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Panconnectivity of Cartesian product graphs

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Abstract A graph G of order n (≥ 2) is said to be panconnected if for each pair (x, y) of vertices of G there exists an xy-path of length ℓ for each ℓ such that $d_G(x, y) \leq \ell \leq n-1$, where $d_G(x, y)$ denotes the length of a shortest xy-path in G. In this paper, we consider the panconnectivity of Cartesian product graphs. As a consequence of our results, we prove that the n-dimensional generalized hypercube $Q_n(k_1, k_2, \ldots, k_n)$ is panconnected if and only if $k_i \geq 3$ $(i = 1, \ldots, n)$, which generalizes a result of Hsieh et al. that the 3-ary n-cube Q_n^3 is panconnected.

Keywords Graph theory · Interconnection networks · Panconnectivity · Cartesian product · Hypercubes · k-ary n-cubes

1 Introduction

We follow [21, 22] for graph-theoretical terminology and notation not defined here. Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. The open neighborhood of a vertex $v \in V(G)$, denoted by $N_G(v)$, is the set of neighbors of v, and the degree of v is $d_G(v) = |N_G(v)|$. The minimum degree of G is $\delta(G) = \min\{d_G(v) : v \in V(G)\}$. For any $u, v \in V(G)$, the distance $d_G(u, v)$ between vertices u and v is the length of a shortest uv-path in G. And the diameter of G is $d(G) = \max\{d_G(u, v) : u, v \in V(G)\}$.

A graph G of order n (≥ 2) is said to be *panconnected* if for each pair (x, y) of vertices of G there exists an xy-path of length ℓ for each ℓ such that $d_G(x, y) \leq \ell \leq n-1$ in G. A graph is of *panconnectivity* if it is panconnected. Note that any path of length at least three is not of panconnectivity.

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The problem of finding paths of various lengths in interconnection networks has recently received much attention because this is an important measurement for determining if the topology of a network is suitable for an application in which mapping paths of various lengths into the topology are required. A survey on this topic can be found in Xu and Ma [23]. There are several interesting studies on panconnectivity of graphs (see, for example, [1, 4–7, 9–14, 17, 19, 20]). In this paper, we investigate the panconnectivity of the Cartesian product graphs since many popular networks can be constructed by such a product operation.

For graphs G_1 and G_2, the *Cartesian product* $G_1 \times G_2$ is the graph with vertex set $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and edge set $E(G_1 \times G_2) = \{(x_1, x_2)(y_1, y_2) \mid x_1 = y_1 \text{ and } x_2y_2 \in E(G_2) \text{ or } x_2 = y_2 \text{ and } x_1y_1 \in E(G_1)\}$.

It is easy to observe that as an operation of graph theory, the Cartesian products satisfy commutative and associative laws if we identify isomorphic graphs. By the commutative and associative laws of the Cartesian products, we may write $G_1 \times G_2 \times \cdots \times G_n$ for the Cartesian product of $G_1, G_2, \ldots, G_n$, where $V(G_1 \times G_2 \times \cdots \times G_n) = V_1 \times V_2 \times \cdots \times V_n$. Two vertices $x_1x_2 \cdots x_n$ and $y_1y_2 \cdots y_n$ are linked by an edge in $G_1 \times G_2 \times \cdots \times G_n$ if and only if two vectors $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$ differ exactly in one coordinate, say the ith, and there is an edge $x_iy_i \in E(G_i)$.

Several important classes of graphs can be defined in terms of the Cartesian products. The *n-dimensional hypercube* Q_n is $K_2 \times K_2 \times \cdots \times K_2$ of n identical complete graphs K_2. The hypercube is an important class of topological structures of interconnection networks. The panconnectivity of Q_n has been investigated by several authors (see, for example, [8–10, 15, 18]). A quite natural generalization of Q_n is the *k-ary n-cube* Q_n^k, which can be defined as $C_k \times C_k \times \cdots \times C_k$ of n identical cycles C_k of length k (≥ 3). The study on panconnectivity of Q_n^k has received much attention [3, 6, 7, 13, 16] recently. In particular, Hsieh, Lin and Huang [6] proved that Q_n^3 is panconnected.

In this paper, we consider panconnectivity of Cartesian product graphs and show that $G_1 \times G_2$ is panconnected if both G_1 and G_2 are two panconnected graphs of order at least three. As a consequence, we prove that the n-dimensional generalized hypercube $Q_n(k_1, k_2, \ldots, k_n)$ is panconnected if and only if $k_i \geq 3$ $(i = 1, \ldots, n)$, which generalizes the above-mentioned result of Hsieh et al. [6].

2 Main results

For a vertex x in G_1 and a subgraph $H \subseteq G_2$, we use xH to denote the subgraph $\{x\} \times H$ of $G_1 \times G_2$. Similarly, for a subgraph $H \subseteq G_1$ and a vertex y in G_2, Hy denotes the subgraph $H \times \{y\}$ of $G_1 \times G_2$. For a path $P = (x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ in a graph G, $P(x_i, x_j)$ denotes the section $(x_i, \ldots, x_j)$ of P. For the sake of convenience, we will express P as

$$P = x_1 \xrightarrow{P(x_1,x_i)} x_i \xrightarrow{P(x_i,x_j)} x_j \xrightarrow{P(x_j,x_n)} x_n.$$

The symbol $\varepsilon(P)$ denotes the length of P, which is the number of edges in P.

Lemma 1 (Theorem 2.3.3 in Xu [21]) *For any two distinct vertices* $x = (x_1, x_2)$ *and* $y = (y_1, y_2)$ *in* $G_1 \times G_2$, *where* $x_1, y_1 \in V(G_1)$ *and* $x_2, y_2 \in V(G_2)$, *we have* $d_{G_1 \times G_2}(x, y) = d_{G_1}(x_1, y_1) + d_{G_2}(x_2, y_2)$.

Lemma 2 *If* G *is a panconnected graph of order* $n \geq 3$, *then* $\delta(G) \geq 2$ *and* $d(G) \leq n - 2$.

Proof Suppose that $\delta(G) = 1$, then there is a vertex $a \in V(G)$ with $d_G(a) = \delta(G) = 1$. Let b be the unique neighbor of a in G. Clearly, for any ℓ with $2 \leq \ell \leq n - 1$, G contains no ab-paths of length ℓ, which contradicts to the panconnectivity of G. Hence $\delta(G) \geq 2$.

We now show that $d(G) \leq n - 2$. Let $d(G) = k$ and let u and v be two vertices in G with $d_G(u, v) = k$. Let P be a shortest uv-path in G. Since $d_G(u) \geq \delta(G) \geq 2$ and the shortness of P, there is some vertex $w \in N_G(u)$ that is not in P. Thus, P contains at most $n - 1$ vertices, which implies $d(G) = k \leq n - 2$. □

Lemma 3 *Let* $P = (x_1, x_2, \ldots, x_n)$ *be a path of order* $n \geq 2$ *and let* G_2 *be a panconnected graph of order* $m \geq 3$. *Then for any* $y \in V(G_2)$, $P \times G_2$ *contains* $(x_1, y)(x_n, y)$*-paths of every length from* $n + 1$ *to* $nm - 1$.

Proof To prove the lemma, for a given integer ℓ with $n + 1 \leq \ell \leq nm - 1$, we only need to find an $(x_1, y)(x_n, y)$-path L of length ℓ in $P \times G_2$.

By the panconnectivity of G_2, for any two distinct vertices u and v in G_2 and every integer i with $d_{G_2}(u, v) \leq i \leq m - 1$, there exists a uv-path of length i in G_2, denoted by $U_i(u, v)$. Since G_2 is panconnected, $\delta(G_2) \geq 2$ by Lemma 2. Let y_1 and y_2 be two distinct neighbors of y in G_2.

Case 1. If $n + 1 \leq \ell \leq 2m + n - 3$, then there are two integers i and j with $1 \leq i, j \leq m - 1$ such that $\ell = n - 1 + i + j$. So

$$L = (x_1, y) \xrightarrow{x_1 U_i(y, y_1)} (x_1, y_1) \xrightarrow{P y_1} (x_n, y_1) \xrightarrow{x_n U_j(y_1, y)} (x_n, y)$$

is an $(x_1, y)(x_n, y)$-path of length ℓ in $P \times G_2$.

Case 2. If $\ell = 2m + n - 2$, then $n \geq 3$ by $\ell \leq nm - 1$. Then the path

$$L = (x_1, y) \xrightarrow{x_1 U_{m-1}(y, y_1)} (x_1, y_1) \to (x_2, y_1) \xrightarrow{x_2 U_{m-1}(y_1, y_2)} (x_2, y_2)$$
$$\xrightarrow{P(x_2, x_n) y_2} (x_n, y_2) \to (x_n, y)$$

is an $(x_1, y)(x_n, y)$-path in $P \times G_2$, and $\varepsilon(L) = (m - 1) + 1 + (m - 1) + (n - 2) + 1 = \ell$.

Case 3. If $2m + n - 1 \leq \ell \leq nm - 1$, then $n \geq 3$. Since

$$0 \leq nm - 1 - (2m + n - 1) = (n - 3)(m - 1) + m - 3,$$

there are such two integers q and r with $0 \leq q \leq n - 3$ and $0 \leq r \leq m - 2$ that

$$\ell = (2m + n - 1) + q(m - 1) + r. \tag{2.1}$$

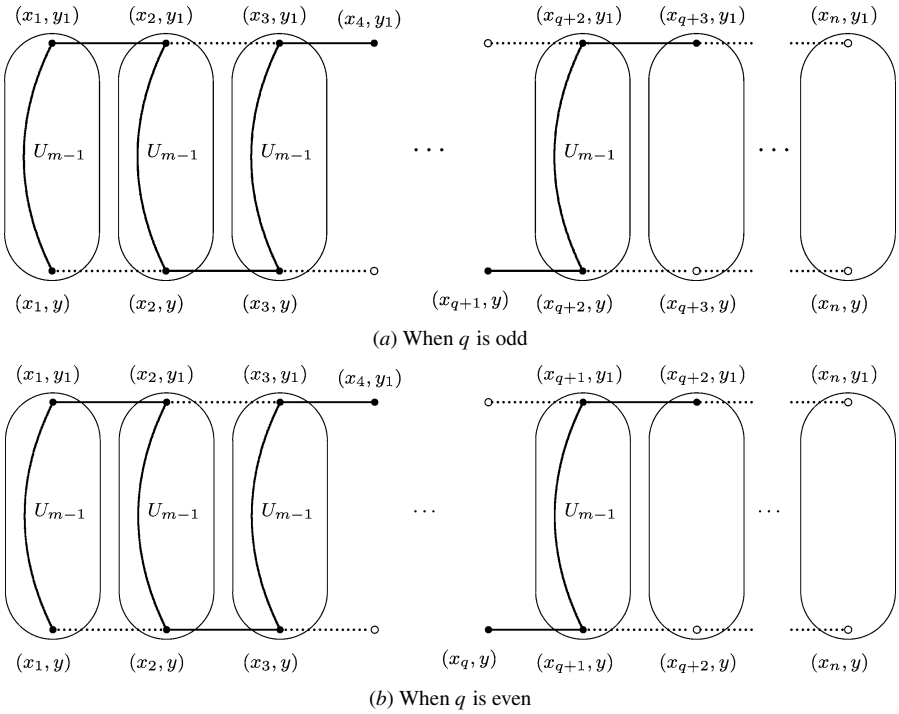

Fig. 1 Illustrations of the path R_q constructed in (2.2)

For a fixed q with $0 \le q \le n-3$, define a path R_q in $P \times G_2$ as follows:

$$R_q = \begin{cases} (x_1, y) \xrightarrow{x_1 U_{m-1}(y, y_1)} (x_1, y_1) \to (x_2, y_1) \xrightarrow{x_2 U_{m-1}(y_1, y)} (x_2, y) \\ \qquad \to \cdots \xrightarrow{x_{q+2} U_{m-1}(y, y_1)} (x_{q+2}, y_1) \to (x_{q+3}, y_1) \quad \text{if } q \text{ is odd;} \\ (x_1, y) \xrightarrow{x_1 U_{m-1}(y, y_1)} (x_1, y_1) \to (x_2, y_1) \xrightarrow{x_2 U_{m-1}(y_1, y)} (x_2, y) \\ \qquad \to \cdots \xrightarrow{x_{q+1} U_{m-1}(y, y_1)} (x_{q+1}, y_1) \to (x_{q+2}, y_1) \quad \text{if } q \text{ is even.} \end{cases} \tag{2.2}$$

Illustrations of the constructed path R_q in (2.2) are shown in Fig. 1.
Then the length of R_q is

$$\varepsilon(R_q) = \begin{cases} (q+2)m & \text{if } q \text{ is odd;} \\ (q+1)m & \text{if } q \text{ is even.} \end{cases} \tag{2.3}$$

Case 3.1. If $(n-1)m + 2 \le \ell \le nm - 1$, then, by (2.1), $q = n-3$ and

$$0 \le r = \ell - (2m + n - 1) - (n-3)(m-1) = \ell - (n-1)m - 2 \le m - 3.$$

Let

$$L = \begin{cases} (x_1, y) \xrightarrow{R_{n-3}} (x_n, y_1) \xrightarrow{x_n U_{r+2}(y_1, y)} (x_n, y) & \text{if } n \text{ is even;} \\ (x_1, y) \xrightarrow{R_{n-3}} (x_{n-1}, y_1) \xrightarrow{x_{n-1} U_{m-1}(y_1, y_2)} (x_{n-1}, y_2) & \\ \qquad \to (x_n, y_2) \xrightarrow{x_n U_{r+2}(y_2, y)} (x_n, y) & \text{if } n \text{ is odd.} \end{cases}$$

Then L is an $(x_1, y)(x_n, y)$-path in $P \times G_2$, and is of length, by (2.3),

$$\varepsilon(L) = \begin{cases} (n-1)m + r + 2 = \ell & \text{if } n \text{ is even;} \\ (n-2)m + (m-1) + 1 + (r+2) = \ell & \text{if } n \text{ is odd.} \end{cases}$$

Case 3.2. If $2m + n - 1 \le \ell \le (n-1)m + 1$, then $0 \le q \le n-4$ and $0 \le r \le m-2$ by (2.1).

If q is odd and $0 \le r \le m-3$, then the path

$$L = (x_1, y) \xrightarrow{R_q} (x_{q+3}, y_1) \xrightarrow{P(x_{q+3}, x_n) y_1} (x_n, y_1) \xrightarrow{x_n U_{r+2}(y_1, y)} (x_n, y)$$

is an $(x_1, y)(x_n, y)$-path in $P \times G_2$, and is of length, by (2.1) and (2.3),

$$\begin{aligned} \varepsilon(L) &= (q+2)m + (n-q-3) + (r+2) \\ &= 2m + n - 1 + q(m-1) + r \\ &= \ell. \end{aligned}$$

If q is odd and $r = m-2$, then

$$\begin{aligned} L = (x_1, y) &\xrightarrow{R_q} (x_{q+3}, y_1) \xrightarrow{x_{q+3} U_{m-1}(y_1, y_2)} (x_{q+3}, y_2) \xrightarrow{P(x_{q+3}, x_n) y_2} (x_n, y_2) \\ &\to (x_n, y) \end{aligned}$$

is an $(x_1, y)(x_n, y)$-path in $P \times G_2$, and is of length, by (2.1) and (2.3),

$$\begin{aligned} \varepsilon(L) &= (q+2)m + (m-1) + (n-q-3) + 1 \\ &= 2m + n - 1 + q(m-1) + (m-2) \\ &= \ell. \end{aligned}$$

If q is even, then we can choose such two integers k and j with $1 \le k,\ j \le m-1$ that $k + j = r + 2$ since $0 \le r \le m-2$. So

$$\begin{aligned} L = (x_1, y) &\xrightarrow{R_q} (x_{q+2}, y_1) \xrightarrow{x_{q+2} U_{m-1}(y_1, y)} (x_{q+2}, y) \to (x_{q+3}, y) \\ &\xrightarrow{x_{q+3} U_k(y, y_1)} (x_{q+3}, y_1) \xrightarrow{P(x_{q+3}, x_n) y_1} (x_n, y_1) \xrightarrow{x_n U_j(y_1, y)} (x_n, y) \end{aligned}$$

is an $(x_1, y)(x_n, y)$-path in $P \times G_2$, and is of length, by (2.3),

$$\begin{aligned}\varepsilon(L) &= (q+1)m + (m-1) + 1 + k + (n-q-3) + j \\ &= 2m + n - 1 + q(m-1) + k + j - 2 \\ &= 2m + n - 1 + q(m-1) + r \\ &= \ell.\end{aligned}$$

The lemma follows. □

Theorem 4 *If G_1 and G_2 are two panconnected graphs of order at least three, then $G_1 \times G_2$ is panconnected.*

Proof Let $u = (x_1, y_1)$ and $v = (x_2, y_2)$ be two arbitrary vertices of $G_1 \times G_2$. Let $n = |V(G_1)|$, $m = |V(G_2)|$, $d = d_{G_1 \times G_2}(u, v)$ and ℓ be any positive integer with $d \le \ell \le nm - 1$. To prove the theorem, we only need to show that there is a uv-path of length ℓ in $G_1 \times G_2$.

Since G_1 is panconnected, for any two distinct vertices x and y in G_1 and every k with $d_{G_1}(x, y) \le k \le n - 1$, there is an xy-path of length k in G_1, denoted by $P_k(x, y)$. Similarly, since G_2 is panconnected, for any two distinct vertices x and y in G_2 and every j with $d_{G_2}(x, y) \le j \le m - 1$, there is an xy-path of length j in G_2, denoted by $U_j(x, y)$.

Case 1. $x_1 = x_2$ or $y_1 = y_2$. Without loss of generality, assume that $y_1 = y_2 = y$. By Lemmas 1 and 2, $d_{G_1}(x_1, x_2) = d \le n - 2$.

If $d \le \ell \le n - 1$, then the path $P_\ell(x_1, x_2)y$ is a uv-path of length ℓ in $G_1 \times G_2$.

If $\ell = n$, let z be a neighbor of y in G_2; then the path

$$u = (x_1, y_1) \to (x_1, z) \xrightarrow{P_{n-2}(x_1,x_2)z} (x_2, z) \to (x_2, y_2) = v$$

is a uv-path of length ℓ in $G_1 \times G_2$.

If $n + 1 \le \ell \le nm - 1$, then, by Lemma 3, there is an $(x_1, y)(x_2, y)$-path of length ℓ in $P_{n-1}(x_1, x_2) \times G_2$ ($\subseteq G_1 \times G_2$).

Case 2. $x_1 \ne x_2$ and $y_1 \ne y_2$. Let $d_{G_1}(x_1, x_2) = d_1$ and $d_{G_2}(y_1, y_2) = d_2$. Then $d = d_1 + d_2$ by Lemma 1, and $d_1 \le n - 2$ and $d_2 \le m - 2$ by Lemma 2.

If $d \le \ell \le n + m - 2$, we can choose such two integers k and t with $d_1 \le k \le n - 1$ and $d_2 \le t \le m - 1$ that $k + t = \ell$, then the path

$$u = (x_1, y_1) \xrightarrow{x_1 U_t(y_1,y_2)} (x_1, y_2) \xrightarrow{P_k(x_1,x_2)y_2} (x_2, y_2) = v$$

is a uv-path of length ℓ in $G_1 \times G_2$.

Let z be the neighbor of x_1 in $P_{n-1}(x_1, x_2)$. Then we can let $P_{n-2}(z, x_2)$ be a zx_2-path of length $n - 2$ in G_1 which does not contain vertex x_1. By Lemma 3, for every integer k with $n \le k \le (n-1)m - 1$, there is a $(z, y_2)(x_2, y_2)$-path L_k of length k in $P_{n-2}(z, x_2) \times G_2$.

If $\ell = n + m - 1$, then

$$u = (x_1, y_1) \xrightarrow{x_1 U_{m-2}(y_1,y_2)} (x_1, y_2) \to (z, y_2) \xrightarrow{L_n} (x_2, y_2) = v$$

is a uv-path in $G_1 \times G_2$ with length $(m - 2) + 1 + n = n + m - 1 = \ell$.

If $n+m \le \ell \le nm-1$, then

$$u=(x_1,y_1) \xrightarrow{x_1U_{m-1}(y_1,y_2)} (x_1,y_2) \to (z,y_2) \xrightarrow{L_{\ell-m}} (x_2,y_2)=v$$

is a uv-path in $G_1 \times G_2$ with length $(m-1)+1+(\ell-m)=\ell$.

The proof of the theorem is complete. □

In [2], Bhuyan and Agrawal introduce the concept of generalized hypercube, which is a natural generalization of hypercube. Let n be a positive integer. An n-dimensional generalized hypercube is a Cartesian product $Q_n(k_1,\ldots,k_n)=K_{k_1}\times K_{k_2}\times\cdots\times K_{k_n}$, where, for $i=1,\ldots,n$, k_i (≥ 2) is an integer and K_{k_i} is a complete graph of order k_i. In particular, $Q_n(2,\ldots,2)$ is the n-dimensional hypercube Q_n and $Q_n(3,\ldots,3)$ is the 3-ary n-cube Q_n^3. Hsieh, Lin and Huang [6] proved that Q_n^3 is panconnected. Next we generalize this results and give a characterization of all panconnected generalized hypercubes.

Corollary 5 *Let $n \ge 2$ and $k_i \ge 2$ $(i=1,2,\ldots,n)$ be integers. Then $Q_n(k_1,k_2,\ldots,k_n)$ is panconnected if and only if $k_i \ge 3$ $(i=1,2,\ldots,n)$.*

Proof By induction on $n \ge 2$ it is clear that if $k_i \ge 3$ for each $i=1,2,\ldots,n$, then, by Theorem 4, $Q_n(k_1,k_2,\ldots,k_n)$ is panconnected since the complete graph K_{k_i} is panconnected for $i=1,2,\ldots,n$.

Conversely, if $Q_n(k_1,k_2\cdots,k_n)$ is panconnected, we will prove that $k_i \ge 3$ for each $i=1,2,\ldots,n$. To the contrary, assume that there exists some $i \in \{1,2,\ldots,n\}$ such that $k_i=2$, without loss of generality, say $k_1=2$, then we denote the vertex set $V(K_{k_1})$ by $\{x_1,x_2\}$. For any $y \in V(K_{k_2}\times\cdots\times K_{k_n})$, it is easy to see that (x_1,y) and (x_2,y) are adjacent, but there does not exist $(x_1,y)(x_2,y)$-path of length 2 in $Q_n(2,k_2,\ldots,k_n)$, which contradicts that $Q_n(2,k_2,\ldots,k_n)$ is panconnected. □

Corollary 6 (Hsieh, Lin and Huang [6]) *$Q_n(3,\ldots,3)$ is panconnected.*

3 Conclusions

A path (linear array) is a fundamental network for parallel and distributed computation, and is suitable for designing simple algorithms with low communication costs. The problem of finding paths of various lengths in networks, i.e. panconnectivity of networks, has recently received much attention. In this paper, we consider the Cartesian product graphs since many popular networks can be constructed by the product operation such as the hypercube Q_n, the k-ary n-cube Q_n^k and the mash networks. We show that the Cartesian product graph is panconnected if its factorial graphs all are panconnected. As a consequence, we prove that the n-dimensional generalized hypercube $Q_n(k_1,k_2,\ldots,k_n)$ is panconnected if and only if $k_i \ge 3$ $(i=1,\ldots,n)$, which generalizes the result of Hsieh et al. [6] that the 3-ary n-cube Q_n^3 is panconnected.

We give an efficient and necessary condition $k_i \ge 3$ $(i=1,\ldots,n)$ for $Q_n(k_1,k_2,\ldots,k_n)$ to be panconnected. However, a natural question is if there is a path of every length except $d+1$ between two vertices with distance d in $Q_n(k_1,k_2,\ldots,k_n)$.

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