Available online at www.sciencedirect.com
SCIENCE DIRECT®
Information Processing Letters 96 (2005) 146–150
Information Processing Letters
www.elsevier.com/locate/ipl

Edge-fault-tolerant edge-bipancyclicity of hypercubes ☆

Jun-Ming Xu *, Zheng-Zhong Du, Min Xu

Department of Mathematics, University of Science and Technology of China, Hefei, Anhui 230026, China

Received 6 September 2004; received in revised form 6 May 2005
Available online 25 August 2005
Communicated by M. Yamashita

Abstract

In this paper, we consider the problem embedding a cycle into the hypercube Q_n with existence of faulty edges and show that for any edge subset F of Q_n with $|F| \leqslant n-1$ every edge of $Q_n - F$ lies on a cycle of every even length from 6 to 2^n inclusive provided $n \geqslant 4$ and all edges in F are not incident with the same vertex. This result improves some known results.

Keywords: Cycles; Pancyclicity; Hypercube; Fault tolerance

1. Introduction

To find a cycle of given length in graph G is a cycle embedding problem, and to find cycles of all length from 3 to $|V(G)|$ is a pancyclic problem. The cycle embedding problem is investigated in a lot of interconnection networks [4,5,9,10,12,15]. In general, a graph is of pancyclicity if it contains cycles of all length [2]. The pancyclicity is an important property to determine if a topology of network is suitable for an application where mapping cycles of any length into the topology of network is required. The concept of pancyclicity has been extended to vertex-pancyclicity [6] and edge-pancyclicity [1]. A graph is vertex-pancyclic if every vertex lies on a cycle of every length from 3 to $|V(G)|$; and edge-pancyclic if every edge lies on a cycle of every length from 3 to $|V(G)|$. It is clear that if a graph G is edge-pancyclic then it is vertex-pancyclic certainly. Bipancyclicity is essentially a restriction of the concept of pancyclicity to bipartite graphs whose cycles are necessarily of even length. A graph G is edge-bipancyclic if every edge lies on a cycle of every even length from 4 to $|V(G)|$ [13]. A graph G is k-edge-fault-tolerant Hamiltonian (edge-pancyclic) if the resulting graph by deleting any k edges from G is Hamiltonian (edge-pancyclic).

The fault-tolerant Hamiltonicity and pancyclicity of many networks are investigated, for example, Hsieh and Chen [7] for Möbius cubes, and Hsieh et al. [8] for arrangement graphs. In this paper we consider edge-bipancyclicity of hypercubes with faulty edges. Fault-tolerant properties are critical to the performance eval-

☆ The work was supported by NNSF of China (No. 10271114).
* Corresponding author.
E-mail address: xujm@ustc.edu.cn (J.-M. Xu).

0020-0190/$ – see front matter © 2005 Published by Elsevier B.V.
doi:10.1016/j.ipl.2005.06.006

uation of the network topology. As a topology for an interconnection network of a multiprocessor system, the hypercube Q_n $(n \geqslant 2)$ is a widely used and well-known interconnection model since it possesses many attractive properties [14,17]. Saad and Schultz [14] proved that Q_n is bipancyclic. Leu and Kuo [10], Litifi et al. [12] and Sen et al. [15], independently, proved that Q_n is $(n-2)$-edge-fault-tolerant Hamiltonian. Sengupta [16] proved that Q_n is $(n-1)$-edge-fault-tolerant Hamiltonian if $n \geqslant 4$ and all the faulty edges are not incident with the same vertex. Recently, Li et al. [11] have further showed that Q_n is $(n-2)$-edge-fault-tolerant edge-bipancyclic. In this paper, we obtain the following result.

Theorem. *For any subset F of $E(Q_n)$ with $|F| \leqslant n-1$, every edge of $Q_n - F$ lies on a cycle of every even length from 6 to 2^n inclusive provided $n \geqslant 4$ and all edges in F are not incident with the same vertex.*

Obviously, a cycle of length 2^n in $Q_n - F$ is a Hamilton cycle, which is the result of Sengupta. If $|F| \leqslant n-2$ then all edges in F are not incident with the same vertex since Q_n is n-regular, which implies that our theorem holds. Also, any edge e in Q_n lies on exactly $n-1$ cycles of length four. If $|F| \leqslant n-2$ then every edge in $Q_n - F$ must lie on a cycle of length four. Thus, our result implies the result of Li et al.

The proof of theorem is in Section 3. In Section 2, some lemmas are given.

2. Some lemmas

We follow [3] or [18] for graph-theoretical terminology and notation not denned here. A graph $G = (V, E)$ always means a simple and connected graph, where $V = V(G)$ is the vertex-set and $E = E(G)$ is the edge-set of G. A uv-path is a sequence of adjacent vertices, written as $\langle v_0, v_1, v_2, \ldots, v_m \rangle$, in which $u = v_0$, $v = v_m$ and all the vertices $v_0, v_1, v_2, \ldots, v_m$ are distinct except possibly $v_0 = v_m$. The length of a path P is the number of edges in P. A cycle is a path with at least three vertices such that the first vertex is the same as the last one. A cycle is called a Hamiltonian cycle if it contains all vertices of G and a uv-path is called a Hamiltonian path if it contains all vertices of G.

An n-dimensional binary hypercube Q_n is a graph with 2^n vertices, each vertex denoted by an n-bit binary string $u = u_n u_{n-1} \ldots u_2 u_1$. Two vertices are adjacent if and only if their strings differ exactly in one bit position. It has been proved that Q_n is a vertex and edge transitive bipartite graph (see, for example, [17]).

By the definition, for any $k \in \{1, 2, \ldots, n\}$, Q_n can be expressed as $Q_n = L_k \odot R_k$, where L_k and R_k are the two $(n-1)$-subcubes of Q_n induced by the vertices with the k bit position is 0 and 1, respectively. We call edges between L_k and R_k to be k-dimensional, which form a perfect matching of Q_n. Clearly, for any edge e of Q_n, there is some $k \in \{1, 2, \ldots, n\}$ such that e is k-dimensional. Use u_L and u_R to denote two vertices in L_k and R_k, respectively, linked by the k-dimensional edge $u_L u_R$ in Q_n.

For a subset F of $E(Q_n)$ and any $k \in \{1, 2, \ldots, n\}$, we always express Q_n as $Q_n = L_k \odot R_k$, and let $F_L = F \cap E(L_k)$, $F_R = F \cap E(R_k)$ and $F_k = F \backslash (F_L \cup F_R)$.

Lemma 2.1. *For any subset F of $E(Q_n)$ and any given $i \in \{1, 2, \ldots, n\}$, if $|F| = n-1$ with $n \geqslant 4$ and all edges in F are not incident with the same vertex, then there is $k \in \{1, \ldots, n\} \backslash \{i\}$ such that $|F_L| \leqslant n-2$ and $|F_R| \leqslant n-2$. Moreover, if $|F_L| = n-2$ (or $|F_R| = n-2$) with $n \geqslant 5$, then all these $n-2$ edges in F_L (or F_R) are not incident with the same vertex.*

Proof. For a given i, let $N_i = \{1, \ldots, n\} \backslash \{i\}$. We first choose $j \in N_i$ such that $|F_j|$ is as large as possible. If $|F_j| \geqslant 2$, then $|F_L| \leqslant n-3$ and $|F_R| \leqslant n-3$. Let $k = j$ and we are done.

If $|F_j| = 1$, then F contains at most one edge of every dimension in N_i. Let $I = \{t\colon |F_t| = 1,\ t \in N_i\}$. Choose $k \in I$ such that the only edge $f_k \in F_k$ is adjacent other edges in F as many as possible. So the former part in the lemma holds clearly. If $|F_L| = n-2$ (or $|F_R| = n-2$) and all edges in F_L are incident with the same vertex, say x, then f_k is not incident with x by the hypothesis of F, which contradicts to the choice of k for $n \geqslant 5$. Therefore, if $|F_L| = n-2$ with $n \geqslant 5$, then all edges in F_L are not incident with the same vertex.

If $|F_j| = 0$, then $F = F_i$. Let uv and xy be two distinct edges in F_i and let $u = u_n \ldots u_i \ldots u_1$ and $x = x_n \ldots x_i \ldots x_1$. Then $v = u_n \ldots u_{i+1} \bar{u}_i u_{i-1} \ldots u_1$ and $y = x_n \ldots x_{i+1} \bar{x}_i x_{i-1} \ldots x_1$. Since $uv \neq xy$ and $F = F_i$, there exists some $k \in N_i$ such that $u_k \neq x_k$.

Without loss of generality, suppose that uv is in L_k. Then xy in R_k and so $|F_L| \leqslant n-2$ and $|F_R| \leqslant n-2$. Also since $F = F_i$, any two distinct edges in F are not incident with the same vertex and so k is required. The lemma follows. □

When we express Q_n as $Q_n = L_k \odot R_k$, for an edge $e_L = u_L v_L$ in L_k, there is an corresponding edge in R_k, denoted by $e_R = u_R v_R$. Similarly, for a path P or a cycle C in L_k, we denote the corresponding path or cycle in R_k by P' or C'.

Lemma 2.2. *Any two edges in Q_n $(n \geqslant 2)$ are included in a Hamiltonian cycle.*

Proof. We prove the lemma by induction on $n \geqslant 2$. Obviously, the lemma is true for $n = 2$. Assume that the lemma is true for every k with $2 \leqslant k < n$. Let e and e' be two edges in Q_n and express $Q_n = L_k \odot R_k$ such that none of e and e' is k-dimensional. Without loss of generality, we may assume $e \in L_k$. Furthermore, we can suppose that e' is in L_k, otherwise consider e'_L instead of e'. By the induction hypothesis, there exists a Hamiltonian cycle C containing e and e' in L_k. Let $u_L v_L$ be an edge on C different from e and e'. The corresponding C' is a Hamiltonian cycle in R_k containing $u_R v_R$, e_R and e'_R. Let $P = C - u_L v_L$ and $P' = C' - u_R v_R$. Then $P + u_L v_L + P' + v_R v_L$ is a Hamiltonian cycle in Q_n containing e and e'. □

Lemma 2.3. *For any edge uv of Q_n, there is a Hamiltonian cycle C such that it contains uv and two neighbors of $\{u, v\}$ on C are adjacent in Q_n.*

Proof. We proof the lemma by induction on $n \geqslant 2$. Obviously, the lemma is true for $n = 2$. Assume that the lemma is true for all $2 \leqslant k < n$ and denote $Q_n = L_k \odot R_k$ such that uv is not k-dimensional edge. Without loss of generality, we may assume $uv \in L_k$. By the induction hypothesis, there exists a Hamiltonian cycle C such that it contains uv and two neighbors $\{w_L, z_L\}$ of $\{u, v\}$ on C are adjacent in L_k, where w_L is a neighbor of u and z_L is a neighbor of v on C. Since the length of C is not less than 4, there exists an edge $x_L y_L$ on C such that $\{x_L, y_L\} \cap \{u, v\} = \emptyset$. Let $P = C - x_L y_L$. The corresponding P' is an $x_R y_R$-Hamiltonian path in R_k. Thus, $P + y_L y_R + P' + x_R x_L$ is a desired Hamiltonian cycle in Q_n. □

Usually, we use P_l and C_l to denote the path and cycle of length l, respectively.

Lemma 2.4 (Li et al. [11]). *Q_3 is 1-edge-fault-tolerant edge-bipancyclic.*

Lemma 2.5. *Every edge of $Q_4 - F$ lies on a cycle of every even length from 6 to 16 inclusive for any $F \subset E(Q_4)$ with $|F| = 3$ provided all edges in F are not incident with the same vertex.*

Proof. Let F be a subset of $E(Q_4)$ with $|F| = 3$ and suppose that all edges in F are not incident with the same vertex. Let e be an edge in $Q_4 - F$ and suppose that e is i-dimensional for some $i \in \{1, 2, 3, 4\}$. By Lemma 2.1 we can choose $k \in \{1, 2, 3, 4\} \backslash \{i\}$ and express $Q_4 = L_k \odot R_k$ such that $|F_L| \leqslant 2$ and $|F_R| \leqslant 2$. Let l be any even integer with $6 \leqslant l \leqslant 16$. To prove the lemma, we need to construct a cycle of length l in $Q_4 - F$ containing e. Since $k \neq i$, without loss of generality, we may assume that $e \in L_k$. There are four cases.

Case 1. $|F_L| \leqslant 1$ and $|F_R| \leqslant 1$.

Since $|F_L| \leqslant 1$, by Lemma 2.4, the edge e lies on a cycle of even length l in $L_k - F_L$, with $4 \leqslant l \leqslant 8$. In particular, we use C_8 to denote such a cycle of length 8.

We now assume $10 \leqslant l \leqslant 16$. Since $|E(C_8 - e)| = 8 - 1 > 2|F|$, there is an edge $u_L v_L$ on $C_8 - e$ such that $\{u_L u_R, v_L v_R, u_R v_R\} \cap F = \emptyset$. Let $P_7 = C_8 - u_L v_L$. Then $P_7 + u_L u_R + u_R v_R + v_R v_L$ is a cycle of length 10 in $Q_4 - F$ containing e. Assume $l \geqslant 12$ below. Since $|F_R| \leqslant 1$, by Lemma 2.4, the edge $u_R v_R$ lies on a cycle C'_{l-8} of even length $l - 8$ in $R_k - F_R$. Let $P'_{l-9} = C'_{l-8} - u_R v_R$. Then $P_7 + v_L v_R + P'_{l-9} + u_R u_L$ is a cycle of even length l in $Q_4 - F$ containing e.

Case 2. $|F_L| = 2$ and $|F_R| \leqslant 1$.

Since $|F_L| = 2$, $|F_k \cup F_R| = 1$ and all edges in F are not incident with the same vertex, there is an edge $u_L v_L \in F_L$ such that $\{u_L u_R, v_L v_R, u_R v_R\} \cap F = \emptyset$. By Lemma 2.4, e lies on a cycle C_{l_0} of even length l_0 in $L_k - (F_L - u_L v_L\}$ for $4 \leqslant l_0 \leqslant 8$.

Suppose that $6 \leqslant l \leqslant 8$. If $u_L v_L \notin C_l$, then the cycle C_l is required. If $u_L v_L \in C_l$ and $u_L v_L \in C_{l-2}$, let $P_{l-3} = C_{l-2} - u_L v_L$. Then $P_{l-3} + u_L u_R + v_L v_R + u_R v_R$ is a cycle of length l in $Q_4 - F$ containing e. If $u_L v_L \in C_l$ and $u_L v_L \notin C_{l-2}$, choose an edge $x_L y_L \in C_{l-2} - e$ such that $\{x_L x_R, y_L y_R, x_R x_R\} \cap F = \emptyset$ for

$|F_k \cup F_R| = 1$. Let $P_{l-3} = C_{l-2} - x_L y_L$, then $P_{l-3} + x_L x_R + x_R y_R + y_R y_L$ is a cycle of length l in $Q_4 - F$ containing e.

Suppose that $10 \leqslant l \leqslant 16$.

If $u_L v_L \in C_8$, let $P_7 = C_8 - u_L v_L$. Then $P_7 + u_L u_R + v_L v_R + u_R v_R$ is a cycle of length 10 in $Q_4 - F$ containing e. Assume $l \geqslant 12$ below. Since $|F_R| \leqslant 1$, by Lemma 2.4, there is a cycle C'_{l-8} of length $l - 8$ in R_k containing $u_R v_R$. Let $P'_{l-9} = C'_{l-8} - u_R v_R$. Then $P_7 + u_L u_R + P'_{l-9} + v_R v_L$ is a cycle of length l with in $Q_4 - F$ containing e.

If $u_L v_L \notin C_8$, choose an edge $x_L y_L \in C_8 - e$ such that $\{x_L x_R, y_L y_R, x_R y_R\} \cap F = \emptyset$ for $|F_k \cup F_R| = 1$. Let $P_7 = C_8 - x_L y_L$. Then $P_7 + x_L x_R + x_R y_R + y_R y_L$ is a cycle of length 10 in $Q_4 - F$ containing e. Assume $l \geqslant 12$ below. Since $|F_R| \leqslant 1$, by Lemma 2.4, let C'_{l-8} be a cycle of even length $l - 8$ in R_k containing $x_R y_R$ and $P'_{l-9} = C'_{l-8} - x_R y_R$. Then $P_7 + x_L x_R + P'_{l-9} + y_R y_L$ is a cycle of even length l in $Q_4 - F$ containing e.

Case 3. $|F_L| = 0$ and $|F_R| = 2$.

By Lemma 2.4, the edge e lies on a cycle of even length l with $4 \leqslant l \leqslant 8$ in L_k. In particular, we use C_8 to denote such a cycle of length 8. Since $|E(C_8 - e)| = 8 - 1 > 2|F|$, we can choose an edge $x_L y_L$ on $C_8 - e$ such that $\{x_L x_R, y_L y_R, x_R y_R\} \cap F = \emptyset$. Let $P_7 = C_8 - x_L y_L$. Then $P_7 + x_L x_R + x_R y_R + y_R y_L$ is a cycle of length 10 in $Q_4 - F$ containing e. Next, we suppose that $12 \leqslant l \leqslant 16$.

Since $|F_R| = 2$, $|F_k| = 1$ and all edges in F are not incident with the same vertex, there is an edge $u_R v_R \in F_R$ such that $\{u_L u_R, v_L v_R, u_L v_L\} \cap F = \emptyset$.

Suppose that $e \neq u_L v_L$. By Lemma 2.2, there is a cycle C_8 of length 8 containing e and $u_L v_L$ in L_k. Let $P_7 = C_8 - u_L v_L$. By Lemma 2.4, the edge $u_R v_R$ lies on a cycle C'_{l-8} of even length $l - 8$ in $R_k - (F_R - u_R v_R)$ and let $P'_{l-9} = C'_{l-8} - u_R v_R$. Then $P_7 + u_L u_R + P'_{l-9} + v_R v_L$ a cycle of length l in $Q_4 - F$ containing e.

Suppose that $e = u_L v_L$. By Lemma 2.3, there is a cycle $C_8 = u_L v_L + v_L v'_L + v'_L v''_L + P_3 + u''_L u'_L + u'_L u_L$ of length 8 in L_k such that $u'_L v'_L \in E(L_k)$. For $|F_k| = 1$, we assume that $\{v'_L v'_R, v''_L v''_R\} \cap F_k = \emptyset$ (or $\{u'_L u'_R, u''_L u''_R\} \cap F_k = \emptyset$). By Lemma 2.4, there is a cycle C'_{l-8} of even length $l - 8$ in $R_k - (F_R - u_R v_R) + v'_R v''_R$ containing $v'_R v''_R$, where the edge $v'_R v''_R$ is added only if $v'_R v''_R \in F_R$. If $u_R v_R \notin C'_{l-8}$, let $P'_{l-9} = C'_{l-8} - v'_R v''_R$. Then we get a cycle $C_l = u_L v_L + v_L v'_L + v'_L v'_R + P'_{l-9} + v''_R v''_L + P_3 + u''_L u'_L + u'_L u_L$ of length l in $Q_4 - F$ containing e. If $u_R v_R \in C'_{l-8}$, we may write C'_{l-8} as $u_R v_R + P'_r + v'_R v''_R + P'_s$ (or $u_R v_R + P'_r + v''_R v'_R + P'_s$) with $r + s = l - 10$. Then we get a cycle $C_l = u_L v_L + v_L v_R + P'_r + v'_R v'_L + v'_L u'_L + u'_L u''_L + P_3 + v''_L v''_R + P'_s + u_R u_L$ (or $u_L v_L + v_L v_R + P'_r + v''_R v''_L + P_3 + u''_L u'_L + u'_L v'_L + v'_L v'_R + P'_s + u_R u_L$) of length l in $Q_4 - F$ containing e.

Case 4. $|F_L| = 1$ and $|F_R| = 2$.

Since $|F_L| = 1$, by Lemma 2.4, e lies on a cycle C_4 of length four. By the choice of k in the proof of Lemma 2.1, we deduce $F = F_i$ from $|F_k| = 0$ and $|F_k| \geqslant |F_j|$ for all $j \in \{1, 2, 3, 4\} \setminus \{i\}$. Express $Q_n = L_i \odot R_i$, then $|F_R| = |F_L| = 0$. Let $e = x_L x_R$ and $C_4 = e + x_L y_L + y_L y_R + y_R x_R$.

Suppose that $6 \leqslant l \leqslant 10$. By Lemma 2.4, there is a cycle C_{l-2} of length $l - 2$ in L_i containing $x_L y_L$. Let $P_{l-3} = C_{l-2} - x_L y_L$. Then $P_{l-3} + e + x_R y_R + y_R y_L$ is a cycle of length l in $Q_4 - F$ containing e. In particular, let $P_7 = C_8 - x_L y_L$.

Suppose that $12 \leqslant l \leqslant 16$. By Lemma 2.4, there is a cycle C'_{l-8} of length $l - 8$ in R_i containing $x_R y_R$. Let $P'_{l-9} = C'_{l-8} - x_R y_R$. Then $P_7 + e + P'_{l-9} + y_R y_L$ is a cycle of length l in $Q_4 - F$ containing e.

The lemma is proved. □

3. Proof of theorem

We prove the theorem stated in Introduction by induction on $n \geqslant 4$. By Lemma 2.5, the theorem is true for $n = 4$. Suppose that the theorem is true for every m with $4 \leqslant m < n$. Let F be a subset of $E(Q_n)$ with $|F| = n - 1$ and suppose that all edges in F are not incident with the same vertex. Let e be an i-dimensional edge in $Q_n - F$ for some $i \in \{1, 2, \ldots, n\}$. By Lemma 2.1 we can choose $k \in \{1, 2, \ldots, n\} \setminus \{i\}$ and express $Q_n = L_k \odot R_k$ such that $|F_L| \leqslant n - 2$ and $|F_R| \leqslant n - 2$. Moreover, if the equality hold, then all these $n - 2$ edges in either L_k or R_k are not incident with the same vertex.

Without loss of generality, assume $e \in L_k$ and let l be any even integer with $6 \leqslant l \leqslant 2^n$. To prove the theorem, we only need to construct a cycle of length l in $Q_n - F$ containing e.

If $6 \leqslant l \leqslant 2^{n-1}$ then, since $|F_L| \leqslant n - 2$ and by the induction hypothesis, e lies on a cycle of even length l in $L_k - F_L$.

In particular, let $C_{2^{n-1}}$ and$C_{2^{n-1}-2}$ denote such a cycle of length 2^{n-1} and $2^{n-1}-2$, respectively. Since $|E(C_{2^{n-1}}-e)| = 2^{n-1}-1 > 2(n-1) = 2|F|$ for $n \geqslant 5$, there is an edge u_Lv_L on $C_{2^{n-1}}$ such that $u_Lv_L \neq e$ and $\{u_Lu_R, v_Lv_R, u_Rv_R\} \cap F = \emptyset$. Let $P_{2^{n-1}-1} = C_{2^{n-1}} - u_Lv_L$. Since $|E(C_{2^{n-1}-2}-e)| = 2^{n-1}-2-1 > 2(n-1) = 2|F|$ for $n \geqslant 5$, there is an edge x_Ly_L on $C_{2^{n-1}-2}$ such that $x_Ly_L \neq e$ and $\{x_Lx_R, y_Ly_R, x_Ry_R\} \cap F = \emptyset$. Let $P_{2^{n-1}-3} = C_{2^{n-1}-2} - x_Ly_L$.

If $l = 2^{n-1}+2$, then $P_{2^{n-1}-1} + u_Lu_R + u_Rv_R + v_Rv_L$ is a cycle of length l in $Q_n - F$ containing e.

If $l = 2^{n-1}+4$ then, since $|F_R| \leqslant n-2$ and by the induction hypothesis, x_Ry_R lies on a cycle C'_6 of length 6 in $R_k - F_R$. Let $P'_5 = C'_6 - x_Ry_R$. Then $P_{2^{n-1}-3} + x_Lx_R + P'_5 + y_Ry_L$ is a cycle of even length l in $Q_n - F$ containing e.

If $2^{n-1}+6 \leqslant l \leqslant 2^n$ then, since $|F_R| \leqslant n-2$ and by the induction hypothesis, u_Rv_R lies on a cycle $C'_{l-2^{n-1}}$ of even length $l-2^{n-1}$ in $R_k - F_R$. Let $P'_{l-2^{n-1}-1} = C'_{l-2^{n-1}} - u_Rv_R$. Then $P_{2^{n-1}-1} + u_Lu_R + P'_{l-2^{n-1}-1} + v_Rv_L$ is a cycle of even length l in $Q_n - F$ containing e.

The theorem is proved.

References

[1] B. Alspach, D. Hare, Edge-pancyclic block-intersection graphs, Discrete Math. 97 (1–3) (1991) 17–24.

[2] J.A. Bondy, Pancyclic graphs, I, J. Combin. Theory 11 (1971) 80–84.

[3] J.A. Bondy, U.S.R. Murty, Graph Theory with Applications, Macmillan Press, London, 1976.

[4] K. Day, A. Tripathi, Embedding of cycles in arrangement graphs, IEEE Trans. Comput. 42 (1993) 1002–1006.

[5] A. Germa, M.C. Heydemann, D. Sotteau, Cycles in the cube-connected cycles graphs, Discrete Appl. Math. 83 (1998) 135–155.

[6] A. Hobbs, The square of a block is vertex pancyclic, J. Combin. Theory B 20 (1) (1976) 1–4.

[7] S.-Y. Hsieh, C.-H. Chen, Pancyclicity on Möbius cubes with maximal edge faults, Parallel Comput. 30 (3) (2004) 407–421.

[8] S.-Y. Hsieh, C.-W. Ho, G.-H. Chen, Fault-free Hamiltonian cycles in faulty arrangement graphs, IEEE Trans. Parallel Distributed Systems 10 (3) (1999) 223–237.

[9] S.C. Hwang, G.H. Chen, Cycles in butterfly graphs, Networks 35 (2) (2000) 161–171.

[10] Y.-R. Leu, S.-Y. Kuo, Distributed fault-tolerant ring embedding and reconfiguration in hypercubes, IEEE Trans. Comput. 48 (1) (1999) 81–88.

[11] T.K. Li, C.H. Tsai, J.J.M. Tan, L.H. Hsu, Bipanconnectivity and edge-fault-tolerant bipancyclicity of hypercubes, Inform. Process. Lett. 87 (2003) 107–110.

[12] S. Litifi, S. Zheng, N. Bagherzadeh, Optimal ring embedding in hypercubes with faulty links, in: Proc. Fault-Tolerant Computing Symp., 1992, pp. 178–184.

[13] J. Mitchem, E. Schmeichel, Pancyclic and bipancyclic graphs—a survey, in: Proc. First Colorado Symp. on Graphs and Applications, Boulder, CO, Wiley–Interscience, New York, 1985, pp. 271–278.

[14] Y. Saad, M.H. Schultz, Topological properties of hypercubes, IEEE Trans. Comput. 37 (7) (1988) 867–872.

[15] S. Sen, A. Sengupta, S. Bandyopadhyay, On some topological properties of hypercube, incomplete hypercube and supercube, in: Proc. Internat. Parallel Processing Symp., 1993, pp. 636–642.

[16] A. Sengupta, On ring in hypercubes with faulty nodes and links, Inform. Process. Lett. 68 (1998) 207–214.

[17] J.-M. Xu, Topological Structure and Analysis of Interconnection Networks, Kluwer Academic Publishers, Dordrecht/Boston/London, 2001.

[18] J.-M. Xu, Theory and Application of Graphs, Kluwer Academic Publishers, Dordrecht/Boston/London, 2003.