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Fault-tolerant pancyclicity of augmented cubes ☆

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Abstract

As an enhancement on the hypercube Q_n, the augmented cube AQ_n, prosed by Choudum and Sunitha [S.A. Choudum, V. Sunitha, Augmented cubes, Networks 40 (2) (2002) 71–84], not only retains some favorable properties of Q_n but also possesses some embedding properties that Q_n does not. For example, AQ_n is pancyclic, that is, AQ_n contains cycles of arbitrary length for $n \geqslant 2$. This paper shows that AQ_n remains pancyclic provided faulty vertices and/or edges do not exceed $2n-3$ and $n \geqslant 4$.

Keywords: Combinatorics; Networks; Augmented cubes; Pancycle; Fault tolerance

1. Introduction

It is well known that interconnection networks play an important role in parallel computing/communication systems. One of the central issues in evaluating a network is to study the embedding problem [14]. When a network is modeled by a graph, the embedding problem asks if a guest graph is a subgraph of a host graph, and an important benefit of graph embedding is that we can apply existing algorithms for guest graphs to host graphs. This problem has attracted a burst of studies in recent years. Cycle networks are suitable for designing simple algorithms with low communication cost. Since some parallel applications, such as those in image and signal processing, are originally designated on a cycle architecture, it is important to have effective cycle embedding in a network.

A graph is pancyclic if it contains cycles of every length from its girth to order inclusive. A graph is of pancyclicity if it is pancyclic. The pancyclicity of many networks has been investigated in the literature (see, for example, [1,4,5,9,7,16,18,19]).

Edge and/or vertex failures are inevitable when a large parallel computer system is put in use. Therefore, the fault-tolerant capacity of a network is a critical issue in parallel computing. A graph $G=(V,E)$ is k (resp. k-edge)-fault-tolerant pancyclic if $G-F$ is still pancyclic for any $F \subset E(G) \cup V(G)$ (resp. $F \subset E(G)$) with $|F| \leqslant k$. Fault-tolerant pancyclicity has been widely studied in many networks, such as [2, 4–8,10,13,15,17].

The hypercube network Q_n has proved to be one of the most popular interconnection networks [11,14]. As a variant of Q_n, Choudum and Sunitha [3] introduced the augmented cube AQ_n and proved AQ_n is pancyclic. Re-

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cently, Ma et al. [12] improved this result by showing that AQ_n is $(2n-3)$-edge-fault-tolerant pancyclic for any $n \geqslant 2$. For the hybrid presence of edge and vertex failures, Hsu et al. [8] showed that AQ_n is $(2n-3)$-Hamiltonian for $n \geqslant 4$. In this paper, we improve the above-mentioned result by showing the following theorem.

Theorem. *AQ_n is $(2n-3)$-fault-tolerant pancyclic for $n \geqslant 4$.*

The proof of the Theorem is in Section 3. Section 2 gives the definition of the augmented cube AQ_n.

2. Structure of augmented cubes

The n-dimensional augmented cube AQ_n $(n \geqslant 1)$ can be defined recursively as follows: AQ_1 is a complete graph K_2 with the vertex set $\{0, 1\}$. For $n \geqslant 2$, AQ_n is obtained by taking two copies of the augmented cube AQ_{n-1}, denoted by AQ^0_{n-1} and AQ^1_{n-1}, and adding 2^n edges between the two copies as follows.

Let $V(AQ^0_{n-1}) = \{0u_{n-1}\ldots u_2u_1\colon\ u_i = 0 \text{ or } 1\}$ and $V(AQ^1_{n-1}) = \{1u_{n-1}\ldots u_2u_1\colon\ u_i = 0 \text{ or } 1\}$. A vertex $u = 0u_{n-1}\ldots u_2u_1$ of AQ^0_{n-1} is joined to a vertex $v = 1v_{n-1}\ldots v_2v_1$ of AQ^1_{n-1} if and only if either

- (i) $u_i = v_i$ for $1 \leqslant i \leqslant n-1$; in this case, setting $v = u^h$ or $u = v^h$, or
- (ii) $u_i = \bar{v}_i$ for $1 \leqslant i \leqslant n-1$; in this case, setting $v = u^c$ or $u = v^c$.

The augmented cubes AQ_1, AQ_2, and AQ_3 are shown in Fig. 1. It is proved in [3] that AQ_n is a vertex transitive, $(2n-1)$-regular, and $(2n-1)$-connected graph with 2^n vertices for any positive integer n.

We call u^h and u^c the out-neighbors of u and call the edges between L and R crossed edges, denoted by E_c. Clearly every vertex u in AQ_n is incident with two edges (u, u^h) and (u, u^c) in E_c. According to the above definition, we write this recursive construction of AQ_n symbolically as $AQ_n = L \oplus R$, where $L \cong AQ^0_{n-1}$ and $R \cong AQ^1_{n-1}$. It is also clear that for two distinct vertices u and v in AQ_n their out-neighbors $u^h \neq v^h$ and $u^c \neq v^c$. Moreover, for any two distinct vertices u and v in AQ^0_{n-1} with $n \geqslant 4$, the four vertices u^h, u^c, v^h and v^c in AQ^1_{n-1} are all distinct provided that $(u, v) \notin \{(u, v) \notin E(AQ^0_{n-1})\colon\ u = 0u_{n-1}\ldots u_1 \text{ and } v = 0\bar{u}_{n-1}\ldots\bar{u}_1\}$.

3. Proof of Theorem

In this section, we give the proof of the Theorem stated in Introduction. For all the terminology and notation not defined here, we follow [14]. A graph G is Hamiltonian connected if there is a Hamiltonian path between any two vertices of G, and is k-fault-tolerant Hamiltonian connected if $G - F$ remains Hamiltonian connected for any $F \subset E(G) \cup V(G)$ with $|F| \leqslant k$. The following two lemmas, due to Hsu et al. [8], are used in our proofs.

Lemma 1. *AQ_n is $(2n-4)$-Hamiltonian connected for $n \geqslant 4$.*

Lemma 2. *For any four distinct vertices u, v, x, y in AQ_n $(n \geqslant 2)$, there exist two disjoint ux-path P_1 and vy-path P_2 such that $P_1 \cup P_2$ contains all vertices of AQ_n.*

Proof of Theorem. We prove the theorem by induction on $n \geqslant 4$. For $n = 4$, we have verified this conclusion with a computer by depth first search method within a polynomial time. Assume that the theorem is true for AQ_{n-1} with $n \geqslant 5$. Let F be any subset in $V(AQ_n) \cup E(AQ_n)$ with $|F| = 2n-3$, $F_v = F \cap V(AQ_n)$, $F_e = F \cap E(AQ_n)$. We prove that there is a cycle of length ℓ in $AQ_n - F$ for every ℓ with $3 \leqslant \ell \leqslant 2^n - |F_v|$. To this end, let us denote $AQ_n = L \oplus R$, where $L \cong AQ^0_{n-1}$ and $R \cong AQ^1_{n-1}$, $F^L = F \cap L$, $F^R = F \cap R$, $F^L_v = F_v \cap V(L)$, $F^R_v = F_v \cap V(R)$ and $F^c_e = F_e \cap E_c$.

Without loss of generality, we may assume $|F^L| \geqslant |F^R|$. Then $|F^R| \leqslant n-2 < 2n-6$ for $n \geqslant 5$, which implies that $R - F^R$ is Hamiltonian connected by

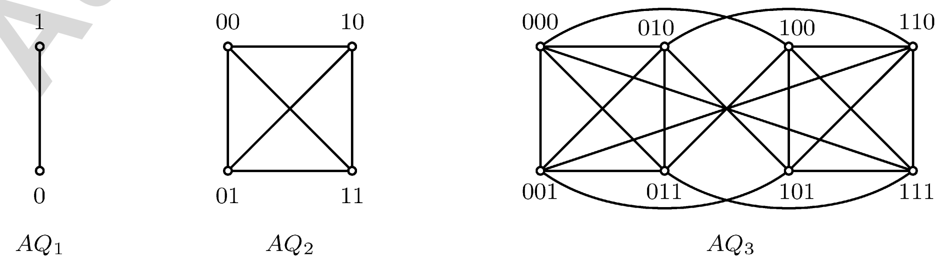

Fig. 1. Three augmented cubes AQ_1, AQ_2, and AQ_3.

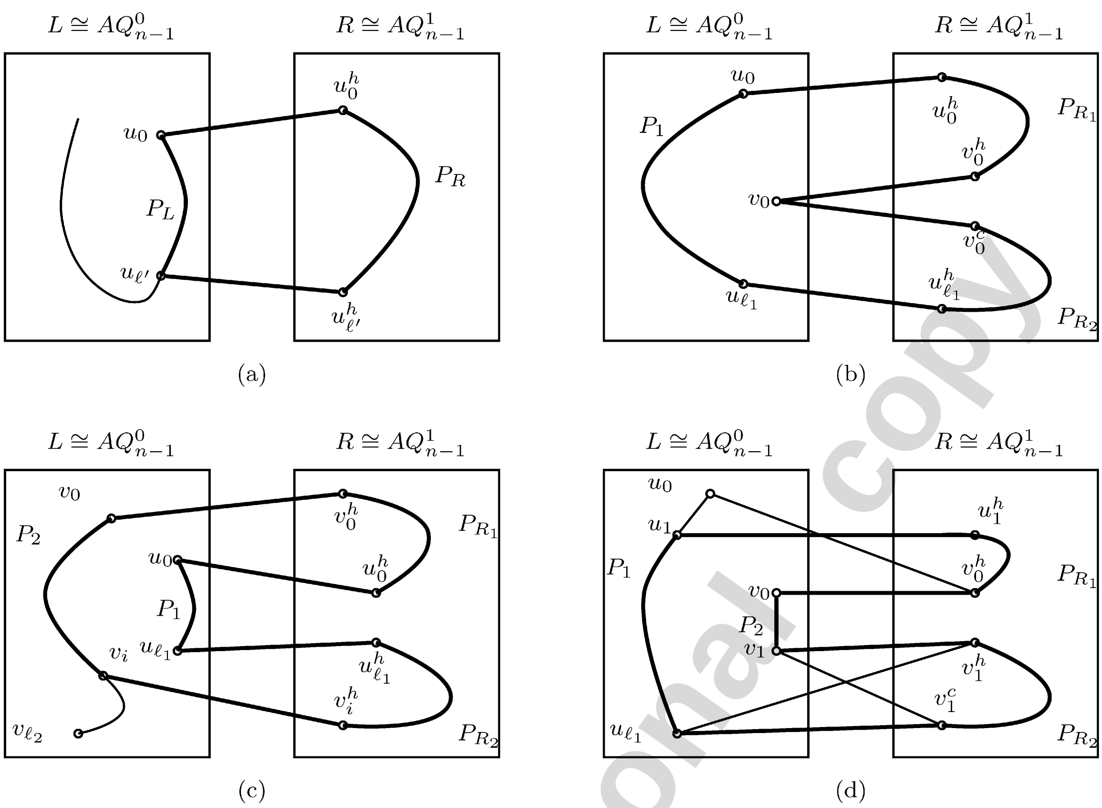

Fig. 2. Illustrations for the proof of Theorem. (A straight line represents an edge and a curve line represents a path between two vertices.)

Lemma 1. By the induction hypothesis, R is $(2n-5)$-fault-tolerant pancyclic, that is, for any ℓ with $3 \leqslant \ell \leqslant 2^{n-1} - |F_v^R|$, there is a cycle of length ℓ in $R - F^R$, and so in $AQ_n - F$.

Note $|F^L| + |F_e^c| \leqslant |F| = 2n-3 < 2^{n-1}$ for $n \geqslant 2$. There is a vertex u in L such that $\{(u, u^h), (u, u^c)\} \subset E_c \setminus F_e^c$. Let P_R be a (u^h, u^c)-path of length $2^{n-1} - |F_v^R| - 1$ in $R - F^R$ since $R - F^R$ is Hamiltonian connected. Then $(u, u^h) \cup P_R \cup (u^c, u)$ is a cycle of length $2^{n-1} - |F_v^R| + 1$ in $AQ_n - F$.

Thus, to complete the proof of the theorem, we only need to prove that there is a cycle of length ℓ in $AQ_n - F$ for every ℓ with $2^{n-1} - |F_v^R| + 2 \leqslant \ell \leqslant 2^n - |F_v|$. For a given ℓ, let $\ell' = \ell - 2^{n-1} + |F_v^R| - 1$. Then $1 \leqslant \ell' \leqslant 2^{n-1} - |F_v^L| - 1$. Consider the following two cases.

Case 1. $|F^L| \leqslant 2n-4$.

We first claim that there is a $(u_0, u_{\ell'})$-path P_L of length ℓ' in $L - F^L$ such that at least one of the two sets of crossed edges $\{(u_0, u_0^h), (u_{\ell'}, u_{\ell'}^h)\}$ and $\{(u_0, u_0^c), (u_{\ell'}, u_{\ell'}^c)\}$ is fault-free.

If $|F^L| \leqslant 2n-5$, then $L - F^L$ contains a cycle C of $2^{n-1} - |F_v^L|$ by the induction hypothesis. Thus there are $2^{n-1} - |F_v^L|$ distinct paths of length ℓ' in C. Suppose to the contrary that there does not exist such a $(u_0, u_{\ell'})$-path P_L in $L - F^L$. Then there are at least $2^{n-1} - |F_v^L|$ faults outside L, that is, $2n-3 = |F| \geqslant 2^{n-1}$ for $n \geqslant 5$, a contradiction.

If $|F^L| = 2n-4$, we claim that there is a Hamiltonian path H in $L - F^L$. In fact, by the induction hypothesis, L is $(2n-5)$-Hamiltonian. Let e be any element in F^L and C a Hamiltonian cycle in $L - \{F^L - \{e\}\}$. Then e must be in C, and so $C - \{e\}$ is a Hamiltonian path in $L - F_L$.

Take any section of H with length ℓ' as the $(u_0, u_{\ell'})$-path P_L. Since $|F_e^c| \leqslant 1$, one of $\{(u_0, u_0^h), (u_{\ell'}, u_{\ell'}^h)\}$ and $\{(u_0, u_0^c), (u_{\ell'}, u_{\ell'}^c)\}$ is fault-free.

Thus, there exists a required $(u_0, u_{\ell'})$-path P_L in $L - F^L$. Without loss of generality, assume $\{(u_0, u_0^h), (u_{\ell'}, u_{\ell'}^h)\}$ is fault-free.

Let P_R be a Hamiltonian $(u_{\ell'}^h, u_0^h)$-path in $R - F^R$ since $R - F^R$ is Hamiltonian connected. Then $P_L \cup (u_{\ell'}, u_{\ell'}^h) \cup P_R \cup (u_0^h, u_0)$ is a cycle in $AQ_n - F$ with length $\ell' + 2 + (2^{n-1} - |F_v^R| - 1) = \ell$ (see Fig. 2(a)).

Case 2. $|F^L| = 2n-3$.

In this case, all faults are in L, that is, $|F^R| = |F_e^c| = 0$ and $\ell' = \ell - 2^{n-1} - 1$.

Suppose that P_L is a $(u_0, u_{\ell'})$-path of length ℓ' in $L - F^L$. Let P_R be a $(u_0^h, u_{\ell'}^h)$-path of length $2^{n-1} - 1$ in R since R is Hamiltonian connected. Then $P_L \cup (u_{\ell'}, u_{\ell'}^h) \cup P_R \cup (u_0^h, u_0)$ is a cycle in $AQ_n - F$ with length $\ell' + 2 + (2^{n-1} - 1) = \ell$ (see Fig. 2(a)).

We now suppose that the length of any path in $L - F^L$ is smaller than ℓ'. Noting that an edge $(x, y) \in F$ can be replaced by the vertex x or the vertex y, we may, without loss of generality, assume $|F^L| = |F_v^L| = 2n - 3$.

Let u, v be in F_v^L. By the induction hypothesis, there is a Hamiltonian cycle C in $L - (F_v^L \setminus \{u, v\})$. Then both u and v are in C and not adjacent, otherwise $C - \{u, v\}$ is a path in $L - F_v^L$ with length $2^{n-1} - |F_v^L| - 1 \geqslant \ell'$, a contradiction. Let P_1 and P_2 be two sections of $C - \{u, v\}$. Then P_1 and P_2 are two disjoint paths in $L - F^L$. Denote the length of P_i as ℓ_i for $i = 1, 2$. Then $\ell_1 + \ell_2 = 2^{n-1} - |F_v| - 2$. Let $P_1 = (u_0, u_1, \ldots, u_{\ell_1})$, $P_2 = (v_0, v_1, \ldots, v_{\ell_2})$ and $C = (u, u_0) \cup P_1 \cup (u_{\ell_1}, v) \cup (v, v_{\ell_2}) \cup P_2 \cup (v_0, u)$. Without loss of generality, we may assume $\ell_1 \geqslant \ell_2$. Then $\ell_1 + 1 \leqslant \ell' \leqslant 2^{n-1} - |F_v| - 1$.

Note that if $(u_0, v_0) \in E(AQ_n)$ then the path $P_1 \cup (u_0, v_0) \cup P_2$ has length $2^{n-1} - |F_v^L| - 1 \geqslant \ell'$, a contradiction, which implies $(u_0, v_0) \notin E(AQ_n)$. Similarly, (u_0, v_{ℓ_2}), (u_{ℓ_1}, v_0), $(u_{\ell_1}, v_{\ell_2}) \notin E(AQ_n)$. This fact implies that the four vertices u_0^h, u_0^c, v_0^h and v_0^c are all distinct, and so are $u_{\ell_1}^h, u_{\ell_1}^c, v_{\ell_2}^h$ and $v_{\ell_2}^c$.

(a) If $\ell_2 = 0$, then P_2 consists of a single vertex v_0. In this case, $\ell_1 = 2^{n-1} - |F_v| - 2$, which implies $\ell' = \ell_1 + 1$ and $\ell = 2^n - |F_v|$. By Lemma 2, there are two disjoint (u_0^h, v_0^h)-path P_{R_1} and $(v_0^c, u_{\ell_1}^h)$-path P_{R_2} in R such that $P_{R_1} \cup P_{R_2}$ contains all vertices in R. Then $(u_0, u_0^h) \cup P_{R_1} \cup (v_0^h, v_0) \cup (v_0, v_0^c) \cup P_{R_2} \cup (u_{\ell_1}^h, u_{\ell_1}) \cup P_1$ is a cycle in $AQ_n - F$, of length $\ell_1 + 4 + 2^{n-1} - 2 = 2^n - |F_v| = \ell$ (see Fig. 2(b)).

(b) If $\ell_2 \geqslant 1$, then $\ell_1 \leqslant 2^{n-1} - |F_v| - 3$.

If $\ell_1 + 2 \leqslant \ell' \leqslant 2^{n-1} - |F_v| - 1$, then $2^{n-1} + \ell_1 + 3 \leqslant \ell \leqslant 2^n - |F_v|$. Let $\ell' = \ell_1 + i + 1$. Then $1 \leqslant i \leqslant \ell_2$ and $\ell = \ell_1 + i + 2^{n-1} + 2$. Let P_2' be the section of P_2 from v_0 to v_i. Clearly, $u_0^h, u_{\ell_1}^h, v_0^h, v_i^h$ are four distinct vertices. By Lemma 2, there are two disjoint (u_0^h, v_0^h)-path P_{R_1} and $(v_i^h, u_{\ell_1}^h)$-path P_{R_2} such that $P_{R_1} \cup P_{R_2}$ contains all vertices of R. So, $(u_0, u_0^h) \cup P_{R_1} \cup (v_0^h, v_0) \cup P_2' \cup (v_i, v_i^h) \cup P_{R_2} \cup (u_{\ell_1}^h, u_{\ell_1}) \cup P_1$ is a cycle in AQ_n, of length $\ell_1 + i + 4 + 2^{n-1} - 2 = \ell_1 + i + 2^{n-1} + 2 = \ell$ (see Fig. 2(c)).

Now assume $\ell' = \ell_1 + 1$. Then $\ell = 2^{n-1} + \ell_1 + 2$.

If there is a vertex v_i on P_2 whose two out-neighbors are different from out-neighbors of u_0 and u_{ℓ_1}, then there are two disjoint (u_0^h, v_i^h)-path P_{R_1} and $(v_i^c, u_{\ell_1}^h)$-path P_{R_2} such that $P_{R_1} \cup P_{R_2}$ contains all vertices of R by Lemma 2. So, $(u_0, u_0^h) \cup P_{R_1} \cup (v_i^h, v_i) \cup (v_i, v_i^c) \cup P_{R_2} \cup (u_{\ell_1}^h, u_{\ell_1}) \cup P_1, u_0$ is a cycle in $AQ_n - F$, of length $2^{n-1} + \ell_1 + 2 = \ell$.

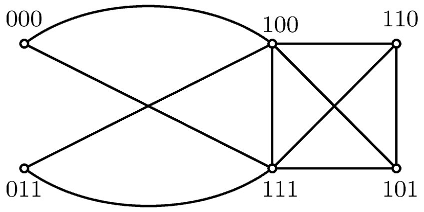

Fig. 3. $G = AQ_3 - F$.

If the two out-neighbors of every vertex in P_2 are the same as the out-neighbors of u_0 or u_{ℓ_1}, then $\ell_2 = 1$, that is, P_2 consists of a single edge (v_0, v_1). Without loss of generality, assume $\{v_0^h, v_0^c\} = \{u_0^h, u_0^c\}$ and $\{v_1^h, v_1^c\} = \{u_{\ell_1}^h, u_{\ell_1}^c\}$. Then $u_1^h \notin \{v_0^h, v_0^c, v_1^h, v_1^c\}$. By Lemma 2, there are disjoint (u_1^h, v_0^h)-path P_{R_1} and (v_1^h, v_1^c)-path P_{R_2} such that $P_{R_1} \cup P_{R_2}$ contains all vertices of R. Let P_1' be the section of P_1 from u_1 to u_{ℓ_1}. Then $(u_1, u_1^h) \cup P_{R_1} \cup (v_0^h, v_0) \cup P_2 \cup (v_1, v_1^h) \cup P_{R_2} \cup (v_1^c, u_{\ell_1}) \cup P_1'$ is a cycle in $AQ_n - F$, of length $2^{n-1} + \ell_1 + 2 = \ell$ (see Fig. 2(d)).

The proof of the theorem is complete. $\square$

Remark. In AQ_3, let $F = \{001, 010, (000, 011)\}$, then $AQ_3 - F$ is a graph G shown in Fig 3. Clearly, there is no cycle of length 6 in G since $G - \{100, 111\}$ has three connected components. Therefore, AQ_3 is not 3-fault-tolerant pancyclic. However, we can verify that AQ_3 is 2-fault-tolerant pancyclic with a computer by depth first search method within a polynomial time.

4. Conclusions

As one of the most fundamental networks for parallel and distributed computation, cycles are suitable for developing simple algorithms with low communication cost. Edge and/or vertex failures are inevitable when a large parallel computer system is put in use. Therefore, the fault-tolerant capacity of a network is a critical issue in parallel computing. The fault-tolerant pancyclicity of an interconnection network is a measure of its capability of implementing ring-structured parallel algorithms in a communication-efficient fashion in the presence of faults. The augmented cube AQ_n, as a variation of the hypercube Q_n, not only retains some favorable properties of Q_n but also possesses some embedding properties that Q_n does not (see [3]). Choudum and Sunitha [3] showed that AQ_n is pancyclic, that is, AQ_n contains cycles of arbitrary length for $n \geqslant 2$. Ma et al. [12] showed that AQ_n remains pancyclic provided faulty edges do not exceed $2n - 3$. For the hybrid presence of edge and vertex failures, Hsu et al. [8] showed that AQ_n remains Hamiltonian provided faulty vertices and/or edges do not exceed $2n-3$ and $n \geqslant 4$. In this paper, we improved these results by proving that AQ_n remains pancyclic provided faulty vertices and/or edges do not exceed $2n-3$ and $n \geqslant 4$.

In view of the fact that the hypercube network Q_n is not pancyclic, AQ_n is superior to Q_n in terms of the fault-tolerant pancyclicity. This shows that, when the augmented cube is used to model the topological structure of a large-scale parallel processing system, our result implies that the system has larger capability of implementing ring-structured parallel algorithms in a communication-efficient fashion in the hybrid presence of edge and vertex failures than one of the hypercube network. Our further work is to determine fault-tolerant panconnectivity of AQ_n, that is, whether there is all paths of every length between any two distinct vertices in AQ_n with failures.

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