On (a, d)-Antimagic Labelings of Generalized Petersen Graphs *

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Abstract

A connected graph $G = (V, E)$ is said to be (a, d)- antimagic, for some positive integers a and d, if its edges admit a labeling by all the integers in the set $\{1, 2, \ldots, |E(G)|\}$ such that the induced vertex labels, obtained by adding all the labels of the edges adjacent to each vertex, consist of an arithmetic progression with the first term a and the common difference d. Mirka Miller and Martin Bača proved that the generalized Petersen graph $P(n, 2)$ is $(\frac{3n+6}{2}, 3)$-antimagic for $n \equiv 0 \pmod 4$, $n \geq 8$ and conjectured that $P(n, k)$ is $(\frac{3n+6}{2}, 3)$-antimagic for even n and $2 \leq k \leq \frac{n}{2} - 1$. The first author of this paper proved that $P(n, 3)$ is $(\frac{3n+6}{2}, 3)$-antimagic for even $n \geq 6$. In this paper, we show that the generalized Petersen graph $P(n, 2)$ is $(\frac{3n+6}{2}, 3)$-antimagic for $n \equiv 2 \pmod 4$, $n \geq 10$.

Keywords: *(a, d)-antimagic labeling, Petersen graph, vertex labeling, edge labeling*

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1 Introduction

Hartsfield and Ringel [2] introduced the concept of arithmetic graphs. An arithmetic graph G is a graph whose edges can be labeled with the integers $1, 2, \ldots, |E(G)|$ so that the sum of the labels at any given vertex is different from the sum of the labels at any other vertex, in other words, no two vertices receive the same weight, where the weight of a vertex is defined in an obvious way. Hartsfield and Ringel conjectured that every tree other than K_2 is antimagic and, more strongly, every connected graph other than K_2 is antimagic.

Bodendiek and Walther[3] defined the concept of an (a, d)-antimagic graph as a special case of an antimagic graph. Let $G = (V, E)$ be a finite, undirected and simple graph with vertex set $V(G)$ and edge set $E(G)$, and let $p = |V(G)|$ and $q = |E(G)|$ be the number of vertices and edges of G, respectively. A connected graph $G = (V, E)$ is called (a, d)- antimagic if there exist positive integers a, d and a bijection $f : E \to \{1, 2, \ldots, q\}$ such that the induced mapping $g_f : V \to N$, defined by $g_f(v) = \sum f(uv),\ uv \in E(G)$, is injective and $g_f(V) = \{a, a + d, \ldots, a + (p - 1)d\}$. In this case f is called an (a, d)-antimagic labeling of G.

Bodendiek and Walther[4] proved that some graphs (including even cycles, paths of even order, stars, $C_3^{(k)}$, $C_7^{(k)}$, $K_{3,3}$ and a tree with odd order $n \geq 5$ and having a vertex adjacent to at least three end vertices) are not (a, d)-antimagic. They also proved that P_{2k+1} is $(k, 1)$-antimagic; C_{2k+1} is $(k+2, 1)$-antimagic; if a tree of odd order $2k+1 (k > 1)$ is (a, d)-antimagic, then $d = 1$ and $a = k$; if K_{4k} $(k \geq 2)$ is (a, d)-antimagic, then d is odd and $d \leq (2k + 1)(4k - 1) + 1$; if K_{2k+1} $(k \geq 2)$ is (a, d)-antimagic, then $d \leq (2k+1)(k-1)+1$. For special graphs called parachutes, (a, d)-antimagic labelings are described in [5, 6].

Let n and k be integers such that $n \geq 3,\ 1 \leq k < n$ and $n \neq 2k$. For such n, k, the generalized Petersen graph $P(n,k)$ is defined by

$$V(P(n,k)) = \{u_i, v_i | 1 \leq i \leq n\},$$
$$E(P(n,k)) = \{u_i u_{i+1},\ u_i v_i,\ v_i v_{i+k} | 1 \leq i \leq n\}$$

where and in the sequel the subscript of a vertex is computed modulo n and taken the least positive residue of n, in other words, we take u_n and v_n instead of u_0 and v_0, respectively.

Since $P(n,k)$'s form an important class of 3-regular graphs with $2n$ vertices and $3n$ edges, it is desirable to determine which $P(n,k)$'s are (a,d)-antimagic.

Bodendiek and Walther[7] conjectured that $P(n,1)$ is $(\frac{7n+4}{2}, 1)$-antimagic for even n and $P(n,1)$ is $(\frac{5n+5}{2}, 2)$-antimagic for odd n. These conjectures were proved in [9], where it was also shown that $P(n,1)$ is $(\frac{3n+6}{2}, 3)$-antimagic for even n.

Mirka Miller and Martin Bača [9] proved that $P(n,2)$ is $(\frac{3n+6}{2}, 3)$-antimagic for $n \equiv 0 \pmod 4), n \geq 8$ and conjectured that $P(n,k)$ is $(\frac{3n+6}{2}, 3)$-antimagic for even n and $2 \leq k \leq \frac{n}{2} - 1$. The first author [10] of this paper proved that $P(n,3)$ is $(\frac{3n+6}{2}, 3)$-antimagic for even $n \geq 6$. In this paper, we show that the generalized Petersen graph $P(n,2)$ is $(\frac{3n+6}{2}, 3)$-antimagic for $n \equiv 2 \pmod 4$, $n \geq 10$.

2 Statement of the Main Result

Theorem 2.1 $P(n,2)$ is $(\frac{3n+6}{2}, 3)$-antimagic for $n \equiv 2 \pmod 4$, $n \geq 10$.

Proof. We consider three cases:
Case 1: $n \equiv 10 \pmod{12}$, $n \geq 10$.

For $n \geq 10$, we define the edge labeling f of $P(n,2)$ as follows:

$$f(u_{i-1}u_i) = \begin{cases} \frac{5n+5-i}{2} & \text{if } 1 \le i \le n-3 \text{ and } i \equiv 1 \pmod 2, \\ 2n - \frac{i}{2} & \text{if } 2 \le i \le n-2 \text{ and } i \equiv 0 \pmod 2, \\ 3n+1-i & \text{if } n-1 \le i \le n, \end{cases}$$

$$f(u_iv_i) = \begin{cases} \frac{3n-1-i}{2} & \text{if } 1 \le i \le n-3 \text{ and } i \equiv 1 \pmod 2, \\ 3n+1-\frac{i}{2} & \text{if } 2 \le i \le n-4 \text{ and } i \equiv 0 \pmod 2, \\ -\frac{n}{2}+1+i & \text{if } i = n-2, \\ n+4+i & \text{if } i = n-1, \\ \frac{n}{2}+i & \text{if } i = n. \end{cases}$$

$$f(v_{i-2}v_i) = \begin{cases} i & \text{if } 1 \le i \le n-1 \text{ and } i \equiv 1 \pmod 2, \\ n+1-\frac{i}{2} & \text{if } 2 \le i \le \frac{n-4}{3} \text{ and } i \equiv 2 \pmod 4, \\ n-\frac{i}{2} & \text{if } \frac{n+2}{3} \le i \le n-2 \text{ and } i \equiv 0 \pmod 4, \\ \frac{n}{2}-1-\frac{i}{2} & \text{if } 4 \le i \le \frac{n-10}{3} \text{ and } i \equiv 0 \pmod 4 \\ & \quad \text{and } n > 10, \\ \frac{n}{2}-\frac{i}{2} & \text{if } \frac{n+8}{3} \le i \le n-4 \text{ and } i \equiv 2 \pmod 4, \\ 2n & \text{if } i = n. \end{cases}$$

For two integers a and b with $a \le b$, by $[a,b]$ we denote the set of consecutive integers from a to b. Set

$$A = \{f(u_{i-1}u_i) | 1 \le i \le n\},$$
$$B = \{f(u_iv_i) \ | 1 \le i \le n\},$$
$$C = \{f(v_{i-2}v_i) | 1 \le i \le n\}.$$

Then we have $A = A_1 \cup A_2 \cup A_3$, where

$$\begin{aligned}
A_1 &= \{f(u_{i-1}u_i)|1 \le i \le n-3 \text{ and } i \equiv 1 \pmod 2\} \\
&= \{\tfrac{5n+5-i}{2} \mid 1 \le i \le n-3 \text{ and } i \equiv 1 \pmod 2\} \\
&= \{\tfrac{5n}{2}+2, \tfrac{5n}{2}+1, \ldots, 2n+5, 2n+4\} \\
&= \{2n+4, 2n+5, \ldots, \tfrac{5n}{2}+1, \tfrac{5n}{2}+2\}, \\
A_2 &= \{f(u_{i-1}u_i)|2 \le i \le n-1 \text{ and } i \equiv 0 \pmod 2\} \\
&= \{2n-\tfrac{i}{2} \mid 2 \le i \le n-1 \text{ and } i \equiv 0 \pmod 2\} \\
&= \{2n-1, 2n-2, \ldots, \tfrac{3n}{2}+2, \tfrac{3n}{2}+1\} \\
&= \{\tfrac{3n}{2}+1, \tfrac{3n}{2}+2, \ldots, 2n-1\}, \\
A_3 &= \{f(u_{i-1}u_i)|n-1 \le i \le n\} = \{3n+1-i|n-1 \le i \le n\} \\
&= \{2n+2, 2n+1\} = \{2n+1, 2n+2\},
\end{aligned}$$

$B = B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5$, where

$$\begin{aligned}
B_1 &= \{f(u_iv_i)|1 \le i \le n-3 \text{ and } i \equiv 1 \pmod 2\} \\
&= \{\tfrac{3n-1-i}{2}|1 \le i \le n-3 \text{ and } i \equiv 1 \pmod 2\} \\
&= \{\tfrac{3n}{2}-1, \tfrac{3n}{2}-2, \ldots, n+2, n+1\} \\
&= \{n+1, n+2, \ldots, \tfrac{3n}{2}-1\}, \\
B_2 &= \{f(u_iv_i)|2 \le i \le n-4 \text{ and } i \equiv 0 \pmod 2\} \\
&= \{3n+1-\tfrac{i}{2}|2 \le i \le n-4 \text{ and } i \equiv 0 \pmod 2\} \\
&= \{3n, 3n-1, \ldots, \tfrac{5n}{2}+4, \tfrac{5n}{2}+3\} \\
&= \{\tfrac{5n}{2}+3, \tfrac{5n}{2}+4, \ldots, 3n-1, 3n\}, \\
B_3 &= \{f(u_iv_i)|i = n-2\} = \{-\tfrac{n}{2}+1+i|i = n-2\} = \{\tfrac{n}{2}-1\}, \\
B_4 &= \{f(u_iv_i)|i = n-1\} = \{n+4+i|i = n-1\} = \{2n+3\}, \\
B_5 &= \{f(u_iv_i)|i = n\} = \{\tfrac{n}{2}+i|i = n\} = \{\tfrac{3n}{2}\},
\end{aligned}$$

and $C = \bigcup_{i=1}^{6} C_i$, where

$$\begin{aligned}
C_1 &= \{f(v_{i-2}v_i) | 1 \le i \le n-1 \text{ and } i \equiv 1 \pmod 2\} \\
&= \{i | 1 \le i \le n-1 \text{ and } i \equiv 1 \pmod 2\} \\
&= \{1, 3, 5, \dots, n-1\}, \\
C_2 &= \{f(v_{i-2}v_i) | 2 \le i \le \tfrac{n-4}{3} \text{ and } i \equiv 2 \pmod 4\} \\
&= \{n+1-\tfrac{i}{2} | 2 \le i \le \tfrac{n-4}{3} \text{ and } i \equiv 2 \pmod 4\} \\
&= \{n, n-2, \dots, n-\tfrac{n-10}{2}\} \\
&= \{n-\tfrac{n-10}{6}, n-\tfrac{n-22}{6}, \dots, n-2, n\}, \\
C_3 &= \{f(v_{i-2}v_i) | \tfrac{n+2}{3} \le i \le n-2 \text{ and } i \equiv 0 \pmod 4\} \\
&= \{n-\tfrac{i}{2} | \tfrac{n+2}{3} \le i \le n-2 \text{ and } i \equiv 0 \pmod 4\} \\
&= \{n-\tfrac{n+2}{6}, n-\tfrac{n+14}{6}, \dots, \tfrac{n}{2}+1\} \\
&= \{\tfrac{n}{2}+1, \tfrac{n}{2}+3, \dots, n-\tfrac{n+14}{6}, n-\tfrac{n+2}{6}\}, \\
C_4 &= \{f(v_{i-2}v_i) | 4 \le i \le \tfrac{n-10}{3} \text{ and } i \equiv 0 \pmod 4\} \\
&= \{\tfrac{n}{2}-1-\tfrac{i}{2} | 4 \le i \le \tfrac{n-10}{3} \text{ and } i \equiv 0 \pmod 4\} \\
&= \{\tfrac{n}{2}-3, \tfrac{n}{2}-5, \dots, \tfrac{n}{2}-\tfrac{n-4}{6}\} \\
&= \{\tfrac{n}{2}-\tfrac{n-4}{6}, \tfrac{n}{2}-\tfrac{n-16}{6}, \dots, \tfrac{n}{2}-5, \tfrac{n}{2}-3\}, \\
C_5 &= \{f(v_{i-2}v_i) | \tfrac{n+8}{3} \le i \le n-4 \text{ and } i \equiv 2 \pmod 4\} \\
&= \{\tfrac{n}{2}-\tfrac{i}{2} | \tfrac{n+8}{3} \le i \le n-4 \text{ and } i \equiv 2 \pmod 4\} \\
&= \{\tfrac{n}{2}-\tfrac{n+8}{6}, \tfrac{n}{2}-\tfrac{n+20}{6}, \dots, 4, 2\} \\
&= \{2, 4, \dots, \tfrac{n}{2}-\tfrac{n+20}{6}, \tfrac{n}{2}-\tfrac{n+8}{6}\}, \\
C_6 &= \{f(v_{i-2}v_i) | i = n\} = \{2n\}.
\end{aligned}$$

It is easy to see that

$$\begin{aligned}
&C_1 \cup C_5 \cup C_4 \cup B_3 \cup C_3 \cup C_2 \\
&= \{1, 3, \dots, n-1\} \cup \{2, 4, \dots, \tfrac{n}{2}-\tfrac{n+20}{6}, \tfrac{n}{2}-\tfrac{n+8}{6}\} \\
&\quad \cup \{\tfrac{n}{2}-\tfrac{n-4}{6}, \tfrac{n}{2}-\tfrac{n-16}{6}, \dots, \tfrac{n}{2}-3\} \cup \{\tfrac{n}{2}-1\} \\
&\quad \cup \{\tfrac{n}{2}+1, \tfrac{n}{2}+3, \dots, n-\tfrac{n+14}{6}, n-\tfrac{n+2}{6}\} \\
&\quad \cup \{n-\tfrac{n-10}{6}, n-\tfrac{n-22}{6}, \dots, n-2, n\} \\
&= \{1, 3, \dots, n-1\} \cup \{2, 4, \dots, \tfrac{n}{2}-1\} \cup \{\tfrac{n}{2}+1, \tfrac{n}{2}+3, \dots, n\} \\
&= \{1, 2, 3, \dots, n\} = [1, n], \\
&B_1 \cup B_5 \cup A_2 \cup C_6 \\
&\ = \{n+1, n+2, \dots, \tfrac{3n}{2}-1\} \cup \{\tfrac{3n}{2}\} \cup \{\tfrac{3n}{2}+1, \tfrac{3n}{2}+2, \dots, 2n-1\} \cup \{2n\} \\
&\ = \{n+1, n+2, \dots, 2n\} = [n+1, 2n]
\end{aligned}$$

and

$$\begin{aligned} & A_3 \cup B_4 \cup A_1 \cup B_2 \\ & \quad = \{2n+1, 2n+2\} \cup \{2n+3\} \cup \{2n+4, 2n+5, \ldots, \tfrac{5n}{2}+1, \tfrac{5n}{2}+2\} \\ & \qquad \cup \{\tfrac{5n}{2}+3, \tfrac{5n}{2}+4, \ldots, 3n-1, 3n\} \\ & \quad = \{2n+1, 2n+2 \ldots, 3n\} = [2n+1, 3n]. \end{aligned}$$

So, we can prove that $f(E(G)) = [1, 3n]$.

Recall that for a vertex $v \in V(G)$, $g_f(v) = \sum_{uv \in E(G)} f(uv)$. We now prove that $g_f(V) = \{g_f(v) | v \in V\} = \{a + 3i | i = 0, 1, \ldots, 2n-1\}$, where $a = \frac{3n+6}{2}$.

For convenience, define $h_f(v) = \frac{1}{3}[g_f(v) - a]$ and write

$$W = \{h_f(v) | v \in V(G)\}.$$

Then, in order to prove $g_f(V) = \{a + 3i | i = 0, 1, \ldots, 2n-1\}$ it suffices to show $W = [0, 2n-1]$ or equivalently $[0, 2n-1] \subseteq W$.

By definition we see that

$$\begin{aligned} h_f(u_i) = \tfrac{1}{3}\left[f(u_{i-1}u_i) + f(u_i u_{i+1}) + f(u_i v_i) - a\right], \ 1 \le i \le n, \\ h_f(v_i) = \tfrac{1}{3}\left[f(v_{i-2}v_i) + f(v_i v_{i+2}) + f(u_i v_i) - a\right], \ 1 \le i \le n. \end{aligned}$$

(1) For $1 \le i \le n-3$ and $i \equiv 1 \pmod 2$ we have

$$\begin{aligned} h_f(v_i) &= \tfrac{1}{3}\left[f(v_{i-2}v_i) + f(v_i v_{i+2}) + f(u_i v_i) - a\right] \\ &= \tfrac{1}{3}\left[i + i + 2 + (\tfrac{3n-1-i}{2}) - a\right] = \tfrac{i-1}{2}, \end{aligned}$$

(2) For $i = n-2$ we have that

$$\begin{aligned} h_f(v_{n-2}) &= \tfrac{1}{3}\left[f(v_{n-4}v_{n-2}) + f(v_{n-2}v_n) + f(u_{n-2}v_{n-2}) - a\right] \\ &= \tfrac{1}{3}\left[(n - \tfrac{n-2}{2} + n - 2) + 2n + (-\tfrac{n}{2} + 1 + n - 2) - a\right] = \tfrac{n-2}{2}. \end{aligned}$$

which and (1) imply $[0, \frac{n-2}{2}] \subseteq W$.

(3) For $i = n-1$ we have that

$$\begin{aligned} h_f(v_{n-1}) &= \tfrac{1}{3}\left[f(v_{n-3}v_{n-1}) + f(v_{n-1}v_1) + f(u_{n-1}v_{n-1}) - a\right] \\ &= \tfrac{1}{3}\left[(n-1) + 1 + (n + 4 + n - 1) - a\right] = \tfrac{n}{2}. \end{aligned}$$

which and (2) imply $[0, \frac{n}{2}] \subseteq W$.

(4) For $\frac{n+2}{3} \leq i \leq n-4$ and $i \equiv 0 \pmod 2$ we have

$$h_f(v_i) \quad = \quad \frac{1}{3}\left[(n-\frac{i}{2})+(\frac{n}{2}-\frac{i+2}{2})+(3n+1-\frac{i}{2})-a\right]=n-1-\frac{i}{2},$$

which and (3) imply $[0, n-\frac{n+8}{6}] \subseteq W$.

(5) For $i = n-2$ we have

$$\begin{aligned} h_f(u_{n-2}) &= \frac{1}{3}(f(u_{i-1}u_i)+f(u_iu_{i+1})+f(u_iv_i)-a) \\ &= \frac{1}{3}\left[(2n-\frac{i}{2})+3n+1-(i+1)-\frac{n}{2}+1+i-a\right]=n-\frac{n+2}{6}, \end{aligned}$$

which and (4) imply $[0, n-\frac{n+2}{6}] \subseteq W$.

(6) For $2 \leq i \leq \frac{n-10}{3}$ and $i \equiv 0 \pmod 2$ we have

$$h_f(v_i) \quad = \quad \frac{1}{3}\left[(n+1-\frac{i}{2})+(\frac{n}{2}-1-\frac{i+2}{2})+(3n+1-\frac{i}{2})-a\right]=n-1-\frac{i}{2},$$

which and (5) imply $[0, n-2] \subseteq W$.

(7) For $i = n, \frac{n-4}{3}$ we have

$$h_f(v_i) \quad = \quad \begin{cases} \frac{1}{3}\left[2n+n+\frac{3n}{2}-a\right]=n-1, & \text{if } i=n \\ \frac{1}{3}\left[(n+1-\frac{n-4}{6})+(n-\frac{n+2}{6})\right] & \\ +\frac{1}{3}\left[(3n+1-\frac{n-4}{6})-a\right]=n, & \text{if } i=\frac{n-4}{3} \end{cases}$$

which and (6) imply $[0, n] \subseteq W$.

(8) For $1 \leq i \leq n-3$ and $i \equiv 1 \pmod 2$ we have

$$h_f(u_i) \quad = \quad \frac{1}{3}\left[\frac{5n+5-i}{2}+(2n-\frac{i+1}{2})+\frac{3n-1-i}{2}-a\right]=\frac{3n}{2}-\frac{i+1}{2},$$

which and (7) imply $[0, \frac{3n}{2}-1] \subseteq W$.

(9) For $n-1 \leq i \leq n$ we have

$$h_f(u_i) \quad = \quad \begin{cases} \frac{1}{3}\left[(2n+2)+(2n+1)+(2n+3)-a\right] & \\ =\frac{3n}{2}+1, & \text{if } i=n-1 \\ \frac{1}{3}\left[(2n+1)+(\frac{5n}{2}+2)+\frac{3n}{2}-a\right]=\frac{3n}{2}, & \text{if } i=n \end{cases}$$

which and (8) imply $[0, \frac{3n}{2}+1] \subseteq W$.

(10) For $2 \leq i \leq n-4$ and $i \equiv 0 \pmod 2$ we have

$$h_f(u_i) = \frac{1}{3}\left[(2n - \tfrac{i}{2}) + (\tfrac{5n+5-(i+1)}{2}) + (3n+1-\tfrac{i}{2}) - a\right] = 2n - \tfrac{i}{2},$$

which and (9) imply $[0, 2n-1] \subseteq W$.

We complete the proof of Case 1.

Case 2: $n \equiv 6 \pmod{12}$ and $n \geq 18$.

For $n \geq 18$, we define the edge labeling f of $P(n,2)$ as follows:

$$f(u_{i-1}u_i) = \begin{cases} 2n - \frac{i-3}{2} & \text{if } 1 \leq i \leq n-3 \text{ and } i \equiv 1 \pmod 2, \\ \frac{5n}{2} + 1 - \frac{i}{2} & \text{if } 2 \leq i \leq n-2 \text{ and } i \equiv 0 \pmod 2, \\ 2 & \text{if } i = n-1, \\ n-2 & \text{if } i = n. \end{cases}$$

$$f(u_iv_i) = \begin{cases} \frac{3n+5-i}{2} & \text{if } 1 \leq i \leq n-3 \text{ and } i \equiv 1 \pmod 2, \\ 3n+1-\frac{i}{2} & \text{if } 2 \leq i \leq n-4 \text{ and } i \equiv 0 \pmod 2, \\ n-4 & \text{if } i = n-2, \\ \frac{n}{2}+3 & \text{if } i = n-1, \\ n+1 & \text{if } i = n. \end{cases}$$

$$f(v_{i-2}v_i) = \begin{cases} i & \text{if } 1 \leq i \leq n-3 \text{ and } i \equiv 1 \pmod 2, \\ n-1 & \text{if } i = 2, \\ 4 & \text{if } i = 4, \\ 6 & \text{if } i = 6, \\ \frac{n}{2}+1+\frac{i}{2} & \text{if } 8 \leq i \leq n-22 \text{ and } i \equiv 8 \pmod{12}, \\ \frac{n}{2}+2+\frac{i}{2} & \text{if } 10 \leq i \leq n-20 \text{ and } i \equiv 10 \pmod{12}, \\ \frac{n}{2}+3+\frac{i}{2} & \text{if } 12 \leq i \leq n-18 \text{ and } i \equiv 0 \pmod{12}, \\ 1+\frac{i}{2} & \text{if } 14 \leq i \leq n-16 \text{ and } i \equiv 2 \pmod{12}, \\ 2+\frac{i}{2} & \text{if } 16 \leq i \leq n-14 \text{ and } i \equiv 4 \pmod{12}, \\ 3+\frac{i}{2} & \text{if } 18 \leq i \leq n-12 \text{ and } i \equiv 6 \pmod{12}, \\ \frac{n}{2}-1 & \text{if } i = n-10, \\ \frac{n}{2}+1 & \text{if } i = n-8, \\ n & \text{if } i = n-6, \\ n+2 & \text{if } i = n-4, \\ \frac{5n}{2}+1 & \text{if } i = n-2, \\ \frac{5n}{2}+2 & \text{if } i = n-1, \\ n+3 & \text{if } i = n. \end{cases}$$

In a similar way in Case 1, we can prove that $f(E(G)) = [1, 3n]$ and $g_f(V) = \{a + 3i | i = 0, 1, \ldots, 2n-1\}$. We omit the proof for short.

Case 3: $n \equiv 2 \pmod{12}$ and $n \geq 14$.

For $n \geq 14$, we define the edge labeling f of $P(n, 2)$ as follows:

$$f(u_{i-1}u_i) = \begin{cases} \frac{5n+5-i}{2} & \text{if } 1 \leq i \leq n-3 \text{ and } i \equiv 1 \pmod 2, \\ 2n-\frac{i}{2} & \text{if } 2 \leq i \leq n-2 \text{ and } i \equiv 0 \pmod 2, \\ 3n+1-i & \text{if } n-1 \leq i \leq n, \end{cases}$$

$$f(u_iv_i) = \begin{cases} \frac{3n-1-i}{2} & \text{if } 1 \leq i \leq n-3 \text{ and } i \equiv 1 \pmod 2, \\ 3n+1-\frac{i}{2} & \text{if } 2 \leq i \leq n-4 \text{ and } i \equiv 0 \pmod 2, \\ \frac{n}{2}+1 & \text{if } i = n-2, \\ 2n+3 & \text{if } i = n-1, \\ \frac{3n}{2} & \text{if } i = n. \end{cases}$$

$$f(v_{i-2}v_i) = \begin{cases} i & \text{if } 1 \le i \le n-1 \text{ and } i \equiv 1 \pmod 2, \\ i & \text{if } 2 \le i \le \frac{n-2}{3} \text{ and } i \equiv 0 \pmod 2, \\ n+1-\frac{i}{2} & \text{if } \frac{n+4}{3} \le i \le n-4 \text{ and } i \equiv 2 \pmod 4, \\ \frac{3n+6+i}{4} & (\text{ if } \frac{n+10}{3} \le i \le n-6 \text{ and } i \equiv 0 \pmod 8 \\ & \text{and } n \equiv 14 \pmod{24}\ ) \\ & \text{or (if } \frac{n+10}{3} \le i \le n-6 \text{ and } i \equiv 4 \pmod 8 \\ & \text{and } n \equiv 2 \pmod{24}), \\ \frac{n-2+i}{4} & (\text{if } \frac{n+22}{3} \le i \le n-2 \text{ and } i \equiv 4 \pmod 8 \\ & \text{and } n \equiv 14 \pmod{24}), \\ & \text{or (if } \frac{n+22}{3} \le i \le n-2 \text{ and } i \equiv 0 \pmod 8 \\ & \text{and } n \equiv 2 \pmod{24}), \\ 2n & \text{if } i = n. \end{cases}$$

In a similar way in Case 1, we can prove that $f(E(G)) = [1, 3n]$ and $g_f(V) = \{a + 3i | i = 0, 1, \ldots, 2n-1\}$. We omit the proof for short.

According to the proof of Case 1, Case 2 and Case3, we thus conclude that $P(n, 2)$ is a $(\frac{3n+6}{2}, 3)$-antimagic for $n \equiv 2(\text{mod } 4), n \ge 10$. The proof is complete. □

In Figure 2.1, Figure 2.2 and Figure 2.3, we give $(\frac{3n+6}{2}, 3)$-antimagic labeling for $P(18, 2)$, $P(22, 2)$ and $P(26, 2)$.

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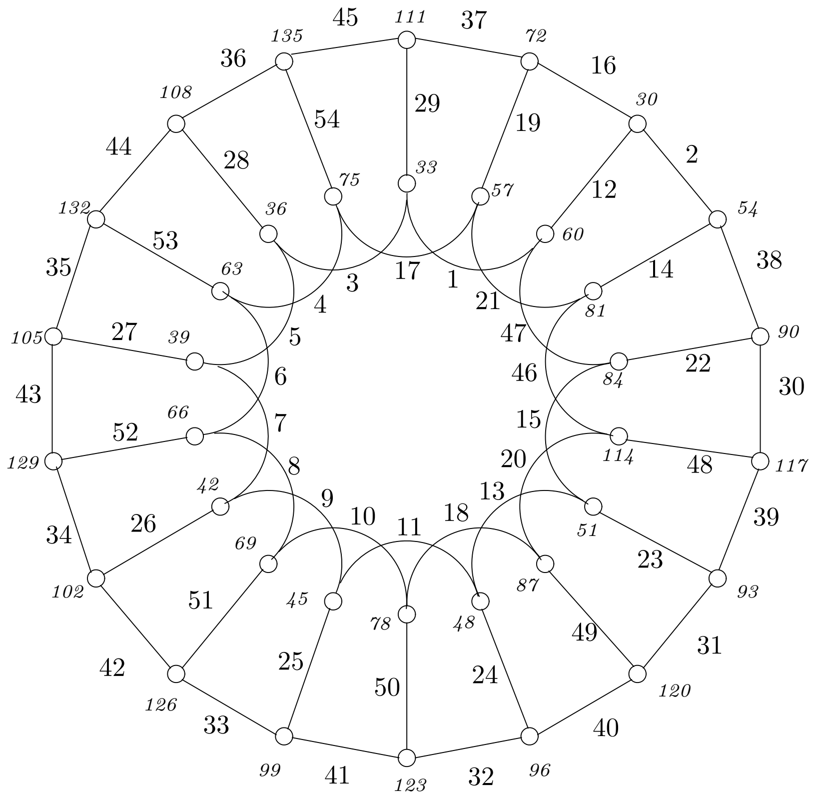

Figure 2.1 : The $(\frac{3n+6}{2}, 3)$-antimagic labeling of the graph $P(18, 2)$.

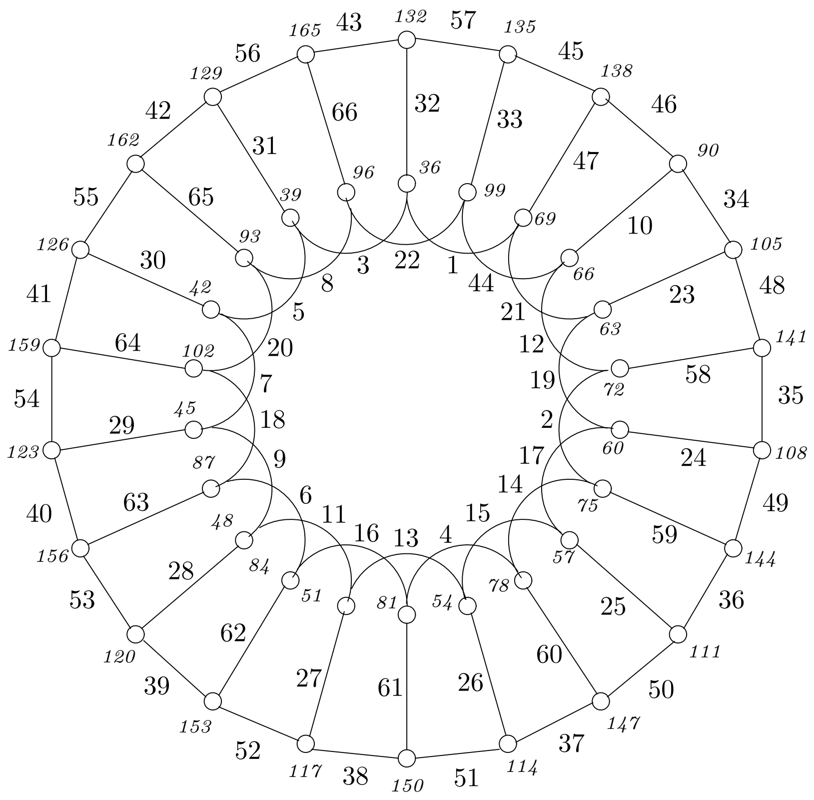

Figure 2.2 : The $(\frac{3n+6}{2}, 3)$-antimagic labeling of the graph $P(22, 2)$.

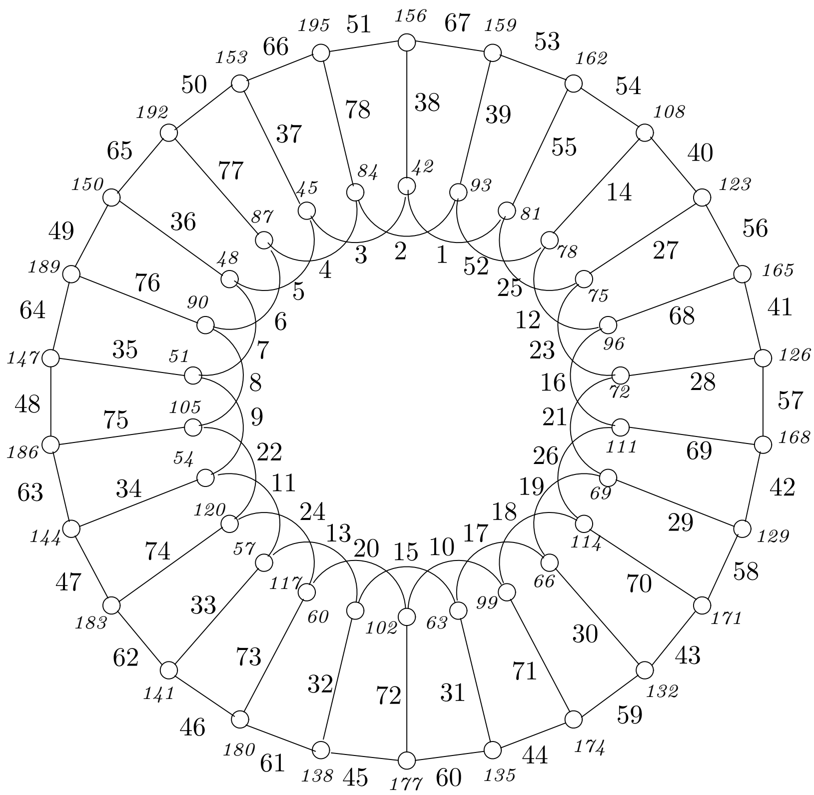

Figure 2.3 : The $(\frac{3n+6}{2}, 3)$-antimagic labeling of the graph $P(26, 2)$.