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Robust guaranteed cost control of discrete-time networked control systems

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SUMMARY

This paper is concerned with the robust guaranteed cost control problem for networked control systems (NCSs). The plant considered is an uncertain linear discrete-time system, where the communication limitations include packet-loss and signal transmission delay. Our purpose is to design a robust state-feedback guaranteed cost controller such that the resulting closed-loop system is robustly stable, and a specified quadratic cost function is upper bound for all admissible uncertainties under such communication limitations. A model of NCSs is established which contains two additive delay components, one being a known constant, and the other unknown constant. By introducing a novel Lyapunov-Krasoviskii function with the idea of delay partitioning, new sufficient conditions for the existence of guaranteed cost controllers are proposed. Numerical examples are provided to demonstrate the usefulness of the developed theory.
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KEY WORDS: delay; guaranteed cost control; networked control systems; packet-loss

1. INTRODUCTION

Networked control systems(NCSs) are completely distributed and so are networked real-time feedback control systems whose sensors, actuators, estimator units, and control units are interconnected through communication networks [1]. Compared with conventional point-to-point interconnected control systems, NCSs have many advantages, such as simple and fast implementation (reduced system wiring and powerful configuration tools), ease of system diagnosis and maintenance, and increased system agility [2]. NCSs have many industrial applications in automobiles, manufacturing plants, aircrafts, and HVAC systems. Consequently, considerable attention has been paid to the studying of NCSs recently, and many useful results are reported in the literature,

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see, for example, [1, 3–7] and references therein. Among these recent results, the stability problem for NCSs has been investigated in [1, 8], the stabilization and control problems have been studied in [4, 7, 9], the $\mathscr{H}_\infty$ performance-based control problem has been considered in [6, 10].

Unlike conventional control systems, in an NCS, the insertion of the communication channels creates discrepancies between the data records to be transmitted and the data records transmitted, and hence, raises new interesting and challenging problems, such as quantization, signal transmission delay, and packet losses. The first issue is the signal transmission delay (sensor-to-controller delay and controller-to-actuator delay), which is usually caused by the limited bit rate of the communication channel, by a node waiting to send out a packet via a busy channel, or by signal processing and propagation. The network-induced delay may degrade the performance of the NCSs and even result in instability. There have been many methodologies available to deal with the signal transmission delay, for example, [7, 9]. The other issue raised in NCSs is the packet losses, which are caused by the limited bandwidth and the large amount of data packets transmitted over a single channel [11]. The packet losses are usually unavoidable though many networked systems employ Automatic Repeat reQuest mechanisms. So far, there have been many results reported that deal with the issue of packet losses. For example, Zhang *et al.* [1] proposed a criterion to check whether the NCS is stable at a certain rate of packet losses, and searched for the maximum packet-loss rate under which the overall system remains stable. Gao *et al.* [4] considered the packet loss problem and modelled the NCS as a new system with two additive time-varying delay components.

In the past few years, there have been growing theoretical interests in the fields of stability analysis, control, filtering, and model reduction of time-delay systems, see, for example, [12–19] and references therein. Although there have been numerous results on robust control of uncertain delay systems, much effort has been made toward finding a controller that guarantees robust stability only. However, when controlling a real plant, it is often desirable to construct a controller that guarantees not only robust stability but also an adequate level of the performance. One approach to this problem is the so-called guaranteed cost control approach [20, 21]. The guaranteed cost control of an uncertain system aims at designing a robust controller to stabilize the uncertain system and to guarantee a specified level of the performance index of a closed-loop system for all possible uncertainties. Based on this idea, many results have been presented, for example, for continuous-time systems [21–23], discrete-time systems [23–25]. All the above results are delay-independent, which is regarded as more conservative than the delay-dependent ones. Recently, Chen *et al.* [26] considered the delay-dependent approach to solve the guaranteed cost control problem for uncertain discrete-time systems with delay, and Xu *et al.* [19] investigated the same problem for a class of uncertain continuous-time systems with state and input delays, and the obtained results were delay-dependent.

This paper is concerned with the robust guaranteed cost control problem for NCSs. The physical plant considered here is an uncertain linear discrete-time system, the uncertainties are assumed to be norm-bounded and the communication limitations include packet-loss and signal transmission delay, which typically appear in a networked environment. Our purpose is to design a robust state-feedback guaranteed cost controller such that the resulting closed-loop system is robustly stable, and a specified quadratic cost function has an upper bound for all admissible uncertainties under the above communication limitations. A new model of NCS is first established with two additive delay components, one a known constant, and the other unknown. Second, by constructing a new Lyapunov-Krasovskii function combined with the idea of delay-partitioning, a new sufficient criterion for the robust stability of the NCSs is proposed in terms of linear matrix inequality (LMI), and it is shown to be less conservative via an illustrative example. Then, based on the obtained stability condition, the stabilization problem and the guaranteed cost control problem of NCS are both investigated. Sufficient conditions are established for existence of stabilization controller and guaranteed cost controller, respectively. Numerical examples are provided to demonstrate the usefulness of the developed theory.

The remainder of this paper is organized as follows. The robust guaranteed cost control with limited communication capacity is formulated in Section 2. Section 3 presents our main results. Section 4 provides an illustrative example and we conclude the paper in Section 5.

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Notation. The superscript 'T' represents matrix transposition; $\mathbb{Z}_+$ denotes the set of nonnegative integers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $P>0$ means that P is real symmetric and positive definite; I and 0 represent the identity matrix and zero matrix, respectively, and trace(M) denotes the trace of matrix M. In symmetric block matrices or long matrix expressions, we use a star ($\star$) to represent a term that is induced by symmetry. The spectral norm of a matrix is denoted by $\|\cdot\|$ and the maximum eigenvalue of a real symmetric matrix is denoted by $\lambda_{\max}\{\cdot\}$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

The framework of NCSs considered in the paper is depicted in Figure 1. Suppose the physical plant is denoted by an uncertain linear discrete-time system

$$x(k+1)=(A+\Delta A(k))x(k)+(B+\Delta B(k))u(k), \tag{1}$$

where $k\in\mathbb{Z}_+$ is the time step, $x(k)\in\mathbb{R}^n$ is the system state vector, $u(k)\in\mathbb{R}^l$ is the control input and $x_0\triangleq x(0)$ is the initial state. A and B are two constant matrices of appropriate dimensions. $\Delta A(k)$ and $\Delta B(k)$ are time-varying matrices representing system parameter uncertainties, which are assumed to be norm-bounded and can be described by

$$[\Delta A(k) \;\; \Delta B(k)]\triangleq GF(k)[H_1 \;\; H_2], \tag{2}$$

where G, H_1 and H_2 are constant matrices of appropriate dimensions, $F(k)$ is an unknown matrix, which is Lebesque measurable and satisfies $F^{\mathrm{T}}(k)F(k)\leqslant I$. There are networks that exist between the sampler and the controller, and between the controller and the zero-order holder (ZOH). The sampler is assumed to be clock driven, the controller and ZOH are event driven and the data are transmitted in a single packet at each time step. The networked controller is a state-feedback controller given by

$$u(k)=Kx(k), \tag{3}$$

where $K\in\mathbb{R}^{m\times n}$ is state-feedback control gain matrix to be designed later.

From Figure 1, we can see that there are two communication channels, that is, one is between the sampler and the controller, and the other is between the controller and the ZOH. As discussed above, the presence of the network may often lead to signal transmitting delay and data dropout which can degrade the performance of the closed-loop system. To model these two features, we denote the updating instants of the ZOH (successfully transmitted signal from the sampler to the

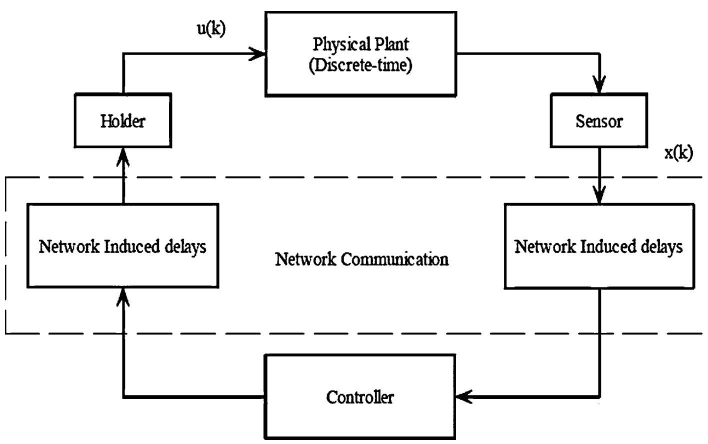

Figure 1. Networked control systems with network-induced delays and packet losses.

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ZOH) as t_k, $k\in\mathbb{Z}_+$, and let $\mathscr{P}\triangleq\{t_1,t_2,\ldots\}$ be a subsequence of $\{1,2,\ldots\}$. Let $d(k)$ represent the experienced time-delay of the sampled data of the plant output received by the actuator at time instant t_k. It is supposed that the signal transmission delay $d(k)$ satisfies

$$d_{\mathrm{m}}\leqslant d(k)\leqslant d_{\mathrm{M}}, \tag{4}$$

where d_{m} and d_{M} denote the minimum and maximum delays, respectively.

From the viewpoint of the zero-order hold, the control input is

$$u(k)=Kx(t_k-d(k)),\quad t_k\leqslant k\leqslant t_{k+1}-1 \tag{5}$$

with t_{k+1} being the next updating instant of the actuator after t_k, and the initial condition of control input is set to be zero, that is, $u(l)=0$, $0\leqslant l\leqslant t_1-1$.

On the other hand, suppose the number of accumulated data packet dropouts at the updating instant t_k since the last updating instant t_{k-1} is $\eta(k)$. Moreover, let $\eta\triangleq\max_{t_k\in\mathscr{P}}\{\eta(k)\}$ be the maximum packet-loss upper bound, that is, $0\leqslant\eta(k)\leqslant\eta$. Then, based on the above analysis and (4) it can be seen that

$$1+d_{\mathrm{m}}\leqslant t_{k+1}-t_k\leqslant\eta+1+d_{\mathrm{M}}, \tag{6}$$

which implies that the interval between any two successive updating instants is upper bound by $(\eta+1+d_{\mathrm{M}})$ and lower bound by $(1+d_{\mathrm{m}})$. Therefore, the closed-loop system can be described by

$$x(k+1)=(A+\Delta A(k))x(k)+(B+\Delta B(k))Kx(t_k-d(k)), \tag{7}$$

where $t_k\leqslant k\leqslant t_{k+1}-1$, $t_k\in\mathscr{P}$. Let us represent $t_k-d(k)$ as

$$t_k-d(k)=k-d_{\mathrm{m}}-\tau(k), \tag{8}$$

where $\tau(k)=k-t_k+(d(k)-d_{\mathrm{m}})$, and we can see from (6) that

$$0\leqslant\tau(k)\leqslant\bar{d}, \tag{9}$$

where $\bar{d}\triangleq 2d_{\mathrm{M}}-d_{\mathrm{m}}+\eta$. Therefore, the closed-loop system can be further described by

$$x(k+1)=(A+\Delta A(k))x(k)+(B+\Delta B(k))Kx(k-d_{\mathrm{m}}-\tau(k)), \tag{10}$$

which has two delays in the system state, that is, one is d_{m} which is constant, the other is $\tau(k)$ which is time-varying with upper bound in (9). Associated with the system in (1), we define the following quadratic cost function:

$$\mathscr{J}\triangleq\sum_{k=0}^{\infty}\{x^{\mathrm{T}}(k)\mathbb{Q}x(k)+u^{\mathrm{T}}(k)\mathbb{R}u(k)\}, \tag{11}$$

where $\mathbb{Q}$ and $\mathbb{R}$ are given positive-definite symmetric matrices.

Definition 1

Consider uncertain system (1) with the communication limitations (that is, signal transmission delay and packet-losses), if there exist a networked controller $u^*(k)$ and a positive scalar $\mathscr{J}^*$ such that for all admissible uncertainties, the closed-loop system (10) is robustly stable and the cost function (11) satisfies $\mathscr{J}\leqslant\mathscr{J}^*$, then $\mathscr{J}^*$ is said to be a guaranteed cost and $u^*(k)$ is said to be a networked guaranteed cost controller for the uncertain system in (1).

Therefore, the problem of robust guaranteed cost control with limited communication capacity to be addressed in this paper can be expressed as follows.

Problem RGCCLCC (Robust Guaranteed Cost Control with Limited Communication Capacity): Consider the uncertain discrete-time system in (1), design a memoryless state feedback networked guaranteed cost controller $u(k)=Kx(k)$ such that the closed-loop system is robustly stable and a specified quadratic cost function has an upper bound for all admissible uncertainties under the above communication limitations.

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Before proceeding further, we give the following lemma that will be used to deal with the uncertainties.

Lemma 1 (*Wang et al.* [27])
Let Σ_1, Σ_2 and Σ_3 be real matrices of appropriate dimensions, with $\Sigma_1 = \Sigma_1^{\mathrm{T}}$, then

$$\Sigma_1 + \Sigma_2 \Delta(k) \Sigma_3 + \Sigma_3^{\mathrm{T}} \Delta^{\mathrm{T}}(k) \Sigma_2^{\mathrm{T}} < 0 \tag{12}$$

holds for all $\Delta(k)$ satisfying $\Delta^{\mathrm{T}}(k)\Delta(k) \leqslant I$ if and only if for some $\varepsilon > 0$,

$$\Sigma_1 + \varepsilon^{-1} \Sigma_2 \Sigma_2^{\mathrm{T}} + \varepsilon \Sigma_3^{\mathrm{T}} \Sigma_3 < 0. \tag{13}$$

3. STABILIZATION PROBLEM

Before proceeding, we first establish a new stability condition for the following nominal linear discrete-time system with two additive time-delays (one is a constant and the other is unknown but bound by (9)).

$$x(k+1) = Ax(k) + A_d x(k - d_{\mathrm{m}} - \tau(k)), \tag{14}$$

where $x(k) \in \mathbb{R}^n$ is the system state vector; A and A_d are two constant matrices of appropriate dimensions; $x(\theta) = \phi(\theta)$, $\theta \in [-d_{\mathrm{m}} - \bar{d},\ 0]$ denotes the initial condition.

Remark 1
Notice that there are two additive delays in system (14), the conventional method to deal with this time-delay system is to combine d_{m} and $\tau(k)$ into one delay $h(k)$, that is, $h(k) = d_{\mathrm{m}} + \tau(k)$, thus system (14) becomes

$$x(k+1) = Ax(k) + A_d x(k - h(k)). \tag{15}$$

So far, there have been many results reported for system (15), for example, delay-independent approach [28, 29], delay-dependent approach [29, 30]. However, these approaches do not make full use of the information about d_{m} and $\tau(k)$, it would be inevitably conservative for some situations; see [31] for the details. In this paper, we will consider the two additive delays d_{m} and $\tau(k)$ independently to fully utilize the information about time-delay, and a new stability criterion will be presented.

In the following, we will use the delay partitioning approach [32] to derive the stability condition of system (14). We assume that the constant part d_{m} can always be described by $d_{\mathrm{m}} = \tau m$, where τ and m are both integers. We define the following vector:

$$\Upsilon(i) \triangleq [x^{\mathrm{T}}(i)\ \ x^{\mathrm{T}}(i-\tau)\ \ x^{\mathrm{T}}(i-2\tau)\ \ \cdots\ \ x^{\mathrm{T}}(i - \tau m + \tau)]^{\mathrm{T}}.$$

Theorem 1
Given positive integers τ and m, the time-delay system in (14) is asymptotically stable if there exist real matrices $P > 0$, $R > 0$, $Q_i > 0$, $S_i > 0$, $i = 1, 2$, M_j, $j = 1, 2, \ldots, m$, $M > 0$, $N > 0$, X, Y and Z satisfying

$$\begin{bmatrix} \Phi_1 + \Phi_2 + \Phi_3 + \Psi + \Psi^{\mathrm{T}} & \Xi \\ \star & -\mathrm{diag}\{P, S_1, S_2\} \end{bmatrix} < 0, \tag{16}$$

$$\Pi_1 \triangleq \begin{bmatrix} M & X \\ \star & S_1 \end{bmatrix} \geqslant 0, \quad \Pi_2 \triangleq \begin{bmatrix} N & Y \\ \star & S_2 \end{bmatrix} \geqslant 0, \tag{17}$$

$$\Pi_3 \triangleq \begin{bmatrix} N & Z \\ \star & S_2 \end{bmatrix} \geqslant 0, \tag{18}$$

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where

$$\Phi_1 \triangleq -\Xi_2^{\mathrm{T}} P \Xi_2 + (\bar{d}+1)\Xi_2^{\mathrm{T}} R \Xi_2 + \Xi_2^{\mathrm{T}} Q_2 \Xi_2,$$

$$\Phi_2 \triangleq W_{Q_1}^{\mathrm{T}} \bar{Q}_1 W_{Q_1} - W_R^{\mathrm{T}} R W_R - W_{Q_2}^{\mathrm{T}} Q_2 W_{Q_2} + \tau M + \bar{d} N,$$

$$\Phi_3 \triangleq \mathrm{diag}\{M_1, M_2 - M_1, \ldots, M_m - M_{m-1}, -M_m, 0_{n\times n}, 0_{n\times n}\},$$

$$\Psi \triangleq [X \quad Y \quad Z] \begin{bmatrix} I_n & -I_n & 0_{n\times(m+1)n} & & \\ 0_{n\times mn} & I_n & -I_n & 0_n \\ 0_{n\times(m+1)n} & I_n & -I_n & \end{bmatrix},$$

$$W_R \triangleq \begin{bmatrix} 0_{n\times(m+1)n} & I_n & 0_n \end{bmatrix}, \quad W_{Q_2} \triangleq [0_{n\times(m+2)n} \quad I_n],$$

$$\bar{Q}_1 \triangleq \begin{bmatrix} Q_1 & 0 \\ 0 & -Q_1 \end{bmatrix}, \quad W_{Q_1} \triangleq \begin{bmatrix} I_{mn} & 0_{mn\times 3n} & \\ 0_{mn\times n} & I_{mn} & 0_{mn\times 2n} \end{bmatrix},$$

$$\Xi \triangleq \begin{bmatrix} \Xi_1^{\mathrm{T}} P & \sqrt{\tau}\Xi_3^{\mathrm{T}} S_1 & \sqrt{\bar{d}}\Xi_3^{\mathrm{T}} S_2 \end{bmatrix}, \quad \Xi_3 \triangleq \Xi_1 - \Xi_2,$$

$$\Xi_1 \triangleq [A \quad 0_{n\times mn} \quad A_d \quad 0_n], \quad \Xi_2 \triangleq [I_n \quad 0_{n\times(m+2)n}].$$

Proof

Define the following Lyapunov-Krasovskii function:

$$V(k) \triangleq V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k) \tag{19}$$

with

$$V_1(k) \triangleq x^{\mathrm{T}}(k) P x(k),$$

$$V_2(k) \triangleq \sum_{i=k-\tau}^{k-1} \Upsilon^{\mathrm{T}}(i) Q_1 \Upsilon(i) + \sum_{i=k-\tau m-\bar{d}}^{k-1} x^{\mathrm{T}}(i) Q_2 x(i),$$

$$V_3(k) \triangleq \sum_{i=-\tau}^{-1} \sum_{j=k+i}^{k-1} \delta^{\mathrm{T}}(j) S_1 \delta(j) + \sum_{i=-\tau m-\bar{d}}^{-\tau m-1} \sum_{j=k+i}^{k-1} \delta^{\mathrm{T}}(j) S_2 \delta(j),$$

$$V_4(k) \triangleq \sum_{i=-\tau m-\bar{d}+1}^{-\tau m+1} \sum_{j=k-1+i}^{k-1} x^{\mathrm{T}}(j) R x(j),$$

$$V_5(k) \triangleq \sum_{i=1}^{m} \sum_{j=k-i\tau}^{k-(i-1)\tau-1} x^{\mathrm{T}}(j) M_i x(j),$$

$$\delta(j) \triangleq x(j+1) - x(j),$$

where $P>0$, $R>0$, $Q_i>0$, $S_i>0$ $(i=1,2)$ and M_j $(j=1,2,\ldots,m)$ are the matrices to be determined. Then, along with the solution of system (14), the increment of $V(k)$ is given by

$$\begin{aligned} \Delta V_1(k) &= x^{\mathrm{T}}(k+1) P x(k+1) - x^{\mathrm{T}}(k) P x(k) \\ &= [Ax(k) + A_d x(k-h(k))]^{\mathrm{T}} P [Ax(k) + A_d x(k-h(k))] - x^{\mathrm{T}}(k) P x(k), \\ \Delta V_2(k) &= \Upsilon^{\mathrm{T}}(k) Q_1 \Upsilon(k) - \Upsilon^{\mathrm{T}}(k-\tau) Q_1 \Upsilon(k-\tau) + x^{\mathrm{T}}(k) Q_2 x(k) \\ &\quad - x^{\mathrm{T}}(k-\tau m-\bar{d}) Q_2 x(k-\tau m-\bar{d}), \end{aligned}$$

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$$\Delta V_3(k) = \tau\delta^{\mathrm{T}}(k)S_1\delta(k)+\bar{d}\delta^{\mathrm{T}}(k)S_2\delta(k)-\sum_{j=k-\tau}^{k-1}\delta^{\mathrm{T}}(j)S_1\delta(j)$$
$$-\sum_{j=k-h(k)}^{k-\tau m-1}\delta^{\mathrm{T}}(j)S_2\delta(j)-\sum_{j=k-\tau m-\bar{d}}^{k-h(k)-1}\delta^{\mathrm{T}}(j)S_2\delta(j), \tag{20}$$

$$\begin{aligned}\Delta V_4(k) &= (\bar{d}+1)x^{\mathrm{T}}(k)Rx(k)-\sum_{i=k-\tau m-\bar{d}}^{k-\tau m}x^{\mathrm{T}}(i)Rx(i)\\ &\leqslant (\bar{d}+1)x^{\mathrm{T}}(k)Rx(k)-x^{\mathrm{T}}(k-h(k))Rx(k-h(k))\\ &= (\bar{d}+1)\zeta^{\mathrm{T}}(k)\Xi_2^{\mathrm{T}}R\Xi_2\zeta(k)-x^{\mathrm{T}}(k-h(k))Rx(k-h(k)),\end{aligned}$$

$$\begin{aligned}\Delta V_5(k) &= \sum_{i=1}^{m}x^{\mathrm{T}}(k-(i-1)\tau)M_i x(k-(i-1)\tau)-\sum_{i=1}^{m}x^{\mathrm{T}}(k-i\tau)M_i x(k-i\tau)\\ &= \zeta^{\mathrm{T}}(k)\Phi_3\zeta(k),\end{aligned}$$

where $\zeta(k)\triangleq[\Upsilon^{\mathrm{T}}(k) \quad x^{\mathrm{T}}(k-\tau m) \quad x^{\mathrm{T}}(k-h(k)) \quad x^{\mathrm{T}}(k-\tau m-\bar{d})]^{\mathrm{T}}$.

By using the technique in [18], for any matrices X, Y and Z the following equations always hold:

$$2\zeta^{\mathrm{T}}(k)X\left[x(k)-x(k-\tau)-\sum_{j=k-\tau}^{k-1}\delta(j)\right]=0,$$

$$2\zeta^{\mathrm{T}}(k)Y\left[x(k-\tau m)-x(k-h(k))-\sum_{j=k-h(k)}^{k-\tau m-1}\delta(j)\right]=0,$$

$$2\zeta^{\mathrm{T}}(k)Z\left[x(k-h(k))-x(k-d_{\mathrm{m}}-\bar{d})-\sum_{j=k-\tau m-\bar{d}}^{k-h(k)}\delta(j)\right]=0.$$

On the other hand, for any appropriately dimensioned matrices $M>0$ and $N>0$, the following identities hold:

$$\begin{aligned}0 &= \tau\zeta^{\mathrm{T}}(k)M\zeta(k)-\sum_{j=k-\tau}^{k-1}\zeta^{\mathrm{T}}(k)M\zeta(k),\\ 0 &= \bar{d}\zeta^{\mathrm{T}}(k)N\zeta(k)-\sum_{j=k-h(k)}^{k-\tau m-1}\zeta^{\mathrm{T}}(k)N\zeta(k)-\sum_{j=k-\tau m-\bar{d}}^{k-h(k)-1}\zeta^{\mathrm{T}}(k)N\zeta(k).\end{aligned} \tag{21}$$

Then, from (20) to (21) we have

$$\begin{aligned}\Delta V(k) &= \Delta V_1(k)+\Delta V_2(k)+\Delta V_3(k)+\Delta V_3(k)\\ &\leqslant \zeta^{\mathrm{T}}(k)(\Psi+\Psi^{\mathrm{T}}+\Xi_1^{\mathrm{T}}P\Xi_1)\zeta(k)+\zeta^{\mathrm{T}}(k)(\Phi_1+\Phi_2+\Phi_3)\zeta(k)+\zeta^{\mathrm{T}}(k)\Xi_3^{\mathrm{T}}(\tau S_1+\bar{d}S_2)\Xi_3\zeta(k)\\ &\quad -\sum_{j=k-\tau}^{k-1}\xi^{\mathrm{T}}(k,j)\Pi_1\xi(k,j)-\sum_{j=k-h(k)}^{k-\tau m-1}\xi^{\mathrm{T}}(k,j)\Pi_2\xi(k,j)-\sum_{j=k-\tau m-\bar{d}}^{k-h(k)-1}\xi^{\mathrm{T}}(k,j)\Pi_3\xi(k,j),\end{aligned}$$

where $\xi(k,j)\triangleq[\zeta^{\mathrm{T}}(k) \quad \delta^{\mathrm{T}}(j)]^{\mathrm{T}}$. By Schur complement, (16) implies $\Phi_1+\Phi_2+\Phi_3+\Psi+\Psi^{\mathrm{T}}+\Xi_1^{\mathrm{T}}P\Xi_1+\Xi_3^{\mathrm{T}}\tau S_1\Xi_3+\bar{d}\Xi_3^{\mathrm{T}}S_2\Xi_3<0$, and together with $\Pi_i\geqslant 0$ $(i=1,2,3)$, we have $\Delta V<-\varepsilon\|\zeta\|^2$ with $\varepsilon>0$. Then by Lyapunov's stability theory, we can see that system (1) is asymptotically stable. □

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Based on Theorem 1, we can easily solve the networked stabilization problem for uncertain system (1). The following theorem gives the main result of the networked stabilization problem.

Theorem 2
Consider uncertain linear discrete-time system (1), there exists a state-feedback controller (3), such that the NCS (10) is robustly stable if there exist matrices $\mathscr{P}>0$, $\mathscr{R}>0$, $\mathscr{M}>0$, $\mathscr{N}>0$, $\mathscr{Q}_i>0$, $\mathscr{S}_i>0$, $i=1,2$, $\mathscr{M}_j>0$, $j=1,2,\ldots,m$, $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$, $\mathscr{K}$ and a scalar $\varepsilon>0$ satisfying the following LMIs:

$$\begin{bmatrix} \breve{\Pi}_{11} & \breve{\Pi}_{12} & \breve{\Pi}_{13} & \breve{\Pi}_{14} & \Sigma^{\mathrm{T}} \\ \star & \breve{\Pi}_{22} & \sqrt{\tau}\varepsilon GG^{\mathrm{T}} & \sqrt{\bar{d}}\varepsilon GG^{\mathrm{T}} & 0 \\ \star & \star & \breve{\Pi}_{33} & \sqrt{\tau\bar{d}}\varepsilon GG^{\mathrm{T}} & 0 \\ \star & \star & \star & \breve{\Pi}_{44} & 0 \\ \star & \star & \star & \star & -\varepsilon I \end{bmatrix}<0, \tag{22}$$

$$\Pi_4 \triangleq \begin{bmatrix} \mathscr{M} & \mathscr{X} \\ \star & 2\mathscr{P}-\mathscr{S}_1 \end{bmatrix} \geqslant 0, \quad \Pi_5 \triangleq \begin{bmatrix} \mathscr{N} & \mathscr{Y} \\ \star & 2\mathscr{P}-\mathscr{S}_2 \end{bmatrix} \geqslant 0, \tag{23}$$

$$\Pi_6 \triangleq \begin{bmatrix} \mathscr{N} & \mathscr{Z} \\ \star & 2\mathscr{P}-\mathscr{S}_2 \end{bmatrix} \geqslant 0, \tag{24}$$

where

$$\breve{\Pi}_{12} \triangleq \begin{bmatrix} \mathscr{P}A^{\mathrm{T}} \\ 0_{mn\times n} \\ \mathscr{K}^{\mathrm{T}}B^{\mathrm{T}} \\ 0_{n\times n} \end{bmatrix}, \quad \breve{\Pi}_{13} \triangleq \sqrt{\tau} \begin{bmatrix} \mathscr{P}(A-I)^{\mathrm{T}} \\ 0_{mn\times n} \\ \mathscr{K}^{\mathrm{T}}B^{\mathrm{T}} \\ 0_{n\times n} \end{bmatrix}, \quad \breve{\Pi}_{14} \triangleq \sqrt{\bar{d}} \begin{bmatrix} \mathscr{P}(A-I)^{\mathrm{T}} \\ 0_{mn\times n} \\ \mathscr{K}^{\mathrm{T}}B^{\mathrm{T}} \\ 0_{n\times n} \end{bmatrix},$$

$$\breve{\Pi}_{22} \triangleq -\mathscr{P}+\varepsilon GG^{\mathrm{T}}, \quad \breve{\Pi}_{33} \triangleq -\mathscr{S}_1+\tau\varepsilon GG^{\mathrm{T}}, \quad \breve{\Pi}_{44} \triangleq -\mathscr{S}_2+\bar{d}\varepsilon GG^{\mathrm{T}},$$

$$\breve{\Pi}_{11} \triangleq T_1+T_2+T_2^{\mathrm{T}}+T_3+T_4+T_5+\tau\mathscr{M}+\bar{d}\mathscr{N}, \quad \Sigma \triangleq \begin{bmatrix} H_1\mathscr{P} & 0_{n\times mn} & H_2\mathscr{K} & 0_{n\times n} \end{bmatrix},$$

$$T_4 \triangleq \begin{bmatrix} & 0_{(m+1)n\times(m+3)n} & \\ 0_{n\times(m+1)n} & -\mathscr{R} & 0_{n\times n} \\ & 0_{n\times(m+3)n} & \end{bmatrix} + \begin{bmatrix} & 0_{(m+2)n\times(m+3)n} \\ 0_{n\times(m+2)n} & -\mathscr{Q}_2 \end{bmatrix},$$

$$T_1 \triangleq \begin{bmatrix} -\mathscr{P}+(\bar{d}+1)\mathscr{R}+\mathscr{Q}_2 & 0_{n\times(m+2)n} \\ & 0_{(m+2)n\times(m+3)n} \end{bmatrix},$$

$$T_2 \triangleq \begin{cases} \begin{bmatrix} \mathscr{X} & -\mathscr{X} & 0_{(m+3)n\times(m-2)n} & \mathscr{Y} & -\mathscr{Y}+\mathscr{Z} & -\mathscr{Z} \end{bmatrix} & (m\geqslant 2) \\ \begin{bmatrix} \mathscr{X} & -\mathscr{X}+\mathscr{Y} & -\mathscr{Y}+\mathscr{Z} & -\mathscr{Z} \end{bmatrix} & (m=1), \end{cases}$$

$$T_3 \triangleq \begin{bmatrix} \mathscr{Q}_1 & 0_{mn\times 3n} \\ & 0_{3n\times(m+3)n} \end{bmatrix} - \begin{bmatrix} & 0_{n\times(m+3)n} & \\ 0_{mn\times n} & \mathscr{Q}_1 & 0_{mn\times 2n} \\ & 0_{2n\times(m+3)n} & \end{bmatrix},$$

$$T_5 \triangleq \mathrm{diag}\{\mathscr{M}_1, \mathscr{M}_2-\mathscr{M}_1, \ldots, \mathscr{M}_m-\mathscr{M}_{m-1}, -\mathscr{M}_m, 0_{n\times n}, 0_{n\times n}\}.$$

In this case, the controller is given by $K=\mathscr{K}\mathscr{P}^{-1}$.

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Proof

After the matrices A and A_d in LMI (16) are replaced by $\tilde{A} \triangleq A + \Delta A(k)$ and $\tilde{B} \triangleq BK + \Delta B(k)K$, respectively, the closed-loop NCS is robustly stable, according to Theorem 1, if there exist real matrices $P>0$, $R>0$, $Q_i>0$, $S_i>0$, $i=1,2$, $M_j>0$, $j=1,2,\ldots,m$, $M>0$, $N>0$, X, Y and Z such that for all admissible uncertainties, (17)–(18) and the following LMI holds:

$$\begin{bmatrix} \Phi_1+\Phi_2+\Phi_3+\Psi+\Psi^{\mathrm{T}} & \tilde{\Xi} \\ \star & -\mathrm{diag}\{P, S_1, S_2\} \end{bmatrix} < 0, \tag{25}$$

where $\tilde{\Xi} \triangleq [\tilde{\Xi}_1^{\mathrm{T}} P \quad \sqrt{\tau}\tilde{\Xi}_3^{\mathrm{T}} S_1 \quad \sqrt{\bar{d}}\tilde{\Xi}_3^{\mathrm{T}} S_2]$ with $\tilde{\Xi}_1 \triangleq [\tilde{A} \quad 0_{n\times mn} \quad \tilde{B} \quad 0_n]$, $\tilde{\Xi}_3 \triangleq \tilde{\Xi}_1 - \Xi_2$, and Ξ_2 defined in Theorem 1. Then, LMI (25) can be further rewritten as

$$\Sigma_1 + \Sigma_2 F(k)\Sigma_3 + \Sigma_3^{\mathrm{T}} F^{\mathrm{T}}(k)\Sigma_2^{\mathrm{T}} < 0, \tag{26}$$

where

$$\Sigma_1 \triangleq \begin{bmatrix} \Phi_1+\Phi_2+\Phi_3+\Psi+\Psi^{\mathrm{T}} & \bar{\Xi} \\ \star & -\mathrm{diag}\{P, S_1, S_2\} \end{bmatrix},$$

$$\Sigma_2 \triangleq \begin{bmatrix} 0_{n\times(m+3)n} & G^{\mathrm{T}}P & \sqrt{\tau}G^{\mathrm{T}}S_1 & \sqrt{\bar{d}}G^{\mathrm{T}}S_2 \end{bmatrix}^{\mathrm{T}},$$

$$\Sigma_3 \triangleq \begin{bmatrix} H_1 & 0_{n\times mn} & H_2K & 0_{n\times 4n} \end{bmatrix},$$

$$\bar{\Xi} \triangleq \begin{bmatrix} \bar{\Xi}_1^{\mathrm{T}}P & \sqrt{\tau}\bar{\Xi}_3^{\mathrm{T}}S_1 & \sqrt{\bar{d}}\bar{\Xi}_3^{\mathrm{T}}S_2 \end{bmatrix},$$

$$\bar{\Xi}_1 \triangleq \begin{bmatrix} A & 0_{n\times mn} & BK & 0_n \end{bmatrix}, \quad \bar{\Xi}_3 \triangleq \bar{\Xi}_1 - \Xi_2.$$

Pre- and post-multiplying (26) by $\mathrm{diag}\left\{P^{-1}, P^{-1}, \ldots, P^{-1}, S_1^{-1}, S_2^{-1}\right\}$ and defining

$$\mathscr{P} \triangleq P^{-1}, \quad \mathscr{K} \triangleq K\mathscr{P}, \quad \mathscr{S}_1 \triangleq S_1^{-1}, \quad \mathscr{S}_2 \triangleq S_2^{-1},$$

$$\mathscr{R} \triangleq \mathscr{P}R\mathscr{P}, \quad \mathscr{Q}_2 \triangleq \mathscr{P}Q_2\mathscr{P},$$

$$\mathscr{Q}_1 \triangleq \mathscr{P}_1 Q_1 \mathscr{P}_1, \quad \mathscr{M} \triangleq \mathscr{P}_2 M \mathscr{P}_2, \quad \mathscr{N} \triangleq \mathscr{P}_2 N \mathscr{P}_2,$$

$$\mathscr{X} \triangleq \mathscr{P}_2 X \mathscr{P}, \quad \mathscr{Y} \triangleq \mathscr{P}_2 Y \mathscr{P}, \quad \mathscr{Z} \triangleq \mathscr{P}_2 Z \mathscr{P},$$

$$\mathscr{M}_1 \triangleq \mathscr{P}M_1\mathscr{P}, \quad \mathscr{M}_2 \triangleq \mathscr{P}M_2\mathscr{P}, \quad \ldots, \quad \mathscr{M}_m \triangleq \mathscr{P}M_m\mathscr{P},$$

$$\mathscr{P}_1 \triangleq \begin{bmatrix} \mathscr{P} & 0 & \cdots & 0 \\ \star & \mathscr{P} & \cdots & 0 \\ \star & \star & \ddots & \vdots \\ \star & \star & \star & \mathscr{P} \end{bmatrix}_{mn\times mn},$$

$$\mathscr{P}_2 \triangleq \begin{bmatrix} \mathscr{P} & 0 & \cdots & 0 \\ \star & \mathscr{P} & \cdots & 0 \\ \star & \star & \ddots & \vdots \\ \star & \star & \star & \mathscr{P} \end{bmatrix}_{(m+3)n\times(m+3)n},$$

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we have

$$\breve{\Pi}+\begin{bmatrix}0_{(m+3)n\times n}\\ G\\ \sqrt{\tau}G\\ \sqrt{\bar{d}}G\end{bmatrix}F(k)\begin{bmatrix}\mathscr{P}H_1^{\mathrm{T}}\\ 0_{mn\times n}\\ \mathscr{K}^{\mathrm{T}}H_2^{\mathrm{T}}\\ 0_{4n\times n}\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix}\mathscr{P}H_1^{\mathrm{T}}\\ 0_{mn\times n}\\ \mathscr{K}^{\mathrm{T}}H_2^{\mathrm{T}}\\ 0_{4n\times n}\end{bmatrix}F^{\mathrm{T}}(k)\begin{bmatrix}0_{(m+3)n\times n}\\ G\\ \sqrt{\tau}G\\ \sqrt{\bar{d}}G\end{bmatrix}^{\mathrm{T}}<0, \tag{27}$$

where

$$\breve{\Pi}\triangleq\begin{bmatrix}\breve{\Pi}_{11} & \breve{\Pi}_{12} & \breve{\Pi}_{13} & \breve{\Pi}_{14}\\ \star & -\mathscr{P} & 0 & 0\\ \star & \star & -\mathscr{S}_1 & 0\\ \star & \star & \star & -\mathscr{S}_2\end{bmatrix},$$

By Lemma 1 and the Schur complement of (27), we know that (22) holds.

Pre- and post-multiplying Π_1, Π_2 and Π_3 in (17) and (18) by

$$\mathrm{diag}\{P^{-1},\ldots,P^{-1},P^{-1}\},$$
$$\mathrm{diag}\{P^{-1},\ldots,P^{-1},P^{-1}\}$$

and

$$\mathrm{diag}\{P^{-1},\ldots,P^{-1},P^{-1}\},$$

respectively, yields

$$\begin{bmatrix}\mathscr{M} & \mathscr{X}\\ \star & \mathscr{P}S_1\mathscr{P}\end{bmatrix}\geqslant 0,\quad \begin{bmatrix}\mathscr{N} & \mathscr{Y}\\ \star & \mathscr{P}S_2\mathscr{P}\end{bmatrix}\geqslant 0,\quad \begin{bmatrix}\mathscr{N} & \mathscr{Z}\\ \star & \mathscr{P}S_2\mathscr{P}\end{bmatrix}\geqslant 0. \tag{28}$$

Notice that (28) is not in LMI form because of the nonlinear terms $\mathscr{P}S_1\mathscr{P}$ and $\mathscr{P}S_2\mathscr{P}$. By noticing $S_1>0$ and $S_2>0$, we have $(\mathscr{S}_1-\mathscr{P})S_1(\mathscr{S}_1-\mathscr{P})\geqslant 0$ and $(\mathscr{S}_2-\mathscr{P})S_2(\mathscr{S}_2-\mathscr{P})\geqslant 0$, which is equivalent to

$$\mathscr{P}S_1\mathscr{P}\geqslant 2\mathscr{P}-\mathscr{S}_1,\quad \mathscr{P}S_2\mathscr{P}\geqslant 2\mathscr{P}-\mathscr{S}_2. \tag{29}$$

Then, by considering (28) and (29), (23) and (24) can be obtained. This completes the proof. □

4. GUARANTEED COST CONTROL

In this section, we will consider the guaranteed cost control problem for NCSs based on the results obtained in the previous section.

Theorem 3

Given positive integers τ, m, η and d_{M}, the state-feedback controller in (3) is a networked guaranteed cost controller for the uncertain system in (1), if there exist matrices $P>0$, $Q_i>0$, $S_i>0$, $i=1,2$, $M_j>0$, $j=1,2,\ldots,m$, $R>0$, $M>0$, $N>0$, X, Y and Z such that for all admissible uncertainties both the LMIs in (17), (18) and the following LMI hold:

$$\begin{bmatrix}\Phi_1+\Phi_2+\Phi_3+\Psi+\Psi^{\mathrm{T}}+\Gamma_1+\Gamma_2 & \tilde{\Xi}\\ \star & -\mathrm{diag}\{P,S_1,S_2\}\end{bmatrix}<0, \tag{30}$$

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where $\tilde{\Xi}$ is defined in (25) and

$$\Gamma_1 \triangleq \begin{bmatrix} \mathbb{Q} & 0_{n\times(m+2)n} \\ 0_{(m+2)n\times(m+3)n} & \end{bmatrix}, \quad \Gamma_2 \triangleq W_R^{\mathrm{T}} K^{\mathrm{T}} \mathbb{R} K W_R.$$

Proof
According to Theorem 1 and (25), it can be easily seen that the closed-loop system is robustly stable if condition (30) holds. On the other hand, choose the same Lyapunov-Krasovskii functional as in (19) and along the same line as in the proof of Theorem 1, if (30) holds, then we have

$$\Delta V(k) \leqslant -x^{\mathrm{T}}(k)\mathbb{Q}x(k) - x^{\mathrm{T}}(k-d_{\mathrm{m}}-d(k))K^{\mathrm{T}}\mathbb{R}Kx(k-d_{\mathrm{m}}-d(k)) < 0. \tag{31}$$

Furthermore, by summating both sides of the inequality (31) from 0 to N and using the initial condition, we obtain

$$\sum_{k=0}^{N}(V(k+1)-V(k)) \leqslant -\sum_{k=0}^{N}\{x^{\mathrm{T}}(k)\mathbb{Q}x(k) + x^{\mathrm{T}}(k-d_{\mathrm{m}}-d(k))K^{\mathrm{T}}\mathbb{R}Kx(k-d_{\mathrm{m}}-d(k))\}, \tag{32}$$

which implies

$$\begin{aligned} \mathscr{J} &= \lim_{N\to+\infty}\sum_{k=0}^{N}\{x^{\mathrm{T}}(k)\mathbb{Q}x(k)+u^{\mathrm{T}}(k)\mathbb{R}u(k)\} \\ &= \lim_{N\to+\infty}\sum_{k=0}^{N}\{x^{\mathrm{T}}(k)\mathbb{Q}x(k)+x^{\mathrm{T}}(k-d_{\mathrm{m}}-d(k))K^{\mathrm{T}}\mathbb{R}Kx(k-d_{\mathrm{m}}-d(k))\} \\ &\leqslant \lim_{N\to+\infty}\sum_{k=0}^{N}(V(k)-V(k+1)) = V(0) - \lim_{N\to+\infty}V(N+1), \end{aligned} \tag{33}$$

Notice that the closed-loop NCS (10) is asymptotically stable, therefore, when $N\to+\infty$ it follows that $V(N+1)\to 0$. Then, according to (19), we have

$$\begin{aligned} \mathscr{J} &= \sum_{k=0}^{\infty}\left\{x^{\mathrm{T}}(k)\mathbb{Q}x(k)+u^{\mathrm{T}}(k)\mathbb{R}u(k)\right\}. \\ &\leqslant V(0). \\ &= x^{\mathrm{T}}(0)Px(0)+\sum_{i=-\tau}^{-1}\Upsilon^{\mathrm{T}}(i)Q_1\Upsilon(i)+\sum_{i=-\tau m-\bar{d}}^{-1}x^{\mathrm{T}}(i)Q_2x(i)+\sum_{i=-\tau}^{-1}\sum_{j=i}^{-1}\delta^{\mathrm{T}}(j)S_1\delta(j) \\ &\quad+\sum_{i=-\tau m-\bar{d}}^{-\tau m-1}\sum_{j=i}^{-1}\delta^{\mathrm{T}}(j)S_2\delta(j)+\sum_{i=-\tau m-\bar{d}+1}^{-\tau m+1}\sum_{j=-1+i}^{-1}x^{\mathrm{T}}(j)Rx(j)+\sum_{i=1}^{m}\sum_{j=-i\tau}^{-(i-1)\tau-1}x^{\mathrm{T}}(j)M_ix(j), \\ &= x^{\mathrm{T}}(0)\mathscr{P}^{-1}x(0)+\sum_{i=-\tau}^{-1}\Upsilon^{\mathrm{T}}(i)\mathscr{P}_1^{-1}\mathscr{Q}_1\mathscr{P}_1^{-1}\Upsilon(i)+\sum_{i=-\tau m-\bar{d}}^{-1}x^{\mathrm{T}}(i)\mathscr{P}^{-1}\mathscr{Q}_2\mathscr{P}^{-1}x(i) \\ &\quad+\sum_{i=-\tau}^{-1}\sum_{j=i}^{-1}\delta^{\mathrm{T}}(j)\mathscr{S}_1^{-1}\delta(j)+\sum_{i=-\tau m-\bar{d}}^{-\tau m-1}\sum_{j=i}^{-1}\delta^{\mathrm{T}}(j)\mathscr{S}_2^{-1}\delta(j) \\ &\quad+\sum_{i=-\tau m-\bar{d}+1}^{-\tau m+1}\sum_{j=-1+i}^{-1}x^{\mathrm{T}}(j)\mathscr{P}^{-1}\mathscr{R}\mathscr{P}^{-1}x(j)+\sum_{i=1}^{m}\sum_{j=-i\tau}^{-(i-1)\tau-1}x^{\mathrm{T}}(j)\mathscr{P}^{-1}\mathscr{M}_i\mathscr{P}^{-1}x(j). \end{aligned}$$

From Definition 1 the result follows. □

We are in a position to present a solution to design the networked guaranteed cost controller in (3).

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Theorem 4

Consider uncertain linear discrete-time system in (1), there exists a state-feedback networked guaranteed cost controller in (3), if there exist matrices $\mathscr{P}>0$, $\mathscr{R}>0$, $\mathscr{M}>0$, $\mathscr{N}>0$, $\mathscr{Q}_i>0$, $\mathscr{S}_i>0$, $i=1,2$, $\mathscr{M}_j>0$, $j=1,2,\ldots,m$, $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$, $\mathscr{K}$ and a scalar $\varepsilon>0$ satisfying the following LMIs:

$$\begin{bmatrix} \breve{\Pi}_{11} & \breve{\Pi}_{12} & \breve{\Pi}_{13} & \breve{\Pi}_{14} & \Sigma^{\mathrm{T}} & \Omega_1 & \Omega_2 \\ \star & \breve{\Pi}_{22} & \sqrt{\tau}\varepsilon GG^{\mathrm{T}} & \sqrt{\bar{d}}\varepsilon GG^{\mathrm{T}} & 0 & 0 & 0 \\ \star & \star & \breve{\Pi}_{33} & \sqrt{\tau\bar{d}}\varepsilon GG^{\mathrm{T}} & 0 & 0 & 0 \\ \star & \star & \star & \breve{\Pi}_{44} & 0 & 0 & 0 \\ \star & \star & \star & \star & -\varepsilon I & 0 & 0 \\ \star & \star & \star & \star & \star & -\mathbb{Q}^{-1} & 0 \\ \star & \star & \star & \star & \star & \star & -\mathbb{R}^{-1} \end{bmatrix}<0, \tag{34}$$

$$\Pi_4 \triangleq \begin{bmatrix} \mathscr{M} & \mathscr{X} \\ \star & 2\mathscr{P}-\mathscr{S}_1 \end{bmatrix}\geqslant 0, \quad \Pi_5 \triangleq \begin{bmatrix} \mathscr{N} & \mathscr{Y} \\ \star & 2\mathscr{P}-\mathscr{S}_2 \end{bmatrix}\geqslant 0, \tag{35}$$

$$\Pi_6 \triangleq \begin{bmatrix} \mathscr{N} & \mathscr{Z} \\ \star & 2\mathscr{P}-\mathscr{S}_2 \end{bmatrix}\geqslant 0 \tag{36}$$

with

$$\Omega_1 \triangleq \begin{bmatrix} \mathscr{P} \\ 0_{(m+2)n\times n} \end{bmatrix}, \quad \Omega_2 \triangleq \begin{bmatrix} 0_{(m+1)n\times n} \\ \mathscr{K}^{\mathrm{T}} \\ 0_{n\times n} \end{bmatrix}.$$

Moreover, if the above conditions have a feasible solution, then the state-feedback networked guaranteed cost controller is given by $K=\mathscr{K}\mathscr{P}^{-1}$, and the cost function (11) satisfies

$$\begin{aligned} \mathscr{J}\leqslant \mathscr{J}^* \triangleq{}& x^{\mathrm{T}}(0)\mathscr{P}^{-1}x(0)+\sum_{i=-\tau}^{-1}\Upsilon^{\mathrm{T}}(i)\mathscr{P}_1^{-1}\mathscr{Q}_1\mathscr{P}_1^{-1}\Upsilon(i)+\sum_{i=-\tau m-\bar{d}}^{-1}x^{\mathrm{T}}(i)\mathscr{P}^{-1}\mathscr{Q}_2\mathscr{P}^{-1}x(i) \\ &+\sum_{i=-\tau}^{-1}\sum_{j=i}^{-1}\delta^{\mathrm{T}}(j)\mathscr{S}_1^{-1}\delta(j)+\sum_{i=-\tau m-\bar{d}}^{-\tau m-1}\sum_{j=i}^{-1}\delta^{\mathrm{T}}(j)\mathscr{S}_2^{-1}\delta(j) \\ &+\sum_{i=-\tau m-\bar{d}+1}^{-\tau m+1}\sum_{j=-1+i}^{-1}x^{\mathrm{T}}(j)\mathscr{P}^{-1}\mathscr{R}\mathscr{P}^{-1}x(j)+\sum_{i=1}^{m}\sum_{j=-i\tau}^{-(i-1)\tau-1}x^{\mathrm{T}}(j)\mathscr{P}^{-1}\mathscr{M}_i\mathscr{P}^{-1}x(j). \end{aligned} \tag{37}$$

The result can be obtained by employing the same techniques as used in Theorems 2 and 3.

Remark 2

It is worth noting that Theorem 4 gives a set of networked guaranteed cost controllers characterized in terms of the solutions to (34)–(36). Each networked guaranteed cost controller ensures the robust stability of the resulting closed-loop system and an upper bound on the cost function given by (37). However, the bound (37) obtained in Theorem 4 depends on the initial condition of system (1). It is desirable to remove this dependence. If the initial state of system (1) is assumed to be arbitrary but belong to the set $\mathscr{Y}\triangleq\{x(i)\in\mathbb{R}^n : x(i)\triangleq Uv_i,\, v_i^{\mathrm{T}}v_i\leqslant 1,\, i=-\tau m-\bar{d}, -\tau m-\bar{d}+1,\ldots,1,0\}$, where

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U is a given matrix. Then, the cost function (11) satisfies

$$\begin{aligned}\mathscr{J} \leqslant\, & \lambda_{\max}\{U^{\mathrm{T}}\mathscr{P}^{-1}U\}+\tau\lambda_{\max}\{\mathscr{U}^{\mathrm{T}}\mathscr{P}_1^{-1}\mathscr{Q}_1\mathscr{P}_1^{-1}\mathscr{U}\}+(\tau m+\bar{d})\lambda_{\max}\{U^{\mathrm{T}}\mathscr{P}^{-1}\mathscr{Q}_2\mathscr{P}^{-1}U\}\\ &+4\tau^2\lambda_{\max}\{U^{\mathrm{T}}\mathscr{S}_1^{-1}U\}+4\bar{d}(\tau m+\bar{d})\lambda_{\max}\{U^{\mathrm{T}}\mathscr{S}_2^{-1}U\}+(\bar{d}+1)(\tau m+\bar{d})\lambda_{\max}\{U^{\mathrm{T}}\mathscr{P}^{-1}\mathscr{R}\mathscr{P}^{-1}U\}\\ &+m\tau\max_i\lambda_{\max}\{U^{\mathrm{T}}\mathscr{P}^{-1}\mathscr{M}_i\mathscr{P}^{-1}U\}, \qquad (38)\end{aligned}$$

where $\mathscr{U}\triangleq[U^{\mathrm{T}} \ \ \ldots \ \ U^{\mathrm{T}}]^{\mathrm{T}}_{mn\times n}$.

5. NUMERICAL EXAMPLES

In this section, we will use three numerical examples to illustrate the stabilization and the guaranteed cost control problems for the NCS based on the new model, respectively.

Example 1 (*Stability*)
Consider system (14) with

$$A=\begin{bmatrix}0.8 & 0\\ 0.05 & 0.9\end{bmatrix},\quad A_d=\begin{bmatrix}-0.1 & 0\\ -0.2 & -0.1\end{bmatrix},$$

which has also been considered in [12].

For a given lower delay bound d_{m}, we are interested in the upper delay d_{M} for which the above system is asymptotically stable for all $d_{\mathrm{m}}\leqslant d(k)\leqslant d_{\mathrm{M}}$. Table I presents a comparison between the results obtained in [12] and Theorem 1.

It can be seen from Table I that our results are identical to the existing results of [12] when $m=1$. However, when m increases, the conservatism is reduced.

Example 2 (*Stabilization*)
Consider the system (1) with the following system matrices:

$$A=\begin{bmatrix}0.85 & 0 & 0.1\\ 0.01 & 0.96 & 0\\ 0 & 0 & 1.0\end{bmatrix},\quad B=\begin{bmatrix}-0.1 & 0 & 0\\ -0.2 & -0.1 & 0\\ -0.1 & 0.1 & 0.1\end{bmatrix},\quad G=\begin{bmatrix}0.1 & 0.1 & 0\\ 0 & 0.1 & 0.1\\ 0 & 0 & 0.1\end{bmatrix},$$

$$H_1=\begin{bmatrix}0.1 & 0 & 0.1\\ 0.1 & 0.1 & 0.1\\ 0 & 0 & 0.1\end{bmatrix},\quad H_2=\begin{bmatrix}0.1 & 0 & 0.1\\ 0.1 & 0.1 & 0\\ 0 & 0.1 & 0.1\end{bmatrix}$$

It is assumed that the network induced delay bound in (4) are given by $d_{\mathrm{m}}=1$ $(m=1, \tau=1)$ and $d_{\mathrm{M}}=5$, the maximum number of data packet loss $\eta=6$. Then, from (9) we have $\bar{d}=15$. The eigenvalues of A are 0.9600, 0.8500 and 1.0000, thus the above system is not stable. Our purpose is to design a state-feedback controller in (3) over a networked environment with limited

Table I. Allowable upper bound of d_{M} for different values of d_{m}.

d_{m}	10	12	15
[12]	$d_{\mathrm{M}}=19$	$d_{\mathrm{M}}=21$	$d_{\mathrm{M}}=23$
Theorem 1, $m=1$	$d_{\mathrm{M}}=19$	$d_{\mathrm{M}}=21$	$d_{\mathrm{M}}=23$
Theorem 1	$m=2,\ d_{\mathrm{M}}=20$	$m=2,\ d_{\mathrm{M}}=22$	$m=3,\ d_{\mathrm{M}}=24$

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communication capacity such that the resulting closed-loop system is robustly stable. By solving Theorem 2, we have

$$\mathscr{P}=\begin{bmatrix}4.1997 & 0.0564 & 0.8306\\ 0.0564 & 3.6882 & 0.5212\\ 0.8306 & 0.5212 & 1.8503\end{bmatrix},\quad \mathscr{K}=\begin{bmatrix}-0.4765 & 0.1616 & 0.1582\\ 1.1022 & 0.0575 & -0.0395\\ -1.9844 & -0.2425 & -0.5810\end{bmatrix},$$

thus a state-feedback controller matrix is given by

$$K=\begin{bmatrix}-0.1419 & 0.0259 & 0.1419\\ 0.2942 & 0.0341 & -0.1630\\ -0.4525 & -0.0450 & -0.0982\end{bmatrix}.$$

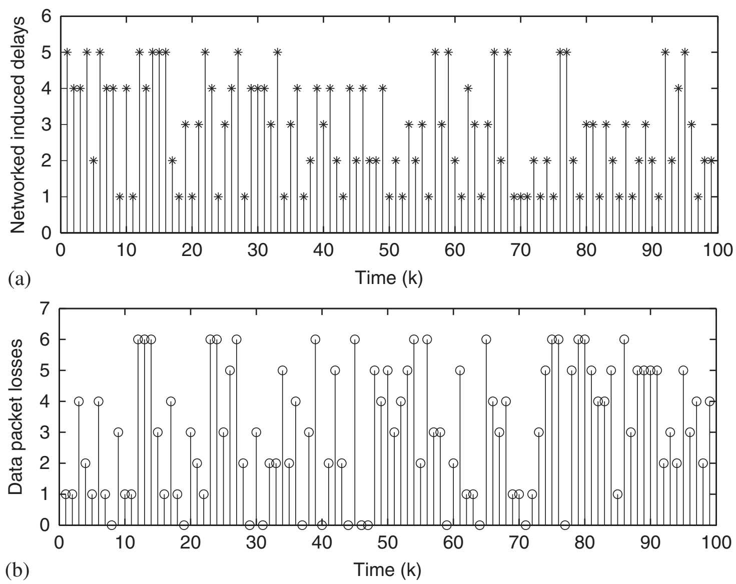

Figure 2. Network-induced delays and data packet losses of Example 2.

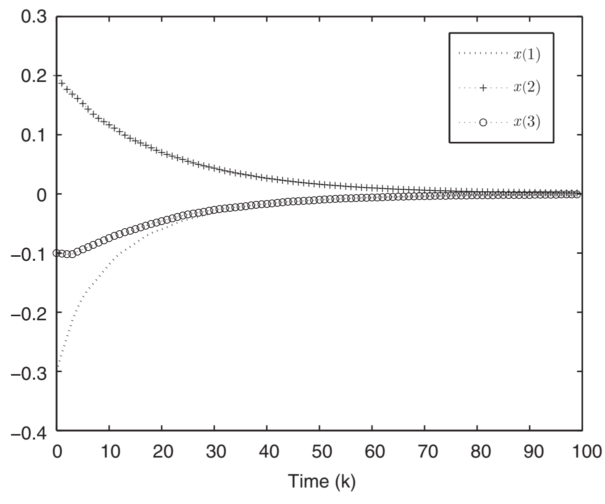

Figure 3. Initial response of the closed-loop system of Example 2.

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To further illustrate the effectiveness of the proposed design scheme above, we will give the state response of the closed-loop system with the above-obtained controller. The initial condition is assumed to be

$$x(0) = [-0.3 \;\; 0.2 \;\; -0.1]^{\mathrm{T}},$$
$$x(-j) = [0 \;\; 0 \;\; 0 \;\; 0]^{\mathrm{T}} \quad j = 1, 2, \ldots, 16.$$

In the simulation, the network induced delays $d(k)$ and the data packet dropouts $\eta(k)$ are generated randomly (evenly distributed within their ranges), and shown in (A) and (B) of Figure 2, respectively. The state variables of the closed-loop system are depicted in Figure 3. The control input is shown in Figure 4.

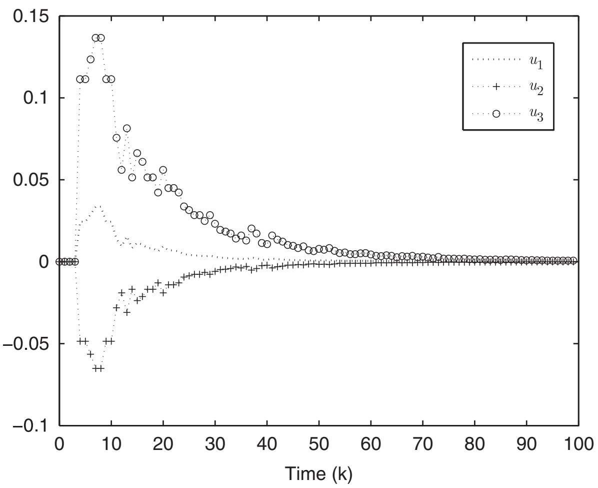

Figure 4. Control input of Example 2.

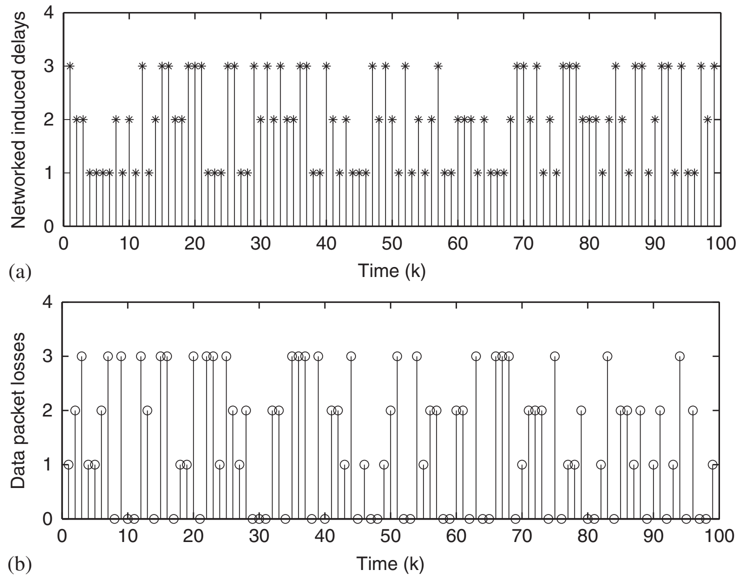

Figure 5. Network-induced delays and data packet losses of Example 3.

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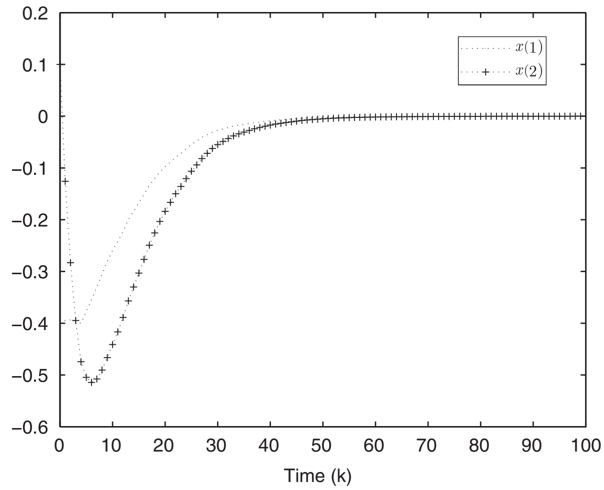

Figure 6. Initial response of the closed-loop system of Example 3.

Example 3 (*guaranteed cost control*)
Consider system (1) with the following system matrices:

$$A=\begin{bmatrix}1.0 & 0.01\\ 0.5 & 0.7\end{bmatrix},\quad B=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},\quad G=\begin{bmatrix}0.1 & 0.1\end{bmatrix},\quad H_1=\begin{bmatrix}-0.1 & 0.1\end{bmatrix}$$

$$H_2=0.1,\quad \mathscr{Q}=\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix},\quad \mathscr{R}=0.01,\quad U=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

and $x(0)=[-0.4 \quad 0.1]^{\mathrm{T}}$. It is assumed that the network-induced delay bound in (4) are given by $d_{\mathrm{m}}=1$ $(m=1, \tau=1)$ and $d_{\mathrm{M}}=3$, the maximum number of data packet loss $\eta=3$. The eigenvalues of A are 1.0158 and 0.6842, thus the original system is unstable. Our purpose in this example is to design a state-feedback guaranteed cost controller in the form of (3) over a networked environment with limited communication capacity, such that the resulting closed-loop system is robustly stable and a specified quadratic cost function has an upper bound for all admissible uncertainties. By solving Theorem 4, it follows that the upper bound of the closed-loop cost function (38) is 350.0510, and

$$\mathscr{P}=\begin{bmatrix}1.3267 & 1.8369\\ 1.8369 & 6.1786\end{bmatrix},\quad \mathscr{K}=\begin{bmatrix}-1.0328 & -1.6736\end{bmatrix},$$

thus a state-feedback controller matrix is given by

$$K=[-0.6857 \quad -0.0670].$$

The network-induced delays $d(k)$ and the data packet dropouts $\eta(k)$ are generated randomly, and shown in (A) and (B) of Figure 5, respectively. The states of the resulting closed-loop system are depicted in Figure 6. The control input is shown in Figure 7.

6. CONCLUSIONS

This paper has investigated the robust guaranteed cost control problem of NCS. First, a model of NCS has been established for NCS with two additive delay components, one constant, and the other also constant, but unknown. Based on a new Lyapunov-Krasovskii functional combined

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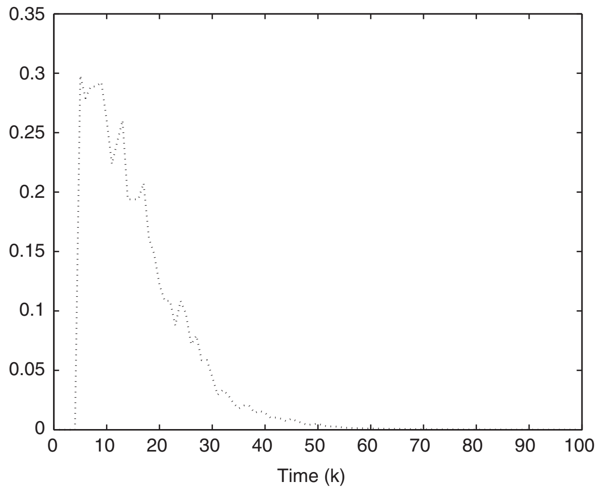

Figure 7. Control input of Example 3.

with the idea of delay-partitioning, an improved sufficient condition for the robust stability of the NCS model has been proposed in terms of LMIs. Then, the stabilization and the guaranteed cost control problems of NCS have been investigated. Sufficient conditions have been established for the existence of a stabilizing controller and a guaranteed cost controller, respectively. Finally, numerical examples have been provided to demonstrate the usefulness of the developed theory.

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REFERENCES

1. Zhang W, Branicky MS, Phillips SM. Stability of networked control systems. *IEEE Control Systems Magazine* 2001; **21**(1):84–99.
2. Yang T. Networked control system: a brief survey. *IEE Proceedings Part D*: *Control Theory and Applications* 2006; **153**(4):403–412.
3. He X, Wang Z, Zhou D. Robust $\mathscr{H}_\infty$ filtering for networked systems with multiple state delays. *International Journal of Control* 2007; **80**(8):1217–1232.
4. Gao H, Chen T, Lam J. A new delay system approach to network-based control. *Automatica* 2008; **44**(1):39–52.
5. Wang Z, Yang F, Ho DWC, Liu X. Robust $\mathscr{H}_\infty$ control for networked systems with random packet losses. *IEEE Transactions on Systems*, *Man*, *and Cybernetics-Part B* 2007; **37**(4):916–924.
6. Yue D, Han QL, Lam J. Network-based robust $\mathscr{H}_\infty$ control of systems with uncertainty. *Automatica* 2005; **41**(6):999–1007.
7. Yue D, Han QL, Peng C. State feedback controller design of networked control systems. *IEEE Transactions on Circuits and Systems* (*II*) 2004; **51**(11):640–644.
8. Walsh GC, Ye H, Bushnell L. Stability analysis of networked control systems. *IEEE Transactions on Control Systems Technology* 2002; **10**(3):438–446.
9. Zhang L, Shi Y, Chen T, Huang B. A new method for stabilization of networked control systems with random delays. *IEEE Transactions on Automatic Control* 2005; **50**(8):1177–1181.
10. Seiler P, Sengupta R. An $\mathscr{H}_\infty$ approach to networked control. *IEEE Transactions on Automatic Control* 2005; **50**(3):356–364.
11. Yang F, Wang Z, Ho DWC, Gani M. Robust $\mathscr{H}_\infty$ control with missing measurements and time-delays. *IEEE Transactions on Automatic Control* 2007; **52**(9):1666–1672.
12. He Y, Wu M, Liu G, She J. Output feedback stabilization for a discrete-time system with a time-varying delay. *IEEE Transactions on Automatic Control* 2008; **53**(10):2372–2377.
13. He Y, Wu M, She J, Liu G. Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays. *Systems and Control Letters* 2004; **51**:57–65.

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14. Wang Z, Liu Y, Liu X. $\mathscr{H}_\infty$ filtering for uncertain stochastic time-delay systems with sector-bounded nonlinearities. *Automatica* 2008; **44**(5):1268–1277.
15. Wu L, Shi P, Gao H, Wang C. Delay-dependent robust $\mathscr{H}_\infty$ and $\mathscr{L}_2$-$\mathscr{L}_\infty$ filtering for LPV systems with both discrete and distributed delays. *IEE Proceedings Part D*: *Control Theory and Applications* 2006; **153**(4):483–492.
16. Wu L, Shi P, Gao H, Wang J. $\mathscr{H}_\infty$ model reduction for linear parameter-varying systems with distributed delay. *International Journal of Control* 2009; **82**(3):408–422.
17. Wu L, Zheng W. Weighted $\mathscr{H}_\infty$ model reduction for linear switched systems with time-varying delay. *Automatica* 2009; **45**(1):186–193.
18. Wu M, He Y, She JH, Liu GP. Delay-dependent criteria for robust stability of time-varying delay systems. *Automatica* 2004; **40**(8):1435–1439.
19. Xu S, Lam J, Zou Y. Delay-dependent guaranteed cost control for uncertain systems with state and input delays. *IEE Proceedings Part D*: *Control Theory and Applications* 2006; **153**(3):35–53.
20. Boukas E-K, Shi P. Stochastic stability and guaranteed cost control of discrete-time uncertain systems with Markovian jumping parameters. *International Journal of Robust and Nonlinear Control* 1998; **8**(13):1155–1167.
21. Wu L, Wang Z. Guaranteed cost control for linear switched systems with neutral delay via dynamic output feedback. *International Journal of Systems Science* 2009; **40**(7):717–728.
22. Mukaidani H. An LMI approach to guaranteed cost control for uncertain delay systems. *IEEE Transactions on Circuits and Systems* (*I*) 2003; **50**(6):795–800.
23. Shi P, Boukas EK, Shi Y, Agarwal RK. Optimal guaranteed cost control of uncertain discrete time-delay systems. *Journal of Computational and Applied Mathematics* 2003; **157**(2):435–451.
24. Guan X, Lin Z, Duan G. Robust guaranteed cost control for discrete-time uncertain systems with delay. *IEE Proceedings Part D*: *Control Theory and Applications* 1999; **146**(6):598–602.
25. Yang G, Wang J, Soh YC. Guaranteed cost control for discrete-time linear systems under controller gain perturbations. *Linear Algebra and its Applications* 2000; **312**(1–3):161–180.
26. Chen W, Guan Z, Lu X. Delay-dependent guaranteed cost control for uncertain discretetime systems with delay. *IEE Proceedings Part D*: *Control Theory and Applications* 2003; **150**(4):412–416.
27. Wang Y, Xie L, de Souza CE. Robust control of a class of uncertain nonlinear system. *Systems and Control Letters* 1992; **19**:139–149.
28. Palhares RM, de Souza CE, Peres PLD. Robust $\mathscr{H}_\infty$ filtering for uncertain discrete-time state-delayed systems. *IEEE Transactions on Signal Processing* 2001; **49**(8):1696–1703.
29. Xu S, Lam J. A survey of linear matrix inequality techniques in stability analysis of delay systems. *International Journal of Systems Science* 2008; **39**(12):1095–1113.
30. Gao H, Wang C. A delay-dependent approach to robust $\mathscr{H}_\infty$ filtering for uncertain discrete-time state-delayed systems. *IEEE Transactions on Signal Processing* 2004; **52**(6):1631–1640.
31. Lam J, Gao H, Wang C. Stability analysis for continuous systems with two additive time-varying delay components. *Systems and Control Letters* 2007; **56**:16–24.
32. Du B, Lam J. Stability analysis of static recurrent neural networks using delay-partitioning and projection. *Neural Networks* 2009; **22**(4):343–347.

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