Staining algorithm for full waveform inversion
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Summary

Full waveform inversion (FWI) has the potential to provide high resolution velocity parameters. FWI is an iterative algorithm that modifies the velocity model each step. Each step of iteration consists of a reverse time migration process, which calculates the gradient of misfit function by correlating the forward wavefield of the actual sources and the backward wavefield of the receiver data residuals. If some structures such as those in subsalt shadow zones cannot be imaged well by reverse time migration, these areas can hardly be well reconstructed by full waveform inversion either. Staining algorithm is introduced to improve the signal-to-noise ratio of poorly illuminated subsurface structures. Staining algorithm has a significant advantage that it is amplitude-preserved for the locally recovered wavefield, which makes it feasible to be adopted to waveform inversion process. Combination of waveform inversion and staining algorithm is promising to get better resolution of velocity parameter in poorly illuminated areas.

Introduction

Full waveform inversion is based on the generalized least-squares criterion, which minimizes the misfit between recorded and modeled data. The gradient of the misfit function is estimated by correlating the forward wavefield of the actual sources and the backward wavefield of the receiver data residuals (Taratola, 1984). The only difference between this process and reverse time migration is that receiver data misfit is back propagated in full waveform inversion, while receiver data is back propagated in reverse time migration. However, the data misfit function is strongly nonlinear, in most cases, the inversion will fall into a local minimum due to circle skipping. Bunks et al. (1995) proposes multiscale waveform inversion, which processes subdata sets from low frequencies to high frequencies because low frequency contents are more linear with respect to model perturbation. Boonyasiriwat et al. (2009) improves the efficiency of time domain multiscale strategy by using some low-pass filters for choosing optimal frequency bands.

In order to enhance the image quality of poorly illuminated areas in reverse time migration, staining algorithm is introduced (Chen and Jia, 2014). By using staining algorithm, we can stain any target structure in the velocity model and excite an imaginary wavefield, which is synchronized with the real wavefield but only contains the response relevant to the target structure; therefore, we can trace and identify the reflections from a certain desired structure in the wavefield and the recorded data. A generalized staining algorithm (Li and Jia, 2015) which is amplitude-preserved for the stained wavefield is proposed and applied to migration, which guarantees the amplitude of migration image remains the same magnitude between using the imaginary wavefield and the real wavefield.

In this paper, we apply staining algorithm in full waveform inversion. Since staining algorithm is proved practically effective for enhancing RTM image in local areas, we use the algorithm to improve the gradient in full waveform inversion. As the iteration goes on, we get a more accurate inverted image which is locally enhanced.

Theory

In full waveform inversion, we define the misfit function as

\[ E(\nu) = \frac{1}{2} (L(\nu) - d_{obs})^T (L(\nu) - d_{obs}) \]

where \( L \) is the modeling operator, and \( d_{obs} \) is the observed data. The optimization problem is to minimize the misfit function. Expanding the misfit function near the model using Taylor Expansion, we obtain

\[ \nu^{(k+1)} = \nu^{(k)} - H^{-1}(k) \nabla E^{(k)} \]

in which \( k \) denotes the \( k \)-th iteration, \( \nu \) denotes the iterated model, \( H \) is the Hessian matrix and \( \nabla E \) is the gradient of the misfit function.

We calculate the gradient by applying variation principle to the acoustic wave equation, and obtain

\[ g(\nu) = \frac{\delta E}{\delta \nu} = \frac{2}{\nu} \int_0^T \frac{\partial^2 p}{\partial \nu^2} \, dt \]

in this formula, \( p \) represents the forward propagated source wavefield, \( \lambda \) represents the back propagated residual data wavefield. The residual data is calculated by subtraction of observed receiver data and synthetic receiver data. The integral part represents the correlation of source wavefield and residual wavefield. The process is quite similar to reverse time migration.

The Hessian matrix is usually neglected due to its high computational cost. We use some approximation instead of directly calculating the Hessian matrix. Equation (2) is then replaced with

\[ \nu^{(k+1)} = \nu^{(k)} - \alpha_k d_k \]

\( \alpha_k \) is the step length calculated by a line search algorithm, and \( d_k \) is the searching direction, which is related to the gradient but calculated by different inversion method, such
as steepest-descent, conjugate-gradient or quasi-Newton methods.

We use Wiener filter to obtain low frequency contents for multiscale inversion (Boonyasirivat et al., 2009). Wiener filter is computed by

\[ f_{\text{Wiener}}(\omega) = \frac{W_{\text{target}}(\omega)W^*_\text{original}(\omega)}{|W_{\text{original}}(\omega)|^2} \]  

where \( W_{\text{original}} \) denotes the original wavelet, \( W_{\text{target}} \) denotes the low frequency target wavelet, and symbol * denotes the complex conjugate. Both source wavelet and observed data have to be low filtered by Wiener filter.

In multiscale inversion, the low frequencies yield the large scale structure, and the high frequencies yield high-resolution details. On the other hand, staining algorithm is a promising processing technique in seismic migration that improves signal-to-noise ratio especially in poorly illuminated areas (Chen and Jia, 2014). A generalized staining algorithm is given as follows

\[ \frac{\partial^2 \tilde{p}}{\partial t^2} = \tilde{v}^2 \Delta \tilde{p} + \tilde{s} \]  
\[ \frac{\partial^2 p}{\partial t^2} = \hat{v}^2 \Delta p \]  
\[ p|_{\hat{v}=1} = \tilde{p} \]  

where \( \tilde{p} \) and \( \hat{v} \) represent the real wavefield and the real velocity, \( \tilde{s} \) represents the real source, \( \tilde{p} \) and \( \tilde{v} \) represent the imaginary wavefield and the imaginary velocity, and \( \hat{v} = 1 \) represents the stained area. Equations (6) and (7) describe the real wavefield and the imaginary wavefield respectively. The real wavefield and the imaginary wavefield are propagated separately, and the only connection is to inject the real wavefield at stained area as the source of the imaginary wavefield. Furthermore, the generalized staining algorithm is amplitude-preserved.

In our research, staining algorithm is implemented in the high frequencies contents of multiscale to achieve a more detailed gradient. Staining algorithm is able to provide a better migration image, so it provides a better gradient for full waveform inversion. We can apply staining algorithm in the determination of the gradient. We calculate the correlation in the stained area using the imaginary wavefield, while in other areas using the real wavefield. The image using the imaginary wavefield has a higher signal-to-noise ratio than the real wavefield. By doing this we obtain a new gradient which has a higher resolution in the stained area. Figure 1 illustrates the workflow of full waveform inversion with staining algorithm.

**Numerical examples**

Staining algorithm has the advantage of enhancing the image quality in poorly illuminated areas. The algorithm has been well implemented in reverse time migration. Some tests of this algorithm in BP model is shown in figure 2.

The overhanging structure in BP model is poorly illuminated, which becomes a great challenge for most migration methods. In the staining algorithm, the stained area works like a lamp which lights up the adjacent structures. Compared with the real wavefield, the imaginary wavefield has a relatively smaller amplitude, while the phase and waveform are almost the same. Nevertheless, the imaginary wavefield only contains the response close to the stained area, and less reflections from the unstained areas. Therefore, we achieve a more accurate local image using the imaginary wavefield. Figure 2 shows that it is very effective to get better local image by staining algorithm.
We apply full waveform inversion in Marmousi model. Our full waveform inversion uses a limited memory quasi-Newton method, which we call the l-BFGS method (Deng et al., 2012). The initial velocity model is given by blurring the standard Marmousi velocity model. Our goal is to obtain an enhanced image in the black frame in figure 3(a). The blue line is the stained area. When we apply staining algorithm, we calculate local gradient using the imaginary wavefield. That is, the difference whether we use staining algorithm or not is the value of gradient in the black frame. Figure 3 shows the conventional FWI without staining algorithm. Figure 4 is the extraction of the gradient in the black frame. Figure 4(a) shows the local gradient without staining algorithm, while figure 4(b) shows the local gradient with staining algorithm. We can see that in the black frame, gradient with staining algorithm is more consistent with the true model, especially in the area marked in red. However, figures 4(a) and 4(b) do not show much difference, because the illumination is adequate in Marmousi model. Only where it is poorly illuminated, the staining algorithm gives a obviously higher signal-to-noise ratio.

If some areas are poorly illuminated and hard for migration, it is also a challenge to calculate the gradient in these areas. Consequently, the final velocity model after inversion is of poor quality. By applying staining algorithm, we get a more accurate gradient, thus getting better inverted result. Further tests will be done with SEG and BP models, which is promising to obtain better results in poorly illuminated areas.

Conclusion

We take advantage of amplitude-preservation in generalized staining algorithm, and apply it to full waveform inversion to calculate the gradient of the misfit function. Wavefield calculated by staining algorithm only contains the response close to the stained structure; hence the gradient using staining algorithm has a higher signal-to-noise ratio, especially in poorly illuminated areas. Therefore, the quality of inversion result in the stained area gets improved.

Acknowledgments

This study received support from National Natural Science Foundation of China (41374006, 41274117).

Figure 2. (a) BP 2004 benchmark velocity model. The white dashed line and solid line are stained structures for 2(c) and 2(d) respectively. (b) The conventional RTM image. (c) Image by staining algorithm corresponding to the dashed line. (d) Image by staining algorithm corresponding to the solid line.
Figure 3. (a) The true model. (b) The initial model. (c) The gradient in the first iteration. (d) The inverted model after 100 iterations.

Figure 4. Extraction of the gradient in the black frame in figure 3. (a) Gradient of misfit function in the black frame using the real wavefield. (b) Gradient in the black frame using the imaginary wavefield. (c) Marmousi velocity in the black frame for comparison.
EDITED REFERENCES
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