,\chi) = (\theta - h_1)\begin{bmatrix}\chi_7 \\ \chi_8\end{bmatrix} - \begin{bmatrix}\chi_{11} \\ \chi_{12}\end{bmatrix},$$

$$g_2(\theta,\chi) = (h_2 - \theta)\begin{bmatrix}\chi_9 \\ \chi_{10}\end{bmatrix} - \begin{bmatrix}\chi_{13} \\ \chi_{14}\end{bmatrix}.$$

Consequently, the derivative of $V_P(x(t))$ is given as

$$\begin{aligned}\frac{d}{dt}V_P\big(x(t)\big) = \varphi^T\mathrm{He}\Big(&G_1^T(h,e)(W\otimes P)\vec{G}_0 + (W\otimes\mathcal{M})g_1(h,e) \\ &+ (W\otimes\mathcal{N})g_2(h,e)\Big)\varphi.\end{aligned} \tag{13}$$

By deriving the time derivative of functional $V_S(x(t))$, it can be found that

$$\frac{d}{dt}V_S\big(x(t)\big) = \varphi^T\vec{S}\varphi, \tag{14}$$

where $\vec{S} = \mathrm{diag}(W \otimes S_1, W \otimes (-S_1 + S_2), 0_{nN\times nN}, -W \otimes S_2, 0_{10nN\times 10nN})$.

To obtain the derivative of functional $V_R\big(x(t), \dot{x}(t)\big)$, we can see that

$$\frac{d}{dt}V_R\big(x(t), \dot{x}(t)\big) = \dot{x}^T(t)\Big(h_1^2(W \otimes R_1) + h_{12}^2(W \otimes R_2)\Big)\dot{x}(t) + \Upsilon_1 + \Upsilon_2,$$

where

$$\Upsilon_1 = -h_1 \int_{t-h_1}^{t} \dot{x}^T(u)(W \otimes R_1)\dot{x}(u)du,$$
$$\Upsilon_2 = -h_{12}\left(\int_{t-h}^{t-h_1} + \int_{t-h_2}^{t-h}\right)\dot{x}^T(u)(W \otimes R_2)\dot{x}(u)du.$$

From Lemma 3, it can be found that

$$\Upsilon_1 \le -\varphi^T G_2(e)^T \vec{R}_1 G_2(e)\varphi,$$
$$\Upsilon_2 \le -\frac{1}{\beta}\varphi^T G_3(e)^T \vec{R}_2 G_3(e)\varphi - \frac{1}{1-\beta}\varphi^T G_4(e)^T \vec{R}_2 G_4(e)\varphi$$
$$= -\varphi^T \Gamma(e)^T \begin{bmatrix} \frac{1}{\beta}\vec{R}_2 & 0 \\ 0 & \frac{1}{1-\beta}\vec{R}_2 \end{bmatrix} \Gamma(e)\varphi,$$

where $\vec{R}_i = \mathrm{diag}(W \otimes R_i, 3W \otimes R_i, 5W \otimes R_i)$, $i = 1, 2$, and $\beta = \frac{h-h_1}{h_{12}}$. Next, it can be noticed from Lemma 4 that

$$\Upsilon_2 \le -\varphi^T \Gamma(e)^T \vec{\Psi} \Gamma(e)\varphi,$$

where

$$\vec{\Psi} = \begin{bmatrix} \vec{R}_2 & \vec{X} \\ \vec{X}^T & \vec{R}_2 \end{bmatrix} \le 0.$$

Consequently, it can be summarized that

$$\frac{d}{dt}V_R\big(x(t), \dot{x}(t)\big) \le \varphi^T\Big[\vec{\Sigma}^T\Big(h_1^2(W \otimes R_1) + h_{12}^2(W \otimes R_2)\Big)\vec{\Sigma} - G_2(e)^T \vec{R}_1 G_2(e) - \Gamma(e)^T \vec{\Psi}\Gamma(e)\Big]\varphi. \tag{15}$$

Finally, putting the previous formulas (13)–(15) together, and recalling Lemmas 1 and 2, then we have

$$\frac{d}{dt}V\big(x(t), \dot{x}(t)\big) \le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \varrho_i\varrho_j \tilde{\varphi}^T \Phi_{ij}(h)\tilde{\varphi}$$

where

$$\tilde{\varphi} = \varphi_i - \varphi_j,$$
$$\Phi_{ij}(\theta) = \Phi_0(\theta) - \Gamma^T(w)\Psi\Gamma(w),$$
$$\Phi_0(\theta) = \mathrm{He}\Big(G_1^T(\theta, w)PG_0 + \mathcal{M}g_1(\theta, w) + \mathcal{N}g_2(\theta, w)\Big) + \hat{S} + \Sigma_{ij}^T\Big(h_1^2 R_1 + h_{12}^2 R_2\Big)\Sigma_{ij} - G_2(w)^T \tilde{R}_1 G_2(w),$$
$$G_0 = \big[\Sigma_{ij}^T \;\; w_1^T - w_2^T \;\; w_1^T + w_2^T - 2w_5^T \;\; w_2^T - w_4^T \;\; \hat{G}_0^T(w)\big]^T,$$
$$\hat{S} = \mathrm{diag}(S_1,\ -S_1 + S_2,\ 0_{n\times n},\ -S_2,\ 0_{10n\times 10n}),$$
$$\tilde{R}_i = \mathrm{diag}(R_i,\ 3R_i,\ 5R_i),$$
$$\Psi = \begin{bmatrix} \tilde{R}_2 & X \\ X^T & \tilde{R}_2 \end{bmatrix} \le 0.$$

It can be noticed the fact that $\Phi_{ij}(h)$ is affine respecting h, in order that $\Phi_{ij}(h)$ is convex. In consequence, the two inequalities $\Phi_{ij}(h_1) < 0$ and $\Phi_{ij}(h_2) < 0$ indicate $\Phi_{ij}(h) < 0$ for all $h \in [h_1, h_2]$. Afterwards, it can be found that $\frac{d}{dt}V\big(x(t), \dot{x}(t)\big) < 0$ and $V\big(x(t), \dot{x}(t)\big) \le V\big(x(0), \dot{x}(0)\big)$, which indicates that $V\big(x(t), \dot{x}(t)\big)$ is bounded. Hence, $\vec{x}^T(t)(W \otimes P)\vec{x}(t)$ is bounded as well and

$$\varrho_i\varrho_j\lambda_{\min}(P)\|\vec{x}_i(t) - \vec{x}_j(t)\|^2 \le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \varrho_i\varrho_j\Big(\vec{x}_i(t) - \vec{x}_j(t)\Big)^T P\Big(\vec{x}_i(t) - \vec{x}_j(t)\Big) = \vec{x}^T(t)(W \otimes P)\vec{x}(t) = O(e^{-\varrho t}).$$

Therefore, based on Definition 1, synchronization in network systems (2) suffering time variant delay $\eta(t)$ can be guaranteed.

Theorem 2. *It is assumed that the oriented graph* **G** *is strong. Given time-varing delay $0 \le \eta_1 \le \eta(t) \le \eta_2$, upper bound of sampling intervals $h_b > 0$ and parameters ϑ_g, $(g = 1, 2, 3)$, if there exist $\bar{P}_2 \in \mathbb{S}_+^{4n}$, $\bar{S}_1, \bar{S}_2 \in \mathbb{S}_+^n$, $\bar{\mathcal{M}}, \bar{\mathcal{N}} \in \mathbb{R}^{14n\times 2n}$, $\bar{X} \in \mathbb{R}^{3n\times 3n}$, $U \in \mathbb{R}^{n\times n}$ and a diagonal matrix $\bar{J} \in \mathbb{S}_+^n$ so that $\forall i, j = 1, 2, \ldots, N$ and any $\kappa = 1, 2$, the following inequalities hold:*

$$\bar{\Psi} = \begin{bmatrix} \bar{R}_2 & \bar{X} \\ \bar{X}^T & \bar{R}_2 \end{bmatrix} \le 0, \tag{16}$$

$$\begin{bmatrix} \bar{\Omega}_1 + \bar{\Omega}_2(h_\kappa) & \bar{Z}^T \\ \bar{Z} & -\bar{\delta}\bar{J} \end{bmatrix} < 0, \tag{17}$$

where $h_1 = \eta_1$, $h_2 = h_b + \eta_2$, and for any $\theta \in \mathbb{R}$,

$$\bar{\Omega}_1 = \mathrm{He}\Big(w_1^T\delta_1(A\bar{J}w_1 + \bar{\alpha}_{ij}BUw_3)\Big),$$
$$\bar{\Omega}_2(\theta) = \mathrm{He}\Big(G_{12}^T(\theta)\bar{P}_2 G_{02} + \bar{\mathcal{M}}g_1(\theta) + \bar{\mathcal{N}}g_2(\theta)\Big) + \bar{S} - G_2^T(w)\bar{R}_1 G_2(w) - \Gamma^T(w)\bar{\Psi}\Gamma(w),$$
$$G_{12}(\theta) = \big[h_1 w_5^T \;\; h_1 w_6^T \;\; w_{11}^T + w_{13}^T \;\; \hat{G}_1^T(\theta)\big]^T,$$
$$\hat{G}_1(\theta) = (h_2 - \theta)(w_{11} + w_{14}) + (\theta - h_1)(w_{12} - w_{13}),$$
$$G_{02} = \big[w_1^T - w_2^T \;\; w_1^T + w_2^T - 2w_5^T \;\; w_2^T - w_4^T \;\; \hat{G}_0^T\big]^T,$$
$$\hat{G}_0 = h_{12}(w_2 + w_4) - 2(w_{11} + w_{13}),$$
$$\bar{S} = \mathrm{diag}(\bar{S}_1,\ -\bar{S}_1 + \bar{S}_2,\ 0_{n\times n},\ -\bar{S}_2,\ 0_{10n\times 10n}),$$
$$\bar{R}_i = \delta_{i+1}\mathrm{diag}(\bar{J},\ 3\bar{J},\ 5\bar{J}),$$
$$\bar{Z} = \bar{\delta}(A\bar{J}w_1 + \bar{\alpha}_{ij}BUw_3),$$
$$\bar{\delta} = h_1^2\delta_2 + h_{12}^2\delta_3,$$

and

$$g_1(\theta) = (\theta - h_1)\begin{bmatrix} w_7 \\ w_8 \end{bmatrix} - \begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix},$$
$$g_2(\theta) = (h_2 - \theta)\begin{bmatrix} w_9 \\ w_{10} \end{bmatrix} - \begin{bmatrix} w_{13} \\ w_{14} \end{bmatrix}.$$

Then, the network systems (2) can accomplish the synchronization with the control gain $K = U\bar{J}^{-1}$ in (5).

Proof. Based on Theorem 1, we choose $P = \mathrm{diag}(P_1,\ P_2)$, where $P_1 \in \mathbb{S}_+^n$ and $P_2 \in \mathbb{S}_+^{4n}$. By selecting $P_1 = \delta_1 J$, $R_1 = \delta_2 J$ and $R_2 = \delta_3 J$, where $J \in \mathbb{S}_+^n$ is a diagonal matrix, it can be seen that

$$G_1^T(\theta)PG_0 = \big[w_1^T \;\; G_{12}^T(\theta)\big]\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} \Sigma_{ij} \\ G_{02} \end{bmatrix} = w_1^T\delta_1 J(Aw_1 + \bar{\alpha}_{ij}BKw_3) + G_{12}^T(\theta)P_2 G_{02}.$$

Furthermore, we set

$\bar{J} = J^{-1}$, $U = K\bar{J}$, $\bar{P}_2 = \mathrm{diag}^4(\bar{J})P_2\mathrm{diag}^4(\bar{J})$, $\bar{S}_1 = \bar{J}S_1\bar{J}$,
$\bar{S}_2 = \bar{J}S_2\bar{J}$, $\bar{X} = \bar{J}X\bar{J}$, $\bar{\mathcal{M}} = \mathrm{diag}^{14}(\bar{J})\mathcal{M}\mathrm{diag}^2(\bar{J})$,
$\bar{\mathcal{N}} = \mathrm{diag}^{14}(\bar{J})\mathcal{N}\mathrm{diag}^2(\bar{J})$.

Pre- and post-multiplying Ψ by $\mathrm{diag}^6(\bar{J})$ directly leads to (16). Pre- and post-multiplying $\Phi_{ij}(h_\kappa)$ by $\mathrm{diag}^{14}(\bar{J})$ and applying Schur complement, it can be found that (17) holds. □

3.3. Another synchronization result

In the last section, Theorem 1 is obtained based on Lemmas 3 and 4. By substituting Lemma 5 for Lemma 4, an analogous analysis of synchronization can be accomplished alternatively, which results in the following conditions.

Theorem 3. *Assume that the oriented graph* **G** *is strong. Given time-varing delay $0 \le \eta_1 \le \eta(t) \le \eta_2$, controller gain K and upper bound of sampling intervals $h_b > 0$, if there exist $P \in \mathbb{S}_+^{5n}$, $S_1, S_2, R_1, R_2 \in \mathbb{S}_+^n$, $\mathcal{M}, \mathcal{N} \in \mathbb{R}^{14n\times 2n}$ and two matrices $Y_1, Y_2 \in \mathbb{R}^{14n\times 3n}$ so that $\forall i, j = 1, \dots, N$ and any $\kappa = 1, 2$, $\lambda = 1, 2$, the following inequality holds:*

$$\begin{bmatrix} \Phi_0(h_\kappa) - \mathrm{He}\Big(Y_2G_3(w) + Y_1G_4(w)\Big) & Y_\lambda \\ Y_\lambda^T & -\tilde{R}_2 \end{bmatrix} < 0,$$

where $h_1 = \eta_1$, $h_2 = h_b + \eta_2$, and Φ_0, $\tilde{R}_2$ are given in Theorem 1. Accordingly, the network systems (2) can realize the synchronization.

Proof. Some proof steps are ignored due to the fact that the proof process is similar to the technique in Theorem 1. Thereinto, the major difference consists in the utilization of Lemma 4 superseded by Lemma 5. From Lemma 4, it can be found that

$$\begin{aligned}\Upsilon_2 &\le -\varphi^T\Gamma(e)^T\begin{bmatrix} \frac{1}{\beta}\tilde{R}_2 & 0 \\ 0 & \frac{1}{1-\beta}\tilde{R}_2 \end{bmatrix}\Gamma(e)\varphi \\ &\le -\varphi^T\Gamma(e)^T\vec{\Theta}_M(\beta)\Gamma(e)\varphi,\end{aligned}$$

where

$$\vec{\Theta}_M(\beta) = \mathrm{He}\Big(\begin{bmatrix}\vec{X}_1 & \vec{X}_2\end{bmatrix}\Big) - \beta\vec{X}_1\tilde{R}_2^{-1}\vec{X}_1^T - (1-\beta)\vec{X}_2\tilde{R}_2^{-1}\vec{X}_2^T.$$

Consequently, we conclude that

$$\begin{aligned}\frac{d}{dt}V_R\big(x(t), \dot{x}(t)\big) \le \varphi^T\Big[&\vec{\Sigma}^T\Big(h_1^2(W\otimes R_1) + h_{12}^2(W\otimes R_2)\Big)\vec{\Sigma} \\ &- G_2(e)^T\tilde{R}_1G_2(e) - \Gamma(e)^T\vec{\Theta}_M(\beta)\Gamma(e)\Big]\varphi.\end{aligned} \tag{18}$$

Finally, putting the previous formulas (13)–(18) together, and recalling Lemmas 1 and 2, then we have

$$\frac{d}{dt}V\big(x(t), \dot{x}(t)\big) \le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1, j\ne i}^{N}\varrho_i\varrho_j\tilde{\varphi}^T\Xi_{ij}(h)\tilde{\varphi}$$

where

$$\begin{aligned}\Xi_{ij}(\theta) &= \Phi_0(\theta) - \Gamma^T(w)\Theta_M(\beta)\Gamma(w), \\ \Theta_M(\beta) &= \mathrm{He}\Big(\begin{bmatrix} X_1 & X_2 \end{bmatrix}\Big) - \beta X_1\tilde{R}_2^{-1}X_1^T - (1-\beta)X_2\tilde{R}_2^{-1}X_2^T.\end{aligned}$$

Then, denoting $\Gamma(w)$ by Γ, it can be seen that

$$\begin{aligned}\Xi_{ij}(h) = \Phi_0(h) &- \mathrm{He}\Big(\Gamma^T\begin{bmatrix} X_1 & X_2 \end{bmatrix}\Gamma\Big) + \beta\Gamma^TX_1\tilde{R}_2^{-1}X_1^T\Gamma \\ &+ (1-\beta)\Gamma^TX_2\tilde{R}_2^{-1}X_2^T\Gamma.\end{aligned}$$

Denoting $\Gamma^TX_1 = Y_2$ and $\Gamma^TX_2 = Y_1$ leads to

$$\begin{aligned}\Xi_{ij}(h) &= \Lambda + \beta Y_2\tilde{R}_2^{-1}Y_2^T + (1-\beta)Y_1\tilde{R}_2^{-1}Y_1^T \\ &= \beta(\Lambda + Y_2\tilde{R}_2^{-1}Y_2^T) + (1-\beta)(\Lambda + Y_1\tilde{R}_2^{-1}Y_1^T),\end{aligned}$$

where

$$\Lambda = \Phi_0(h) - \mathrm{He}\left(\begin{bmatrix} Y_2 & Y_1 \end{bmatrix}\begin{bmatrix} G_3(w) \\ G_4(w) \end{bmatrix}\right).$$

Applying Schur complement, it can be obtained that

$$\begin{bmatrix} \Lambda & Y_\lambda \\ Y_\lambda^T & -\tilde{R}_2 \end{bmatrix} < 0,$$

for $\lambda = 1, 2$, can lead to $\Xi_{ij}(h) < 0$. Therefore, referring to the proof in Theorem 1 and according to Definition 1, synchronization in network systems (2) with time-varying delay $\eta(t)$ can be guaranteed. □

Remark 6. To compare the conservatism of Theorems 1 and 3, we assume that the conditions given in Theorem 1 are satisfied. By applying Schur complement, it can be noticed that $\Psi \ge 0$ leads to

$$\tilde{R}_2 - X\tilde{R}_2^{-1}X^T \ge 0, \quad \tilde{R}_2 - X^T\tilde{R}_2^{-1}X \ge 0.$$

Then, by choosing

$$Y_1^T = \tilde{R}_2G_4(w) + X^TG_3(w), \quad Y_2^T = \tilde{R}_2G_3(w) + XG_4(w),$$

it can be found that

$$\begin{aligned}&\Phi_0(h_\kappa) - \mathrm{He}\Big(Y_2G_3(w) + Y_1G_4(w)\Big) + Y_1\tilde{R}_2^{-1}Y_1^T \\ &= \Phi_0(h_\kappa) - \Gamma^T(w)\left(\Psi + \begin{bmatrix} \tilde{R}_2 - X\tilde{R}_2^{-1}X^T & 0 \\ 0 & 0 \end{bmatrix}\right)\Gamma(w) \\ &\le \Phi_0(h_\kappa) - \Gamma^T(w)\Psi\Gamma(w),\end{aligned}$$

and

$$\begin{aligned}&\Phi_0(h_\kappa) - \mathrm{He}\Big(Y_2G_3(w) + Y_1G_4(w)\Big) + Y_2\tilde{R}_2^{-1}Y_2^T \\ &= \Phi_0(h_\kappa) - \Gamma^T(w)\left(\Psi + \begin{bmatrix} 0 & 0 \\ 0 & \tilde{R}_2 - X^T\tilde{R}_2^{-1}X \end{bmatrix}\right)\Gamma(w) \\ &\le \Phi_0(h_\kappa) - \Gamma^T(w)\Psi\Gamma(w).\end{aligned}$$

Therefore, the comparison demonstrates that Theorem 3 always brings about better (or at least the same) outcomes than Theorem 1. Generally, the underlying improvement of Theorem 3 over Theorem 1, at the cost of a visible growth of the quantity of decision variables, is the revealment of a tradeoff between the numerical complexity and the reduction of the conservatism.

Theorem 4. *Assume that the oriented graph* **G** *is strong. Given time-varing delay $0 \le \eta_1 \le \eta(t) \le \eta_2$, upper bound of sampling intervals $h_b > 0$ and parameters ϑ_g, $(g = 1, 2, 3)$, if there exist matrices $\bar{P}_2 \in \mathbb{S}_+^{4n}$, $\bar{S}_1, \bar{S}_2 \in \mathbb{S}_+^n$, $\bar{\mathcal{M}}, \bar{\mathcal{N}} \in \mathbb{R}^{14n\times 2n}$, $\bar{Y}_1, \bar{Y}_2 \in \mathbb{R}^{14n\times 3n}$, $U \in \mathbb{R}^{n\times n}$ and a diagonal matrix $\bar{J} \in \mathbb{S}_+^n$ such that for any $i, j = 1, \dots, N$ and any $\kappa = 1, 2$, $\lambda = 1, 2$, the following inequality holds:*

$$\begin{bmatrix} \bar{\Omega}_1 + \bar{\Omega}_3(h_\kappa) & \bar{Z}^T & \bar{Y}_\lambda \\ \bar{Z} & -\bar{\delta}\bar{J} & 0 \\ \bar{Y}_\lambda^T & 0 & -\bar{R}_2 \end{bmatrix} < 0,$$

where $h_1 = \eta_1$, $h_2 = h_b + \eta_2$, and for any $\theta \in \mathbb{R}$,

$$\begin{aligned}\bar{\Omega}_3(\theta) = \mathrm{He}\Big(&G_{12}^T(\theta)\bar{P}_2G_{02} + \bar{\mathcal{M}}g_1(\theta) + \bar{\mathcal{N}}g_2(\theta)\Big) + \bar{S} \\ &- G_2^T(w)\bar{R}_1G_2(w) - \mathrm{He}\Big(\bar{Y}_2G_3(w) + \bar{Y}_1G_4(w)\Big),\end{aligned}$$

and other variables or matrices can be found in Theorem 2. Accordingly, the network systems (2) can realize the synchronization with the control gain matrix $K = U\bar{J}^{-1}$ in (5).

Proof: Referring to the proof in Theorem 2, we can find the way to prove Theorem 4 in like manner.

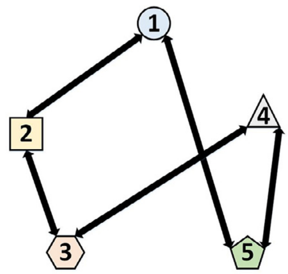

Fig. 1. Topology graph **G** of the network systems.

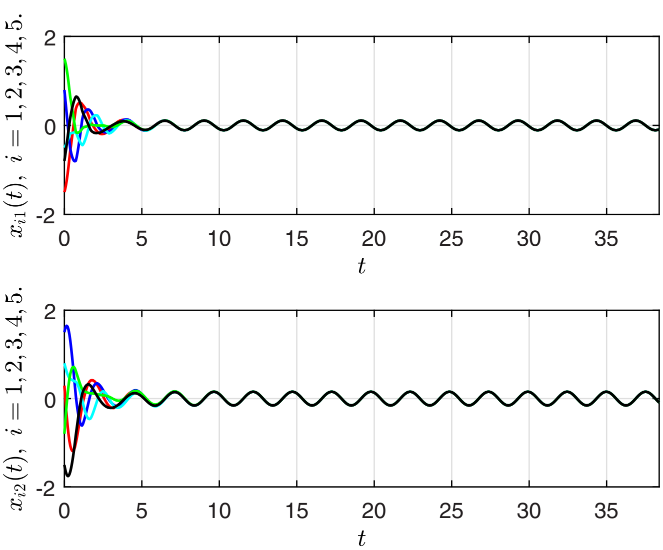

Fig. 2. State trajectories of the network systems.

4. Numerical example

In Section 4, a numerical simulation is provided to demonstrate the availability of the designed approach. The topology graph **G** among the network systems is provided in Fig. 1. The adjacent matrix **A** and the Laplacian matrix L of the network systems are given as

$$\mathbf{A}=\begin{bmatrix}0&1&0&0&1\\1&0&1&0&0\\0&1&0&1&0\\0&0&1&0&1\\1&0&0&1&0\end{bmatrix},\quad L=\begin{bmatrix}2&-1&0&0&-1\\-1&2&-1&0&0\\0&-1&2&-1&0\\0&0&-1&2&-1\\-1&0&0&-1&2\end{bmatrix}.$$

Therefore, it can be found that the directed communication graph **G** of network systems is strong. Correspondingly, the normalized left eigenvector to the eigenvalue 0 can be calculated as $\varrho=[1/5\ \ 1/5\ \ 1/5\ \ 1/5\ \ 1/5]^T$.

The dynamic characteristics of the ith system can be described by Eq. (2) corresponding to the following parameter matrices,

$$A=\begin{bmatrix}0&-1.7458\\3.5539&0\end{bmatrix},\quad B=\begin{bmatrix}-2.0458&-1.3388\\-1.5095&1.3022\end{bmatrix},$$

where $x_i(t)=\left[x_{i1}(t)\quad x_{i2}(t)\right]^T$, $i=1,2,3,4,5.$

The initial states of the network systems are given as follows:

$$x_1(0)=\begin{bmatrix}-1.5\\0.3\end{bmatrix}, x_2(0)=\begin{bmatrix}0.8\\1.5\end{bmatrix}, x_3(0)=\begin{bmatrix}-0.5\\0.8\end{bmatrix},$$
$$x_4(0)=\begin{bmatrix}1.5\\-0.8\end{bmatrix}, x_5(0)=\begin{bmatrix}-0.8\\-1.5\end{bmatrix}.$$

Recalling the framework of the sampling controller (5), we suppose that the coupling strength is $\alpha=0.5$. The time-varying delays satisfy $0.1\le\eta(t)\le 0.2$, and upper bound of sampling lags is $h_b=0.5$. Therefore, it can be seen that $h_1=0.1$ and $h_2=h_b+\eta_2=0.7$. According to Theorem 2, we can find the gain K of the proposed sampling controller by

$$K=\begin{bmatrix}-0.3391&0.1329\\-0.1630&0.4565\end{bmatrix}.$$

Filling the gain matrix K and time-varying delays $\eta(t_k)$ into the sampling controller (5), the state trajectories of the network systems under such sampled-data control law are demonstrated in Fig. 2 which signifies that the network systems can eventually realize synchronization.

The corresponding aperiodic sampling intervals are shown in Fig. 3. The x-axis value of each stem stands for a sampling time $t_k, k=0,1,\ldots$. The height of every stem represents the time span

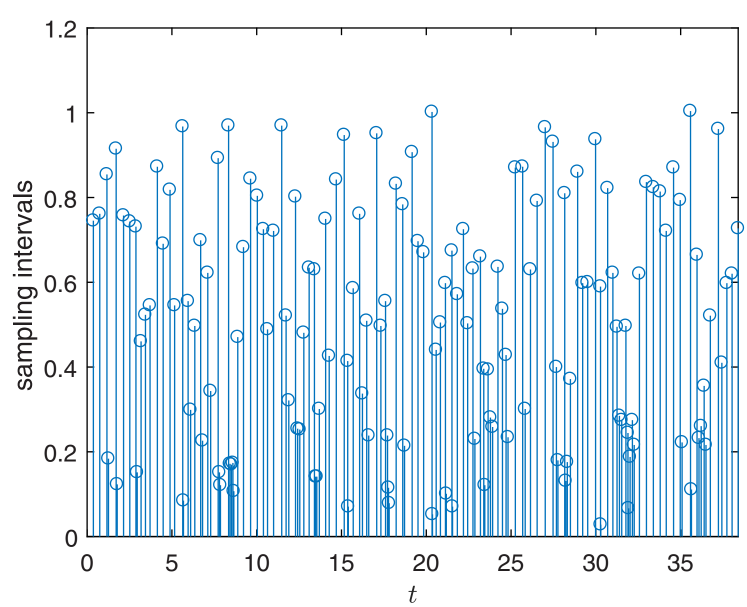

Fig. 3. Aperiodic sampling intervals with upper bound h_b.

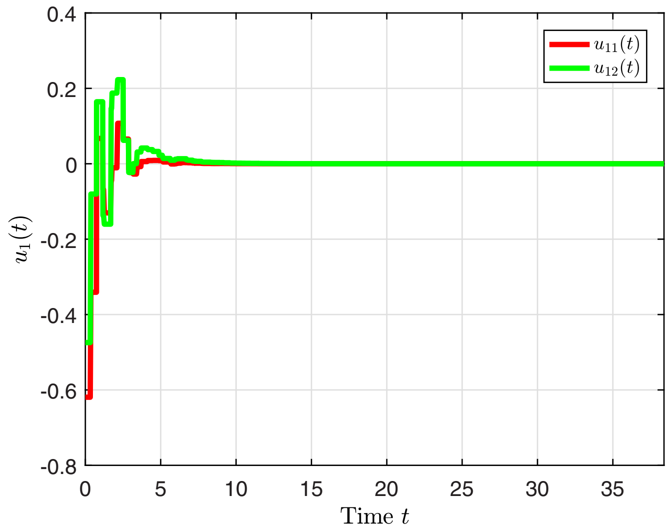

Fig. 4. Sampled-data control signal $u_i(t)$ for system 1.

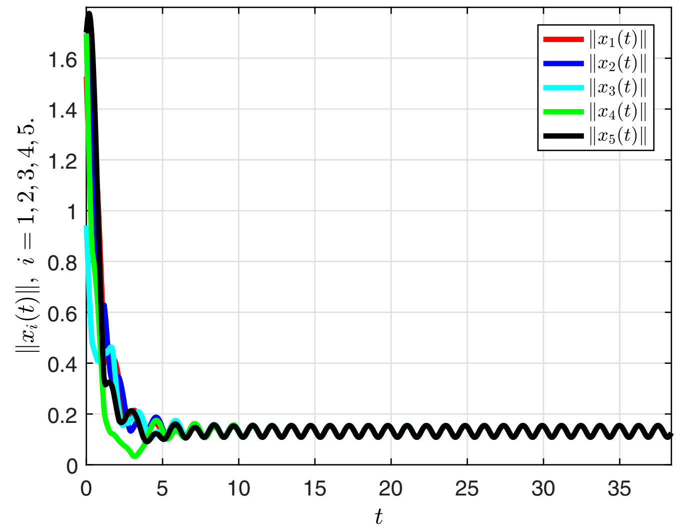

Fig. 6. Norm of the state trajectories $\|x_i(t)\|$ of the network systems.

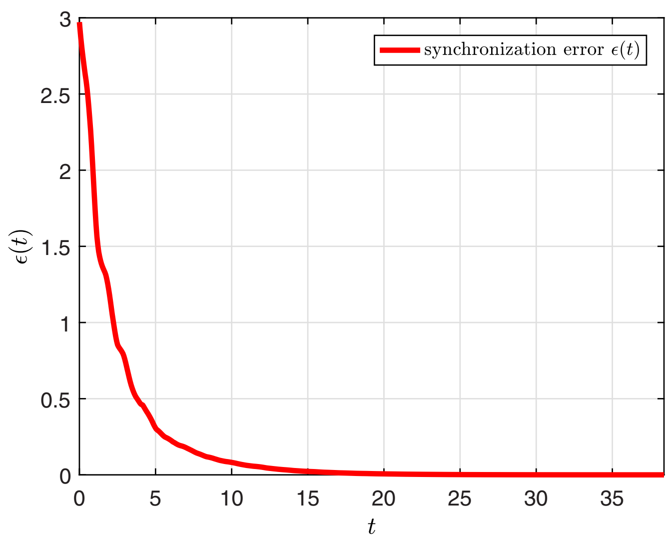

Fig. 5. Synchronization error $\epsilon(t)$ attenuate exponentially to 0.

of the aperiodic sampling interval $t_{k+1} - t_k$ related to the maximum sampling interval h_b. From Fig. 3, we can observe that the sampling controller needs less data transmission. Therefore, communication burden and energy consumption can be reduced by this way intuitively.

In Fig. 4, the sampling control signals $u_i(t)$ for (2) are given. From (5), it can be found that $u_i(t), t \in [t_k, t_{k+1})$ use the information of the states $x_i(t_k - \eta(t_k))$ with time-varying delays $\eta(t_k)$. Furthermore, the signals $u_i(t), t \in [t_k, t_{k+1})$ keep a constant during the sampling intervals $[t_k, t_{k+1})$ due to the ZOH.

For the purpose of demonstrating the availability of our designed algorithm further, the synchronization error of each system is defined as $\epsilon(t) = \sqrt{\sum_{j=2}^{5} \|x_j(t) - x_1(t)\|}$. The synchronization error $\epsilon(t)$ is illustrated in Fig. 5 with a fast convergence rate which can be seen that the synchronization of network systems is achieved within a short time.

The responses of the norm of the state trajectories $\|x_i(t)\|$ of the network systems are presented in Fig. 6, from which we can observe that the network systems achieve the synchronization with the sampled-data controller which suffers time-varying delays.

5. Conclusions

In the paper, utilizing aperiodic sampled data control approach, the synchronization issue of network systems suffering time variant delays has been analyzed. Adopting input delay method, the original sampled data systems have been rebuilt as continuous systems with novel time variant delay terms in the control signals. So as to derive the synchronization conditions of network systems, the Bessel–Legendre inequality has been adopted. The characteristics of this inequality have been adequately adapted to the establishment of extended Lyapunov functionals. Two novel sufficient conditions for synchronizability of network systems and the corresponding controller design methods have been presented. By contrast, it has been identified that the latter sufficient condition brings better (or at least the same) outcomes than the former. At last, a numerical simulation has been shown to verify the availability and advantage of the designed approach.

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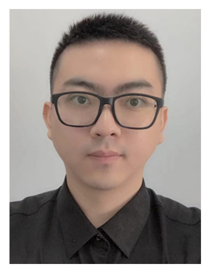

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