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Stability and stabilization of switched stochastic systems under asynchronous switching ☆

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ABSTRACT

This paper studies the stability and stabilization problems for a class of switched stochastic systems under asynchronous switching. The asynchronous switching refers to that the switching of the candidate controllers does not coincide with the switching of system modes. Two situations are considered: (1) time-delayed switching situation, that is, the switching of the candidate controllers has a lag to the switching of the system modes; (2) mismatched switching situation, the switching of the candidate controllers does not match the switching of the system modes. Using average dwell time and Lyapunov-like function, sufficient conditions are established for stochastic input-to-state stability of the whole system. Also, the stabilizing controller design approach is proposed for switched stochastic linear systems. The minimal average dwell time and the controller gain are achieved. Finally, a numerical example is used to demonstrate the validity of the developed results.

1. Introduction

Switched systems are a special class of hybrid systems, and consist of a family of subsystems (also called system modes) and a switching law that orchestrates the switching among the system modes; see [1,2]. In practice, there are numerous physical systems that could be modeled as the switched systems, such as fermentation processes [1], networked control systems [3,4] and scalable video coding systems [2]. Because of practical application and theoretical development, switched systems have been given considerable attention in the last few decades. The readers are referred to [5–9] for a general introduction and the recent progresses in the field of switched systems.

In the practical systems, disturbances are inevitable and have impacts on the stability and the performances of the dynamical systems including switched systems. Furthermore, the stochastic disturbances lead to stochastic modeling and control for the control systems, which leads to switched stochastic systems. In the literature, there are some salient results on switched stochastic systems, such as stability [10–12], fault detection filtering [13], passivity and passification [14], H_∞ control [15], sliding mode control [16]. In the established methods, there are two widely applied approaches to study switched systems, i.e., average dwell time (ADT) approach [8,17] and Lyapunov function approach [18–20]. Average dwell time characterizes the switching rate that guarantees stability of the closed-loop system. In Lyapunov function approach, multiple Lyapunov function is an essential Lyapunov function. Combining ADT and multiple Lyapunov function, stability analyses and control syntheses of switched systems have been investigated; see [12,14,16,19,21].

In the previous works [8,10,11], there is a general assumption: the switching of the candidate controllers and the system modes is coincident, which is called synchronous switching. However, asynchronous switching, which is opposed to the synchronous switching, is more practical. Asynchronous phenomena like time delays can be found in many fields, such as networked control systems [4,22], chemical systems [23], Markovian jump systems [24] and neural systems [25]. For the switched systems, asynchronous switching may be caused by disturbances, identification of the system modes, implementation of the matched controller, time delays in information transmission and even the requirements of the switching law. Because the switched systems do not necessarily inherit the stability properties of the subsystems, asynchronous switching may further deteriorate the performances of switched systems. Some studies have been reported in the literature. For instance, asynchronous control problem of switched linear systems was addressed in [21]. The stability conditions were established in terms of ADT and Lyapunov-like conditions. Stability of

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switched nonlinear systems was considered in [26] by analyzing the Lie derivative of Lyapunov function. If time delays and asynchronous switching were considered, then Lyapunov–Krasovskii functional method was used in [25] to derive the stability conditions for switched nonlinear systems.

In this paper, we study the stability and stabilization problems for switched stochastic systems under asynchronous switching. Sufficient conditions are established for stochastic stability and controller design. Based on the different causes of asynchronous switching, two cases are considered. The first case is time-delayed switching, i.e., there are time delays between the switches of the candidate controllers and the system modes. The second one is mismatched switching, that is, there are no time delays but switching mismatches at the switching times. Under these two cases, stochastic stability of switched stochastic systems is studied in continuous-time context and discrete-time context. Using ADT and Lyapunov function approach, sufficient conditions are established to guarantee stochastic input-to-state stability (SISS). Furthermore, for switched stochastic linear systems, the stabilizing controllers design approach is proposed. Finally, a numerical example is used to demonstrate the effectiveness of the designed controllers. Compared with the previous works in [19,24–26,12], the contributions of this paper are three-fold. First, two asynchronous switching cases are studied, whereas only the time-delayed switching case was considered in the previous works [19,25,26,21]. Especially, the mismatched switching case is first studied in this paper. Second, for above two asynchronous switching cases, the stability conditions are established, which extends the previous results for the deterministic/linear/synchronous switched systems [12,19,21]. Moreover, both the continuous-time systems and the discrete-time systems are considered. Third, for switched stochastic linear systems with asynchronous switching, the stabilizing switched controller is designed, which recovers many previous works [21,27] as the special cases.

This paper is organized as follows. In Section 2, the considered problem is formulated and some preliminaries are given. Using average dwell-time and multiple Lyapunov-like function, sufficient conditions for SISS of switched stochastic systems are derived in Section 3. Both the time-delayed switching case and the mismatched switching case are considered. For these two cases, the stabilizing switched controllers are designed for switched stochastic linear systems in Section 4. In Section 5, a numerical example is used to illustrate the obtained results. Conclusions and future works are stated in Section 6.

Notation: The notation used in this paper is fairly standard. $\mathbb{N}^+$ stands for the set of nonnegative integers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $|\cdot|$ represents the Euclidean vector norm. $\mathbb{P}\{\cdot\}$ denotes the probability measure; $\mathbb{E}[\cdot]$ denotes the mathematical expectation. $\mathcal{C}^{1,2}$ stands for the space of the functions that are continuously differentiable on the first augment and continuously twice differentiable on the second augment. A function $\alpha(t) : \mathbb{R}^+ \to \mathbb{R}^+$ is of class $\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing; $\alpha(t)$ is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. A function $\beta(s,t) : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is of class $\mathcal{KL}$ if $\beta(s,t)$ is of class $\mathcal{K}$ for each fixed $t \geq 0$ and $\beta(s,t)$ decreases to zero as $t \to 0$ for each fixed $s \geq 0$. $\mathcal{L}^n_\infty$ denotes the set of all the measurable and locally essentially bounded signal $x \in \mathbb{R}^n$ on $\mathbb{R}^+$ with norm $\|x\| := \sup_{t\geq t_0} \inf_{\{\mathcal{A}\subset\Omega,\mathbb{P}\{\mathcal{A}\}=0\}} \sup\{|x(t,w)| | w \in \Omega \setminus \mathcal{A}\}$. In addition, the symbols $tr[\cdot]$ and $diag\{\cdot\}$ denote trace operator and block diagonal matrix operator, respectively. The superscript "T" denotes the transpose, and the symmetric term in a matrix is denoted by $*$. $A > 0$ ($A \geq 0$) means that the matrix A is positive definite (positive semidefinite). For simplicity, denote $\alpha_1 \circ \alpha_2(s) := \alpha_1(\alpha_2(s))$ for all $\alpha_1, \alpha_2 : \mathbb{R} \to \mathbb{R}$ and $s \geq 0$.

2. Problem formulation

Consider the switched stochastic nonlinear control system of the form

$$dx(t) = f_{\sigma(t)}(t,x,u,v)dt + g_{\sigma(t)}(t,x,u,v)dw(t) \tag{1}$$

for the continuous-time domain or

$$x(l+1) = f_{\sigma(l)}(l,x,u,v) + g_{\sigma(l)}(l,x,u,v)w(l) \tag{2}$$

for the discrete-time domain, where $x \in \mathbb{R}^{n_x}$ is the system state initializing at $x(t_0) = x_0$ and $t_0 \geq 0$, $u \in \mathbb{R}^{n_u}$ is the control input which is assumed to be measurable and locally bounded, and $v \in \mathcal{L}^{n_v}_\infty$ is the exogenous disturbance. A piecewise constant and right continuous function $\sigma : \mathbb{R}^+ \to \mathcal{M}$ is a switching signal specifying the index of the active subsystem, where $\mathcal{M} = \{1,\ldots,M\}$ is an index set. For the continuous-time version (1), $w(t)$ is an n_w-dimensional independent standard Wiener process (or Brownian motion) defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathbb{P})$; for the discrete-time version (2), $w(l)$ is a scalar Gaussian white noise with $\mathbb{E}[w(l)] = 0$ and $\mathbb{E}[w^2(l)] = \theta$. For each $i \in \mathcal{M}$, both $f_i : [t_0,\infty) \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathcal{L}^{n_v}_\infty \to \mathbb{R}^{n_x}$ and $g_i : [t_0,\infty) \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathcal{L}^{n_v}_\infty \to \mathbb{R}^{n_x \times n_w}$ are continuous with respect to t, x, u and v, and uniformly locally Lipschitz with respect to x and v; $f_i(\cdot,0,0,0) \equiv 0$ and $g_i(\cdot,0,0,0) \equiv 0$. For simplicity of notation, the solution process of the switched stochastic system (1) or (2) is assumed to be existent and unique for all the time; see [9,19]. Otherwise, the solution process is only defined on certain finite interval $[t_0, t_{\max})$ and $t_{\max} > t_0$. However, all the subsequent results are still valid for this case.

Definition 1 (*[8]*). For a switching signal σ and any $t_2 > t_1 \geq t_0$, let $N_\sigma(t_2,t_1)$ be the switching number of σ over the interval $[t_1,t_2)$. If there exist constants $N_0 \geq 1$ and $\tau_a > 0$ such that

$$N_\sigma(t_2,t_1) \leq N_0 + \frac{t_2 - t_1}{\tau_a}, \tag{3}$$

then N_0 and τ_a are called the chatter bound and the average dwell time, respectively.

In the following, the stability definitions are introduced for the continuous-time system (1). For the discrete-time version (2), the stability definitions are obtained similarly.

Definition 2 (*[12]*). The switched stochastic nonlinear system (1) is stochastically input-to-state stable (SISS), if for any $\varepsilon > 0$, there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ such that for all $x_0 \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $v \in \mathcal{L}^{n_v}_\infty$,

$$\mathbb{P}\{|x(t)| \leq \beta(|x(t_0)|, t - t_0) + \gamma(\|v\|)\} \geq 1 - \varepsilon, \quad t \geq t_0. \tag{4}$$

If the inequality (4) holds for $v \equiv 0$, then the system (1) with $v \equiv 0$ is stochastically globally asymptotically stable (SGAS).

To stabilize the switched stochastic nonlinear system (1) and (2), the candidate mode-dependent controllers are designed as $u(t) = \kappa_{\sigma(t)}(x(t))$ for the continuous-time version (1) or $u(l) = \kappa_{\sigma(l)}(x(l))$ for the discrete-time version (2). In the literature, there is a common assumption for the candidate controllers: the switching of the candidate controllers is coincident with the switching of the system modes. In practice, this assumption is hard to be satisfied, whereas the asynchronous switching exists extensively in the physical systems [22–25]. However, the asynchronous switching deteriorates the stability and the performances of the switched stochastic control systems.

Therefore, the objectives of this paper are to establish the sufficient conditions to guarantee stochastic input-to-state stability of switched stochastic systems and to design the mode-dependent controllers under asynchronous switching. Based on the different reasons of the asynchronous switching, two cases are studied in this paper. The first case is the time-delayed switching case, which is induced by the identification of the system modes, the implementation of the matched controller, the time delays of the switched controllers to the system modes and so forth. In the time-delayed switching case, the candidate controllers become $u(t) = \kappa_{\sigma(t-d)}(x(t))$ for the continuous-time context and $u(l) = \kappa_{\sigma(l-d)}(x(l))$ for the discrete-time context. The time delay d is smaller than the corresponding switching interval. That is, if the ith subsystem is active in $[t_k, t_{k+1})$, $k \in \mathbb{N}^+$, then the delay is $d_k < t_{k+1} - t_k$ and the ith controller is activated in $[t_k + d_k, t_{k+1})$. The second case is the mismatched switching case, which is caused by the disturbances, the possible faults, some requirements of the switching law, etc. In the mismatched switching case, there is no delay in $[t_k, t_{k+1})$ but the active controller does not match the active subsystem. That is, if the ith subsystem is activated at t_k, then it is not ith controller but jth controller that is activated at t_k, where $i, j \in \mathcal{M}$ and $i \neq j$.

To analyze the stability properties of the switched stochastic nonlinear systems (1) under the above two cases, the following differential operator is defined.

Definition 3 ([28]). Given any $\mathcal{C}^{1,2}$ functions $V_i : \mathbb{R}^+ \times \mathbb{R}^{n_x} \to \mathbb{R}^+$, $i \in \{1, 2, \ldots, M\}$, the differential operator $\mathscr{L}$, which is associated with the continuous-time stochastic differential equation (1), is defined as

$$\mathscr{L}V_i(t,x) := \frac{\partial V_i(t,x)}{\partial t} + \frac{\partial V_i(t,x)}{\partial x} f_i(t,x,u,v) + \frac{1}{2} tr\left[g_i^T(t,x,u,v)\frac{\partial^2 V_i(t,x)}{\partial x^2} g_i(t,x,u,v)\right]. \quad (5)$$

By Itô's formula [29, Chapter IV. 3], the differential of the function $V_i(t, x)$ for the continuous-time system (1) is

$$dV_i(t,x) = \mathscr{L}V_i(t,x)dt + \frac{\partial V_i(t,x)}{\partial x} g_i(t,x,u,v)dw(t).$$

Furthermore, it follows from the proof of Theorem 1 in [12] that

$$d\mathbb{E}[V_i(t,x)] = \mathbb{E}[\mathscr{L}V_i(t,x)]dt.$$

For the discrete-time version (2), instead of the differential of $V_i(t, x)$, the difference of $V_i(l, x)$, that is, $\Delta V_i(l, x) := V_i(l + 1, x_{l+1}) - V_i(l, x)$, will be used.

3. Stability analysis of switched stochastic nonlinear systems

In this section, based on the multiple Lyapunov-like function and the average dwell time, sufficient conditions are established for SISS of the switched stochastic nonlinear systems (1) and (2) with asynchronous switching. Both the time-delayed switching case and the mismatched switching case are considered.

3.1. Time-delayed switching case

First, some notations are introduced. If the kth subsystem is activated, then t_k and t_{k+1} represent the starting time and the ending time, respectively. Because of the time delays, the corresponding Lyapunov function may be not decreasing in $[t_k, t_{k+1})$, $k \in \mathbb{N}^+$. Thus, $\mathcal{T}_\downarrow(t_k, t_{k+1})$ and $\mathcal{T}_\uparrow(t_k, t_{k+1})$ denote separately the unions of dispersed intervals in $[t_k, t_{k+1})$ where the Lyapunov function is decreasing and increasing. That is, $[t_k, t_{k+1}) = \mathcal{T}_\downarrow(t_k, t_{k+1}) \cup \mathcal{T}_\uparrow(t_k, t_{k+1})$. Moreover, $\mathcal{T}_\downarrow(t_{k+1} - t_k)$ and $\mathcal{T}_\uparrow(t_{k+1} - t_k)$ stand for the lengths of $\mathcal{T}_\downarrow(t_k, t_{k+1})$ and $\mathcal{T}_\uparrow(t_k, t_{k+1})$, respectively.

Based on the aforementioned notations, we are ready to establish the stability conditions for switched stochastic nonlinear systems with time-delayed switching.

Theorem 1. *Consider the continuous-time switched stochastic nonlinear system (1), if there exist $\mathcal{C}^{1,2}$ Lyapunov functions $V_i : \mathbb{R}^+ \times \mathbb{R}^{n_x} \to \mathbb{R}^+$, functions $\alpha_{1i}, \alpha_{2i}, \rho_i \in \mathcal{K}_\infty$ and constants $\varphi, \eta > 0$, $\mu \geq 1$ such that for all $i \in \mathcal{M}$, $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $v \in \mathcal{L}_\infty^{n_v}$,*

$$\alpha_{1i}(|x|) \leq V_i(t,x) \leq \alpha_{2i}(|x|), \quad (6)$$

$$|x| \geq \rho_i(\|v\|) \Rightarrow \mathscr{L}V_i(t,x) \leq \begin{cases} -\varphi V_i(t,x), & t \in \mathcal{T}_\downarrow(t_k, t_{k+1}), \\ \eta V_i(t,x), & t \in \mathcal{T}_\uparrow(t_k, t_{k+1}), \end{cases} \quad (7)$$

$$\mathbb{E}[V_{\sigma(t_k)}(t_k, x(t_k))] \leq \mu \mathbb{E}[V_{\sigma(t_{k-1})}(t_k, x(t_k))], \quad (8)$$

$$\tau_a > \frac{(\varphi+\eta)\mathcal{T}_m + \ln\mu}{\varphi}, \quad (9)$$

where $\mathcal{T}_m := \max\{\mathcal{T}_\uparrow(t_{k+1} - t_k) | k \in \mathbb{N}^+\}$, then the system (1) is SISS.

Proof. Define $\alpha_1(s) := \min_{i\in\mathcal{M}} \alpha_{1i}(s)$, $\alpha_2(s) := \max_{i\in\mathcal{M}} \alpha_{2i}(s)$ and $\rho(s) := \max_{i\in\mathcal{M}} \rho_i(s)$. Thus, (6) and (7) are rewritten as

$$\alpha_1(|x|) \leq V_i(t,x) \leq \alpha_2(|x|),$$

$$|x| \geq \rho(\|v\|) \Rightarrow \mathscr{L}V_i(t,x) \leq \begin{cases} -\varphi V_i(t,x), & t \in \mathcal{T}_\downarrow(t_k, t_{k+1}), \\ \eta V_i(t,x), & t \in \mathcal{T}_\uparrow(t_k, t_{k+1}). \end{cases}$$

Define the set $\mathbf{B} := \{x \in \mathbb{R}^{n_x} | |x| \leq \rho(\|v\|)\}$ and $\bar{t} := \inf\{t \geq t_0 | x(t) \in \mathbf{B}\}$. Therefore, the following two cases are considered.

Case 1: $x(t_0) \notin \mathbf{B}$. For all $t \in [t_0, \bar{t}] \cap [t_k, t_{k+1})$, integrating (7) leads to the fact that

$$\begin{aligned}\mathbb{E}[V_{\sigma(t)}(t, x(t))] &\leq e^{-\varphi\mathcal{T}_\downarrow(t_k,t)+\eta\mathcal{T}_\uparrow(t_k,t)}\mathbb{E}[V_{\sigma(t_k)}(t_k, x(t_k))] \\ &= e^{-\varphi(t-t_k)}e^{(\varphi+\eta)\mathcal{T}_\uparrow(t_k,t)}\mathbb{E}[V_{\sigma(t_k)}(t_k, x(t_k))].\end{aligned}$$

Because of (8) and the definition of $\mathcal{T}_m$, it follows that

$$\begin{aligned}&\mathbb{E}[V_{\sigma(t_{k+1})}(t_{k+1}, x(t_{k+1}))] \\ &\leq e^{-\varphi(t_{k+1}-t_k)}e^{(\varphi+\eta)\mathcal{T}_\uparrow(t_k,t_{k+1})}\mu\mathbb{E}[V_{\sigma(t_k)}(t_k, x(t_k))] \\ &\leq e^{-\varphi(t_{k+1}-t_k)}e^{(\varphi+\eta)\mathcal{T}_m}\mu\mathbb{E}[V_{\sigma(t_k)}(t_k, x(t_k))].\end{aligned} \quad (10)$$

Iterating (10) implies that

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq e^{-\varphi(t-t_0)}e^{(\varphi+\eta)\mathcal{T}_m N_\sigma(t,t_0)} \times \mu^{N_\sigma(t,t_0)}V_{\sigma(t_0)}(t_0, x(t_0)). \quad (11)$$

Combining (11) with the definition of $N_\sigma(t, t_0)$, it holds that

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq e^{-[\varphi-(\varphi+\eta)\mathcal{T}_m/\tau_a-\ln\mu/\tau_a](t-t_0)} \times e^{N_0[(\varphi+\eta)\mathcal{T}_m+\ln\mu]}V_{\sigma(t_0)}(t_0, x(t_0)), \quad t \in [t_0, \bar{t}).$$

That is, one has

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq \beta_1(V_{\sigma(t_0)}(t_0, x(t_0)), t-t_0), \quad t \in [t_0, \bar{t}), \quad (12)$$

where $\beta_1(r, s) := e^{N_0[(\varphi+\eta)\mathcal{T}_m+\ln\mu]}e^{-[\varphi-(\varphi+\eta)\mathcal{T}_m/\tau_a-\ln\mu/\tau_a]s}r$, which is a class $\mathcal{KL}$ function since (9) holds.

Define $\beta_2(r, s) := \beta_1(r, s)/\varepsilon_1$ for arbitrarily small $\varepsilon_1 \in (0, 1)$. Applying Markov's inequality [29, Chapter II, 18.1] to (12), it obtains that

$$\mathbb{P}\{V_{\sigma(t)}(t, x(t)) \geq \beta_2(V_{\sigma(t_0)}(t_0, x(t_0)), t-t_0)\} \leq \frac{\mathbb{E}[V_{\sigma(t)}(t, x(t))]}{\beta_2(V_{\sigma(t_0)}(t_0, x(t_0)), t-t_0)} < \varepsilon_1.$$

Denote $\beta_3(r, s) := \alpha_1^{-1} \circ \beta_2(\alpha_2(r), s)$, it follows that

$$\mathbb{P}\{|x(t)| < \beta_3(|x(t_0)|, t-t_0)\} \geq 1-\varepsilon_1, \quad t \in [t_0, \bar{t}). \quad (13)$$

Let $\hat{t} := \inf\{t > \bar{t} | x(t) \notin \mathbf{B}\}$. If $\hat{t}$ does not exist, define $\hat{t} := \infty$. For all $t \in [\bar{t}, \hat{t})$, $|x| \leq \rho(\|v\|)$, which implies that

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq \alpha_2 \circ \rho(\|v\|), \quad t \in (\bar{t}, \hat{t}). \quad (14)$$

Applying Markov's inequality [29, Chapter, 18.1] to (14), for arbitrarily small $\varepsilon_2 \in (0, 1)$, there exists a $\delta_1(\varepsilon_2) \in \mathcal{K}_\infty$ such that

$$\mathbb{P}\{V_{\sigma(t)}(t, x(t)) \geq \delta_1 \circ \alpha_2 \circ \rho(\|v\|)\} \leq \frac{\mathbb{E}[V_{\sigma(t)}(t, x(t))]}{\delta_1 \circ \alpha_2 \circ \rho(\|v\|)} < \varepsilon_2.$$

Define $\gamma_1(s) := \alpha_1^{-1} \circ \delta_1 \circ \alpha_2 \circ \rho(s)$, it holds that

$$\mathbb{P}\{|x(t)| < \gamma_1(\|v\|)\} \geq 1 - \varepsilon_2, \quad t \in (\bar{t}, \hat{t}).$$

At the time instant $\hat{t}$, $|x(\hat{t})| = \rho(\|v\|)$ holds for the continuity of the system state. If $\hat{t} < \infty$, then define $\tilde{t} := \inf\{t > \hat{t} | x(t) \in \mathbf{B}\}$. Similar to the trajectory from the initial time t_0, for all $t \in (\hat{t}, \tilde{t})$, it holds that

$$\begin{aligned}\mathbb{E}[V_{\sigma(t)}(t, x(t))] &\leq \beta_1(V_{\sigma(\hat{t})}(\hat{t}, x(\hat{t})), t - \hat{t}) \\ &\leq e^{N_0[(\varphi+\eta)\mathcal{T}_m + \ln\mu]} \\ &\quad \times e^{-[\varphi-(\varphi+\eta)\mathcal{T}_m/\tau_a - \ln\mu/\tau_a](\tilde{t}-\hat{t})} \alpha_2 \circ \rho(\|v\|) \\ &\leq e^{N_0[(\varphi+\eta)\mathcal{T}_m + \ln\mu]} \alpha_2 \circ \rho(\|v\|).\end{aligned}$$

Repeating the above analysis, it follows that for arbitrarily small $\varepsilon_2 \in (0, 1)$, there exists a $\delta_2(\varepsilon_2) \in \mathcal{K}_\infty$ such that

$$\mathbb{P}\{|x(t)| < \gamma_2(\|v\|)\} \geq 1 - \varepsilon_2, \quad t \in (\bar{t}, \infty), \tag{15}$$

where $\gamma_2(s) := \max\{\alpha_1^{-1} \circ \delta_2 \circ e^{N_0[(\varphi+\eta)\mathcal{T}_m + \ln\mu]} \alpha_2 \circ \rho(s), \alpha_1^{-1} \circ \delta_1 \circ \alpha_2 \circ \rho(s)\}$.

Combining (13) and (15), we arrive at

$$\mathbb{P}\{|x(t)| < \bar{\beta}(|x(t_0)|, t - t_0) + \bar{\gamma}(\|v\|)\} \geq 1 - \varepsilon, \tag{16}$$

where $\bar{\beta}(r, s) := \beta_3(r, s)$, $\bar{\gamma}(r) := \gamma_2(r)$ and $\varepsilon := \min\{\varepsilon_1, \varepsilon_2\}$.

Case 2: $x(t_0) \in \mathbf{B}\backslash\{0\}$. In this case, $\bar{t} = t_0$. Following the similar proof of Case 1, it holds that

$$\begin{aligned}\mathbb{P}\{|x(t)| < \bar{\beta}(|x(t_0)|, t - t_0) + \bar{\gamma}(\|v\|)\} &\geq \mathbb{P}\{|x(t)| < \bar{\gamma}(\|v\|)\} \\ &\geq 1 - \varepsilon_2.\end{aligned}$$

Especially, when $t = t_0$,

$$\mathbb{P}\{|x(t)| < \bar{\beta}(|x(t_0)|, 0) + \bar{\gamma}(\|v\|)\} = 1.$$

Therefore, based on the aforementioned analysis of two cases, the switched stochastic nonlinear system (1) is SISS. ■

Remark 1. In Theorem 1, the condition (7) is equivalent to

$$\mathscr{L}V_i(t, x) \leq \begin{cases} -\varphi V_i(t, x) + \hat{\rho}_i(\|v\|), & t \in \mathcal{T}_\downarrow(t_k, t_{k+1}), \\ \eta V_i(t, x) + \hat{\rho}_i(\|v\|), & t \in \mathcal{T}_\uparrow(t_k, t_{k+1}), \end{cases}$$

where $\hat{\rho}_i(s) := \alpha_{2i} \circ \rho_i(s)$; see [30,31]. This equivalent relationship will be used in Section 4. On the other hand, if there is no disturbance, SGAS of the switched stochastic system (1) is established along the same fashion.

Remark 2. Theorem 1 recovers the stability results for deterministic switched systems [21] and synchronous switched systems [12] as special cases. If the considered system is deterministic and has no disturbances, then Theorem 1 is reduced to be Lemma 3 in [21]. If the switching is synchronous and there is no disturbance, then Theorem 1 is similar to Theorem 5 in [12].

Remark 3. In this paper, the switching signal σ is deterministic. If $\{\sigma(t), t \geq 0\}$ is a right-continuous Markov process on the probability space taking values in $\mathcal{M}$ (see [6,24]), similar result is also obtained. In this case, the generator of $\{\sigma(t), t \geq 0\}$ is $\Pi = (\pi_{ij}), i, j \in \mathcal{M}$, given by

$$\mathbb{P}\{\sigma(t + \Delta t) = j | \sigma(t) = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where Δt is a small time increment and $\lim_{\Delta t \to 0^+}(o(\Delta t)/\Delta t) = 0$, $\pi_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ and $\pi_{ii} = -\sum_{j\neq i}\pi_{ij}$. The differential operator $\mathscr{L}$ is redefined as

$$\begin{aligned}\mathscr{L}V_i(t, x) := \frac{\partial V_i(t, x)}{\partial t} + \frac{\partial V_i(t, x)}{\partial x} f_i(t, x, u, v) + \sum_{j=1}^{M} \pi_{ij} V_j(t, x) \\ + \frac{1}{2} tr\left[g_i^T(t, x, u, v) \frac{\partial^2 V_i(t, x)}{\partial x^2} g_i(t, x, u, v)\right].\end{aligned}$$

Therefore, Theorem 1 can be extended to the case of switched stochastic systems with asynchronous Markovian switching.

For the discrete-time version (2), the counterpart theorem is presented as follows. Its proof is similar to the proof of Theorem 1, hence omitted here. Likewise, if the switching signal is a Markov chain, the following theorem can also be extended to the case of discrete-time switched stochastic systems with asynchronous Markovian switching.

Theorem 2. *Consider the discrete-time switched stochastic nonlinear system (2), if there exist Lyapunov functions $V_i : \mathbb{N}^+ \times \mathbb{R}^{n_x} \to \mathbb{R}^+$, functions $\alpha_{1i}, \alpha_{2i}, \rho_i \in \mathcal{K}_\infty$ and constants $\varphi \in (0, 1)$, $\eta > -1$, $\mu \geq 1$ such that for all $i \in \mathcal{M}$, $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $v \in \mathcal{L}_\infty^{n_v}$,*

$$\alpha_{1i}(|x_l|) \leq V_i(l, x_l) \leq \alpha_{2i}(|x_l|), \tag{17}$$

$$|x_l| \geq \rho_i(\|v\|) \Rightarrow \mathbb{E}[\Delta V_i(l, x_l)] \leq \begin{cases} -\varphi V_i(l, x_l), & l \in \mathcal{T}_\downarrow(l_k, l_{k+1}), \\ \eta V_i(l, x_l), & l \in \mathcal{T}_\uparrow(l_k, l_{k+1}), \end{cases} \tag{18}$$

$$\mathbb{E}[V_{\sigma(l_k)}(l_k, x_{l_k})] \leq \mu\mathbb{E}[V_{\sigma(l_{k-1})}(l_k, x_{l_k})], \tag{19}$$

$$\tau_a > \frac{\mathcal{T}_m[\ln\bar{\eta} - \ln\bar{\varphi}] + \ln\mu}{-\ln\bar{\varphi}}, \tag{20}$$

where $\mathcal{T}_m := \max\{\mathcal{T}_\uparrow(l_{k+1} - l_k) | k \in \mathbb{N}^+\}$, $\bar{\eta} := 1 + \eta$ and $\bar{\varphi} := 1 - \varphi$, then the system (2) is SISS.

3.2. Mismatched switching case

In this subsection, the mismatched switching case is studied. For the mismatched switching case, sufficient conditions are derived to guarantee the stochastic stability of the whole system.

First, the index set $\mathcal{M}$ is divided into two parts: the coincident switching part $\mathcal{M}_s$ (i.e., the set of the switching times at which there are no mismatched switches between the candidate controllers and the system modes) and the mismatched switching part $\mathcal{M}_u$ (i.e., the set of the switching times at which mismatched switches occur). It follows that $\mathcal{M} = \mathcal{M}_s \cup \mathcal{M}_u$ and $\mathcal{M}_s \cap \mathcal{M}_s = \emptyset$. Given any interval $[\tau_1, \tau_2)$, $\tau_2 \geq \tau_1 \geq t_0$, $\mathcal{I}_s(\tau_2, \tau_1)$ and $\mathcal{I}_u(\tau_2, \tau_1)$ denote the total activation time of the subsystems in $\mathcal{M}_s$ and $\mathcal{M}_u$, respectively. That is, $\tau_2 - \tau_1 = \mathcal{I}_s(\tau_2, \tau_1) + \mathcal{I}_u(\tau_2, \tau_1)$.

Theorem 3. *Consider the continuous-time switched stochastic nonlinear system (1), if there exist $\mathcal{C}^{1,2}$ Lyapunov functions $V_i : \mathbb{R}^+ \times \mathbb{R}^{n_x} \to \mathbb{R}^+$, functions $\alpha_{1i}, \alpha_{2i}, \rho_i \in \mathcal{K}_\infty$ and constants $\varphi, \eta, \lambda, \mathcal{I}_0 > 0$ and $\mu \geq 1$ such that for all $i, j \in \mathcal{M}$, $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $v \in \mathcal{L}_\infty^{n_v}$, (6) holds and*

$$|x| \geq \rho_i(\|v\|) \Rightarrow \mathscr{L}V_i(t, x) \leq \begin{cases} -\varphi V_i(t, x), & i \in \mathcal{M}_s, \\ \eta V_i(t, x), & i \in \mathcal{M}_u, \end{cases} \tag{21}$$

$$\mathbb{E}[V_i(t, x(t))] \leq \mu\mathbb{E}[V_j(t, x(t))], \tag{22}$$

$$\tau_a > \frac{\ln\mu}{(1 - \lambda)\varphi - \lambda\eta}, \quad \lambda < \frac{\varphi}{\varphi + \eta}, \tag{23}$$

$$\mathcal{I}_u(\tau_2, \tau_1) \leq \mathcal{I}_0 + \lambda(\tau_2 - \tau_1), \tag{24}$$

then the system (1) is SISS.

Proof. Define $\alpha_1(s) := \min_{i\in\mathcal{M}} \alpha_{1i}(s)$, $\alpha_2(s) := \max_{i\in\mathcal{M}} \alpha_{2i}(s)$, $\rho(s) := \max_{i\in\mathcal{M}} \rho_i(s)$ and $W(t) := e^{\varphi t}V_{\sigma(t)}(t, x(t))$. If $|x(t)| \geq \rho(\|v\|)$ holds in $[t^1, t^2]$, where $t^2 > t^1 \geq t_0$, then for all $t \in [t^1, t^2] \cap [t_k, t_{k+1})$, it holds from (21) that

$$\mathscr{L}W(t) \leq 0, \quad i_k \in \mathcal{M}_s,$$
$$\mathscr{L}W(t) \leq (\varphi + \eta)W(t), \quad i_k \in \mathcal{M}_u,$$

which implies that for $i_k \in \mathcal{M}_s$,

$$\mathbb{E}[W(t_{k+1})] \leq \mu\mathbb{E}[W(t_{k+1}^-)] \leq \mu\mathbb{E}[W(t_k)], \tag{25}$$

and for $i_k \in \mathcal{M}_u$,

$$\mathbb{E}[W(t_{k+1})] \leq \mu\mathbb{E}[W(t_{k+1}^-)] \leq \mu e^{(\varphi+\eta)(t_{k+1}-t_k)}\mathbb{E}[W(t_k)]. \tag{26}$$

Thus, for any $t \in [t^1, t^2]$, iterating (25) and (26) gives that

$$\mathbb{E}[W(t)] \leq \mu^{N_\sigma(t,t^1)}e^{(\varphi+\eta)\mathcal{I}_u(t,t^1)}\mathbb{E}[W(t^1)],$$

which can be rewritten as

$$\begin{aligned}\mathbb{E}[V_{\sigma(t)}(t, x(t))] &\leq e^{-\varphi(t-t^1)}\mu^{N_\sigma(t,t^1)}e^{(\varphi+\eta)\mathcal{I}_u(t,t^1)}\mathbb{E}[V_{\sigma(t^1)}(t^1, x(t^1))] \\ &\leq e^{[\ln\mu/\tau_a+(\varphi+\eta)\lambda-\varphi](t-t^1)}e^{N_0\ln\mu+(\varphi+\eta)\mathcal{I}_0}\mathbb{E}[V_{\sigma(t^1)}(t^1, x(t^1))].\end{aligned}$$

Denote $\varpi := \ln\mu/\tau_a + (\varphi + \eta)\lambda - \varphi$. It implies from (23) that $\varpi < 0$ and

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq e^{N_0\ln\mu+(\varphi+\eta)\mathcal{I}_0}e^{\varpi(t-t^1)}\mathbb{E}[V_{\sigma(t^1)}(t^1, x(t^1))].$$

Therefore, if $|x(t_0)| \geq \rho(\|v\|)$, then for all $t \in [t_0, \bar{t}]$, where $\bar{t}$ is defined in the proof of Theorem 1, it follows that

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq \beta_4(V_{\sigma(t_0)}(t_0, x(t_0)), t - t_0),$$

where $\beta_4(r, s) := e^{N_0\ln\mu+(\varphi+\eta)\mathcal{I}_0}e^{\varpi s}r \in \mathcal{KL}$.

Similar to the proof of Theorem 1, it obtains from Markov's inequality that

$$\mathbb{P}\{|x(t)| < \beta_5(|x(t_0)|, t - t_0)\} \geq 1 - \varepsilon_1, \quad t \in [t_0, \bar{t}), \tag{27}$$

where $\beta_5(r, s) := \alpha_1^{-1} \circ \beta_4(\alpha_2(r), s)$.

When $t > \bar{t}$, define $\hat{t} := \inf\{t > \bar{t}\,|\,|x| > \rho(\|v\|)\}$, if not exists, $\hat{t} := \infty$. Thus, for all $t \in [\bar{t}, \hat{t}]$, $|x(t)| \leq \rho(\|v\|)$, which implies that

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq \alpha_2 \circ \rho(\|v\|).$$

According to the continuity of the trajectory of the system state, we have that $|x(\hat{t})| = \rho(\|v\|)$. For all $t > \hat{t}$, if $|x(t)| > \rho(\|v\|)$, define $\tilde{t} := \sup\{\tau < t, |x(\tau)| \leq \rho(\|v\|)\}$. For all $t \in [\hat{t}, \tilde{t})$, we have

$$\begin{aligned}\mathbb{E}[V_{\sigma(t)}(t, x(t))] &\leq \beta_4(V_{\sigma(\hat{t})}(\hat{t}, x(\hat{t})), t - \hat{t}) \\ &\leq e^{N_0\ln\mu+(\varphi+\eta)\mathcal{I}_0}\alpha_2 \circ \rho(\|v\|).\end{aligned}$$

Thus, the above analysis gives that

$$\mathbb{E}[V_{\sigma(t)}(t, x(t))] \leq \gamma_1(\|v\|), \quad t > \bar{t}, \tag{28}$$

where $\gamma_1(s) := \max\{1, e^{N_0\ln\mu+(\varphi+\eta)\mathcal{I}_0}\}\alpha_2 \circ \rho(s)$. Applying Markov's inequality to (28), for arbitrarily small $\varepsilon_3 \in (0, 1)$, there exists a function $\delta_3(\varepsilon_3) \in \mathcal{K}_\infty$ such that

$$\mathbb{P}\{|x(t)| < \gamma_2(\|v\|)\} \geq 1 - \varepsilon_3, \quad t > \bar{t},$$

where $\gamma_2(s) := \alpha_1^{-1} \circ \delta_3 \circ \gamma_1(s)$.

Combining all above analyses in $[t_0, \bar{t})$ and $[\bar{t}, \infty)$ implies that

$$\mathbb{P}\{|x(t)| < \beta_5(|x(t_0)|, t - t_0) + \gamma_2(\|v\|)\} \geq 1 - \varepsilon,$$

where $\varepsilon := \min\{\varepsilon_1, \varepsilon_3\}$. Similar to the proof of Theorem 1, the above inequality still holds even if $|x(t_0)| < \rho(\|v\|)$. Therefore, the proof is completed. ■

Remark 4. In Theorem 3, mismatched switching leads to instability of the corresponding subsystem in the switching interval, which is similar to the case that not all the subsystems are stable in [19]. From this perspective, Theorem 3 can be thought of as an extension and reinterpretation of Theorem 2 in [19]. More generally, when the ith subsystem and the corresponding jth controller are active, if the Lyapunov function is increasing and decreasing in different and dispersed intervals of $[t_k, t_{k+1})$, then the stability analysis is a combination of the strategies of Theorems 1 and 3.

For the discrete-time system (2) with mismatched switching, the counterpart result is presented as follows. Its proof is similar to the proof of Theorem 3, hence omitted here.

Theorem 4. *Consider the discrete-time switched stochastic nonlinear system (2), if there exist Lyapunov functions $V_i : \mathbb{N}^+ \times \mathbb{R}^{n_x} \to \mathbb{R}^+$, functions $\alpha_{1i}, \alpha_{2i}, \rho_i \in \mathcal{K}_\infty$ and constants $\varphi \in (0, 1)$, $\eta > -1$, $\lambda, \mathcal{I}_0 \geq 0$ and $\mu \geq 1$ such that for all $i, j \in \mathcal{M}$, $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $v \in \mathcal{L}_\infty^{n_v}$, (17) holds and*

$$|x_k| \geq \rho_i(\|v\|) \Rightarrow \mathbb{E}[\Delta V_i(l, x)] \leq \begin{cases}-\varphi V_i(l, x), & i \in \mathcal{M}_s,\\ \eta V_i(l, x), & i \in \mathcal{M}_u,\end{cases}$$

$$\mathbb{E}[V_i(l, x_l)] \leq \mu\mathbb{E}[V_j(l, x_l)],$$

$$\tau_a > \frac{\ln\mu}{1 - (1-\lambda)\bar{\varphi} - \lambda\bar{\eta}}, \quad \lambda < \frac{1 - \bar{\varphi}}{\bar{\eta} - \bar{\varphi}}, \tag{29}$$

$$\mathcal{I}_u(\tau_2, \tau_1) \leq \mathcal{I}_0 + \lambda(\tau_2 - \tau_1), \tag{30}$$

where $\bar{\varphi} = \ln(1 - \varphi)$ and $\bar{\eta} = \ln(1 + \eta)$, then the system (2) is SISS.

4. Stabilization of switched stochastic linear systems

In this section, the stabilization problem for switched stochastic linear systems is considered. Sufficient conditions are established for the existence of the stabilizing switched controllers for switched stochastic linear systems.

Consider the switched stochastic linear system of the form

$$\begin{aligned}dx(t) = {}&[A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + C_{\sigma(t)}v(t)]dt \\ &+ [D_{\sigma(t)}x(t) + E_{\sigma(t)}u(t) + F_{\sigma(t)}v(t)]dw(t)\end{aligned} \tag{31}$$

for the continuous-time domain, where $w(t)$ is a one-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathbb{P})$ and satisfies $\mathbb{E}[dw(t)] = 0$ and $\mathbb{E}[dw^2(t)] = dt$; and

$$\begin{aligned}x(l+1) = {}&A_{\sigma(l)}x(l) + B_{\sigma(t)}u(l) + C_{\sigma(l)}v(l) \\ &+ [D_{\sigma(l)}x(l) + E_{\sigma(l)}u(l) + F_{\sigma(l)}v(l)]w(l)\end{aligned} \tag{32}$$

for the discrete-time domain, where $w(l)$ is a scalar Gaussian white noise with $\mathbb{E}[w(l)] = 0$ and $\mathbb{E}[w^2(l)] = \theta$. The controller is $u(t) = K_{\sigma(t-d)}x(t)$ for the continuous-time version (31) and $u(l) = K_{\sigma(l-d)}x(l)$ for the discrete-time version (32), respectively.

We are now in a position to state the following theorem concerning the stabilizing switched controllers for the switched stochastic linear systems in the time-delayed switching case.

Theorem 5. *Consider the continuous-time switched stochastic linear system (31), and the constants $\varphi, \eta > 0$ and $\mu > 1$ are given. If there exist matrices $X_i = X_i^T > 0, X_j = X_j^T > 0, Y_i, Y_j$ with appropriate dimensions and constants $\lambda_i > 0$, $i \in \mathcal{M}$, such that for all $(i, j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$,*

$$\begin{bmatrix} G(X_i, Y_i) + \varphi X_i & X_iD_i^T + Y_i^TE_i^T & C_i \\ * & -X_i & F_i \\ * & * & -\lambda_iI \end{bmatrix} \leq 0, \tag{33}$$

$$\begin{bmatrix} G(X_j, Y_j) - \eta X_j & X_jD_i^T + Y_j^TE_i^T & C_i \\ * & -X_j & F_i \\ * & * & -\lambda_iI \end{bmatrix} \leq 0, \tag{34}$$

$$X_j - \mu X_i \leq 0, \tag{35}$$

where $G(X, Y) := A_iX + B_iY + X^TA_i^T + Y^TB_i^T$, then there exist stabilizing switched controllers with the time delay $\mathcal{T}_{\max}$ such that the system (31) is SISS for the switching signal satisfying the ADT condition (9), and the controller gain is $K_i = Y_iX_i^{-1}$.

Proof. If the ith subsystem is activated in $[t_k, t_{k+1})$, then the jth controller $u(t) = K_jx(t)$ is active in $[t_k, t_k + \mathcal{T}_{\max})$, and the ith controller $u(t) = K_ix(t)$ is active in $[t_k + \mathcal{T}_{\max}, t_{k+1})$. Thus, in the interval $[t_k, t_{k+1})$, the closed-loop system is divided into two parts: for $t \in [t_k, t_k + \mathcal{T}_{\max})$,

$$dx(t) = [\hat{A}_ix(t) + C_iv(t)]dt + [\hat{D}_ix(t) + F_iv(t)]dw(t);$$

for $t \in [t_k + \mathcal{T}_{\max}, t_{k+1})$,

$$dx(t) = [\bar{A}_ix(t) + C_iv(t)]dt + [\bar{D}_ix(t) + F_iv(t)]dw(t),$$

where $\hat{A}_i := A_i + B_iK_j$, $\hat{D}_i := D_i + E_iK_j$, $\bar{A}_i := A_i + B_iK_i$ and $\bar{D}_i := D_i + E_iK_i$.

Choose $\rho_i(\|v\|) := \lambda_iv^Tv$ and the Lyapunov function $V_i(x) := x^TP_ix$ for all $\sigma(t-d) = i \in \mathcal{M}$, where $\lambda_i > 0$ and $P_i^{-1} = X_i$ in (33)–(35). Thus, it follows from Theorem 1 and Remark 1 that, if for all $(i, j) \in \mathcal{M} \times \mathcal{M}$ and $i \neq j$,

$$\alpha_1(|x|) \le V_i(x) \le \alpha_2(|x|), \tag{36}$$

$$\mathcal{L}V_i(t, x(t)) + \varphi V_i(t, x(t)) - \lambda_iv^Tv \le 0, \tag{37}$$

$$\mathcal{L}V_i(t, x(t)) - \eta V_i(t, x(t)) - \lambda_iv^Tv \le 0, \tag{38}$$

$$V_i(t_k, x(t_k)) - \mu V_j(t_k, x(t_k)) \le 0, \tag{39}$$

then the system (31) is SISS for the switching signal with the ADT condition (9). Define $\alpha_1(s) := \lambda_{\min}(P_i)s^2$ and $\alpha_2(s) := \lambda_{\max}(P_i)s^2$, where $\lambda_{\min}(P_i)$ and $\lambda_{\max}(P_i)$ are respectively the smallest and the largest eigenvalue of the matrix P_i, then (36) holds. For all $t \in [t_k + \mathcal{T}_{\max}, t_{k+1})$, we have

$$\begin{aligned}&\mathcal{L}V_i(x) + \varphi V_i(x) - \lambda_iv^Tv \\ &\quad = x^T\left(\bar{A}_i^TP_i + P_i\bar{A}_i + \varphi P_i + \bar{D}_i^TP_i\bar{D}_i\right)x \\ &\qquad + 2x^T\left(P_iC_i + \bar{D}_i^TP_iF_i\right)v + v^T\left(F_i^TP_iF_i - \lambda_iI\right)v \le 0,\end{aligned}$$

which is equivalent to $\begin{bmatrix}x^T & v^T\end{bmatrix}\Phi_1\begin{bmatrix}x^T & v^T\end{bmatrix}^T \le 0$, where

$$\Phi_1 := \begin{bmatrix}\bar{A}_i^TP_i + P_i\bar{A}_i + \varphi P_i + \bar{D}_i^TP_i\bar{D}_i & P_iC_i + \bar{D}_i^TP_iF_i \\ * & F_i^TP_iF_i - \lambda_iI\end{bmatrix}.$$

Applying Schur complement lemma to $\Phi_1 \le 0$ gives that

$$\begin{bmatrix}\bar{A}_i^TP_i + P_i\bar{A}_i + \varphi P_i & \bar{D}_i^TP_i & P_iC_i \\ * & -P_i & P_iF_i \\ * & * & -\lambda_iI\end{bmatrix} \le 0. \tag{40}$$

Pre- and post-multiplying the left-hand side of (40) by $diag\{P_i^{-1}, P_i^{-1}, I\}$, respectively, yields that

$$\begin{bmatrix}P_i^{-1}\bar{A}_i^T + \bar{A}_iP_i^{-1} + \varphi P_i^{-1} & P_i^{-1}\bar{D}_i^T & C_i \\ * & -P_i^{-1} & F_i \\ * & * & -\lambda_iI\end{bmatrix} \le 0. \tag{41}$$

Define $X_i := P_i^{-1}$ and $Y_i := K_iP_i^{-1}$. Thus, (33) implies (37). Similarly, for all $t \in [t_k, t_k + \mathcal{T}_{\max})$, (38) is rewritten as

$$\begin{aligned}&\mathcal{L}V_j(x) - \eta V_j(x) - \lambda_iv^Tv \\ &\quad = x^T\left(\hat{A}_i^TP_j + P_j\hat{A}_i - \eta P_j + \hat{D}_i^TP_j\hat{D}_i\right)x \\ &\qquad + 2x^T\left(P_jC_i + \hat{D}_i^TP_jF_i\right)v + v^T\left(F_i^TP_jF_i - \lambda_iI\right)v \le 0.\end{aligned}$$

Along the same fashion as the case that $t \in [t_k + \mathcal{T}_{\max}, t_{k+1})$, it obtains that (38) holds because of (34).

Furthermore, $V_i(t_k, x(t_k)) \le \mu V_j(t_k, x(t_k))$ is equivalent to $P_i - \mu P_j \le 0$. Based on Schur complement lemma, $P_i - \mu P_j \le 0$ holds if and only if

$$\begin{bmatrix}-\mu P_j & I \\ I & -P_i^{-1}\end{bmatrix} \le 0.$$

Pre- and post-multiplying the left-hand side of the above inequality by $diag\{P_j^{-1}, I\}$, it follows that

$$\begin{bmatrix}-\mu X_j & X_j \\ X_j^T & -X_i\end{bmatrix} \le 0.$$

Using Schur complement lemma again and the fact that $-X_i < 0$, it follows that $-X_i + \mu^{-1}X_j \le 0$. That is, if (35) holds, then $P_i - \mu P_j \le 0$. Moreover, if there is a feasible solution satisfying the linear matrix inequalities (LMIs) (33)–(35), then the controller gain is $K_i = Y_iX_i^{-1}$. Therefore, the proof is completed. ■

Theorem 6. *Consider the discrete-time switched stochastic linear system (32), and the constants $\varphi \in (0, 1)$, $\eta > -1$ and $\mu > 1$ are given. If there exist matrices $X_i = X_i^T > 0$, $X_j = X_j^T > 0$, Y_i, Y_j with appropriate dimensions and constants $\lambda > 0$, $i \in \mathcal{M}$, such that for all $(i, j) \in \mathcal{M} \times \mathcal{M}$, $i \neq j$,*

$$\begin{bmatrix}-X_i & 0 & A_iX_i + B_iY_i & C_i \\ * & -\theta^{-1}X_i & D_iX_i + E_iY_i & F_i \\ * & * & (\varphi - 1)X_i & 0 \\ * & * & * & -\lambda_iI\end{bmatrix} \le 0, \tag{42}$$

$$\begin{bmatrix}-X_j & 0 & A_iX_j + B_iY_j & C_i \\ * & -\theta^{-1}X_j & D_iX_j + E_iY_j & F_i \\ * & * & -(1 + \eta)X_j & 0 \\ * & * & * & -\lambda_iI\end{bmatrix} \le 0, \tag{43}$$

$$X_j - \mu X_i \le 0, \tag{44}$$

then there exist stabilizing switched controllers with the time delay $\mathcal{T}_{\max}$ such that the system (32) is SISS for the switching signal satisfying the ADT condition (20), and the controller gain is $K_i = Y_iX_i^{-1}$.

Proof. Similar to the proof of Theorem 5, suppose the ith subsystem is activated in $[l_k, l_{k+1})$, the jth controller $u(l) = K_jx(l)$ is active in $[l_k, l_k + \mathcal{T}_{\max})$, and the ith controller $u(l) = K_ix(l)$ is active in $[l_k + \mathcal{T}_{\max}, l_{k+1})$. Thus, in the interval $[l_k, l_{k+1})$, the closed-loop system has two parts: for $l \in [l_k, l_k + \mathcal{T}_{\max})$,

$$x(l+1) = \hat{A}_ix(l) + C_iv(l) + [\hat{D}_ix(t) + F_iv(t)]w(l);$$

for $l \in [l_k + \mathcal{T}_{\max}, l_{k+1})$,

$$x(l+1) = \bar{A}_ix(l) + C_iv(l) + [\bar{D}_ix(l) + F_iv(l)]w(l).$$

Define $V_i(x) := x^TP_ix$ for all $\sigma(l-d) = i \in \mathcal{M}$, where $P_i^{-1} = X_i$ in (42)–(44). It follows that $\lambda_{\min}(P_i)x^Tx \le V_i(x) \le \lambda_{\max}(P_i)x^Tx$. Based on Theorem 2, if for all $(i, j) \in \mathcal{M} \times \mathcal{M}$ and $i \neq j$,

$$\mathbb{E}[\Delta V_i(l, x(l))] + \varphi V_i(l, x(l)) - \lambda_iv^Tv \le 0, \tag{45}$$

$$\mathbb{E}[\Delta V_i(l, x(l))] - \eta V_i(l, x(l)) - \lambda_iv^Tv \le 0, \tag{46}$$

$$V_i(l_k, x(l_k)) - \mu V_j(l_k, x(l_k)) \le 0, \tag{47}$$

then the system (33) is SISS for the switching signal with the ADT condition (20). Thus, for $l \in [l_k + \mathcal{T}_{\max}, l_{k+1})$, one has

$$\begin{aligned}&\mathbb{E}[\Delta V_i(x(l))] + \varphi V_i(l, x(l)) - \lambda_iv^Tv \\ &\quad = \mathbb{E}[V_i(x(l_{k+1})) - V_i(x(l_k))] + \varphi V_i(l, x(l)) - \lambda_iv^Tv \\ &\quad = \mathbb{E}\left\{\left[\bar{A}_ix + C_iv + (\bar{D}_ix + F_iv)w\right]^TP_i\left[\bar{A}_ix + C_iv\right.\right. \\ &\qquad \left.\left. + (\bar{D}_ix + F_iv)w\right] - x^TP_ix\right\} + \varphi V_i(l, x(l)) - \lambda_iv^Tv\end{aligned}$$

$$= \left(\bar{A}_i x + C_i v\right)^T P_i \left(\bar{A}_i x + C_i v\right) + \theta \left(\bar{D}_i x + F_i v\right)^T P_i \times \left(\bar{D}_i x + F_i v\right) - x^T P_i x + \varphi V_i(l, x(l)) - \lambda_i v^T v \le 0.$$

which implies that (45) is equivalent to $\left[x^T \quad v^T\right] \Phi_2 \left[x^T \quad v^T\right]^T \le 0$, where Φ_2 is

$$\begin{bmatrix} \bar{A}_i^T P_i \bar{A}_i + (\varphi - 1)P_i + \theta \bar{D}_i^T P_i \bar{D}_i & \bar{A}_i^T P_i C_i + \theta \bar{D}_i^T P_i F_i \\ * & C_i^T P_i C_i + \theta F_i^T P_i F_i - \lambda_i I \end{bmatrix}.$$

Using Schur complement lemma twice to $\Phi_2 \le 0$ gives that

$$\begin{bmatrix} -P_i & 0 & P_i \bar{A}_i & P_i C_i \\ * & -\theta^{-1} P_i & P_i \bar{D}_i & P_i F_i \\ * & * & (\varphi - 1)P_i & 0 \\ * & * & * & -\lambda_i I \end{bmatrix} \le 0. \tag{48}$$

Pre- and post-multiplying the left-hand side of (48) by $diag\{P_i^{-1}, P_i^{-1}, P_i^{-1}, I\}$ and define $X_i := P_i^{-1}$ and $Y_i := K_i P_i^{-1}$, it obtains that (45) is achieved from (42). Analogously, for the interval $l \in [l_k, l_k + \mathcal{T}_{\max})$, (46) is rewritten to be

$$\mathbb{E}[\Delta V_j(x)] - \eta V_j(l, x(l)) - \lambda_i v^T v$$
$$= \mathbb{E}[V_j(x(l_{k+1})) - V_j(x(l_k))] - \eta V_j(l, x(l)) - \lambda_i v^T v$$
$$= \left(\hat{A}_i x + C_i v\right)^T P_j \left(\hat{A}_i x + C_i v\right) + \theta \left(\hat{D}_i x + F_i v\right)^T P_j \times \left(\hat{D}_i x + F_i v\right) - x^T P_j x - \eta V_j(l, x(l)) - \lambda_i v^T v \le 0.$$

Similar to the case that $l \in [l_k + \mathcal{T}_{\max}, l_{k+1})$, if (43) holds, then (46) is established. Moreover, along the same fashion as the proof of Theorem 5, (44) implies (47). By solving the LMIs (42)–(44), we have that the controller gain is $K_i = Y_i X_i^{-1}$. Therefore, the proof is completed. ■

Remark 5. In Theorems 5 and 6, all the time delays are the same and set to be $\mathcal{T}_{\max}$, which facilitates the stability analysis and simplifies the practical application of the obtained results. However, the setting of the time delays is not a constraint for our results. The time delay d_k can be artificially enlarged to the bound by implementing a buffer between the system and the controller [32]. If the time delays are arbitrary but bounded, and the increasing interval $\mathcal{T}_{\uparrow}(t_k, t_{k+1})$ and the decreasing interval $\mathcal{T}_{\downarrow}(t_k, t_{k+1})$ are multiple for the Lyapunov function in the switching interval (t_k, t_{k+1}), then the similar results can also be obtained, but more complicated.

Remark 6. Theorems 5 and 6 recover Theorem 1 in [21] and Theorem 2 in [27] as special cases. In the proofs of Theorems 5 and 6, the interval $[t_k, t_{k+1})$ is the analysis unit, and the multiple Lyapunov-like functions are coincident with the switching of the candidate controllers. If the interval $[t_{k-1} + \mathcal{T}_{\max}, t_k + \mathcal{T}_{\max})$ is considered as the analysis unit, the similar result can be obtained. On the other hand, if the multiple Lyapunov-like functions are coincident with the switching of the system modes, the analogous results could also be derived. However, the derived matrix inequalities may not be linear and hard to be solved.

For the mismatched switching case, the existence conditions for the stabilizing switched controllers are presented.

Theorem 7. *Consider the continuous-time switched stochastic linear system (31), and the constants $\varphi, \eta > 0$ and $\mu \ge 1$ are given. Suppose the switching signal satisfies (23)–(24). If there exist matrices $X_i = X_i^T > 0, X_j = X_j^T > 0, Y_i, Y_j$ with appropriate dimensions and constants $\lambda_i > 0, i \in \mathcal{M}$, such that (33) holds for all $i \in \mathcal{M}_s$, (34) for all $i \in \mathcal{M}_u$, (35) for all $i, j \in \mathcal{M}$, then there exist switched controllers such that the system (31) is SISS, and the controller gains are $K_i = Y_i X_i^{-1}$, where $i \in \mathcal{M}$.*

The Sketch of Proof. Assume that the ith subsystem is activated in $[t_k, t_{k+1})$. If the ith controller $u(t) = K_i x(t)$ is active in $[t_k, t_{k+1})$, then the controller and the subsystem match. Thus, the closed-loop system is

$$dx(t) = [\bar{A}_i x(t) + C_i v(t)]dt + [\bar{D}_i x(t) + F_i v(t)]dw(t).$$

If the jth controller $u(t) = K_j x(t)$ is active in $[t_k, t_{k+1})$, then the controller does not match the subsystem, and the closed-loop system is

$$dx(t) = [\hat{A}_i x(t) + C_i v(t)]dt + [\hat{D}_i x(t) + F_i v(t)]dw(t).$$

In the following, based on Theorem 3 and along the same fashion as in the proof of Theorem 5, the sufficient conditions are obtained and the stabilizing switched controllers could be derived to guarantee that the closed-loop system is stable. ■

Theorem 8. *Consider the discrete-time switched stochastic linear system (32), and the constants $\varphi \in (0, 1)$, $\eta > -1$ and $\mu \ge 1$ are given. Suppose the switching signal satisfies (29)–(30). If there exist matrices $X_i = X_i^T > 0, X_j = X_j^T > 0, Y_i, Y_j$ with appropriate dimensions and constants $\lambda > 0$, $i \in \mathcal{M}$, such that (42) holds for all $i \in \mathcal{M}_s$, (43) for all $i \in \mathcal{M}_u$, (44) for all $i, j \in \mathcal{M}$, then there exist switched controllers such that the system (32) is SISS, and the controller gains are $K_i = Y_i X_i^{-1}$, where $i \in \mathcal{M}$.*

Remark 7. In Theorems 7 and 8, the existence of the stabilizing switched controllers requires the exact information of the switching law or the switching times that belong to $\mathcal{M}_s$ and $\mathcal{M}_u$. However, the switching signal is not necessarily known *a priori* or detected immediately in practice, which leads to the invalidity of this assumption. Under the condition that the switching signal is not fully known *a priori* (the average dwell time condition is provided), how to construct the stabilizing switched controllers needs to further study.

5. Illustrative example

In this section, a continuous-time numerical example is presented to illustrate the validity of the obtained results. Similarly, the discrete-time results are also verified and omitted here.

Consider a continuous-time switched linear system consisting of two subsystems

$$dx = (A_\sigma x + B_\sigma u + C_\sigma v)dt + (D_\sigma x + E_\sigma u + F_\sigma v)dw,$$

where $\sigma \in \{1, 2\}$ and

$$A_1 = \begin{bmatrix} 0.2 & -0.5 \\ 0.5 & -0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.2 & 0.3 \\ -1 & 0.4 \end{bmatrix},$$
$$B_1 = E_1 = \begin{bmatrix} -0.4 \\ 1.8 \end{bmatrix},$$
$$B_2 = E_2 = \begin{bmatrix} 0.1 \\ 1.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.2 & 0.1 \\ -0.2 & 0.3 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 & -0.1 \\ 0.2 & 0.3 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \quad v(t) = 0.4e^{-0.4t},$$

and w is a scalar Gaussian white noise with zero-mean and variance of 5. Each subsystem represents an operational mode. It is obvious that above two subsystems are unstable. Given $\mathcal{T}_m = 0.5$ s, $\varphi = 0.75$, $\eta = 1.5$, $\mu = 1.02$, $\lambda_1 = \lambda_2 = 0.5$, by solving the

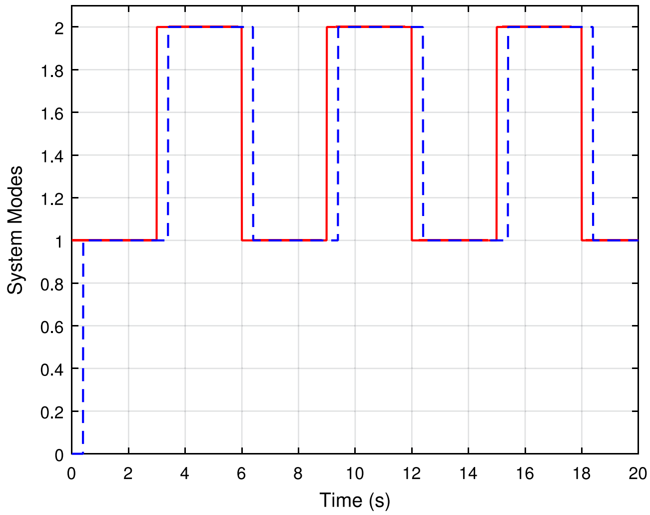

Fig. 1. Switching signals. The red solid line represents the system switching signal; the blue dotted line denotes the controller switching signal. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

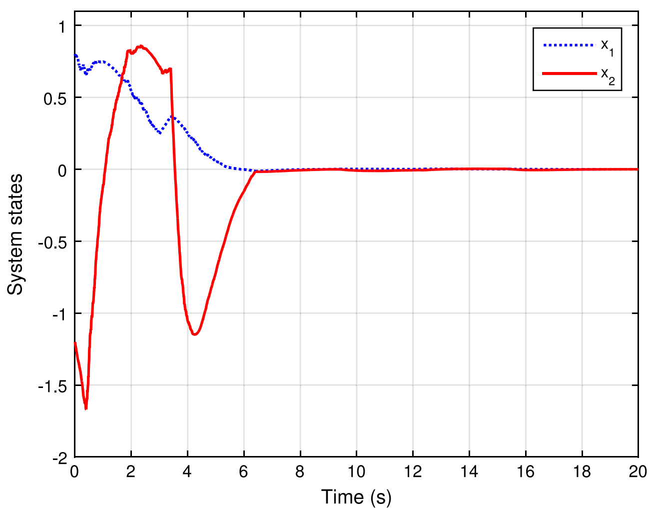

Fig. 2. Under the time-delayed switching case, state responses of the closed-loop system with $d = 0.4$, $\tau_a = 1.5264$.

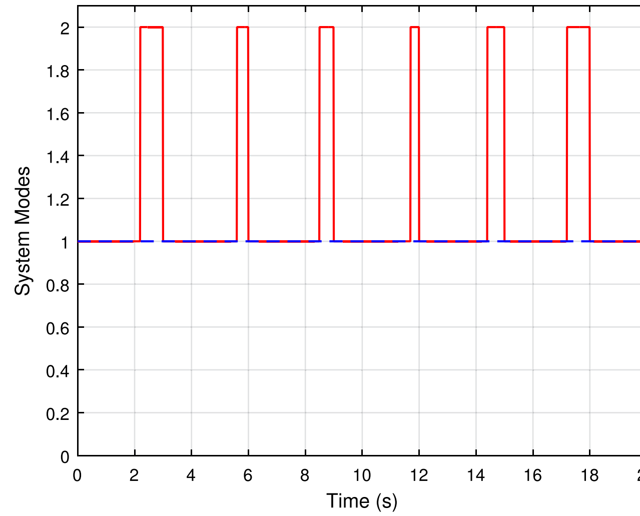

Fig. 3. Switching signals. The red solid line represents the system switching signal; the blue dotted line denotes the controller switching signal. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

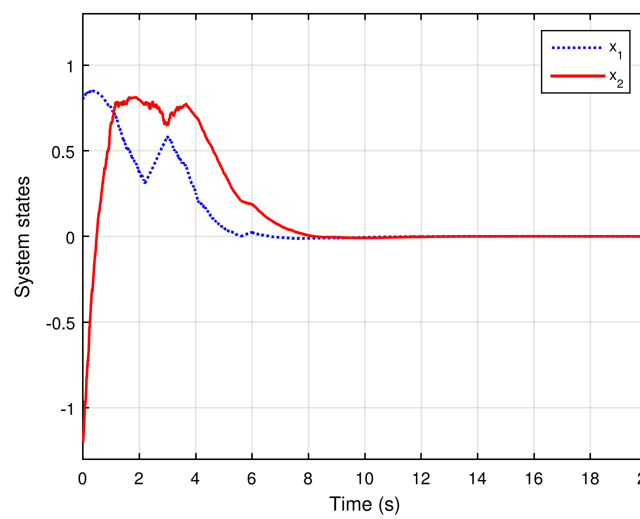

Fig. 4. Under the mismatched switching case, state responses of the closed-loop system.

LMIs in Theorem 5, it obtains that $\tau_a > 1.5264$ and the feasible controllers with the following gains

$$K_1 = \begin{bmatrix} 0.8550 & -0.4772 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -7.0069 & -1.5627 \end{bmatrix}.$$

For the time-delayed switching case, to verify the effectiveness of the proposed method, the switching signals of the system modes and the controllers are presented in Fig. 1. The switching intervals of the system modes and the switched controller are 3 s, and the time delay between switches of the system modes and the switched controllers is 0.4 s $< \mathcal{T}_m$. Under the above conditions, the state response of the closed-loop system is presented in Fig. 2. Observe that the designed switched controllers can guarantee the stochastic stability of the switched stochastic systems with time-delayed switching.

On the other hand, for the mismatched switching case, the controller 1 is activated. Thus, the subsystem 1 is the matched subsystem and that the subsystem 2 is the mismatched subsystem. Based on the average dwell time condition in Theorem 3, it obtains that $\lambda < \frac{1}{3}$. Set $\lambda = 0.3$ and $\mathcal{I}_0 = 0.1$, then $\tau_a > 0.2640$ and $\mathcal{I}_u(0, t) \leq 0.1 + 0.3t$. Thus, the switching signal that satisfies the average dwell time condition is given in Fig. 3, which is aperiodic. Furthermore, by solving the LMIs in Theorem 7, the feasible controllers are established with the following gains

$$K_1 = \begin{bmatrix} 0.8531 & -0.4801 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -1.6092 & -1.1681 \end{bmatrix}.$$

According to the designed switched controllers, the state response of the closed-loop system is illustrated in Fig. 4, which suggests that the closed-loop system is stochastically stable in the mismatched switching case.

6. Conclusion

In this paper, stability and stabilization problems were considered for switched stochastic systems under asynchronous switching. Both the time-delayed switching case and the mismatched switching case were studied. Using average dwell-time condition and Lyapunov approach, sufficient conditions were achieved to guarantee the stochastic input-to-state stability of the whole system. For switched stochastic linear systems, the stabilizing switched controllers were designed for the time-delayed switching case and the mismatched switching case.

Future research could be directed to controller/observer design for switched stochastic nonlinear systems, and stability analysis of impulsive stochastic nonlinear systems with delays, interconnected switched stochastic nonlinear systems.

References

[1] C. Liu, Z. Gong, Optimal Control of Switched Systems Arising in Fermentation Processes, vol. 97, Springer, 2014.
[2] Z. Li, Y. Soh, C. Wen, Switched and Impulsive Systems: Analysis, Design and Applications, vol. 313, Springer Science & Business Media, 2005.
[3] D. Ma, J. Zhao, Stabilization of networked switched linear systems: An asynchronous switching delay system approach, Systems Control Lett. 77 (2015) 46–54.
[4] M. Donkers, W. Heemels, N. Van De Wouw, L. Hetel, Stability analysis of networked control systems using a switched linear systems approach, IEEE Trans. Automat. Control 56 (9) (2011) 2101–2115.
[5] S. Bundfuss, M. Dür, Copositive Lyapunov functions for switched systems over cones, Systems Control Lett. 58 (5) (2009) 342–345.
[6] J. Xiong, J. Lam, Z. Shu, X. Mao, On stability of Markovian switched systems with dwell time, IEEE Trans. Automat. Control 59 (1) (2014) 180–186.
[7] R. Shorten, F. Wirth, O. Mason, K. Wulff, C. King, Stability criteria for switched and hybrid systems, SIAM Rev. 49 (4) (2007) 545–592.
[8] J.P. Hespanha, A.S. Morse, Stability of switched systems with average dwell-time, in: Proceedings of the 38th IEEE Conference on Decision and Control, 1999, pp. 2655–2660.
[9] S. Dashkovskiy, A. Mironchenko, Input-to-state stability of nonlinear impulsive systems, SIAM J. Control Optim. 51 (3) (2013) 1962–1987.
[10] Z. Wu, M. Cui, P. Shi, H.R. Karimi, Stability of stochastic nonlinear systems with state-dependent switching, IEEE Trans. Automat. Control 58 (8) (2013) 1904–1918.
[11] D. Chatterjee, D. Liberzon, Stability analysis of deterministic and stochastic switched systems via a comparison principle and multiple Lyapunov functions, SIAM J. Control Optim. 45 (1) (2006) 174–206.
[12] P. Zhao, W. Feng, Y. Kang, Stochastic input-to-state stability of switched stochastic nonlinear systems, Automatica 48 (10) (2012) 2569–2576.
[13] X. Su, P. Shi, L. Wu, Y. Song, Fault detection filtering for nonlinear switched stochastic systems, IEEE Trans. Automat. Control 61 (5) (2016) 1310–1315.
[14] J. Lian, P. Shi, Z. Feng, Passivity and passification for a class of uncertain switched stochastic time-delay systems, IEEE Trans. Cybern. 43 (1) (2013) 3–13.
[15] U. Shaked, E. Gershon, Robust H_∞ control of stochastic linear switched systems with dwell time, Internat. J. Robust Nonlinear Control 24 (11) (2014) 1664–1676.
[16] L. Wu, W.X. Zheng, H. Gao, Dissipativity-based sliding mode control of switched stochastic systems, IEEE Trans. Automat. Control 58 (3) (2013) 785–791.
[17] G. Zhai, B. Hu, K. Yasuda, A.N. Michel, Stability analysis of switched systems with stable and unstable subsystems: An average dwell time approach, Int. J. Syst. Sci. 32 (8) (2001) 1055–1061.
[18] B. Niu, J. Zhao, Barrier Lyapunov functions for the output tracking control of constrained nonlinear switched systems, Systems Control Lett. 62 (10) (2013) 963–971.
[19] M.A. Müller, D. Liberzon, Input/output-to-state stability and state-norm estimators for switched nonlinear systems, Automatica 48 (9) (2012) 2029–2039.
[20] G. Yang, D. Liberzon, A Lyapunov-based small-gain theorem for interconnected switched systems, Systems Control Lett. 78 (2015) 47–54.
[21] L. Zhang, H. Gao, Asynchronously switched control of switched linear systems with average dwell time, Automatica 46 (5) (2010) 953–958.
[22] D. Antunes, J. Hespanha, C. Silvestre, Stability of networked control systems with asynchronous renewal links: An impulsive systems approach, Automatica 49 (2) (2013) 402–413.
[23] P. Mhaskar, N.H. El-Farra, P.D. Christofides, Robust predictive control of switched systems: satisfying uncertain schedules subject to state and control constraints, Internat. J. Adapt. Control Signal Process. 22 (2) (2008) 161–179.
[24] Z.-G. Wu, P. Shi, H. Su, J. Chu, Asynchronous l_2–l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities, Automatica 50 (1) (2014) 180–186.
[25] Y.-E. Wang, J. Zhao, B. Jiang, Stabilization of a class of switched linear neutral systems under asynchronous switching, IEEE Trans. Automat. Control 58 (8) (2013) 2114–2119.
[26] D.M. de la Pena, P.D. Christofides, Stability of nonlinear asynchronous systems, Systems Control Lett. 57 (6) (2008) 465–473.
[27] L. Zhang, P. Shi, Stability, $\mathcal{L}_2$-gain and asynchronous control of discrete-time switched systems with average dwell time, IEEE Trans. Automat. Control 54 (9) (2009) 2192–2199.
[28] X. Mao, Stochastic Differential Equations and Applications, Elsevier, 2007.
[29] L. Rogers, D. Williams, Diffusions, Markov Processes, and Martingales: Vol. 1, Foundations, Cambridge University Press, 2000.
[30] C. Cai, A.R. Teel, Characterizations of input-to-state stability for hybrid systems, Systems Control Lett. 58 (1) (2009) 47–53.
[31] R.G. Sanfelice, Input-Output-to-State stability tools for hybrid systems and their interconnections, IEEE Trans. Automat. Control 59 (5) (2014) 1360–1366.
[32] J. Zhang, Y. Lin, P. Shi, Output tracking control of networked control systems via delay compensation controllers, Automatica 57 (2015) 85–92.