Frequency-velocity-adaptive Seismic Modeling Using A Generalized Average-derivative Optimal Scheme
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Summary
The computational cost for the frequency-domain finite-difference (FDFD) method is unbearable, which limits its application of dealing with large models. For the FDFD method, simulating seismic waves on the fixed grid in a heterogeneous model requires small grid spacing determined by minimal velocity and maximal frequency, which leads to extra computational cost for the high-velocity area and the high-frequency component. To improve the computational efficiency, we propose a frequency-velocity-adaptive (FVA) gridding strategy to reduce the number of discrete grid points and introduce a generalized average-derivative optimal scheme to implement seismic modeling on the FVA grid. A numerical example is presented to show that seismic modeling on the FVA grid is valid and efficient.

Introduction
Seismic wave numerical modeling is an important part of full waveform inversion (FWI) and migration and also a significant tool to understand wave propagation phenomenon in realistic underground media. The frequency-domain finite-difference (FDFD) method is widely used because it is convenient to manipulate single frequency wavefield. However, a crucial disadvantage of FDFD method is that it would cost memory and time prohibitively when we handle large-scale models. A huge sparse impedance matrix needs to be processed when solving the system of linear equations. The dimension of impedance matrix which is determined by the number of grid points is a key factor that controls the computational efficiency (Hustedt et al., 2004). To reduce the memory and time cost, fewer grid points are used in the numerical computation. The traditional finite-difference method prefers to use fixed grid, and the grid spacing is determined by the maximal frequency in the calculation and the minimal velocity of the entire model to ensure sufficient accuracy. As a result, the model is oversampled when we solve the high-frequency wavefield, and the high-velocity area in a heterogeneous model is also oversampled even if we focus on a single frequency. This leads to unnecessary extra computational costs.

Many kinds of variable grids are employed to avoid this extra computational costs. For the frequency-domain operators, frequency-adaptive (FA) grid is used to perform seismic modeling and migration (Li et al., 2015; Hu and Jia, 2016). They adopt coarse grid for the low frequency and fine grid for the high frequency to reduce the computational costs. Lots of papers on the velocity-adaptive (VA) non-uniform grid are published in recent years (Oprsal and Zahradnik, 1999; Oliveria, 2003; Chu and Stoffa, 2012; Zhang et al., 2013). They adopt coarse grid in the low-velocity area and fine grid in the high-velocity area.

To improve computational efficiency further, it is reasonable to let the grid spacings vary with frequency and velocity at the same time, i.e., frequency-velocity-adaptive (FVA) grid. However, before performing seismic modeling on the FVA grid, we must possess a frequency-domain operator which can model the seismic wave on the VA grid accurately. We develop a generalized average-derivative optimal scheme (GADOS) which is a frequency-domain operator and can be employed to implement seismic modeling on the VA grid (Li et al., 2016). Therefore, the GADOS is adopted to model the seismic wave on the FVA grid.

In this paper, we propose an FVA gridding strategy which can be used to convert the fixed grid to the FVA grid. The formula and idea of the GADOS are introduced briefly. A numerical example is presented to verify that seismic wavefield simulation on the FVA grid is valid and efficient.

Theory and method
Firstly, we present our FVA gridding strategy. The workflow of the FVA gridding strategy is:

Step 1: determine the fixed grid spacing Δfix. For a given model, we first scan it to acquire the minimal velocity in the entire model (vMIN) and the minimal velocities at each depth (vmin(z)) and each distance (vmin(x)). We calculate the minimal wavelength λmin, according to the maximal frequency fmax and the minimal velocity vMIN in the entire model. Then, the fixed grid spacing Δfix is equal to λmin/G (G is the number of grid points per wavelength and is related to the specific operator).
FVA Seismic Modeling with the GADOS

left side in the x-direction and from the surface in the z-direction. For each depth \( z_i \) and each distance \( x_i \), the grid spacing below \( z_i \) and to the right of \( x_i \) is determined as

\[
\Delta z_i = \frac{v_{\text{min}}(z_i)}{v_{\text{MIN}}} \Delta h_{\text{MIN}}, \\
\Delta x_i = \frac{v_{\text{min}}(x_i)}{v_{\text{MIN}}} \Delta h_{\text{MIN}}.
\]  

(2)

We continue this process until we reach the bottom and right side of the model. Compared to the strategy presented by Chu and Stoffa (2012), our gridding strategy does not require the adjustment of the minimal velocity profiles to ensure the minimal velocities never decrease. This allows us to obtain more VA grids.

From equations 1 and 2, we know that the model is sampled adequately when we ensure that the smallest grid spacing (\( \Delta h_{\text{fix}} \)) is sampled enough for the slowest velocity \( v_{\text{MIN}} \) and the maximal frequency \( f_{\text{max}} \). A schematic of the FVA grid is illustrated in Figure 1. Note that either step 2 or step 3 is dispensable and we can omit one of them to obtain the FA grid or the VA grid.

Secondly, we introduce the GADOS briefly and explain why it can be used to implement the seismic modeling on the VA grid.

Consider the 2D acoustic wave equation in frequency domain

\[
p(x, z, \omega) = S(x_0, z_0, \omega),
\]  

(3)

where \( p \) denotes frequency-domain wavefield, \( S \) denotes the source and \((x_0,z_0)\) is the source position. Based on the average-derivative technique (Chen, 2001, 2008), we introduce a GADOS for equation 3

\[
\begin{align*}
\Delta x_i (\tilde{p}_{m+1,n} - \tilde{p}_{m,n}) - \Delta x_k (\tilde{p}_{n+1} - \tilde{p}_{n-1})/2 &= S, \\
\Delta x_i (\tilde{p}_{m+1,n} - \tilde{p}_{m-1,n})/2 + \Delta x_k (\tilde{p}_{n+1} - \tilde{p}_{n-1})/2 &= S, \\
\end{align*}
\]  

(4)

where

\[
\begin{align*}
\tilde{p}_{m+1,n} &= \frac{1}{2} p_{m+1,n+1} + \frac{1}{2} p_{m+1,n-1}, \\
\tilde{p}_{m,n} &= \frac{1}{2} p_{m,n+1} + \frac{1}{2} p_{m,n-1}, \\
\tilde{p}_{m-1,n} &= \frac{1}{2} p_{m-1,n+1} + \frac{1}{2} p_{m-1,n-1}, \\
\tilde{p}_{m,n+1} &= \frac{1}{2} p_{m+1,n+1} + \frac{1}{2} p_{m+1,n-1}, \\
\tilde{p}_{m,n-1} &= \frac{1}{2} p_{m+1,n-1} + \frac{1}{2} p_{m+1,n-1}. \\
\end{align*}
\]
different. The stencils with different grid spacing ratios (i.e., stencils as Figure 1e, in which the grid spacing ratios are The VA grid can be regarded as an assembly of abundant grid. In the VA grid, the minimal grid spacing for each frequency (\(\Delta x\) and \(\Delta z\)) keep constant in the entire model.

Seismic modeling is implemented with the GADOS on the fixed grid, the FA grid, the VA grid and the FVA grid, respectively. The frequency-domain wavefields (f = 30Hz) and snapshots (t = 0.4s) are shown in Figure 3. From top to bottom, they are calculated by the GADOS on the fixed grid, the FA grid, the VA grid and the FVA grid, respectively. We can find that the results calculated on four different grids agree well. This demonstrates that the GADOS is valid and can be applied to implement seismic modeling on the FA grid, the VA grid and the FVA grid.

Numerical Example

We verify the FVA gridding strategy on a theoretical heterogeneous model in which the velocity varies with space gradually (Figure 2a). We establish such a model discretized with a 301 x 316 grid and a fixed grid spacing of 6 m. The source is a Ricker wavelet with the dominant frequency of 20 Hz, and the shot is at \(x = 900\ m, z = 60\ m\). The boundary conditions are the PML conditions, with a thickness of 20 layers. The frequency bandwidth is [2Hz, 60Hz].

To analyze the computational cost, we record the number of discrete grid number (\(nx \times nz\)) for each frequency and computational time required by four different grids and illustrate them in Figure 4. From Figure 4a, we can find that, in the fixed grid and the VA grid, the number of discrete grid number for each frequency is constant because the grid spacings don’t vary with frequency. In the FA and the FVA grid, the number of discrete grid number for each frequency increases with frequency and their upper limit is the number of discrete grid points in the fixed grid and the VA grid, respectively. This is easy to understand in view of our gridding strategy. From the Figure 4b, we can observe that the computational times required by four different grids are remarkably different. Compared to the computational time calculated on the fixed grid, the computational time cost for

\[
\alpha_1, \alpha_2, \beta_1, \beta_2 \text{ and } c_{ij} \text{ are weighted coefficients and } \sum_{j'=1}^{j} c_{ij} = 1. \text{ Figure 1e is shown for more details.}
\]

In equation 4, we formulate a linear combination of grid points to approximate \(\partial^2 p / \partial x^2\) in each column, and the coefficients are shown in Figure 1e. The same technique is employed in the approximation of \(\partial^2 p / \partial z^2\). The mass acceleration term is written as a linear combination of the central grid point and its neighbor grid points (Jo et al., 1996). For every discrete grid point, we select the corresponding optimal coefficients from the "Dictionary" according to the ratio of four grid spacings around it to ensure that the numerical dispersion in wave propagation is always minimal in the entire computational area. This is the key point of our approach to performing seismic modeling on continuous non-uniform grids with the GADOS.

The VA grid can be regarded as an assembly of abundant stencils as Figure 1e, in which the grid spacing ratios are different. The stencils with different grid spacing ratios (i.e., different \(\Delta x_1, \Delta x_2, \Delta z_1\) and \(\Delta z_2\)) lead to different optimal coefficients. We perform the numerical dispersion analysis for many stencils and then obtain a large "Dictionary" including many groups of optimal coefficients (Li et al., 2016). For every discrete grid point, we select the corresponding optimal coefficients from the "Dictionary" according to the ratio of four grid spacings around it to ensure that the numerical dispersion in wave propagation is always minimal in the entire computational area. This is the key point of our approach to performing seismic modeling on continuous non-uniform grids with the GADOS.

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**Numerical Example**

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The FVA gridding strategy is applied on this model to acquire the FVA grid. In the FVA grid, the minimal grid spacing for each frequency (\(\Delta h_{\text{MIN}}(f)\)) varies with frequency as in Figure 2b and the grid spacings (\(\Delta x\) and \(\Delta z\)) keep constant in the entire model.

Seismic modeling is implemented with the GADOS on the fixed grid, the FA grid, the VA grid and the FVA grid, respectively. The frequency-domain wavefields (f = 30Hz) and snapshots (t = 0.4s) are shown in Figure 3. From top to bottom, they are calculated by the GADOS on the fixed grid, the FA grid, the VA grid and the FVA grid, respectively. We can find that the results calculated on four different grids agree well. This demonstrates that the GADOS is valid and can be applied to implement seismic modeling on the FA grid, the VA grid and the FVA grid.

![Figure 2: (a) the velocity model. (b) the minimal grid spacing for each frequency varies with frequency; (c) the grid spacings in the horizontal direction vary with distance (for the maximal frequency); (d) the grid spacings in the vertical direction vary with depth (for the maximal frequency).](image)

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the FA grid, the VA grid and the FVA grid reduces to 44%, 60%, and 30%, respectively. This indicates that performing the seismic modeling using the FVA gridding strategy is more efficient.

**Conclusions**

In this paper, we propose a frequency-velocity-adaptive gridding strategy which can be employed to convert the fixed grid to the FVA grid. We use the GADOS to perform the seismic modeling on the FVA grid. A numerical example is presented to demonstrate that calculating the wavefield on the FVA grid is more efficient than on the fixed grid.

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REFERENCES