

§ 1.5.2 H原子光谱和Bohr原子模型—氢原子光谱线系



Ritz 公式 Rydberg–Ritz combination principle

$$\tilde{\nu} = R_H \left[\frac{1}{m^2} - \frac{1}{n^2} \right] = T(m) - T(n)$$

$$T(m) = \frac{R_H}{m^2} \quad \text{称为光谱项}$$

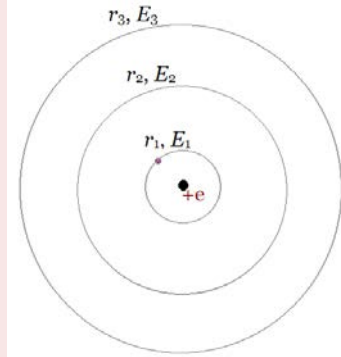


W. Ritz
(1878-1909)

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



Bohr原子模型三部曲



Niels Bohr

7 October 1885 – 18 November 1962

The Nobel Prize in Physics 1922

"for his services in the investigation of the structure of atoms and of the radiation emanating from them"

Niels Bohr (1913). "[On the Constitution of Atoms and Molecules, Part I](#)". *Philosophical Magazine* 26: 1–24.

Niels Bohr (1913). "[On the Constitution of Atoms and Molecules, Part II Systems Containing Only a Single Nucleus](#)". *Philosophical Magazine* 26: 476–502.
<http://web.ihep.su/dbserv/compas/src/bohr13b/eng.pdf>.

Niels Bohr (1913). "On the Constitution of Atoms and Molecules, Part III Systems containing several nuclei". *Philosophical Magazine* 26: 857–875.

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



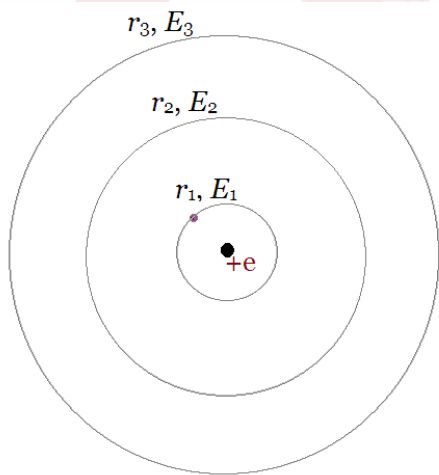
(1) 定态假设 经典轨道+定态条件

经典轨道：H原子中的电子围绕原子核作圆周运动 (Rutherford原子核式模型)。

定态条件：为了避免“原子塌缩”，借鉴了Planck的能量量子化观点。



Max Planck
(1858-1947)



$$E = T + V = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r}$$

假设H原子的能量也是量子化的。



$$E_n = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r_n}$$

H原子核外电子只能在一系列具有确定能量的分立的圆周轨道上运动。

定态条件： 并且假设在这些轨道上运动时不辐射电磁波。

Bohr将这样一些稳定的轨道称为**定态**。

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



(2) 频率条件

如何产生光谱辐射?

从Ritz 公式出发

$$\tilde{\nu} = T(m) - T(n)$$

$$T(n) = \frac{R_H}{n^2}$$



$$hc\tilde{\nu} = h\nu = hcT(m) - hcT(n)$$

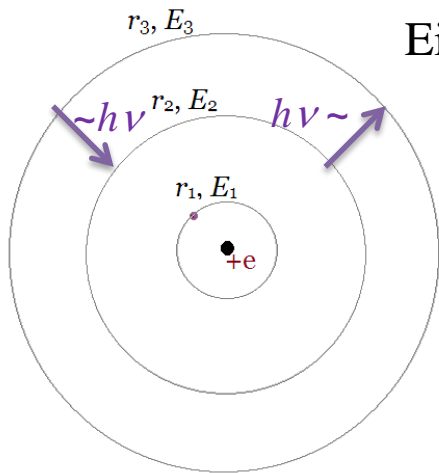
$$E_n = -hcT(n)$$

Einstein的光量子

$$= E_n - E_m$$

定态m的能量

定态n的能量



定态跃迁: 原子能量的改变是从一个定态跃迁(突然变到)到另一个定态。

跃迁过程放出(或吸收)一个光子, 其能量:

$$h\nu = |E_m - E_n|$$

Bohr频率规则

$$\nu = |E_m - E_n| / h$$



J. J. Balmer
(1825-1898)



W. Ritz
(1878-1909)



A. Einstein
(1879-1955)

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



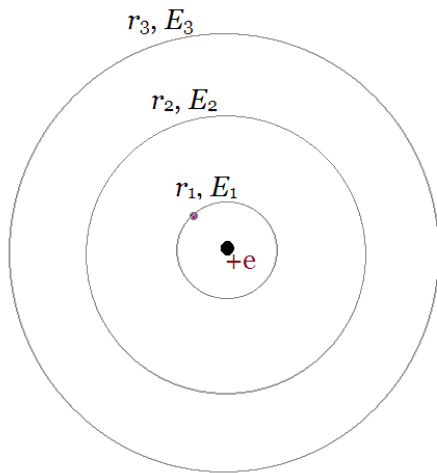
(3) 角动量量子化 (H原子)

$$L = L_n = n\hbar = n \frac{h}{2\pi}, \quad n = 1, 2, 3, \dots$$



N. Bohr
(1885-1962)

Bohr对应原理



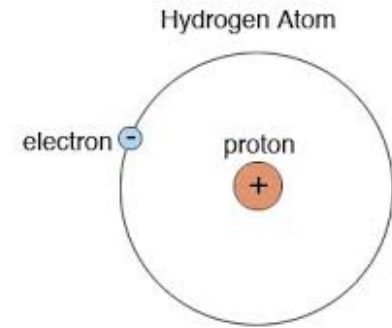
经典规律和量子规律虽然在形式上和内容上均有不同之处，看起来似乎是矛盾的。但这种矛盾只是它们应用在不同条件和场合下才出现，它们在各自的应用领域中均是正确的，因此，经典的和量子的物理量之间应该有一一对应的关系，在极限条件下，彼此趋于一致，即量子规律转化为经典规律，这叫做对应原理。

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



H原子辐射频率

经典理论
$$f = \frac{e}{2\pi} \sqrt{\frac{1}{4\pi\epsilon_0 m_e r^3}}$$



Bohr量子理论

从Rydberg经验公式出发，结合定态假设

H原子定态能量

$$\left. \begin{aligned} E_n &= -hcT(n) = -hc \frac{R_H}{n^2} \\ E_n &= -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r_n} \end{aligned} \right\} r_n = \frac{1}{4\pi\epsilon_0} \frac{e^2}{2R_H hc} n^2$$

H原子的定态能量和轨道半径都是正整数 n 的函数，即是量子化的

$n = 1, 2, 3, \dots$ 称为量子数

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



Bohr频率规则

$$\begin{aligned}\nu &= |E_m - E_n| / h = \left[\left(-hc \frac{R_H}{n^2} \right) - \left(-hc \frac{R_H}{m^2} \right) \right] / h \quad (\text{设 } m < n) \\ &= R_H c \left(\frac{1}{m^2} - \frac{1}{n^2} \right) = R_H c \frac{(n+m)(n-m)}{m^2 n^2}\end{aligned}$$

当 n 很大时，邻能级间跃迁的频率 ($n = m+1 \rightarrow \infty$)

$$\nu = R_H c \frac{(n+n-1)(n-n+1)}{(n-1)^2 n^2} \approx R_H c \frac{2}{n^3}$$

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



按照Bohr对应原理，在大量子数极限下，量子计算与经典计算一致

有
$$\frac{e}{2\pi} \sqrt{\frac{1}{4\pi\epsilon_0 m_e r^3}} = R_H c \frac{2}{n^3} \quad \Rightarrow \quad r = \sqrt[3]{\frac{1}{4\pi\epsilon_0} \frac{e^2}{16\pi^2 R_H^2 c^2 m_e}} n^2$$

又
$$r_n = \frac{1}{4\pi\epsilon_0} \frac{e^2}{2R_H hc} n^2$$

得到
$$R_H = \frac{m_e e^4}{(4\pi\epsilon_0)^2 4\pi\hbar^3 c}$$

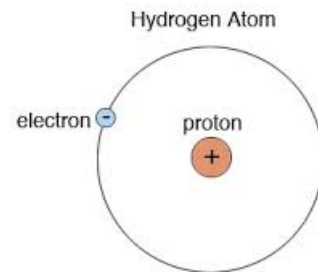
§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



将 $R_H = \frac{m_e e^4}{(4\pi\epsilon_0)^2 4\pi\hbar^3 c}$ 代回 $r_n = \frac{1}{4\pi\epsilon_0} \frac{e^2}{2R_H hc} n^2$

得到 $r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2$

又 $\frac{1}{2} m_e v^2 = \frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r}$ $\Rightarrow v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m_e r}}$



所以，角动量 $L = m_e v r = m_e \sqrt{\frac{e^2}{4\pi\epsilon_0 m_e r}} \cdot r = \sqrt{\frac{e^2}{4\pi\epsilon_0} m_e r} = \sqrt{\frac{e^2}{4\pi\epsilon_0} m_e \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2} = n\hbar$

在大量子数极限下，得到了角动量子化

$$L = n\hbar \quad \text{其中} \quad \hbar = \frac{h}{2\pi}$$

假设在小量子数下，即 n 小的时候也成立。

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论

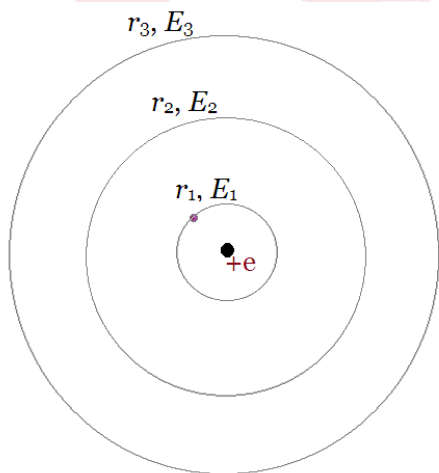


(3) 角动量量子化 (H原子)

$$L = L_n = n\hbar = n \frac{h}{2\pi}, \quad n = 1, 2, 3, \dots$$



N. Bohr
(1885-1962)



§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



H原子的大小

从角动量量子化出发

$$\left. \begin{aligned} L &= m_e v r = n \hbar \\ \frac{1}{2} m_e v^2 &= \frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r} \end{aligned} \right\} r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2 = n^2 a_0$$

复合常数

$$\begin{aligned} \text{其中 } a_0 &= \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = \frac{4\pi\epsilon_0 (\hbar c)^2}{e^2 m_e c^2} \\ &= \frac{1}{1.44 \text{eV} \cdot \text{nm}} \frac{(197 \text{eV} \cdot \text{nm})^2}{0.511 \times 10^6 \text{eV}} \\ &\approx 0.053 \text{nm} = 0.53 \times 10^{-10} \text{m} = 0.53 \text{\AA} \end{aligned}$$

$$\frac{e^2}{4\pi\epsilon_0} = 1.44 \text{eV} \cdot \text{nm}$$

$$\hbar c = 197 \text{eV} \cdot \text{nm}$$

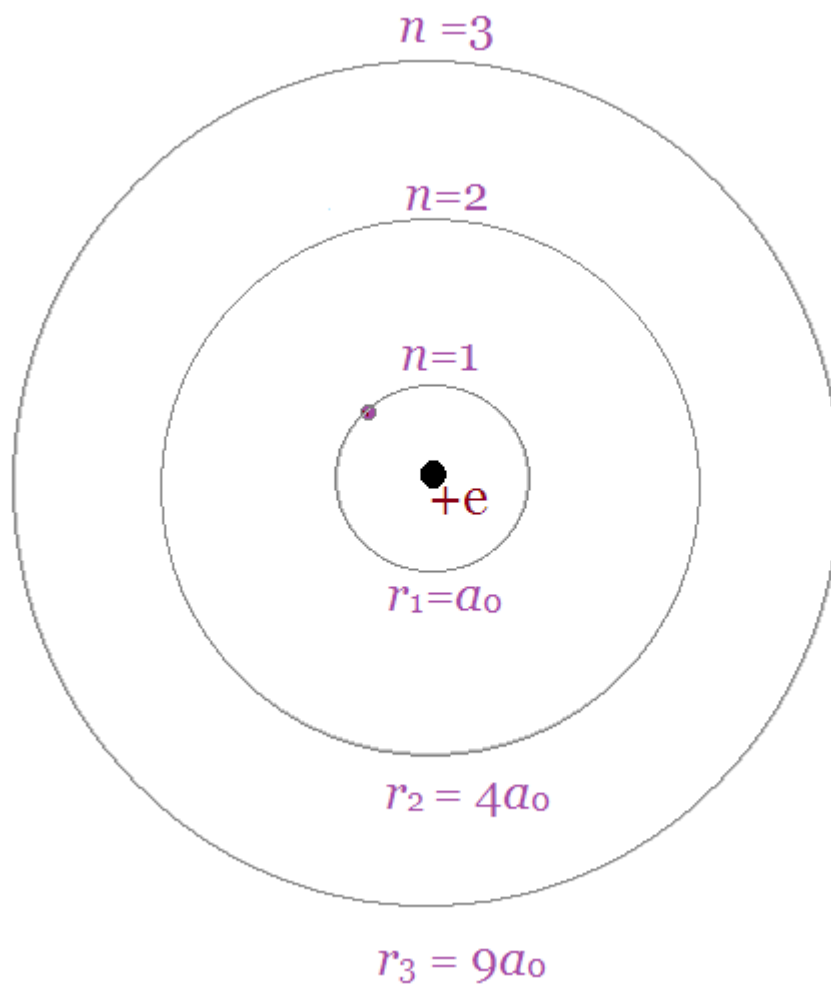
$$m_e c^2 = 0.511 \text{MeV}$$

称为玻尔半径。

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



H原子的大小



➤ 确定的原子大小： 10^{-10}m



H原子的电子运动的速度大小

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} n^2 \quad \text{代入} \quad v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m_e r}}$$

得

$$v = v_n = \frac{e^2}{4\pi\epsilon_0 n \hbar} = \frac{\alpha c}{n}$$

其中 $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$ 称为精细结构常数。

所以，电子的运动速度 $v \ll c$ ，因而不用考虑相对论。

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



H原子的定态能量

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} n^2 \quad \text{代入} \quad E = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r}$$

得

$$E_n = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r_n} = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0} \frac{m_e e^2}{4\pi\epsilon_0 \hbar^2 n^2} = -\frac{1}{(4\pi\epsilon_0)^2} \frac{m_e e^4}{2\hbar^2 n^2}$$
$$= -hcR_\infty \frac{1}{n^2} \quad n = 1, 2, 3, \dots$$

里德伯常数的理论公式

$$R_\infty = \frac{1}{(4\pi\epsilon_0)^2} \frac{m_e e^4}{4\pi\hbar^3 c}$$

$$E_n = -\frac{1}{2} m_e \alpha^2 c^2 \frac{1}{n^2}$$

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



H原子的能级和能级图

$$E_n = -\frac{1}{2} m_e \alpha^2 c^2 \frac{1}{n^2}$$

H原子的能级

基态：能量最低的 $n=1$ 的状态

基态能量：

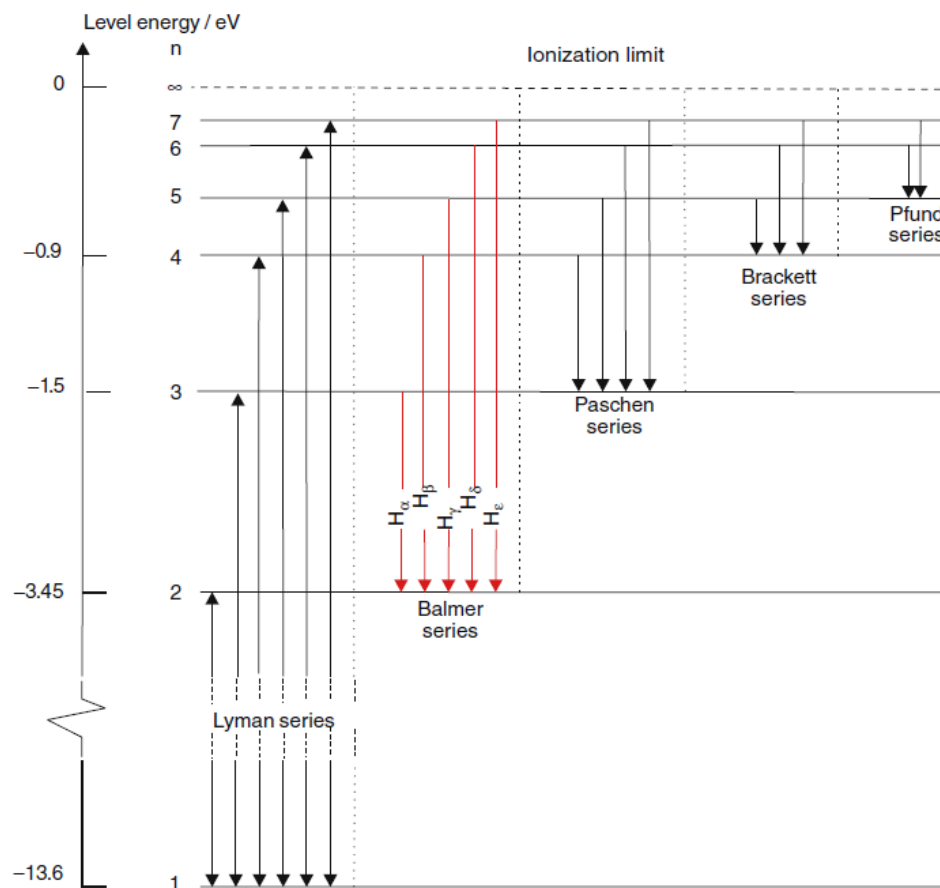
$$\begin{aligned} E_1 &= -\frac{1}{2} m_e \alpha^2 c^2 \frac{1}{1^2} \\ &= -\frac{1}{2} \times \left(\frac{1}{137} \right)^2 \times 0.511 \times 10^6 \text{ eV} \end{aligned}$$

$$\approx -13.6 \text{ eV}$$

激发态： $n > 1$ 的状态

激发态能量：

$$E_n = \frac{E_1}{n^2}$$



H原子的能级图

§ 1.5.3 H原子光谱和Bohr原子模型—Bohr氢原子理论



H原子的能级和能级图

$$E_n = -\frac{1}{2} m_e \alpha^2 c^2 \frac{1}{n^2}$$

H原子的能级

$n \rightarrow \infty$ 时, $E_n \rightarrow 0$

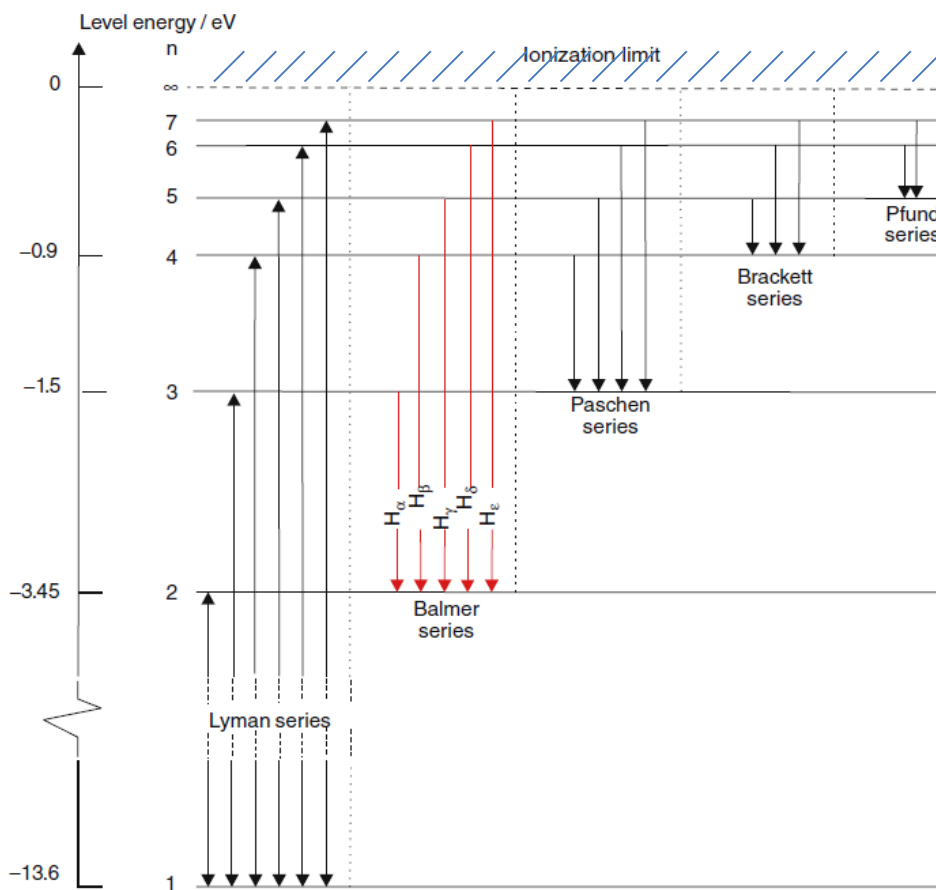
$$E = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r}$$

$r \rightarrow \infty$

电子脱离原子核库仑场的束缚, 成为自由电子, 称为**电离**。

自由电子可以具有一定的动能 T , 而 $r \rightarrow \infty$ 时, 势能 $V(r) = 0$, 所以总能量 $E = T > 0$, 轨道是双曲线的一支, 动能可以为任意值, 不是量子化的。

能级图上是 $n \rightarrow \infty$ 时, $E_n \rightarrow 0$ 之上连续的正能量区。



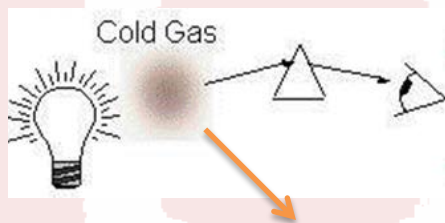
H原子的能级图

§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释

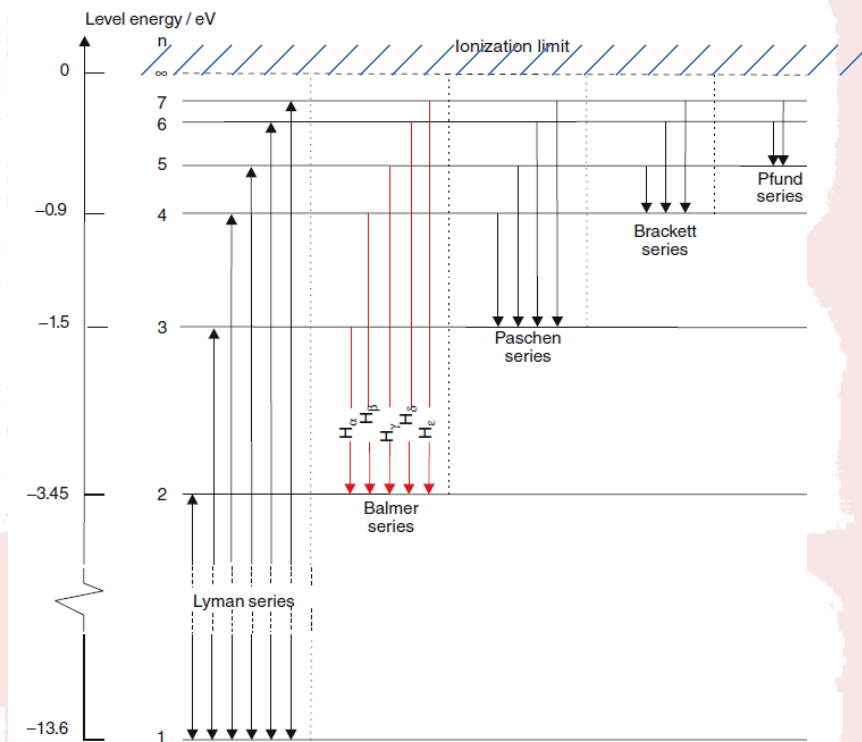


H原子的光谱

冷气体原子的吸收光谱



温度T



H原子的能级图

大量H原子可以处在 $n=1, 2, 3, \dots$ 等不同的能级，例如 N_1 个H原子处在 $n=1$ 能级、 N_2 个H原子处在 $n=2$ 能级、依次类推，服从Boltzmann统计分布。

$$N_i \propto \exp(-E_i / k_B T)$$

§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释



常温 $T=300K$

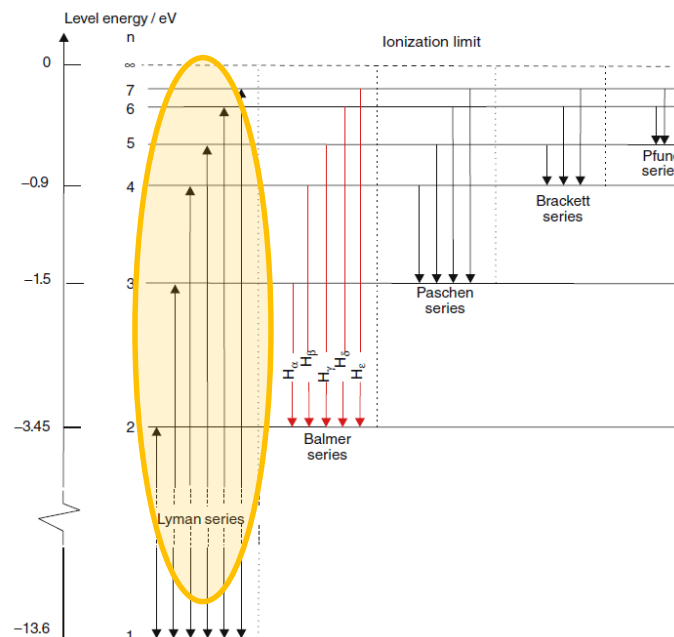
$$\frac{N_2}{N_1} \sim \frac{\exp(-E_2 / k_B T)}{\exp(-E_1 / k_B T)} = e^{-(E_2 - E_1) / k_B T}$$

Boltzmann 常数 $k = 8.62 \times 10^{-5} \text{ eV} \cdot \text{K}^{-1}$

$$E_1 = -13.6 \text{ eV} \quad E_2 = \frac{-13.6 \text{ eV}}{2^2} = -3.4 \text{ eV}$$

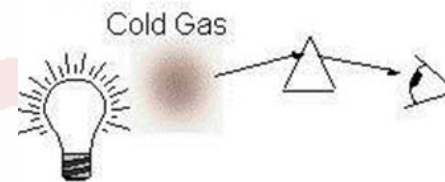
$$\frac{N_2}{N_1} \sim e^{-(-3.4 \text{ eV} + 13.6 \text{ eV}) / (8.62 \times 10^{-5} \text{ eV} \cdot \text{K}^{-1} \cdot 300 \text{ K})}$$

$$= e^{-394.6} \sim 4.2 \times 10^{-172} = 0$$



H原子的能级图

常温下，一团冷气体的原子几乎全部处在**基态**。白光通过，一些H原子吸收光子跃迁到激发态。



只有莱曼线系

$$hc\tilde{\nu} = E_n - E_1 = -\frac{1}{2} m_e \alpha^2 c^2 \frac{1}{n^2} + \frac{1}{2} m_e \alpha^2 c^2 \frac{1}{1^2}$$

§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释



$$\tilde{\nu} = (E_n - E_1) / hc = \frac{m_e \alpha^2 c}{2h} \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

$n = 2, 3, 4, \dots$

比较经验公式

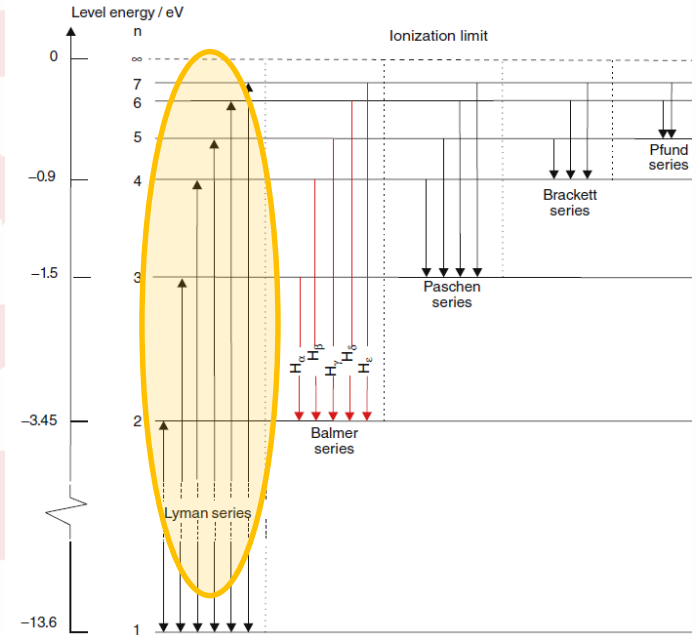
$$\tilde{\nu} = R_H \left[\frac{1}{1^2} - \frac{1}{n^2} \right]$$

里德伯常数的理论公式

$$R_\infty = \frac{1}{(4\pi\epsilon_0)^2} \frac{m_e e^4}{4\pi\hbar^3 c} = 1.097,373,156 \times 10^7 \text{ m}^{-1}$$

光谱实验值

$$R_H = 1.096,775,8 \times 10^7 \text{ m}^{-1}$$



H原子的能级图

理论计算与实验值符合得非常好!

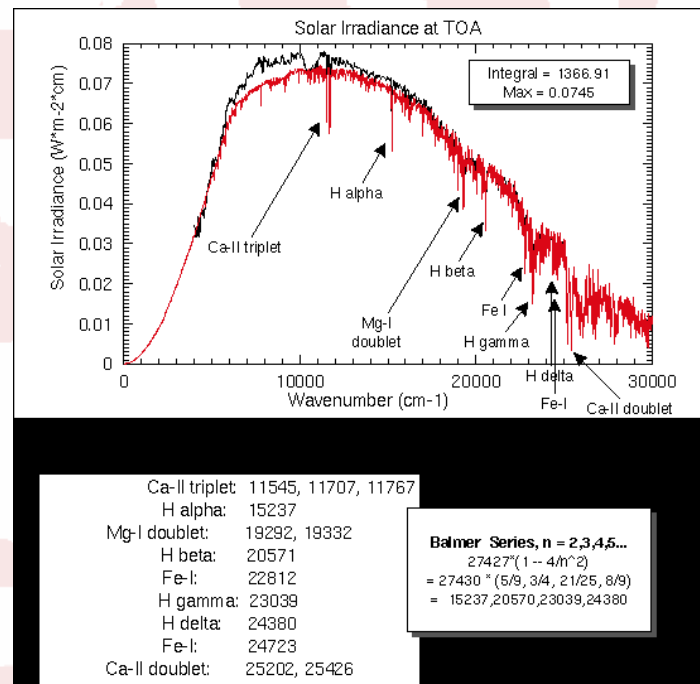
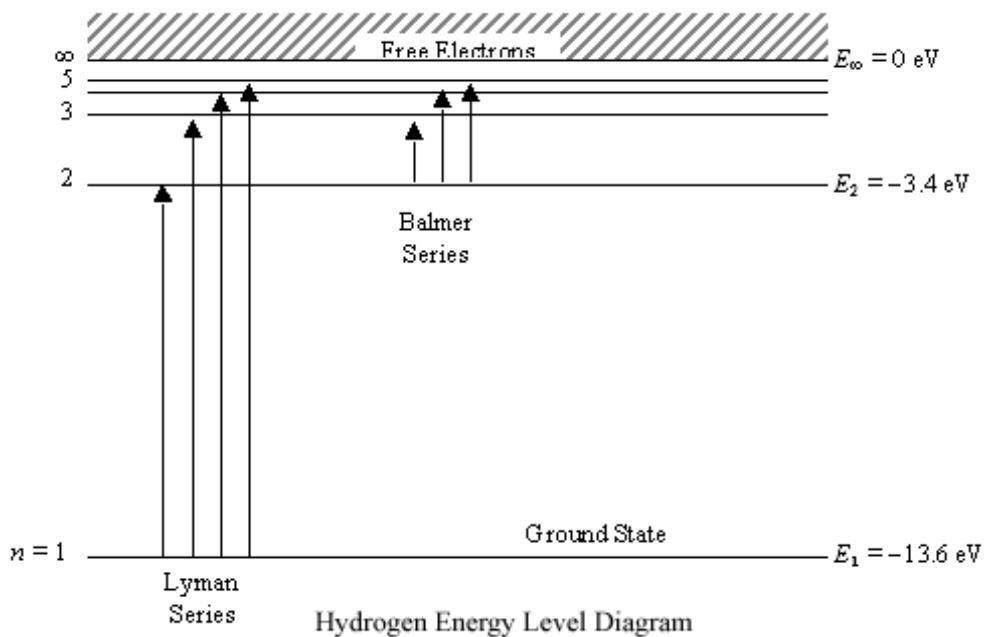
§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释



太阳表面 $T = 6000\text{K}$

$$\frac{N_2}{N_1} \sim e^{-(-3.4\text{eV} + 13.6\text{eV}) / (8.62 \times 10^{-5} \text{eV} \cdot \text{K}^{-1} \cdot 6000\text{K})}$$

$$= e^{-19.73} \sim 2.7 \times 10^{-9}$$

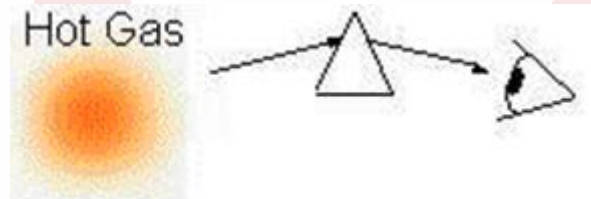


太阳光谱

§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释



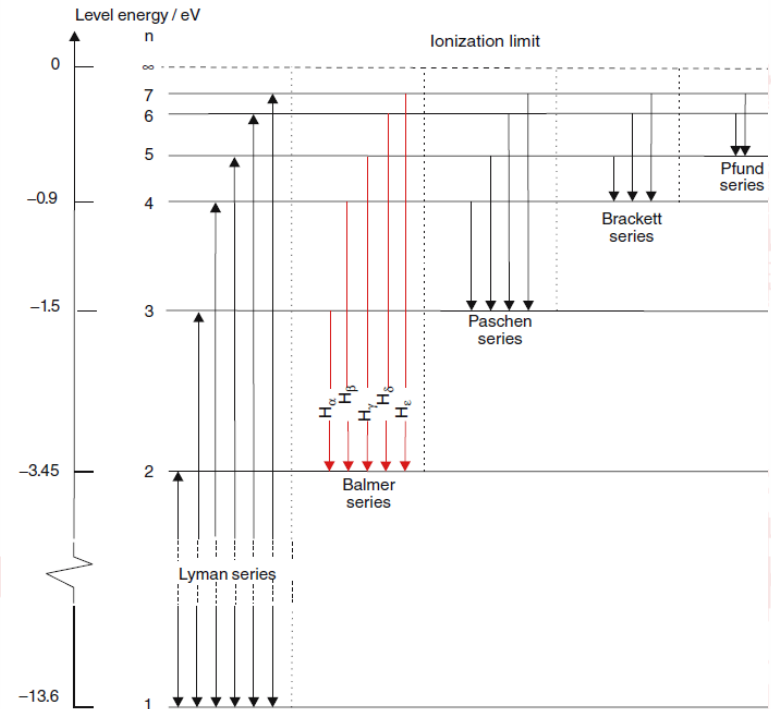
热气体原子的发射光谱



$$hc\tilde{\nu} = E_n - E_m \quad (m < n)$$
$$= -\frac{1}{2}m_e\alpha^2c^2\frac{1}{n^2} + \frac{1}{2}m_e\alpha^2c^2\frac{1}{m^2}$$

$$\tilde{\nu} = \frac{m_e\alpha^2c}{2h} \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

$$= R_\infty \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$



§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释



巴尔末线系 ($m=2$)

$$\tilde{\nu} = R_{\infty} \left[\frac{1}{2^2} - \frac{1}{n^2} \right], \quad n = 3, 4, 5, \dots$$

莱曼系 ($m=1$)

$$\tilde{\nu} = R_{\infty} \left[\frac{1}{1^2} - \frac{1}{n^2} \right], \quad n = 2, 3, 4, \dots$$

帕邢系 ($m=3$)

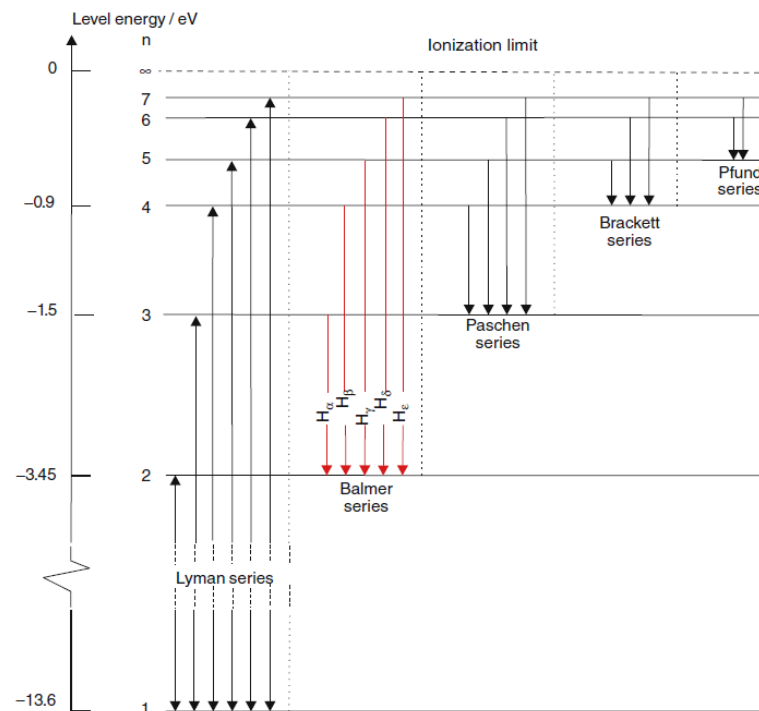
$$\tilde{\nu} = R_{\infty} \left[\frac{1}{3^2} - \frac{1}{n^2} \right], \quad n = 4, 5, 6, \dots$$

布喇开系 ($m=4$)

$$\tilde{\nu} = R_{\infty} \left[\frac{1}{4^2} - \frac{1}{n^2} \right], \quad n = 5, 6, 7, \dots$$

普丰特系 ($m=5$)

$$\tilde{\nu} = R_{\infty} \left[\frac{1}{5^2} - \frac{1}{n^2} \right], \quad n = 6, 7, 8, \dots$$



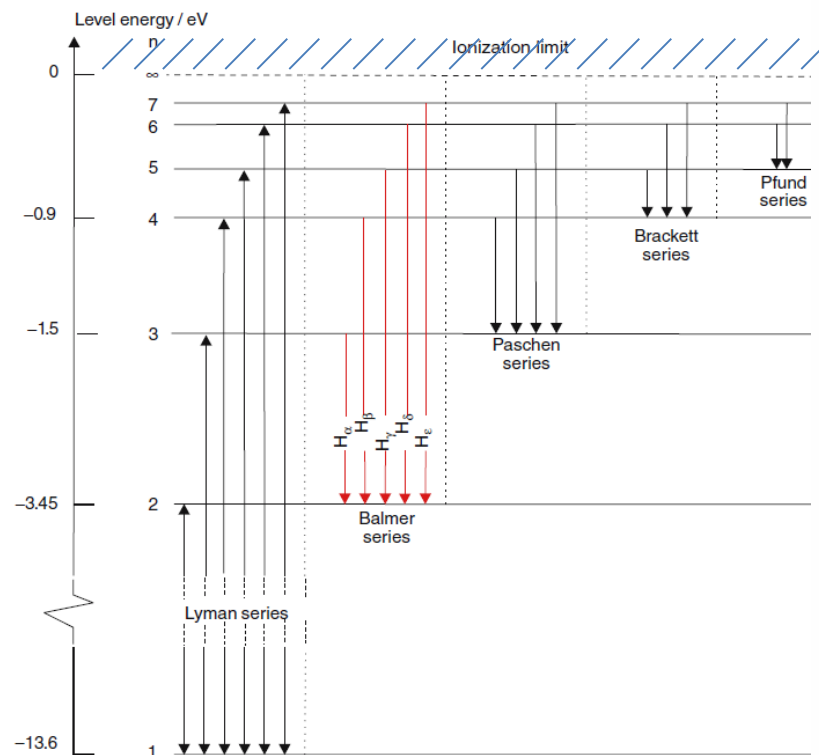
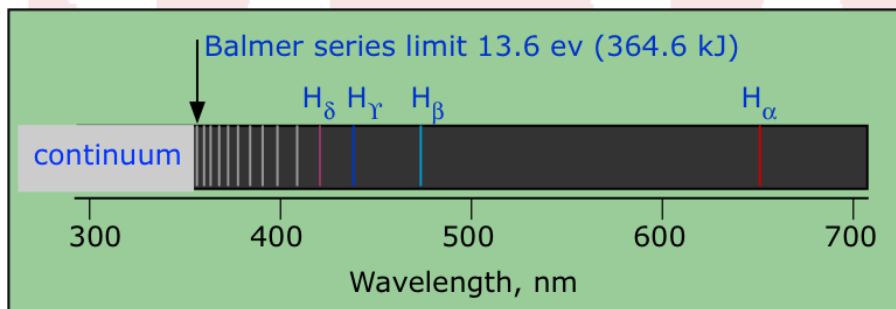
§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释



巴尔末线系 ($m=2$)

$$\tilde{\nu} = R_{\infty} \left[\frac{1}{2^2} - \frac{1}{n^2} \right], \quad n = 3, 4, 5, \dots$$

$$R_{\infty} = 1.097,373,156 \times 10^7 \text{ m}^{-1}$$



3→2 4→2 5→2 6→2

| Name | H-α | H-β | H-γ | H-δ | H-ε | H-ζ | H-η | limit |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Exp. (nm) | 656.3 | 486.1 | 434.1 | 410.2 | 397.0 | 388.9 | 383.5 | 364.6 |
| Theo. (nm) | 656.1 | 486.0 | 433.9 | 410.1 | | | | 364.5 |



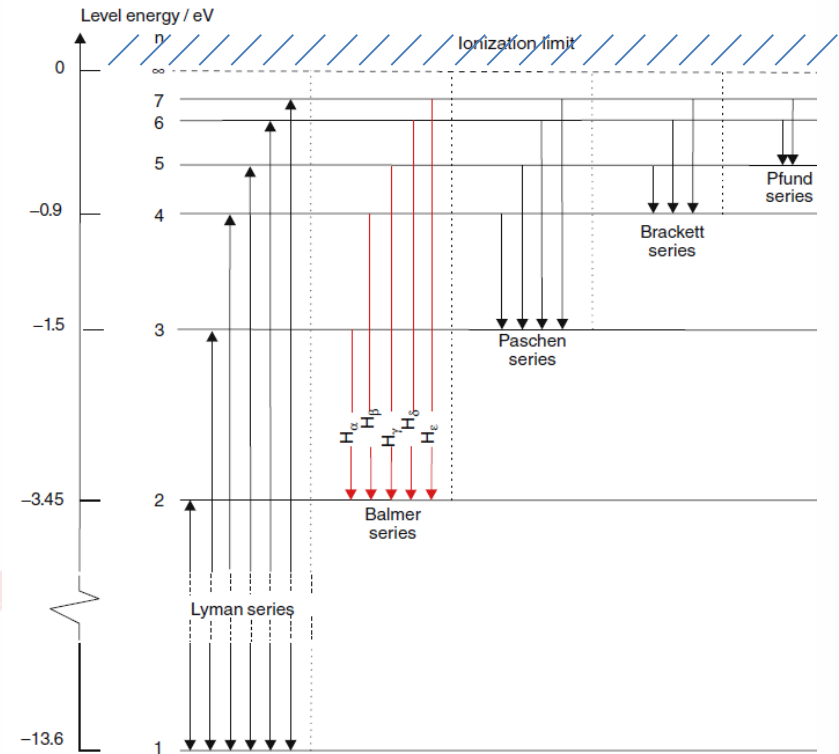
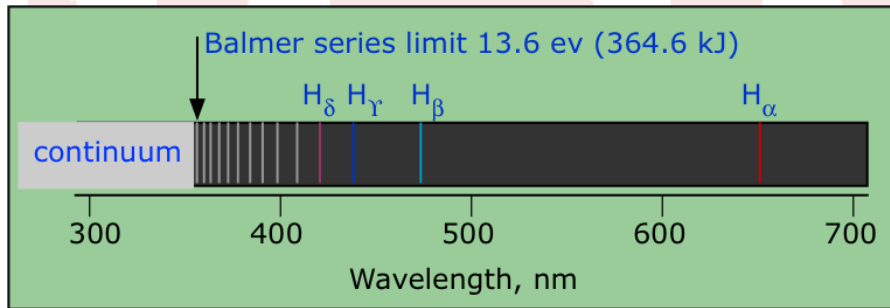
§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释

巴尔末线系的系限

$$n = \infty \rightarrow n = 2$$

$$\tilde{\nu}_{\infty} = R_{\infty} \left[\frac{1}{2^2} - \frac{1}{\infty^2} \right] = \frac{R_{\infty}}{2^2}$$

$$R_{\infty} = 1.097,373,156 \times 10^7 \text{ m}^{-1}$$



3→2 4→2 5→2 6→2 ∞→2

| Name | H- α | H- β | H- γ | H- δ | H- ϵ | H- ζ | H- η | limit |
|------------|-------------|------------|-------------|-------------|---------------|------------|-----------|-------|
| Exp. (nm) | 656.3 | 486.1 | 434.1 | 410.2 | 397.0 | 388.9 | 383.5 | 364.6 |
| Theo. (nm) | 656.1 | 486.0 | 433.9 | 410.1 | | | | 364.5 |

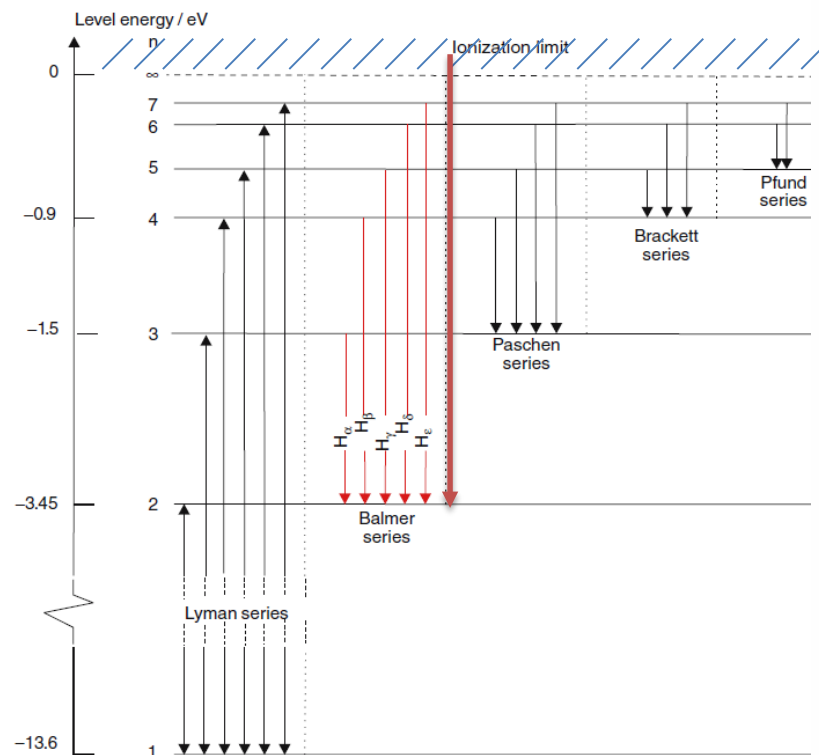
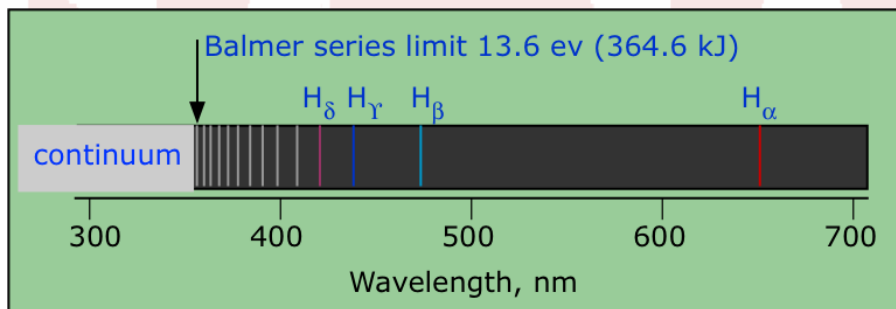
§ 1.5.3 H原子光谱和Bohr原子模型—氢原子光谱解释



系限外连续谱

连续态 $\rightarrow n = 2$

$$\tilde{\nu} = \frac{1}{2} m_e v^2 + \frac{R_\infty}{2^2}$$



3 \rightarrow 2 4 \rightarrow 2 5 \rightarrow 2 6 \rightarrow 2 $\infty \rightarrow$ 2

| Name | H- α | H- β | H- γ | H- δ | H- ϵ | H- ζ | H- η | limit |
|------------|-------------|------------|-------------|-------------|---------------|------------|-----------|-------|
| Exp. (nm) | 656.3 | 486.1 | 434.1 | 410.2 | 397.0 | 388.9 | 383.5 | 364.6 |
| Theo. (nm) | 656.1 | 486.0 | 433.9 | 410.1 | | | | 364.5 |

§ 1.6 氢原子理论的检验



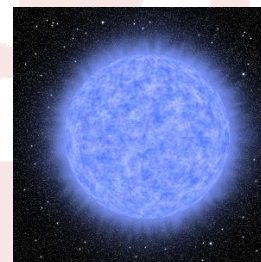
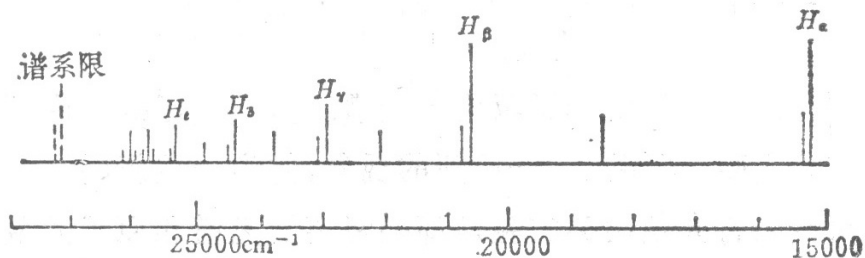
玻尔氢原子理论

- ① 解释了H原子的大小；
- ② 完美地解释了H原子光谱；
- ③ 理论上精确计算了里德伯常数。



E. C. Pickering
(1846-1919)

问题一 1897年，天文学家毕克林在观察船尾座ζ星(中国古天文名：弧矢增二十二)的星光光谱时发现了一个新的线系，称为**毕克林线系**。



Zeta Puppis

$$\tilde{\nu} = R' \left(\frac{1}{2^2} - \frac{1}{k^2} \right) \quad \text{其中, } k = 2.5, 3, 3.5, 4, \dots$$

一直被当作H原子的一个光谱线系



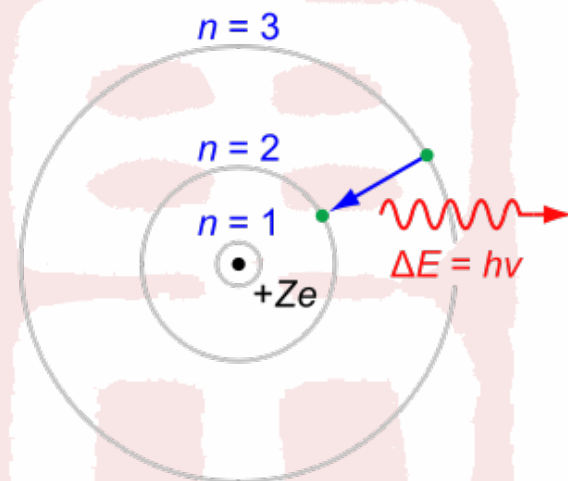
§ 1.6 氢原子理论的检验 - 类氢离子光谱



Bohr将其归属于He的一价离子的光谱。

He⁺, Li⁺⁺, Be⁺⁺⁺, 都是类氢离子:

+Ze的原子核与核外的一个电子组成。



$$E_n = -\frac{1}{2} m_e c^2 \alpha^2 \frac{Z^2}{n^2} = Z^2 E_H, \quad n = 1, 2, 3, \dots$$

$$r_n = \frac{\hbar c n^2}{Z m_e c^2 \alpha} = \frac{r_H}{Z}, \quad n = 1, 2, 3, \dots$$

光谱波数 $\tilde{\nu} = Z^2 R_\infty \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$

对于He⁺

$$\tilde{\nu} = 2^2 R_\infty \left(\frac{1}{m^2} - \frac{1}{n^2} \right) = R_\infty \left(\frac{1}{(m/2)^2} - \frac{1}{(n/2)^2} \right)$$

§ 1.6 氢原子理论的检验 - 类氢离子光谱



Pickering series

$$n = 5, 6, 7, \dots \rightarrow m = 4$$

$$\tilde{\nu} = 2^2 R_{\infty} \left(\frac{1}{m^2} - \frac{1}{n^2} \right) = R_{\infty} \left(\frac{1}{(m/2)^2} - \frac{1}{(n/2)^2} \right)$$

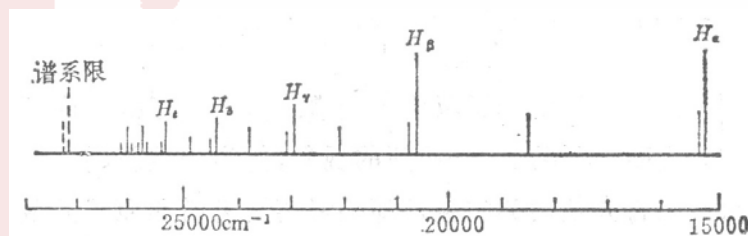
➔
$$\tilde{\nu} = R_{\infty} \left(\frac{1}{2^2} - \frac{1}{k^2} \right)$$

$$k = n/2 = 2.5, 3, 3.5, 4, \dots$$

光谱观测

$$\tilde{\nu} = R' \left(\frac{1}{2^2} - \frac{1}{k^2} \right)$$

$$R' = R_{He^+} = 1.097, 222, 7 \times 10^7 \text{ m}^{-1}$$



§ 1.6 氢原子理论的检验 – 原子核质量的影响



问题二

英国光谱学家福勒指出： R_∞ 与 R_H 差别大于万分之五，但当时的光谱实验精度已经好于万分之一。



A. Fowler
(1868-1940)

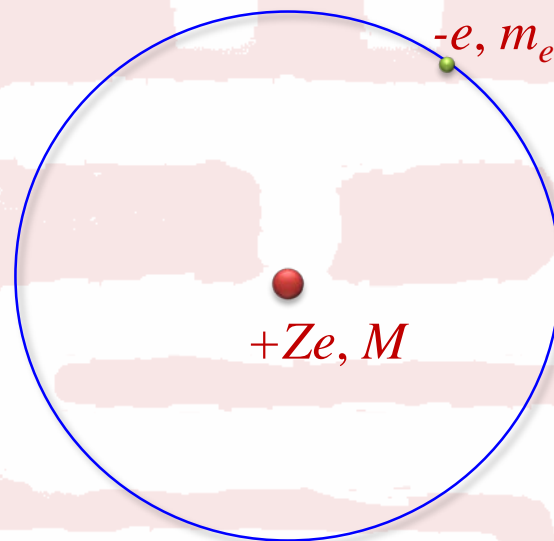
实验值 $R_H = 1.096,775,8 \times 10^7 \text{ m}^{-1}$

$R_{He^+} = 1.097,222,7 \times 10^7 \text{ m}^{-1}$

理论值 $R_\infty = 1.097,373,156 \times 10^7 \text{ m}^{-1}$



近似： $M \gg m_e$



§ 1.6 氢原子理论的检验 - 原子核质量的影响



在质心系中

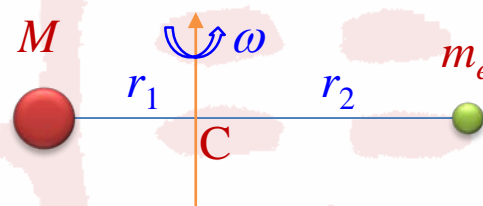
$$r = r_1 + r_2$$

$$Mr_1 = m_e r_2 \quad \rightarrow \quad r_2 = \frac{M}{m_e} r_1$$

$$\rightarrow r_1 = \frac{m_e}{M + m_e} r = \frac{\mu}{M} r$$

$$r_2 = \frac{M}{M + m_e} r = \frac{\mu}{m_e} r$$

其中 $\mu = \frac{m_e M}{M + m_e}$ 是电子和原子核的折合质量(约化质量)



电子和原子核以相同的角速度 ω 绕质心C运动，它们的线速度分别为

$$v_1 = r_1 \omega \quad v_2 = r_2 \omega$$

则，电子和原子核在质心系中的总动能

$$\begin{aligned} T &= \frac{1}{2} M v_1^2 + \frac{1}{2} m_e v_2^2 = \frac{1}{2} M r_1^2 \omega^2 + \frac{1}{2} m_e r_2^2 \omega^2 \\ &= \frac{1}{2} \omega^2 r^2 \left[M \left(\frac{\mu}{M} \right)^2 + m_e \left(\frac{\mu}{m_e} \right)^2 \right] = \frac{1}{2} \mu r^2 \omega^2 \end{aligned}$$



§ 1.6 氢原子理论的检验 - 原子核质量的影响

总动能 $T = \frac{1}{2} \mu r^2 \omega^2$

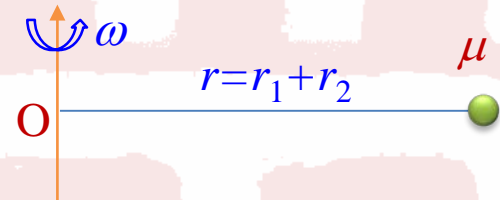
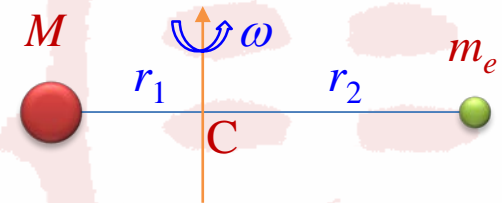
角动量 $L = Mv_1r_1 + m_e v_2 r_2 = \mu \omega r^2 = \mu(r\omega)r$

系统可以等效为：质量为 μ 的粒子、绕固定点、半径为 r 的圆周运动。

$$m_e \rightarrow \mu$$

$$E_n = -\frac{1}{2} \mu c^2 \alpha^2 \frac{Z^2}{n^2}$$

$$r_n = \frac{\hbar c n^2}{Z \mu c^2 \alpha} = \left(\frac{m_e}{Z \mu} \right) n^2 a_0$$



§ 1.6 氢原子理论的检验 – 原子核质量的影响



相应的里德堡常数

$$R_M = \frac{1}{(4\pi\epsilon_0)^2} \frac{\mu e^4}{4\pi\hbar^3 c} = \frac{1}{1 + m_e / M} R_\infty$$

对于H原子

$$R_H^{theo} = \frac{1}{1 + m_e / M_H} R_\infty$$

$$R_\infty = 1.097,373,156 \times 10^7 \text{ m}^{-1}$$

将 $m_e / M_H \approx 1/1836.15$ 代入, 得

$$R_H^{theo} = 1.096,775,832 \times 10^7 \text{ m}^{-1}$$

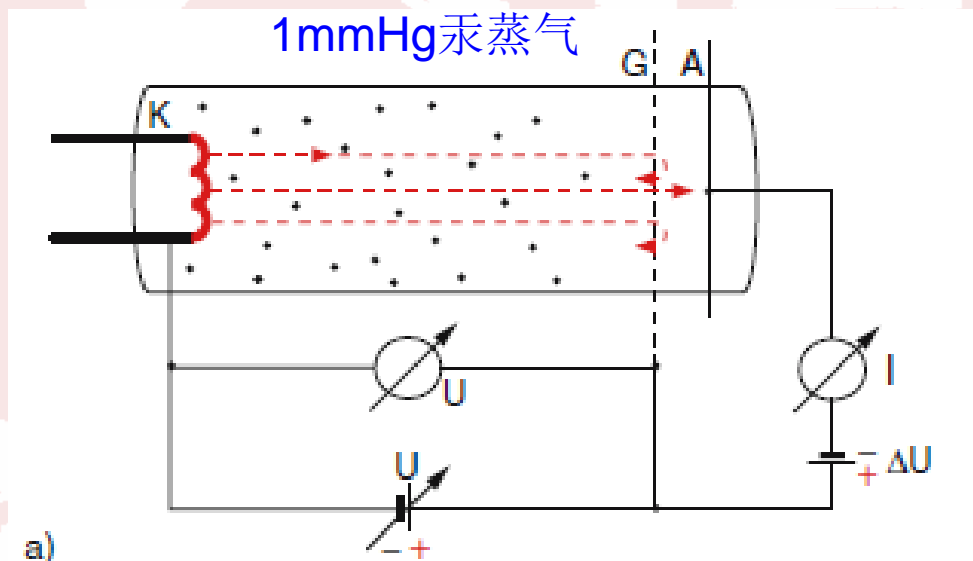
实验值

$$R_H = 1.096,775,8 \times 10^7 \text{ m}^{-1}$$

§ 1.6 氢原子理论的检验 – Franck-Hertz实验



Franck-Hertz Experiment



James Franck
1882 –1964



Gustav Ludwig Hertz
1887 –1975

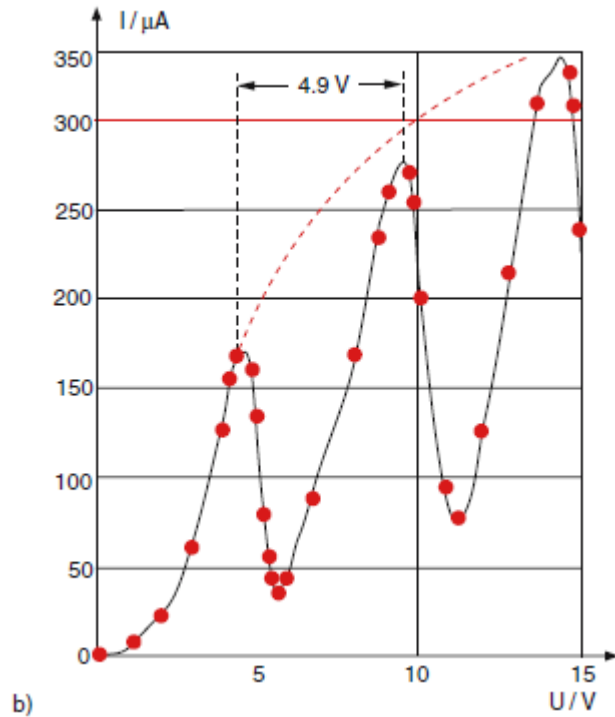
¹J. Franck and G. Hertz, Verh. Dtsch. Phys. Ges. 16, 457 (1914).

²J. Franck and G. Hertz, Verh. Dtsch. Phys. Ges. 16, 512 (1914).



The Nobel Prize in Physics 1925

§ 1.6 氢原子理论的检验 – Franck-Hertz实验



$$i \propto V^{3/2}$$

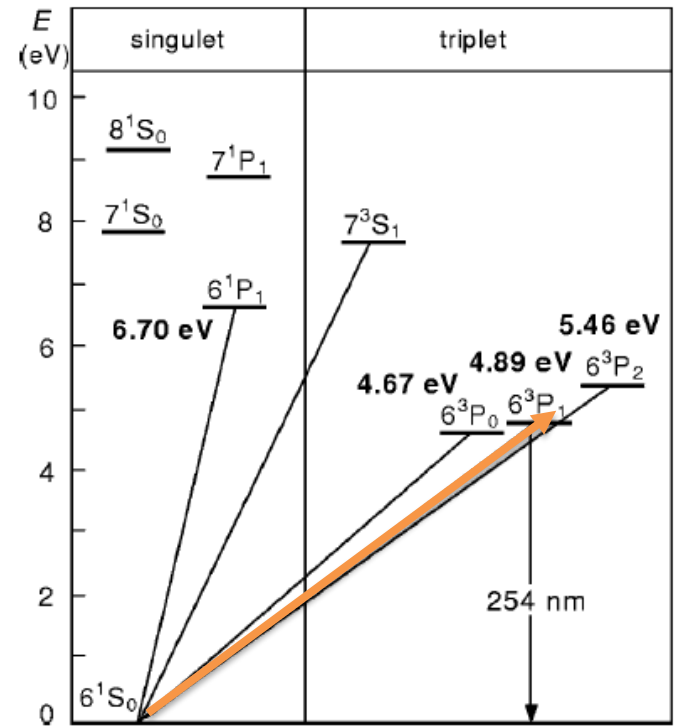
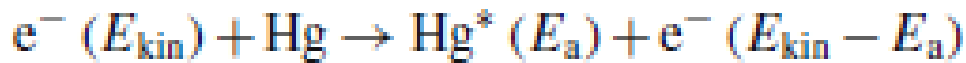


Fig. 2. Lowest energy levels in Hg (Ref. 10).



一个月后

$$\lambda = \frac{hc}{E} = \frac{1240 eV \cdot nm}{4.89 eV} \sim 254 nm$$

§ 1.6 氢原子理论的检验 – Franck-Hertz实验

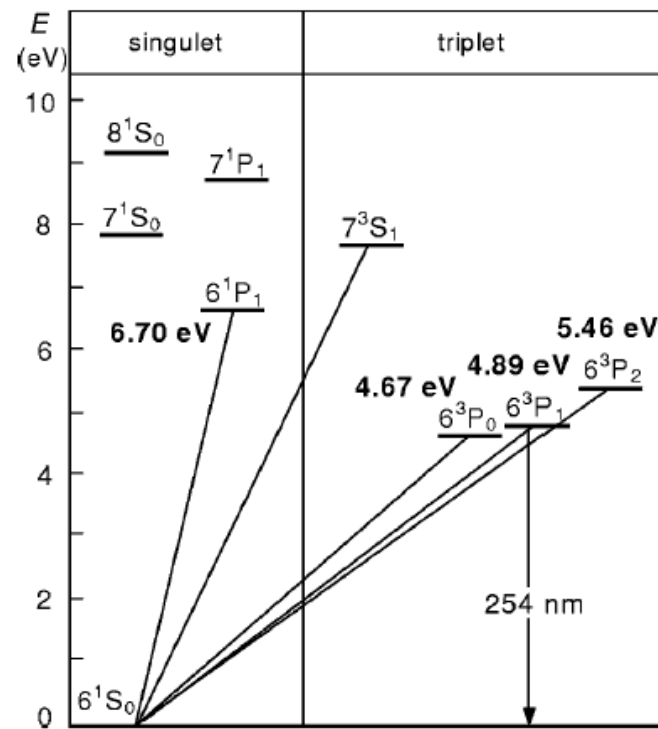
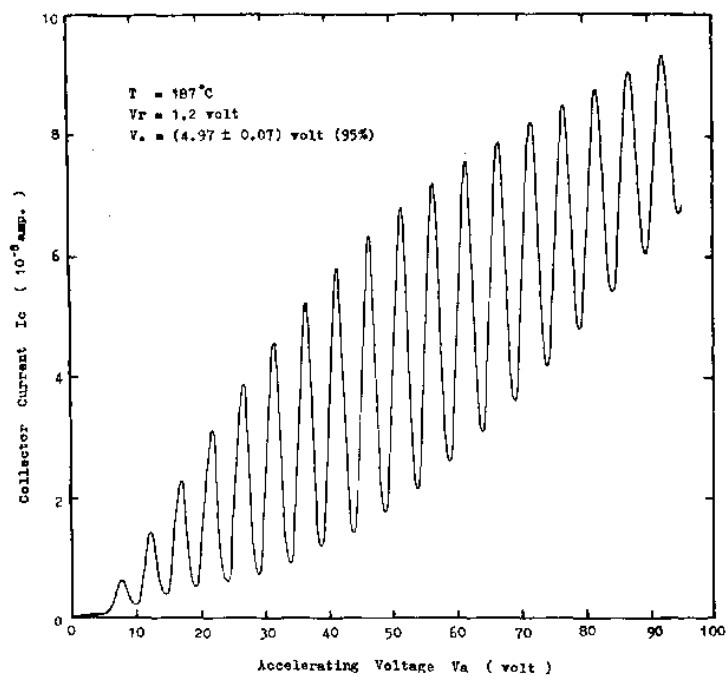


Fig. 2. Lowest energy levels in Hg (Ref. 10).

§ 1.6 氢原子理论的检验 – Franck-Hertz实验

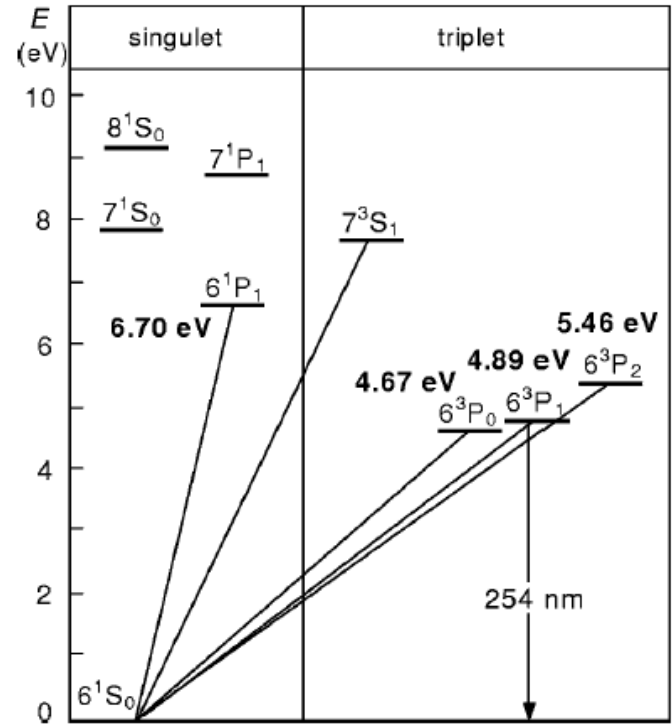
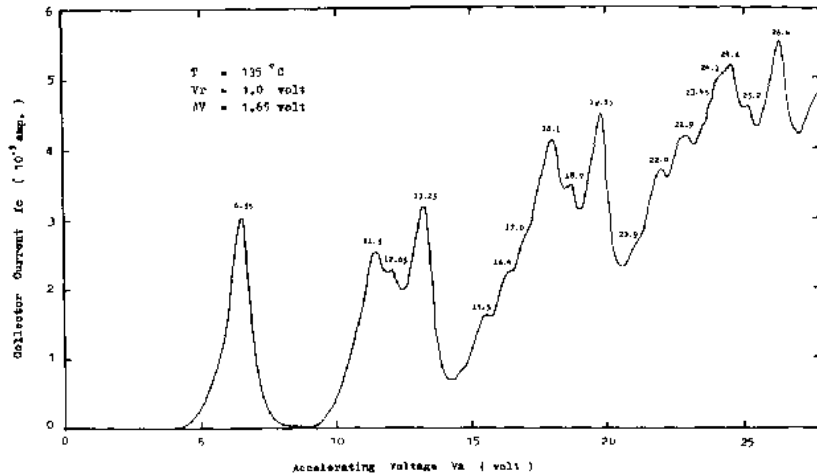
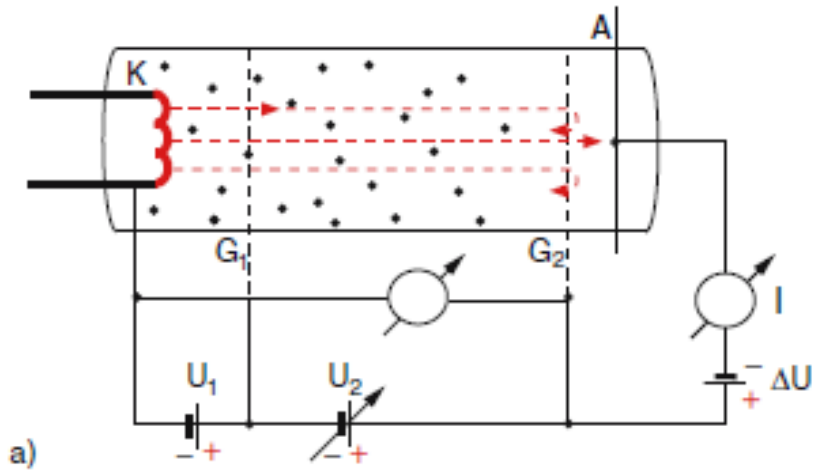


Fig. 2. Lowest energy levels in Hg (Ref. 10).

§ 1.6 氢原子理论的检验 – Franck-Hertz实验



电子能量损失谱仪

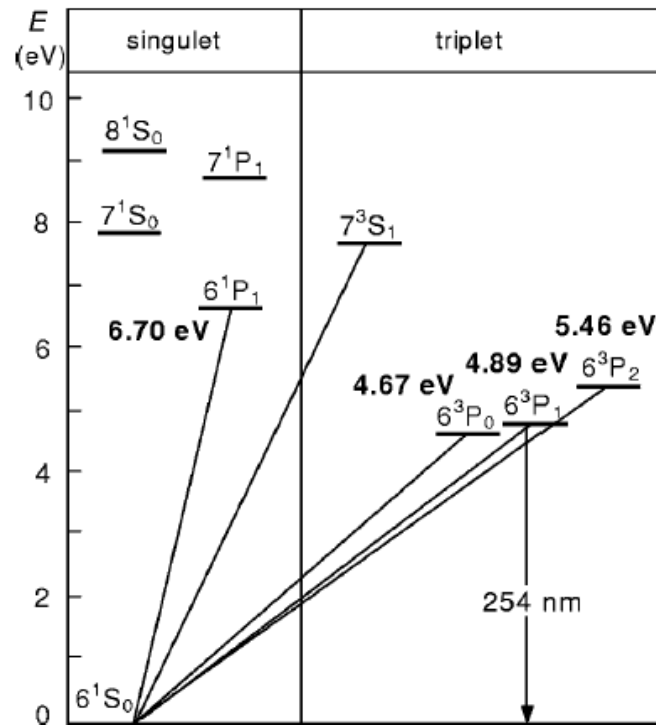
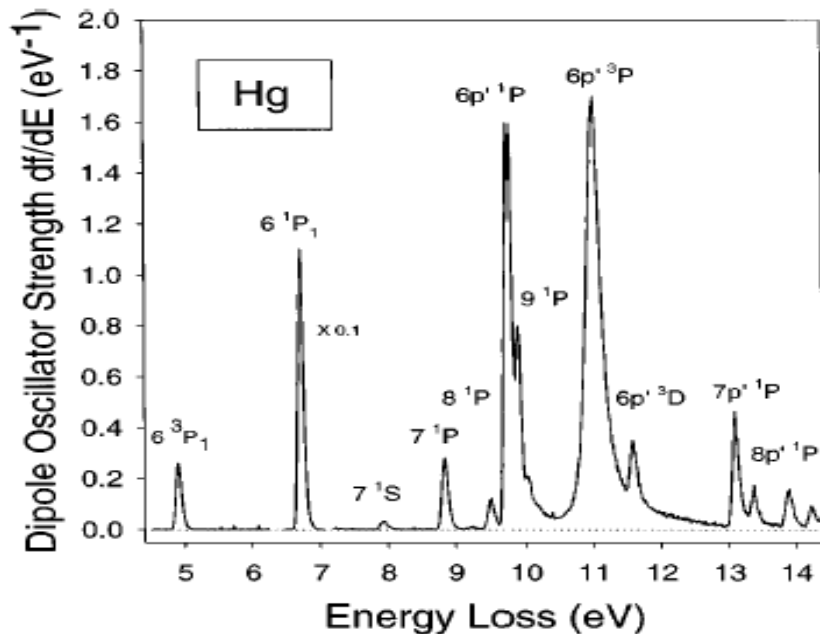
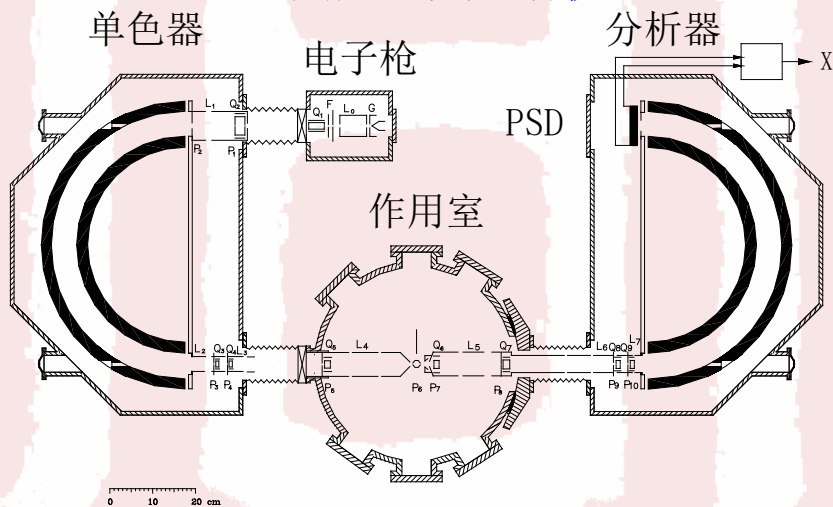


Fig. 2. Lowest energy levels in Hg (Ref. 10).

§ 1.6 氢原子理论的检验 – Franck-Hertz实验



"Niels Bohr was indeed a keen football player and was the goalkeeper in the Danish team Akademisk Boldklub in the beginning of the 20th century," says Nicolaj Egerod. "But even though AB [as the club is commonly known] were, at the time, one of the best clubs in Denmark, he never made it to the national team. However, his brother Harald - a well-known scientist in his own right - who also played at AB, played for the Danish national team and was part of the team that won silver at the 1908 London Olympics."

Nils Refsdal suggests a possible reason why Niels never made it to the international stage: "According to AB, in a match against the German side Mittweida, one of the Germans launched a long shot and the physicist leaning against the post did not react, missing an easy save. After the game he admitted to his team-mates **his thoughts had been on a mathematical problem that was of more interest to him than the game.** He only played for the 1905 season."

