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A note on “The super connectivity of augmented cubes”

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ABSTRACT

The aim of this note is to mend a flaw in the proof of Theorem 2 in our paper [M. Ma, G. Liu, J.-M. Xu, The super connectivity of augmented cubes, Information Processing Letters 106 (2008) 59–63].

For the terminology and notation not given here, the reader is referred to [1]. Theorem 2 in [1] is stated as $\lambda'(AQ_n) = 4n - 4$ for $n \geqslant 5$. However, we find a flaw in the proof, that is, we misstate R's edge-connectivity $2n - 1$ instead of $2n - 3$, which leads to an improper proof. Now we restate our result and give its proof.

Theorem. $\lambda'(AQ_n) = 4n - 4$ *for* $n \geqslant 2$.

Proof. It is clear that $\xi(AQ_n) = 4n - 4$. By Lemma 2 in [1], we only need to prove $\lambda'(AQ_n) \geqslant 4n - 4$ for $n \geqslant 2$. The proof proceeds by induction on $n \geqslant 2$.

It is trivially true for AQ_2. Suppose that the result is true for AQ_{n-1} with $n \geqslant 3$. We will prove the result is true for AQ_n.

Let F be an arbitrary subset of edges in AQ_n such that $|F| \leqslant 4n - 5$. We will prove that if $AQ_n - F$ contains no isolated vertices then $AQ_n - F$ is connected.

Like [1], we write $AQ_n = L \oplus R$, where $L \cong AQ^0_{n-1}$ and $R \cong AQ^1_{n-1}$, and call the edges between L and R crossed edges (see Fig. 1). Every vertex of AQ_n is incident with two crossed edges. Let $F_L = F \cap L$ and $F_R = F \cap R$. Without loss of generality, we may suppose that $|F_L| \geqslant |F_R|$. Then $|F_R| \leqslant \lfloor (4n - 5)/2 \rfloor = 2n - 3$.

Our aim is to prove that if $AQ_n - F$ contains no isolated vertices then $AQ_n - F$ is connected. The proof strongly depends on whether $R - F_R$ is connected or not. However, in [1], we only showed this conclusion for the former, and neglected the later. Now, we replenish our proof. Suppose that $R - F_R$ is not connected. Then, $|F_R| = 2n - 3 = \lambda(R)$, $2n - 3 \leqslant |F_L| \leqslant 2n - 2$, and so there is at most one crossed edge in F. By the induction hypothesis, $\lambda'(R) = 4(n - 1) - 4 = 4n - 8$. Since $\lambda(R) = 2n - 3 < 4n - 8 = \lambda'(R)$ for $n \geqslant 3$, $R - F_R$ certainly contains an isolated vertex U and another connected component R'.

Case 1. $L - F_L$ is connected. Since every vertex is incident with two crossed edges and at most one of them is in F, every vertex in R is connected to a vertex in L via a crossed edge not in F. Hence, $AQ_n - F$ is connected.

Case 2. $L - F_L$ is not connected.

If $|F_L| = 2n - 2$, then every crossed edge is not in F. Every vertex in L is connected to a vertex in R' via a crossed

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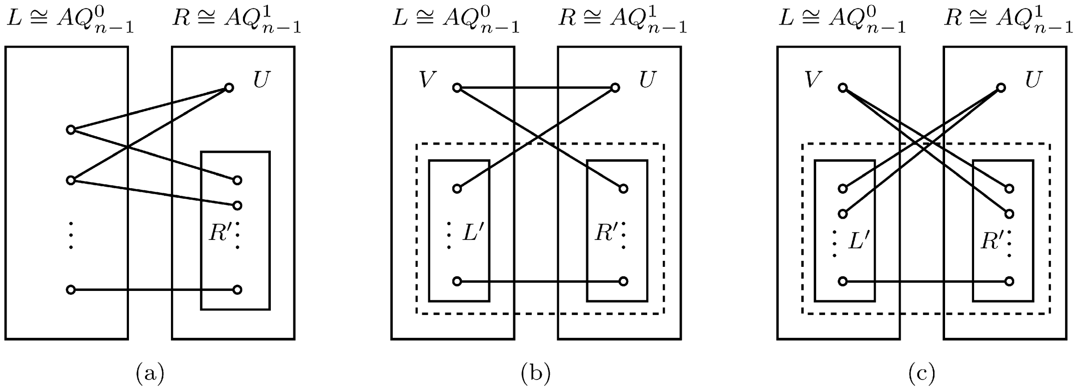

Fig. 1. Illustrations for the proof of the theorem.

edge. The isolated vertex U in R is connected to a vertex in R' via two crossed edges (see Fig. 1(a)). Hence, $AQ_n - F$ is connected.

If $|F_L| = 2n - 3$, then there is at most one crossed edge in F. By the induction hypothesis, $\lambda'(L) = 4(n-1) - 4 = 4n - 8$. Since $\lambda(L) = 2n - 3 < 4n - 8 = \lambda'(L)$ for $n \geqslant 3$, $L - F_L$ certainly contains an isolated vertex V and another connected component L'. It is clear that L' and R' are connected via crossed edges. There are two crossed-edge disjoint paths joining U (respectively, V) to $L' \cup R'$ (see Fig. 1(b) (c)), at least one of which contains no edges in F. Hence, $AQ_n - F$ is connected.

Thus, $AQ_n - F$ is connected, which means $\lambda'(AQ_n) \geqslant 4n - 4$ for $n \geqslant 2$. The theorem follows. □

References

[1] M. Ma, G. Liu, J.-M. Xu, The super connectivity of augmented cubes, Information Processing Letters 106 (2008) 59–63.