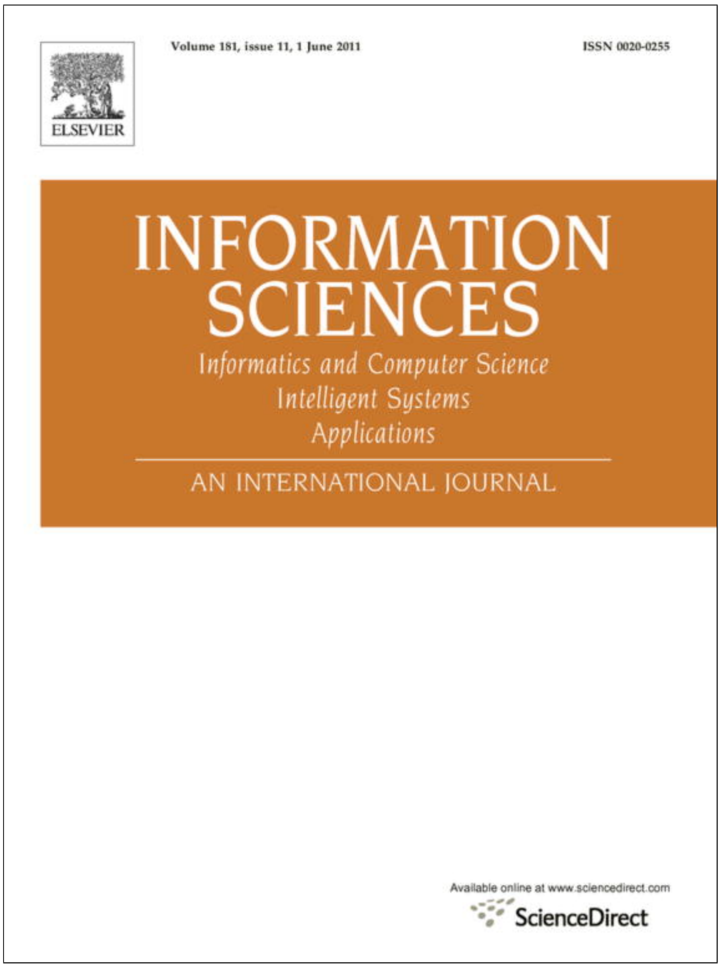
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Fault-tolerant edge-pancyclicity of locally twisted cubes ☆

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The n-dimensional locally twisted cube LTQ_n is a new variant of the hypercube, which possesses some properties superior to the hypercube. This paper investigates the fault-tolerant edge-pancyclicity of LTQ_n, and shows that if LTQ_n ($n \geqslant 3$) contains at most $n-3$ faulty vertices and/or edges then, for any fault-free edge e and any integer ℓ with $6 \leqslant \ell \leqslant 2^n - f_v$, there is a fault-free cycle of length ℓ containing the edge e, where f_v is the number of faulty vertices. The result is optimal in some senses. The proof is based on the recursive structure of LTQ_n.

1. Introduction

As topological structures, interconnection networks play an important role in parallel processing and computing systems, particularly in multicomputer systems, which provide an effective mechanism of exchanging data between processors. The n-dimensional hypercube Q_n, suggested first by Sullivan and Bashkow [32], is one of the most popular, versatile and efficient interconnection networks, which possesses many excellent features such as logarithmic number of links per node, logarithmic diameter, high symmetry, recursive structure, linear bisection width (see [12]) and, thus, becomes the first choice for the topological structure of parallel processing and computing systems (see [22,3]) . The machines based on the hypercube have been implemented commercially such as the Cosmic Cube from Caltech [31], the IPSC/2 from Intel [28] and Connection Machines [14]. Parallel algorithms, and simple but efficient routing and broadcasting algorithms based on the hypercube have been developed [22,29].

It is well known that the diameter of the hypercube Q_n is n. Hillis [14] showed that the hypercube Q_n does not have the smallest possible diameter relative to its number of vertices 2^n. To achieve smaller diameter with the same number of vertices and links as the hypercube, a great number of variants of the hypercube were proposed, such as the Möbius cube [6], the crossed cube [7], the twisted cube [13], the augmented cube [5], the locally twisted cube [40], the bubble-sort graph [2], the star graph [23], and so on (see [37]). All these variants of the hypercube have only about half of the diameter of the hypercube.

It is well known that an interconnection network can be modeled by a connected graph $G = (V, E)$, where $V = V(G)$ is the vertex-set and $E = E(G)$ is the edge-set of G, in which vertices represent processors and edges represent communication links between processors.

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There are a lot of mutually conflicting requirements in designing an interconnection network. It is almost impossible to design an interconnection network which is optimum from all aspects. One has to design a suitable network depending on the requirements and its properties. One of the central issues in designing and evaluating an interconnection network is to study how well other existing networks can be embedded into this network. This problem can be modeled by the following graph embedding problem: given a host graph H, which represents the network into which other networks are to be embedded, and a guest graph G, which represents the network to be embedded, the problem is to find a mapping from $V(G)$ to $V(H)$ such that each edge of G can be mapped to a path in H. Two common measures of effectiveness of an embedding are the dilation, which measures the slowdown in the new architecture, and the load factor, which gauges the processor utilization. The most ideal embedding is an isomorphic embedding, that is, the guest graph is isomorphic to a subgraph of the host graph, since such an embedding has both dilation and load one.

As two common guest graphs, linear arrays (i.e. paths) and rings (i.e. cycles) are fundamental networks for parallel and distributed computation. They are suitable for developing simple algorithms with low communication cost. Many efficient algorithms were originally designed based on linear arrays and rings for solving a variety of algebraic problems, graph problems and some parallel applications, such as those in image and signal processing (see, for example, [1,22]). Thus, it is important to have an effective path and/or cycle embedding in a network, specially in a network with edge (link) and vertex (processor) failures since a massive parallel system has a relatively high probability of failure. The path and/or cycle embedding properties of many interconnection networks, such as the hypercube and its variants, have been deeply investigated in the literature, for example, Fu [9], Tsai et al. [34], Hsieh et al. [16], Kueng et al. [21] and Wang et. al. [35] for hypercubes, Fan et al. [8] and Hsieh et al. [15] for crossed cubes, Yang et al. [42] for twisted cubes, Hsieh et al. [17] for augmented cubes, and so on. Xu and Ma [37] gave a survey of the recent results on these topics.

In this paper, we are interested in the path and/or cycle embedding properties of the n-dimensional locally twisted cube LTQ_n. Yang et al. proposed this new network [41] and proved that LTQ_n contains cycles of all lengths from 4 to 2^n [40]. Ma, Xu [27] and Hu et al.. [20], independently, improved this result by proving that for any edge in LTQ_n there are cycles of all lengths containing it. Ma and Xu [26] further improved this result by showing that for any two different vertices x and y with distance d in LTQ_n, there exist xy-paths of all lengths from d to $2^n - 1$ except for $d + 1$. Even when faulty elements occur, Chang et al. [4] and Park et al. [30], independently, showed that LTQ_n still contains fault-free cycles of all lengths provided that faulty elements do not exceed $n - 2$. Very recently, Han et al. [10] have showed that LTQ_n with at most $n - 3$ faulty elements contains paths of all lengths from $2^{n-1} - 1$ to $2^n - f_v - 1$ between any two distinct fault-free vertices, where f_v is the number of faulty vertices. Hsieh and Wu [19] have considered more faulty edges and showed that LTQ_n contains a fault-free Hamiltonian cycle provided that faulty edges do not exceed $2n - 5$ and each vertex is incident with at least two fault-free edges. This condition is natural since, in practical applications, the probability is small for a vertex x being isolated (all links incident with x are faulty) or pendant (only one link incident with x is fault-free and the others are all faulty).

We, in this paper, improve the related result by proving that if LTQ_n contains at most $n - 3$ faulty elements then, for any fault-free edge e and any integer ℓ with $6 \leqslant \ell \leqslant 2^n - f_v$, there is a fault-free cycle of length ℓ containing the edge e. We also demonstrate that this result is optimal in some senses.

The remainder of this paper is organized as follows. In Section 2, we recall the structure of LTQ_n, and some definitions and notations. In Section 3, we introduce some properties of LTQ_n to used in our proofs. In Section 4, we give the proof of our result. Finally, we give some concluding remarks in Section 5.

2. Preliminaries

We follow Xu [36] for graph-theoretical terminology and notation. A graph $G = (V,E)$ consists of a vertex-set V and an edge-set E, where $V = V(G)$ is a finite set and $E = E(G)$ is a subset of $\{xy|xy$ is an unordered pair of $V\}$. Two vertices x and y are *adjacent* if xy is an edge of G, and are also the *end-vertices* of xy. For a vertex x, we call the vertices adjacent to it the *neighbors* of x. The *degree* of a vertex x is the number of edges incident with it. A graph is called k-regular if each vertex has degree k. For two distinct vertices x and y, an xy-path between x and y is a sequence of distinct vertices in which any two consecutive vertices are adjacent. The length of a path is the number of edges on the path. An xy-path of length at least three is called a *cycle* if $x = y$. A connected subgraph of G is called a *spanning tree* if it contains all vertices of G and no cycles, in which a distinguished vertex is called the *root* of the spanning tree.

The *distance* between two distinct vertices x and y in G is the length of a shortest xy-path in G, and the *diameter* of G is the maximum distance between any two vertices. The *connectivity* of G is the minimum κ for which there are κ vertices whose removal results in a disconnected graph.

We now recall the definition of the *n-dimensional locally twisted cube*, proposed by Yang, Evans and Megson [41], which has 2^n vertices, and each vertex is an n-string on $\{0,1\}$.

Definition 1 [41]. The n-dimensional locally twisted cube, denoted by LTQ_n ($n \geqslant 2$), is recursively defined as follow.

(1) LTQ_2 is a graph isomorphic to Q_2.

(2) For $n \geqslant 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} according to the following steps. Let LTQ^0_{n-1} and LTQ^1_{n-1} denote graphs obtained by prefixing labels of each vertex of one copy of LTQ_{n-1} with 0 and with 1, respectively, and connect a vertex $x = 0x_2,x_3,\ldots,x_n$ of LTQ^0_{n-1} with another vertex $y = 1(x_2 + x_n),x_3,\ldots,x_n$ of LTQ^1_{n-1} by an edge xy, where '+' represents the modulo 2 addition.

The graphs shown in Fig. 1 are LTQ_3 and LTQ_4.

The locally twisted cube LTQ_n can be equivalently defined with the following non-recursive fashion.

Definition 2 [41]. For $n \geqslant 2$, the n-dimensional locally twisted cube LTQ_n is a graph with all n-strings on $\{0,1\}$ as the vertex-set. Two vertices $x = x_1x_2,\ldots,x_{n-1}x_n$ and $y = y_1,y_2,\ldots,y_{n-1}y_n$ of LTQ_n are adjacent if and only if either

(a) $x_i = \bar{y}_i$ and $x_{i+1} = y_{i+1} + x_n$ for some $1 \leqslant i \leqslant n-2$, and $x_j = y_j$ for all the remaining bits, where '+' represents the modulo 2 addition, or
(b) $x_i = \bar{y}_i$ for some $i \in \{n-1, n\}$, and $x_j = y_j$ for all the remaining bits.

According to the above definitions, it is not difficult to see that LTQ_n is n-regular. Furthermore, Yang, Evans and Megson [40,41] proved that the connectivity of LTQ_n is n, the diameter $n-1$ if $n = 3$ or 4, and $\lceil \frac{1}{2}(n+3) \rceil$ if $n \geqslant 5$, and that LTQ_n contains cycles of all lengths from 4 to 2^n. Very recently, Hsieh and Tu [18] have showed that LTQ_n contains n edge-disjoint spanning trees. Lin et al. [24] further proved that any two spanning trees with the same root x and for any other vertex $y \neq x$, the two different paths from x to y, one path in each tree, are internally vertex-disjoint. For more properties of LTQ_n, the reader can refer to [4,10,11,18,19,24–27,30,38–41].

We now make some remarks on the n-dimensional locally twisted cube.

First, like to many variants of the hypercube such as the Möbius cube, the crossed cube, the twisted cube, the augmented cube and otherwise, the locally twisted cube not only keeps many nice properties of the hypercube such as regularity, high connectivity and high recursive constructability, but also has diameter of about half of that of the hypercube of the same size.

Secondly, the locally twisted cube also keeps a nice property of the hypercube, that is, the labels of any two adjacent vertices differ in at most two successive bits. However, a common feature of the above-mentioned variants is that the labels of some neighboring vertices may differ in a large number of bits. As a result, a portion of good properties of hypercube are lost in these variants. For example, the design of efficient parallel algorithms on these variants turns out to be a difficult task [41].

Thirdly, the locally twisted cube LTQ_n contains cycles of all lengths from 4 to 2^n [40], but the hypercube Q_n contains only even cycles since it is a bipartite graph. Thus, LTQ_n is superior to Q_n in cycle embedding property.

Fourthly, the construction of the locally twisted cube LTQ_n is quite different from that of the twisted cube TQ_n. The former is defined for any positive integer n, while the latter only for odd integer.

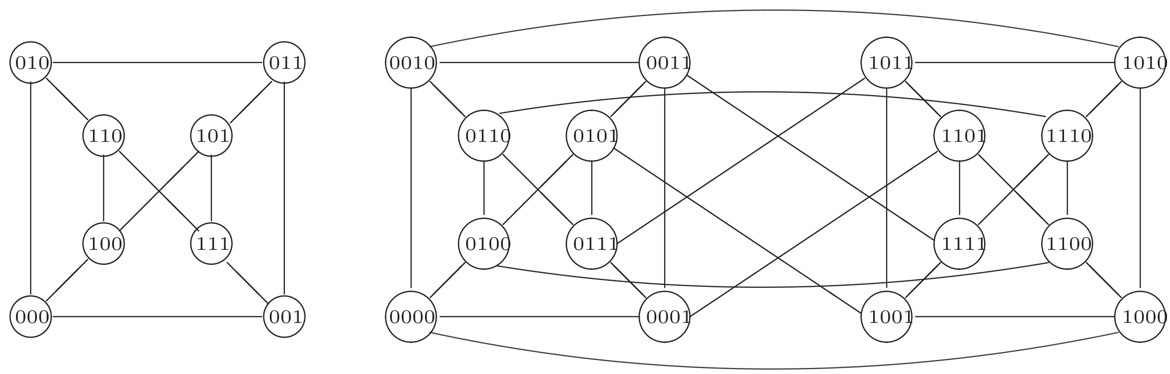
Lastly, it should be noted that, like to many variants of the hypercube, the locally twisted cube LTQ_n is not vertex-transitive for $n \geqslant 4$ proved by Liu et al. [25].

3. Properties

In this section, we introduce some properties of LTQ_n to be used in our proofs in Section 4.

Let $G = (V,E)$ be a graph. A non-empty subset of $E(G)$ is called a *matching* of G if no two of its elements have a common end-vertices in G. A matching M is *perfect* if every vertex of G is an end-vertex of some edge in M.

From Definition 1, LTQ_n can be expressed as the union of two disjoint copies of LTQ_{n-1} by adding a perfect matching between them according to the specified rule. For short, we often write $LTQ_n = L \oplus R$, where $L \cong LTQ^0_{n-1}$ and $R \cong LTQ^1_{n-1}$. Let E_C be the perfect matching, a set of edges connecting L with R. Obviously, $|E_C| = 2^{n-1}$.



(a) LTQ_3 (b) $LTQ_4 = LTQ_3^0 \oplus LTQ_3^1$

Fig. 1. The locally twisted cubes LTQ_3 and LTQ_4.

Yang, Evans and Megson [41] found an isomorphic expression of LTQ_n. For example, two graphs shown in Fig. 2(a) and (b) are other expressions of LTQ_3 and LTQ_4, respectively.

In general, they proved the following result.

Lemma 1 [41]. *Let L' be the graph obtained from Q_{n-1} by suffixing the labels of all vertices with 0, R' be the graph obtained from a graph isomorphic to Q_{n-1} by suffixing the labels of all vertices with 1. Then LTQ_n is isomorphic to the graph obtained from L' and R' by adding a perfect matching M between them, denoted by $LTQ_n = Q_{n-1} \oplus Q_{n-1}$, where M is the set of edges by linking two vertices with difference only suffixes.*

Let $LTQ_n = L \oplus R$ defined in Lemma 1. For convenience, for a vertex u in LTQ_n, if u is in L, we write u_L for u, and use u_R to denote its neighbor in R. Let u_L and v_L be two adjacent vertices in L. We say v_L is a *strong neighbor* of u_L in L if their neighbors u_R and v_R are adjacent in R, and a *weak neighbor* of u_L in L if their neighbors u_R and v_R are not adjacent in R. Similarly, we can define a strong neighbor or a weak neighbor of two adjacent vertices u_R and v_R in R.

Lemma 2. *Let $LTQ_n = L \oplus R$. If $n \geqslant 4$ then, for any vertex u_L in L, there are $n - 2$ strong neighbors and one weak neighbor in L. Moreover, if w_L is the weak neighbor of u_L, then the distance between u_R and w_R is two in R. The same conclusion holds for any vertex u_R in R.*

Proof. Let $u_L = 0x_2,x_3,x_4,\ldots,x_{n-1}x_n$ be a vertex in L. Then its neighbor in R is $u_R = 1(x_2 + x_n),x_3,x_4,\ldots,x_{n-1}x_n$. Let $s_L^2, s_L^3, \ldots, s_L^{n-1}, w_L$ be $n - 1$ neighbors of u_L in L, where

$$\begin{cases} s_L^2 = 0\bar{x}_2(x_3 + x_n)x_4 \ldots x_{n-1}x_n, \\ s_L^3 = 0x_2\bar{x}_3(x_4 + x_n)x_5 \ldots x_{n-1}x_n, \\ \ldots, \\ s_L^{n-2} = 0x_2 \ldots x_{n-3}\bar{x}_{n-2}(x_{n-1} + x_n)x_n, \\ s_L^{n-1} = 0x_2x_3x_4 \ldots x_{n-2}\bar{x}_{n-1}x_n, \\ w_L = 0x_2x_3x_4 \ldots x_{n-1}\bar{x}_n. \end{cases}$$

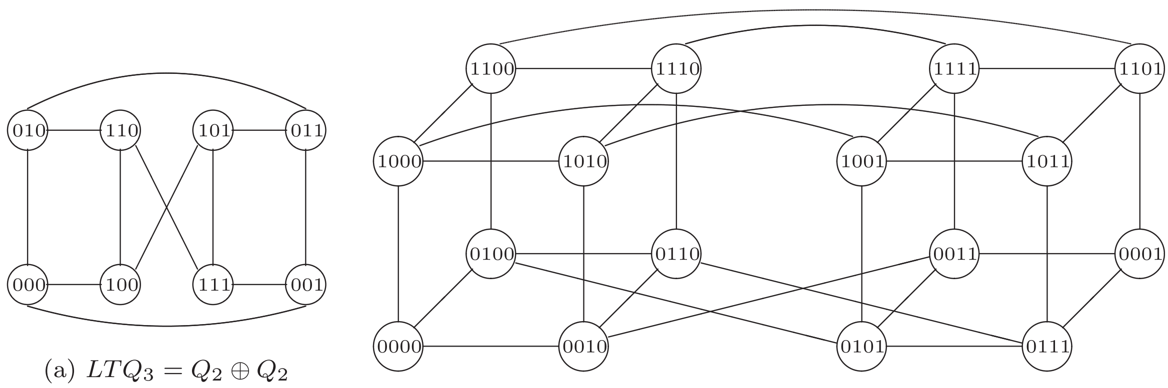
Let $s_R^2, s_R^3, \ldots, s_R^{n-1},\ w_R$ be neighbors of $s_L^2, s_L^3, \ldots, s_L^{n-1},\ w_L$ in R, respectively, where

$$\begin{cases} s_R^2 = 1(\bar{x}_2 + x_n)(x_3 + x_n)x_4 \ldots x_{n-1}x_n, \\ s_R^3 = 1(x_2 + x_n)\bar{x}_3(x_4 + x_n) \ldots x_{n-1}x_n, \\ \ldots, \\ s_R^{n-2} = 1(x_2 + x_n)x_3 \ldots x_{n-3}\bar{x}_{n-2}(x_{n-1} + x_n)x_n, \\ s_R^{n-1} = 1(x_2 + x_n)x_3x_4 \ldots x_{n-2}\bar{x}_{n-1}x_n, \\ w_R = 1(x_2 + \bar{x}_n)x_3x_4 \ldots x_{n-1}\bar{x}_n. \end{cases}$$

Since $\bar{x}_2 + x_n = \overline{x_2 + x_n}$, by definition of LTQ_n, $u_R s_R^i \in E(R)$ for $i = 2,3,\ldots,n - 1$, but u_R and w_R are not adjacent in R. So $s_L^2, s_L^3, \ldots, s_L^{n-1}$ are strong neighbors of u_L and w_L is a weak neighbor of u_L in L. In order to complete the proof of the lemma, we need to find a vertex o_R such that $u_R o_R o_R, w_R \in E(R)$. We can find this vertex w_R as follow.

$$\begin{cases} o_R = 1(x_2 + x_n)(x_3 + \bar{x}_n)x_4 \ldots x_{n-1}\bar{x}_n \quad \text{when} \quad x_n = 1; \\ o_R = 1(x_2 + \bar{x}_n)(x_3 + x_n)x_4 \ldots x_{n-1}x_n \quad \text{when} \quad x_n = 0. \end{cases}$$

It is easy to see that $u_R o_R$, $o_R w_R \in E(R)$, and so the distance between u_R and w_R is two in R. □



(a) $LTQ_3 = Q_2 \oplus Q_2$

(b) $LTQ_4 = Q_3 \oplus Q_3$

Fig. 2. Another painting of LTQ_3 and LTQ_4.

Lemma 3 ([27,20]). *Let e be any edge in LTQ_n and ℓ be any integer with $4 \leqslant \ell \leqslant 2^n$. Then there is a cycle of length ℓ containing the edge e in LTQ_n for $n \geqslant 2$.*

Lemma 4 [26]. *Let x and y be any two different vertices in LTQ_n and the distance between them be d. Then for any integer ℓ with $d \leqslant \ell \leqslant 2^n - 1$ except for $d + 1$, there exists an xy-path of length ℓ in LTQ_n for $n \geqslant 3$.*

Let F be a set of faulty elements in a graph G. An edge or a vertex in G is said to be *fault-free* if it is not in F, and a subgraph H of G to be *fault-free* if H contains no elements in F. Throughout this paper, we use f_v and f_e to denote the numbers of vertices and edges in F, respectively.

Lemma 5 [10]. *If $f_v + f_e \leqslant n - 3$ and $n \geqslant 3$, then for any integer ℓ with $2^{n-1} - 1 \leqslant \ell \leqslant 2^n - f_v - 1$, there is a fault-free path of length ℓ between any two distinct vertices in LTQ_n.*

Lemma 6 [33]. *Every fault-free edge (resp, vertex) of Q_n lies on a fault-free cycle of every even length from 4 to $2^n - 2f_v$ if $f_v + f_e \leqslant n - 2$ and $n \geqslant 3$.*

4. Fault-tolerant edge-pancyclicity

To state and prove our main result, we need some nations and terminologies. Let G be a connected graph, x and y be two distinct vertices in G. We use a sequence of distinct vertices $P = (x_0, x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_k)$, where $x_0 = x$ and $x_k = y$, to denote an xy-path. In addition, a path can be expressed as the union of several subpaths. For example, we can write $P = P(x_0, x_i) + P(x_i, x_{i+1}) + P(x_{i+1}, x_k)$, where $P(x_0, x_i) = (x_0, x_1, \ldots, x_{i-1}, x_i)$, $P(x_i, x_{i+1}) = (x_i, x_{i+1})$ and $P(x_{i+1}, x_k) = (x_{i+1}, \ldots, x_k)$. Since the subpath $P(x_i, x_{i+1})$ is a single edge x_ix_{i+1}, we write x_ix_{i+1} instead of $P(x_i, x_{i+1})$, that is, $P = P(x_0, x_i) + x_ix_{i+1} + P(x_{i+1}, x_k)$.

A graph G of order n is k-pancyclic if it contains cycles of all lengths from k to n, and *edge-k-pancyclic* if each of its edges lies on a cycle of every length from k to n. Clearly, an *edge-k-pancyclic* graph is certainly k-pancyclic. A graph G is f-fault-tolerant *edge-k-pancyclic* if $G - F$ is still *edge-k-pancyclic* for any $F \subset E(G) \cup V(G)$ with $|F| \leqslant f$.

In this section, we investigates the fault-tolerant edge-pancyclicity of LTQ_n and show that LTQ_n is $(n - 3)$-fault-tolerant edge-6-pancyclic. We state this result as the following theorem.

Theorem 1. *If $f_v + f_e \leqslant n - 3$ and $n \geqslant 3$ then, for any fault-free edge e in LTQ_n and any integer ℓ with $6 \leqslant \ell \leqslant 2^n - f_v$, there is a fault-free cycle of length ℓ containing the edge e in LTQ_n.*

Proof. We use the expression $LTQ_n = L \oplus R$, where $L \cong LTQ^0_{n-1}$ and $R \cong LTQ^1_{n-1}$. Let F be a set of faulty elements in LTQ_n with $|F| = f_v + f_e$, $F_L = F \cap L$, $F_R = F \cap R$, $F_C = F \cap E_C$, $F^v = F \cap V(LTQ_n)$, $F^v_L = F_L \cap V(L)$ and $F^v_R = F_R \cap V(R)$. Without loss of generality, we may assume $|F_R| \leqslant |F_L|$.

Let e be an arbitrary fault-free edge in LTQ_n and ℓ be any integer with $6 \leqslant \ell \leqslant 2^n - f_v$. We need to prove that there exists a fault-free cycle of length ℓ containing the edge e in LTQ_n if $|F| \leqslant n - 3$ and $n \geqslant 3$.

We proceed by induction on $n \geqslant 3$. For $n = 3$, there are no faulty vertices or edges in LTQ_3, and so the theorem holds by Lemma 3. Assume that the theorem holds for LTQ_{n-1}. We consider LTQ_n for $n \geqslant 4$.

Case 1. $|F_L| \leqslant n - 4$.

Subcase 1.1. The fault-free edge e is in L or R.

Since $|F_R| \leqslant n - 4$, without loss of generality, assume that the edge e is in L and let $e = u_Lv_L$.

If $6 \leqslant \ell \leqslant 2^{n-1} - |F^v_L|$ then, by the induction hypothesis, there is a fault-free cycle of length ℓ containing the edge e in L, so in LTQ_n. Thus, we only need to consider such an ℓ that satisfies $2^{n-1} - |F^v_L| + 1 \leqslant \ell \leqslant 2^n - f_v$.

If $n = 4$, since $0 \leqslant |F^v_L| \leqslant |F_L| \leqslant n - 4 = 0$, then $2^{4-1} + 1 = 9 \leqslant \ell \leqslant 2^4 - f_v$. Let $\ell = \ell' + 1$. Then $8 \leqslant \ell' \leqslant 2^4 - f_v - 1$. By Lemma 5, there is a fault-free u_Lv_L-path P of length ℓ' in LTQ_4, and so $P + u_Lv_L$ is a fault-free cycle of length ℓ containing the edge e.

Now, assume $n \geqslant 5$ and write $\ell = \ell_1 + 1 + \ell_2$, where $2^{n-2} - |F^v_L| \leqslant \ell_1 \leqslant 2^{n-1} - |F^v_L|$ and $2^{n-2} \leqslant \ell_2 \leqslant 2^{n-1} - |F^v_R| - 1$. Since $2^{n-2} - |F^v_L| \geqslant 2^{n-2} - |F_L| \geqslant 2^{n-2} - n + 4 > 6$ for $n \geqslant 5$, by the induction hypothesis, there is a fault-free cycle of length ℓ_1 containing the edge e in L. Note that a cycle of length ℓ_1 contains a matching M with $|M| = \lfloor \frac{\ell_1}{2} \rfloor$. Consider the following inequality.

$$\left\lfloor \frac{\ell_1}{2} \right\rfloor - |F_C| - |F_R| - |\{e\}| \geqslant \left\lfloor \frac{2^{n-2} - |F^v_L|}{2} \right\rfloor - |F_C| - |F_R| - 1 \geqslant 2^{n-3} - |F| - 1 \geqslant 2^{n-3} - n + 2.$$

Let $f(x) = 2^{x-3} - x + 2$. Since $f'(x) = 2^{x-3}\ln\ 2 - 1 \geqslant 0$ for $x \geqslant 5$, $f(x)$ is an increasing function, which implies that $\lfloor \frac{\ell_1}{2} \rfloor - |F_C| - |F_R| \geqslant f(5) = 2^{5-3} - 5 + 2 = 1$. In other words, there is such an edge, say x_Ly_L, in M that $x_Ly_L \neq e$ and two edges x_Lx_R and y_Ly_R are fault-free (see Fig. 3). Since $|F_R| \leqslant n - 4$, by Lemma 5, there is a fault-free x_Ry_R-path P of length ℓ_2 in R. So $C - x_Ly_L + y_Ly_R + P + x_Rx_L$ is a fault-free cycle of length $\ell (= \ell_1 + 1 + \ell_2)$ containing e (see Fig. 3).

Subcase 1.2. The fault-free edge e is in E_C.

Let $e = u_Lu_R$ and $u_L = 0x_2, x_3, \ldots, x_n$. Then $u_R = 1(x_2 + x_n), x_3, \ldots, x_n$.

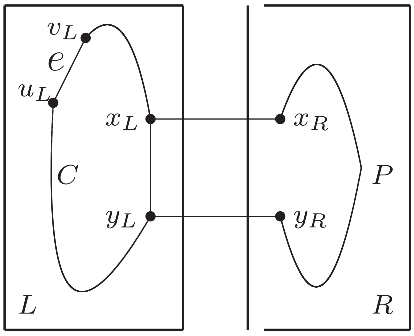


Fig. 3. The illustration of Subcase 1.1.

Assume $\ell = 6$. We use the second expression $LTQ_n = Q_{n-1} \oplus Q_{n-1}$ by Lemma 1. Since u_R and u_L have the same suffix x_n, u_R and u_L are in the same Q_{n-1}. Since $|F| \leqslant n-3 = (n-1)-2$, by Lemma 6, there is a fault-free cycle of length 6 containing the edge e in Q_{n-1}, and so in LTQ_n.

We now consider $\ell = 7$. We can construct $n-2$ cycles of length 7 that are disjoint each other except a common edge $e = u_Lu_R$ by considering two cases depending on $x_n = 0$ or $x_n = 1$.

Subcase 1.2.1. $x_n = 0$.

The required $n-2$ cycles $C_3,\ldots,C_{n-1},C_n$ of length 7 are constructed as follows. For each $i = 3,4,\ldots,n-1$, let $C_i = (u_L, s_L^i, w_L^i, w_R^i, o_R^i, s_R^i, u_R)$, where

$$C_i = \begin{cases} u_L = 0x_2x_3\ldots x_{n-1}0, \\ s_L^i = 0x_2x_3\ldots x_{i-1}\bar{x}_ix_{i+1}\ldots x_{n-1}0, \\ w_L^i = 0x_2x_3\ldots x_{i-1}\bar{x}_ix_{i+1}\ldots x_{n-1}1, \\ w_R^i = 1(x_2+1)x_3\ldots x_{i-1}\bar{x}_ix_{i+1}\ldots x_{n-1}1, \\ o_R^i = 1(x_2+1)x_3\ldots x_{i-1}\bar{x}_ix_{i+1}\ldots x_{n-1}0, \\ s_R^i = 1x_2x_3\ldots x_{i-1}\bar{x}_ix_{i+1}\ldots x_{n-1},0 \\ u_R = 1x_2x_3\ldots x_{n-1}0 \end{cases}$$

where s_L^i is a strong neighbor of u_L in L, w_L^i is the weak neighbor of s_L^i in L, w_R^i and s_R^i are neighbors of w_L^i and s_L^i in R, respectively, while o_R^i is such a vertex (the existence is guaranteed by Lemma 2) that $w_R^io_R^i,\ o_R^is_R^i \in E(R)$ (see Fig. 4(a)).

And let $C_n = (u_L, o_L, s_L, w_L, w_R, s_R, u_R)$, where

$$C_n = \begin{cases} u_L = 0x_2\ldots x_{n-1}0, \\ o_L = 0(x_2+1)x_3\ldots x_{n-1}0, \\ w_L = 0(x_2+1)x_3\ldots x_{n-1}1, \\ s_L = 0(x_2+1)x_3\ldots x_{n-2}\bar{x}_{n-1}1, \\ s_R = 1x_2\ldots x_{n-2}\bar{x}_{n-1}1, \\ w_R = 1x_2\ldots x_{n-1}1, \\ u_R = 1x_2\ldots x_{n-1}0, \end{cases}$$

where w_R is the weak neighbor of u_R in R, while s_R is a strong neighbor of w_R in R, s_L and w_L are neighbors of s_R and w_R in L, respectively, o_L is such a vertex that $u_Lo_L,\ o_Lw_L \in E(L)$ (see Fig. 4(b)).

Since $n \geqslant 4$, such constructed cycles of length 7 are well defined. For example, in LTQ_4, let $e = u_Lu_R$, where $u_L = 0000$ and $u_R = 1000$. Two cycles C_3 and C_4 of length 7 containing the edge e are as follows.

$$C_3 = \{0000, 0010, 0011, 1111, 1110, 1010, 1000\},$$
$$C_4 = \{0000, 0100, 0101, 0111, 1011, 1001, 1000\}.$$

It is easy to see that the cycles $C_3,\ldots,C_{n-1}$, C_n are as required, at least one of them is fault-free since $|F| = n-3 < n-2$.

Subcase 1.2.2. $x_n = 1$.

The required $n-2$ cycles $C_3,\ldots,C_{n-1},C_n$ are constructed by considering two cases depending on $n = 4$ or $n \geqslant 5$.

If $n = 4$ then $|F| = 1$ and $|E_C| = 8$, where

$$E_C = \left\{ \begin{array}{l} (0000, 1000), (0010, 1010), (0100, 1100), (0110, 1110), \\ (0111, 1011), (0101, 1001), (0001, 1101), (0011, 1111). \end{array} \right\}.$$

For each edge of E_C, we find two cycles of length 7, which contain this edge and disjoint except this edge as follows.

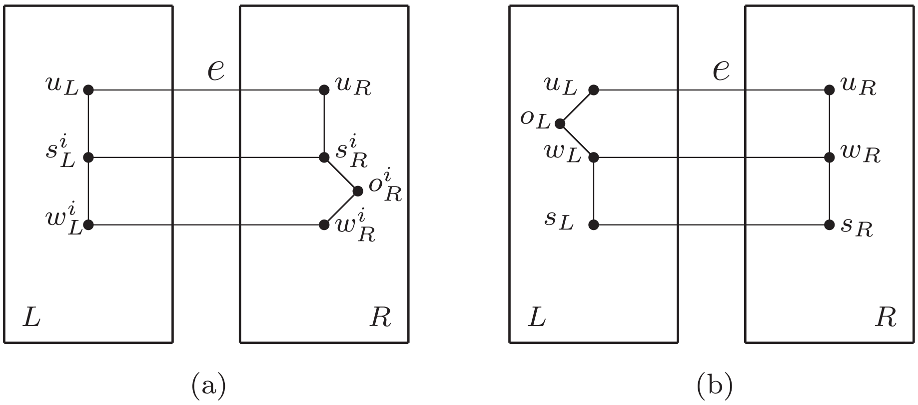


Fig. 4. The illustrations of Subcase 1.2.1.

Edge	Two cycles in common only the edge
(0000, 1000)	(0000, 0001, 1101, 1111, 1110, 1100, 1000), (0000, 0010, 0110, 0100, 0101, 1001, 1000)
(0010, 1010)	(0010, 0011, 0101, 0100, 0000, 1000, 1010), (0010, 0110, 1110, 1111, 1101, 1011, 1010)
(0100, 1100)	(0100, 0110, 0111, 0101, 1001, 1000, 1100), (0100, 0000, 0001, 1101, 1111, 1110, 1100)
(0110, 1110)	(0110, 0111, 0001, 0000, 1000, 1010, 1110), (0110, 0010, 0011, 0101, 100, 1111, 1110)
(0111, 1011)	(0111, 0110, 0010, 0000, 1000, 1001, 1011), (0111, 0001, 1101, 1111, 1110, 1010, 1011)
(0101, 1001)	(0101, 0111, 0110, 0100, 0000, 1000, 1001), (0101, 0011, 1111, 1110, 1010, 1011, 1001)
(0001, 1101)	(0001, 0000, 0100, 0101, 0111, 1011, 1101), (0001, 0011, 1111, 1001, 1000, 1100, 1101)
(0011, 1111)	(0011, 0001, 0000, 0100, 0101, 1001, 1111), (0011, 0010, 1010, 1000, 1100, 1110, 1111)

Now, we assume $n \geqslant 5$.

We construct $n-2$ cycles $C_3, C_4, \ldots, C_{n-2}, C_{n-1}, C_n$ of length 7 as follows. For each $i = 3, 4, \ldots, n-2$, let $C_i = (u_L, s_L^i, w_L^i, w_R^i, o_R^i, s_R^i, u_R)$, where

$$C_i = \begin{cases} u_L = 0x_2x_3\ldots x_{n-1}1, \\ s_L^i = 0x_2x_3\ldots x_{i-1}\bar{x}_i(x_{i+1}+1)x_{i+2}\ldots x_{n-1}1, \\ w_L^i = 0x_2x_3\ldots x_{i-1}\bar{x}_i(x_{i+1}+1)x_{i+2}\ldots x_{n-1}0, \\ w_R^i = 1x_2x_3\ldots x_{i-1}\bar{x}_i(x_{i+1}+1)x_{i+2}\ldots x_{n-1}0, \\ o_R^i = 1(x_2+1)x_3\ldots x_{i-1}\bar{x}_i(x_{i+1}+1)x_{i+2}\ldots x_{n-1}0, \\ s_R^i = 1(x_2+1)x_3\ldots x_{i-1}\bar{x}_i(x_{i+1}+1)x_{i+2}\ldots x_{n-1}1, \\ u_R = 1(x_2+1)x_3\ldots x_{n-1}1, \end{cases}$$

$C_{n-1} = (u_L, s_L^{n-1}, w_L^{n-1}, w_R^{n-1}, o_R^{n-1}, s_R^{n-1}, u_R)$, where

$$C_{n-1} = \begin{cases} u_L = 0x_2x_3\ldots x_{n-1}1, \\ s_L^{n-1} = 0x_2x_3\ldots x_{n-2}\bar{x}_{n-1}1, \\ w_L^{n-1} = 0x_2x_3\ldots x_{n-2}\bar{x}_{n-1}0, \\ w_R^{n-1} = 1x_2x_3\ldots x_{n-2}\bar{x}_{n-1}0, \\ o_R^{n-1} = 1(x_2+1)x_3\ldots x_{n-2}\bar{x}_{n-1}0, \\ s_R^{n-1} = 1(x_2+1)x_3\ldots x_{n-2}\bar{x}_{n-1}1, \\ u_R = 1(x_2+1)x_3\ldots x_{n-1}1 \end{cases}$$

and $C_n = (u_L, o_L, s_L, w_L, w_R, s_R, u_R)$, where

$$C_n = \begin{cases} u_L = 0x_2x_3\ldots x_{n-2}x_{n-1}1, \\ o_L = 0x_2x_3\ldots x_{n-2}x_{n-1}0, \\ s_L = 0(x_2+1)x_3\ldots x_{n-2}x_{n-1}0, \\ w_L = 0(x_2+1)x_3\ldots x_{n-2}\bar{x}_{n-1}0, \\ w_R = 1(x_2+1)x_3\ldots x_{n-2}\bar{x}_{n-1}0, \\ s_R = 1(x_2+1)x_3\ldots x_{n-2}x_{n-1}0, \\ u_R = 1(x_2+1)x_3\ldots x_{n-2}x_{n-1}1. \end{cases}$$

Since $n \geqslant 5$, such constructed cycles of length 7 are well defined. It is easy to check that these $(n-2)$ cycles are disjoint except a common edge u_Lu_R. Since $|F| = n-3 < n-2$, among them there exists a fault-free u_Lu_R-cycle of length $\ell = 7$ containing edge e.

Now assume $8 \leqslant \ell \leqslant 2^n - |F^v|$. If $n = 4$ then let $\ell = \ell' + 1$, where $2^{4-1} - 1 = 7 \leqslant \ell' \leqslant 2^4 - f_v - 1$. By Lemma 5, there is a fault-free u_Lu_R-path P of length ℓ' in LTQ_4. Then $P + u_Lu_R$ is a fault-free cycle of length ℓ containing the edge $e = u_Lu_R$.

Assume $n \geqslant 5$ below. Let s_L be a fault-free strong neighbor of u_L in L. Then u_Rs_R in R.

If $8 \leqslant \ell \leqslant 2^{n-1} - |F_L^v| - 1$, then let $\ell = \ell' + 2$, where $6 \leqslant \ell' \leqslant 2^{n-1} - |F_L^v| - 3$. By the induction hypothesis, there is a fault-free cycle C' of length ℓ' containing the edge u_Ls_L in L. Then $C = C' - u_Ls_L + s_Ls_R + s_Ru_R + u_Ru_L$ is a fault-free cycle of length ℓ containing the edge $e = u_Lu_R$.

If $2^{n-1} - |F_L^v| \leqslant \ell \leqslant 2^n - f_v$, then let $\ell = \ell_1 + \ell_2 + 1$, where $2^{n-2} - |F_L^v| \leqslant \ell_1 \leqslant 2^{n-1} - |F_L^v|$ and $2^{n-2} - 1 \leqslant \ell_2 \leqslant 2^{n-1} - |F_R^v| - 1$. Since $2^{n-2} - |F_L^v| > 6$ for $n \geqslant 5$, by the induction hypothesis, let C_1 be a cycle of length ℓ_1 containing the edge u_Ls_L in L and, by Lemma 5, let P be a fault-free s_Ru_R-path of length ℓ_2 in R. Then $C = C_1 - u_Ls_L + s_Ls_R + P + u_Ru_L$ is a fault-free cycle of length ℓ containing the edge $e = u_Lu_R$ in LTQ_n (see Fig. 5).

Case 2. $|F_L| = n - 3$. In this case $|F_R| = |F_C| = 0$.

Let $e = uv$ be a fault-free edge in LTQ_n. Let $\ell = \ell' + 1$. If $2^{n-1} - 1 \leqslant \ell' \leqslant 2^n - f_v - 1$ then, by Lemma 5, there exists a fault-free uv-path P of length ℓ' in LTQ_n. Thus, $P + uv$ is a fault-free cycle of length ℓ containing the edge e in LTQ_n. Thus, we only need to consider ℓ with $6 \leqslant \ell \leqslant 2^{n-1}$.

If the fault-free edge e is in R then, since $|F_R| = 0$ and by Lemma 3, there is a fault-free cycle of length ℓ containing the edge e for any ℓ with $6 \leqslant \ell \leqslant 2^{n-1}$. Thus, we only need to consider two cases according as the fault-free edge e is in L or E_C.

Subcase 2.1. The fault-free edge e is in L.

Let $e = u_Lv_L$ and let u_R and v_R be neighbors of u_L and v_L in R, respectively.

Suppose that v_L is a strong neighbor of u_L in L. By Lemma 2, $u_Rv_R \in E(R)$. Since $|F_C| = |F_R| = 0$, the cycle (u_L, v_L, v_R, u_R) of length 4 contains the edge e and is fault-free. For any ℓ with $6 \leqslant \ell \leqslant 2^{n-1}$, let $\ell = \ell' + 2$. Then $4 \leqslant \ell' \leqslant 2^{n-1} - 2$. By Lemma 3, there is a fault-free cycle C of length ℓ' containing edge u_Rv_R in R. So $C - u_Rv_R + v_Rv_L + u_Ru_L + u_Lv_L$ is a fault-free cycle of length ℓ containing the edge e.

Now suppose that v_L is the weak neighbor of u_L in L. Let $u_L = 0x_2,x_3,\ldots,x_n$ then, by Lemma 2, $v_L = 0x_2x_3\ldots\bar{x}_n$ and $d(u_R, v_R) = 2$.

If $\ell = 6$, by the definition of LTQ_n, we know $u_R = 1(x_2 + x_n)x_3,\ldots,x_n$ and $v_R = 1(x_2 + \bar{x}_n)x_3,\ldots,x_{n-1}\bar{x}_n$. We define two vertices w_R and m_R in R according to $x_n = 0$ or 1. If $x_n = 0$ then let $w_R = 1x_2\bar{x}_3,\ldots,x_{n-1}0$ and $m_R = 1x_2,(x_3 + 1)x_4,\ldots,x_{n-1}1$. If $x_n = 1$ then let $w_R = 1x_2(x_3 + 1)x_4,\ldots,x_{n-1}1$ and $m_R = 1x_2\bar{x}_3x_4,\ldots,x_{n-1}0$. Then $P = (u_R, w_R, m_R, v_R)$ is a u_Rv_R-path in R. Since $|F_R| = |F_C| = 0$, $u_Lv_L + v_Lv_R + P + u_Ru_L$ is a fault-free cycle of length 6 containing the edge e.

If $7 \leqslant \ell \leqslant 2^{n-1}$, let $\ell = \ell' + 3$, then $4 \leqslant \ell' \leqslant 2^{n-1} - 3$. Since $|F_R| = |F_C| = 0$, by Lemma 4, there is a fault-free u_Rv_R-path P of length ℓ' in R, and so $P + v_Rv_L + u_Ru_L + u_Lv_L$ is a fault-free cycle of length ℓ containing the edge e (see Fig. 6(a)).

Subcase 2.2. The fault-free edge e is in E_C. Let $e = u_R\, u_L$.

For any integer ℓ with $6 \leqslant \ell \leqslant 2^{n-1}$, let $\ell = \ell' + 2$. Then $4 \leqslant \ell' \leqslant 2^{n-1} - 2$. By Lemma 3, there is a fault-free cycle C of length ℓ' containing the edge u_Rv_R in R. So $C - u_Rv_R + u_Ru_L + u_Lv_L + v_Lv_R$ is a fault-free cycle of length ℓ containing the edge $e = u_Ru_L$ (see Fig. 6 (b)).

The proof of the theorem is complete. □

5. Conclusions and remarks

As one of the most fundamental networks for parallel and distributed computation, a cycle is suitable for developing simple algorithms with low communication cost. Edge and/or vertex failures are inevitable when a large parallel computer system is put in use. Therefore, the fault-tolerant capacity of a network is a critical issue in parallel computing. The fault-tolerant pancyclicity of an interconnection network is a measure of its capability of implementing ring-structured parallel algorithms in a communication-efficient fashion in the presence of faults.

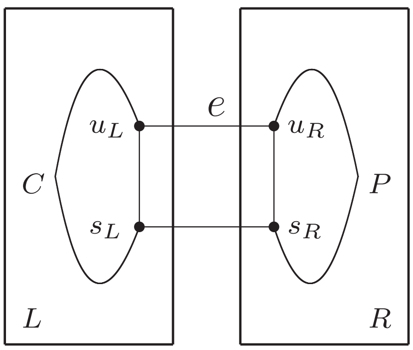


Fig. 5. The illustration of Subcase 1.2 for $\ell = \ell_1 + \ell_2 + 1$.

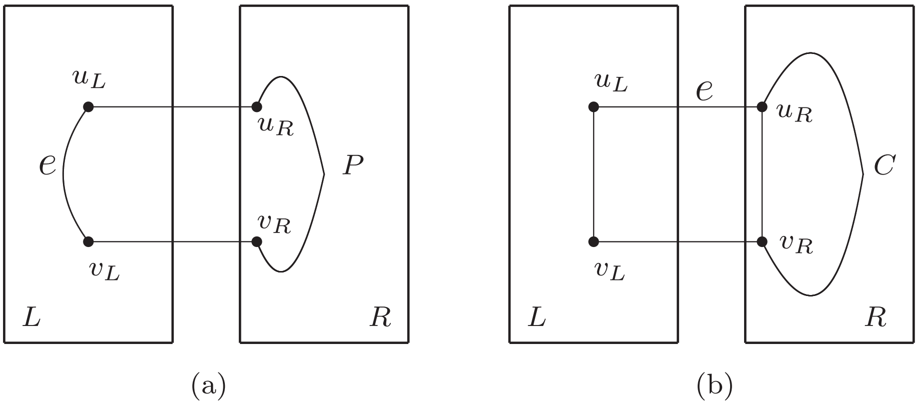


Fig. 6. The illustrations of Case 2.

The locally twisted cube LTQ_n, as a variation of the hypercube Q_n, not only retains some favorable properties of Q_n but also possesses some embedding properties that Q_n does not. For example, the diameter of LTQ_n is only about half of the diameter of Q_n. Yang, Megson and Evans [40] proved that LTQ_n contains cycles of all lengths from 4 to 2^n. Ma and Xu [27], independently, Hu et al.. [20] improved these results by proving that every edge in LTQ_n is contained in cycles of all lengths from 4 to 2^n. In particular, Chang, Ma and Xu [4] further improved the above results by proving that LTQ_n contains fault-free cycles of all lengths from 4 to $2^n - f_v$ provided $f_v + f_e \leqslant n - 2$. In this paper, we improve this result by proving that if $f_v + f_e \leqslant n - 3$ then for any fault-free edge e in $LTQ_n(n \geqslant 3)$ and any integer ℓ with $6 \leqslant \ell \leqslant 2^n - f_v$, there is a fault-free cycle of length ℓ containing the edge e.

In view of the fact that the hypercube network Q_n contains only even cycles, LTQ_n is superior to Q_n in fault-tolerant pancyclicity. This shows that, when the locally twisted cube is used to model the topological structure of a large-scale parallel processing system, our result implies that the system has larger capability of implementing ring-structured parallel algorithms in a communication-efficient fashion in the hybrid presence of edge and vertex failures than one of the hypercube network.

Our result is optimal in the following sense.

(1) Consider the edge uv, where $u = 00\cdots00$ and $v = 00\cdots01$. Then both u and v are in L if we write $LTQ_n = L \oplus R$, and v is the weak neighbor of u. By Lemma 2, it is easy to see that the edge uv is contained in one and only cycle of length 4. If the cycle appears one faulty element except vertices u, v and the edge uv, then there are no fault-free cycles of length 4 containing the edge uv in LTQ_n. For example, in LTQ_4, $u = 0000$ and $v = 0001$, the only cycle of length 4 containing the edge uv is $C = (0000, 0001, 0011, 0010)$ (see Fig. 1). If a faulty element is any vertex except u and v, or any edge except uv in C, then there are no fault-free cycles of length 4 containing the edge uv in LTQ_4.

(2) Consider the edge u_Lu_R, where $u_L \in L$ and $u_R \in R$ if we write $LTQ_n = L \oplus R$. By Lemma 2, there are only two distinct cycles of length 5 containing the edge u_Lu_R, which are obtained by the weak neighbor o_L of u_L in L and the weak neighbor o_R of u_R in R, respectively. It is easy to see that the two cycles contain o_L and o_R. If one of o_L and o_R is faulty, then there are no fault-free cycles of length 5 containing the edge u_Lu_R in LTQ_n. For example, in LTQ_4, taking $u_L = 0011$ and $u_R = 1111$, the only two cycles C_1 and C_2 of length 5 containing the edge uv are as follows.

$$C_1 = (0011, 0010, 0110, 1110, 1111, 0011) \text{ and}$$
$$C_2 = (0011, 0010, 1010, 1110, 1111, 0011).$$

If the vertex $x = \{0010\}$ is faulty, then there are no fault-free cycles of length 5 containing the edge uv in LTQ_4.

(3) As for the condition $f_v + f_e \leqslant n - 3$, we can say that it can be not improved as $n - 2$ at least when n is small. In fact, if so, in LTQ_3, let $\{010\}$ be a faulty vertex, then there are no fault-free cycles of length 6 containing edge $(000,001)$. Our proof for Theorem 1 uses induction on $n \geqslant 3$. The induction is based upon $n = 3$, which does not hold for $f_v + f_e = n - 2$ by the above example. The induction steps strongly depend on Lemma 5 which holds only when $f_v + f_e \leqslant n - 3$. Thus, our method can not improve $n - 3$ as $n - 2$. However, as our further work, we must make it clear whether or not $n - 3$ can be improved as $n - 2$ for more general integer n.

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