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# Breaking symmetry in non-planar bifurcations: Distribution of flow and wall shear stress

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**Abstract.** Non-planarity in blood vessels is known to influence arterial flows and wall shear stress. To gain insight, computational fluid dynamics (CFD) has been used to investigate effects of curvature and out-of-plane geometry on the distribution of fluid flows and wall shear stresses in a hypothetical non-planar bifurcation. Three-dimensional Navier–Stokes equations for a steady state Newtonian fluid were solved numerically using a finite element method. Non-planarity in one of the two daughter vessels is found to deflect flow from the inner wall of the vessel to the outer wall and to cause changes in the distribution of wall shear stresses. Results from this study agree to experimental observations and CFD simulations in the literature, and support the view that non-planarity in blood vessels is a factor with important haemodynamic significance and may play a key role in vascular biology and pathophysiology.

Keywords: Wall shear stress, non-planar bifurcation, CFD modeling

## 1. Introduction

Early atherosclerotic lesions develop preferentially near bifurcations in human arterial systems. The close correlation between the disease and vessel geometry suggests that local haemodynamics plays an important role in the initiation and development of atherosclerosis. There have been studies investigating the influence of steady and non-steady shear stresses, and gradients of shear stress on vascular morphology and biochemistry, e.g., [2,5,8], but most previous studies assume flows to be two-dimensional, whereas flow in large arteries are commonly 3D. Advances in computer capacity and numerical techniques enable us to simulate three-dimensional flows with either idealized or reconstructed geometries, e.g., [3,6]. There are signs that non-planarity in the vascular system is attracting interest from scientists in fundamental research [1,7]. In this paper, we investigate the effects on steady flows of non-planarity in a hypothetical three-dimensional asymmetrical bifurcation. We neglect flexibility and deformation of the vessel walls, itself is an exciting area of study, and assume vessel wall to be rigid. The three-dimensional Navier–Stokes equations for incompressible Newtonian fluid are solved numerically using a finite element method. The focus of the study is to investigate distributions of flows and wall shear stresses in vessels with a non-planar bifurcation.

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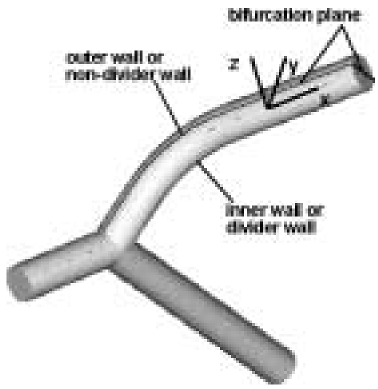


Fig. 1. Schematics of the non-planar bifurcation.

## 2. Mathematical model

We have constructed a hypothetical three-dimensional bifurcation as shown in Fig. 1. In the model, both daughter vessels have the same diameter, $D$, as their mother vessel. The 90 degree bifurcation is initially planar and symmetrical. However, one of the daughter vessels, after a length of $1.5D$, undergoes a 45 degree bending in the direction perpendicular to the plane of the bifurcation with a radius of $4D$. The vessel is then straight for a further length of $4D$. The other daughter vessel is straight and has a length of $8D$. We define the $x$–$y$ plane, shown in Fig. 1, as the bending plane. The $x$–$z$ plane, which is always perpendicular to the bending plane, varies in the direction along the body-fitted $x$ axis of the bending daughter vessel. It is hereafter called the bifurcation plane for easy reference. Comparisons are made to a planar model with a similar geometry except that both daughter vessels in the latter model are straight and stay in the bifurcation plane. Fluid is assumed to be Newtonian and we only consider steady flow conditions. Typical values for the Reynolds number, Re, are a few hundred in arteries, so we may neglect turbulence in the model. Nondimensionalisation using the mean velocity at the inlet and the vessel diameter reduces the Navier–Stokes equations to:

$$\nabla \cdot \vec{u} = 0,$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2 \vec{u},$$

where $\vec{u} = (u, v, w)$ is the fluid velocity and $p$ is the pressure. A finite element scheme based on a velocity correction method is adopted [4], with the following three-step time splitting scheme:

$$\tilde{\vec{u}} = \vec{u}^n - \delta t \cdot \left[ -\frac{1}{\mathrm{Re}}\nabla^2 \cdot \vec{u}^n + (\vec{u}^n \cdot \nabla)\vec{u}^n \right],$$

$$\nabla^2 p^{n+1} = \frac{1}{\delta t}\nabla \tilde{\vec{u}},$$

$$\vec{u}^{n+1} = \tilde{\vec{u}} - \delta t \cdot \nabla p^{n+1}.$$

In our model, a Hagen–Poiseuille parabolic velocity profile is imposed at the inlet of the mother vessel with nondimensional mean velocity of 1. No slip boundary conditions are applied on the tube walls. At the exits of the daughter vessels, constant pressure and zero axial velocity gradients are imposed. To validate the computer code, we have computed fully developed laminar flow in a straight pipe and compared calculated results to the Hagen–Poiseuille solutions. The discrepancy in the wall shear stress (WSS) between the calculated and theoretical results is approximately 0.5%. We have also investigated different grid sizes and time steps in our calculation. For example, extensive grid refinement is carried out using 30,000, 45,000 and 80,000 nodes in our models to determine grid sizes and time steps that yield independent results.

## 3. Results and discussion

In all results presented, Re based on the mean velocity at the entrance and the vessel diameter is 200. Emphasis is given to the comparison between flows and wall shear stresses in the daughter vessels of the planar and non-planar bifurcation. In Fig. 2, we compare axial velocities in different cross-sections along the bending daughter vessel. Distance $x$ is normalized to vessel diameter $D$ and $x = 0$ corresponds to the section where inner walls of the two daughter vessels meet. Axial velocity profiles at different $x$ locations in the bifurcation plane (Fig. 2(a)) are seen to skew towards the flow-dividing wall (i.e., the inner wall of the bifurcation). Flow reversal is evident near the outer wall close to the bifurcation (at $x = 0.5$). Velocity profiles in the $x$–$y$ plane are shown in Fig. 2(b), where typical M-shaped velocity profiles are present. These are consistent with observations obtained in the abdominal aorta and coronary artery [6]. When we compare results from the non-planar model to those from the planar one, we find that near the outer wall of the bifurcation, velocities vary less quickly with the radius in the non-planar vessel than those in the corresponding vessel of the planar model, particularly at sections located at some

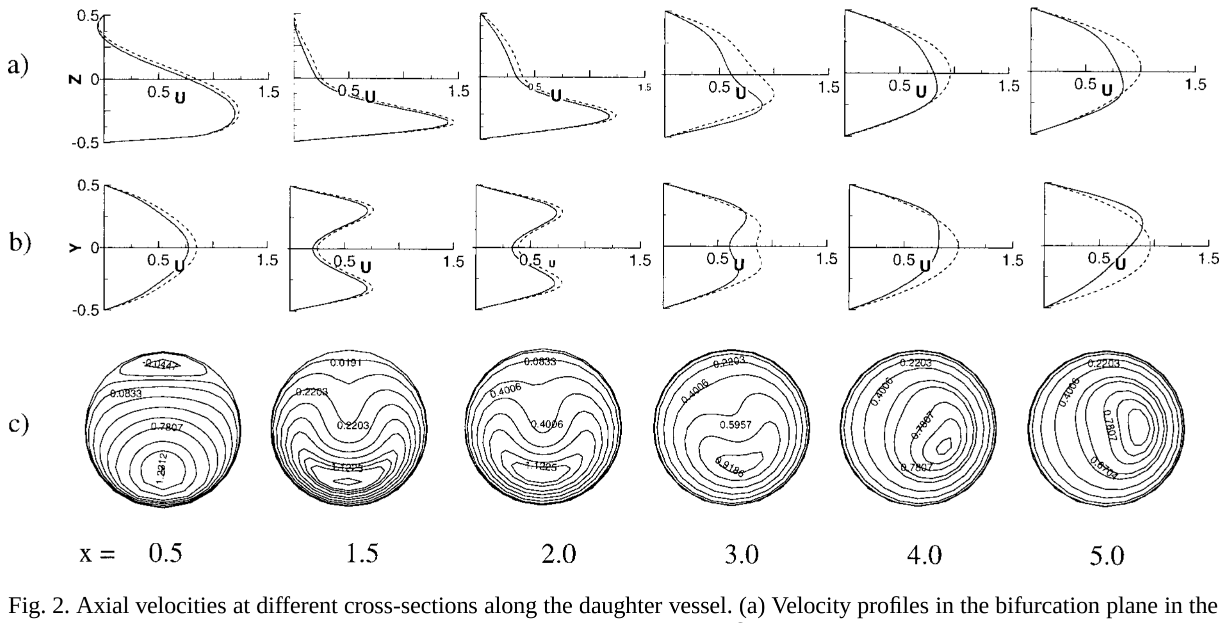


Fig. 2. Axial velocities at different cross-sections along the daughter vessel. (a) Velocity profiles in the bifurcation plane in the planar (dashed line) and the non-planar (solid line) model. (b) Velocity profiles in the bending plane. (c) Velocity contours in the non-planar model.

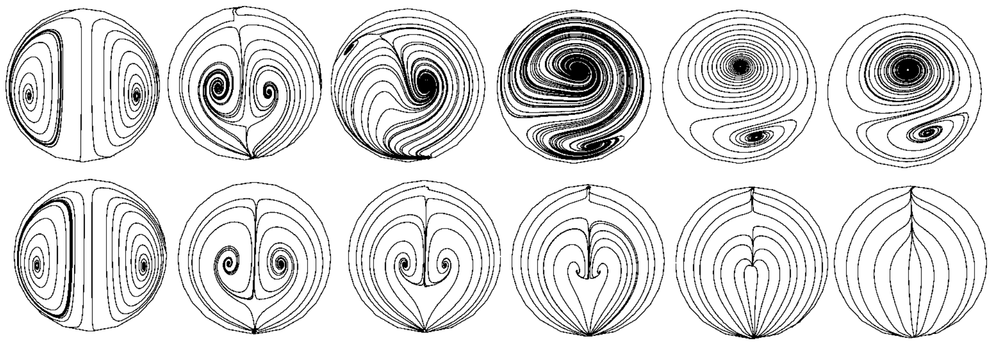


Fig. 3. Secondary flow streamlines in daughter vessels of the non-planar bifurcation (top) and the planar bifurcation (bottom).

distance along the bend, e.g., $x = 3.0$. Curvature in the tube geometry, due to the bifurcation, causes flow in daughter vessels to skew towards the inner wall. Bending of the tube out of the bifurcation plane introduces a second curvature. Superposition of the two results the rotation of the axial flow in the curved tube. The swirling effects are better seen in the axial velocity contours at these sections (Fig. 2(c)). The crescent contours of the axial velocity change from skewed to the inner wall near the bifurcation to a more parabolic profile as flow moves along the vessel. In the bend, flow deviates from the inner wall of the bifurcation to its sidewall. The crescent shape rotates as flow moves along the tube in the anti-clockwise direction viewing from the end of the daughter vessel, which agrees with results by Sherwin et al. [7].

Secondary motions of fluid in the planar and the non-planar bifurcations are presented in Fig. 3, where cross-flow streamlines are plotted at different sections along the daughter vessel. There are pronounced movement of fluid from the outer wall of the bifurcation towards the inner wall. They are caused by a centripetal acceleration (or pressure gradient) introduced by the curvature in the vessel. At sections close to the bifurcation, e.g., at $x = 0.5$ and 1.5, counter-rotating vortices (Dean vortices) are evident in both the planar and the non-planar models. The secondary flows weaken in strength as they travel along the vessel. However, there are noticeable differences between the planar and the non-planar models. In the planar case, the streamlines are symmetrical and there are two counter rotating vortices that weaken together with $x$. In the non-planar model, the two vortices are asymmetrical, which reflects the influence of the curvature due to the out-of-plane bend. The stronger vortex gradually dominates the secondary flow field. A single recirculation zone is a typical feature of non-planar flows due to torsion [10]. It is also found in end-to-side anastomoses and twisted pipes [7,9].

One of the benefits of the computer simulation is its ability to provide detailed distributions of flow, pressure and shear stress. Figure 4 compares the magnitude of wall shear stresses between the planar and the non-planar models at the inner and the outer walls of the bifurcation plane (Fig. 4(a),(b)) and those of the bending plane (Fig. 4(c),(d)) along the daughter vessel. The shear stresses have been normalized to the wall shear stress at the inlet of the mother tube. It is found that peak wall shear stress exists on the inner wall in the bifurcation plane near the entry section of the daughter vessel (Fig. 4(a)). At approximately $x = 1.4$ on the outer wall of the bifurcation plane (Fig. 4(b)), shear stress is close to zero. The out of plane bend introduces changes in WSS in all four figures, but the most significant changes occur in the bending plane. At the inner wall of the bend (Fig. 4(c)), WSS over the section of the bend (approximately between $x = 1.5$ and 4.5) is reduced on average by 10–15% compared to the

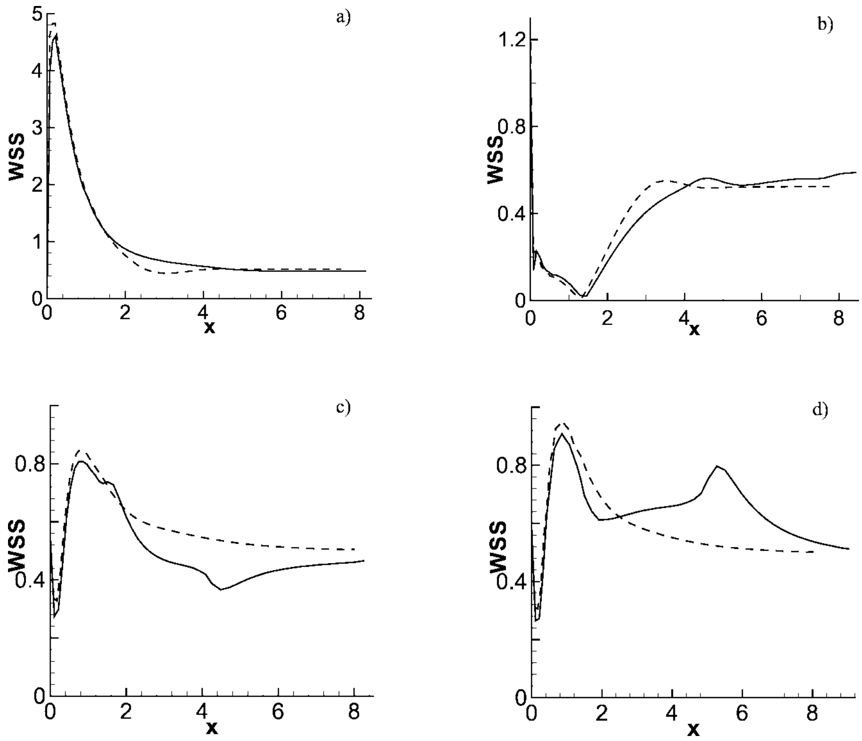


Fig. 4. Distribution of wall shear stresses (WSS) along the daughter vessel in the planar (dashed lines) and non-planar (solid lines) models. (a) Inner wall of the bifurcation plane. (b) Outer wall of the bifurcation plane. (c) Inner wall of the bend plane. (d) Outer wall of the bending plane.

same vessel in a planar model. Differences in WSS increase with the distance, reaching the maximum at approximately $x = 4.5$. At the outer wall of the bend (Fig. 4(d)), the opposite occurs. WSS over the section of the bend is increased on average by 10–15% compared to that in a planar model. Unlike that in the planar model, WSS increases with distance, and the difference in wall shear stresses reaches a maximum at approximately $x = 5.2$. These findings agree with the changes in flow distributions caused by the out of plane bend and correspond to the rotation of the axial flow in the curved branch.

## 4. Conclusions

We have investigated the effects of non-planarity in a daughter vessel of a three-dimensional asymmetrical bifurcation on its flow and wall shear stress distributions. Velocities are solved numerically using a finite element method. We have paid particular attention to computer code validation and ensured results to be grid-size and time-step independent. Velocity profiles and contours are presented at different cross sections along the vessel. We find that non-planarity in the vessel, e.g., an out-of-plane bend following a bifurcation, alters velocity and shear stress distributions in the vessel. Secondary motions become stronger with the addition of the out-of-plane curvature of the vessel introduced by the bend. The flow swirls with a predominate direction (anti-clockwise viewed from the exit of the vessel) in sections of the bend, as shown in the secondary flow streamlines as well as the rotation of the axial velocity patterns. Torsion of the vessel affects its wall shear stress distribution and reduces the peak values of the wall shear stress. All these findings suggest that non-planarity in blood vessels is a factor with important haemodynamic effect. It affects the interaction between flows and the vessel and is likely to alter residence time of particles and biological cells in the close vicinity of the vascular endothelium, and may thus play an important role in vascular biology and pathophysiology.

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