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Generalized measures of fault tolerance in exchanged hypercubes ☆

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ABSTRACT

The exchanged hypercube $EH(s,t)$, proposed by Loh et al. [P.K.K. Loh, W.J. Hsu, Y. Pan, The exchanged hypercube, IEEE Transactions on Parallel and Distributed Systems 16 (9) (2005) 866–874], is obtained by removing edges from a hypercube Q_{s+t+1}. This paper considers a kind of generalized measures $\kappa^{(h)}$ and $\lambda^{(h)}$ of fault tolerance in $EH(s,t)$ with $1 \leqslant s \leqslant t$ and determines $\kappa^{(h)}(EH(s,t)) = \lambda^{(h)}(EH(s,t)) = 2^h(s+1-h)$ for any h with $0 \leqslant h \leqslant s$. The results show that at least $2^h(s+1-h)$ vertices (resp. $2^h(s+1-h)$ edges) of $EH(s,t)$ have to be removed to get a disconnected graph that contains no vertices of degree less than h, and generalizes some known results.

1. Introduction

It is well known that interconnection networks play an important role in parallel computing/communication systems. An interconnection network can be modeled by a graph $G = (V, E)$, where V is the set of processors and E is the set of communication links in the network. For graph terminology and notation not defined here we follow [15].

A subset $S \subset V(G)$ (resp. $F \subset E(G)$) of a connected graph G is called a *vertex-cut* (resp. *edge-cut*) if $G - S$ (resp. $G - F$) is disconnected. The *connectivity* $\kappa(G)$ (resp. *edge-connectivity* $\lambda(G)$) of G is defined as the minimum cardinality over all vertex-cuts (resp. edge-cuts) of G. The connectivity $\kappa(G)$ and edge-connectivity $\lambda(G)$ of a graph G are two important measurements for fault tolerance of the network since the larger $\kappa(G)$ or $\lambda(G)$ is, the more reliable the network is.

Because the connectivity has some shortcomings, Esfahanian [1] proposed the concept of restricted connectivity, Latifi et al. [3] generalized it to restricted h-connectivity which can measure fault tolerance of an interconnection network more accurately than the classical connectivity. The concepts stated here are slightly different from theirs.

A subset $S \subset V(G)$ (resp. $F \subset E(G)$) of a connected graph G, if any, is called an *h-vertex-cut* (resp. *edge-cut*), if $G - S$ (resp. $G - F$) is disconnected and has the minimum degree at least h. The *h-connectivity* (resp. *edge-connectivity*) of G, denoted by $\kappa^{(h)}(G)$ (resp. $\lambda^{(h)}(G)$), is defined as the minimum cardinality over all h-vertex-cuts (resp. h-edge-cut) of G. It is clear that, for $h \geqslant 1$, if $\kappa^{(h)}(G)$ and $\lambda^{(h)}(G)$ exist, then $\kappa^{(h-1)}(G) \leqslant \kappa^{(h)}(G)$ and $\lambda^{(h-1)}(G) \leqslant \lambda^{(h)}(G)$. For any graph G and any integer h, determining $\kappa^{(h)}(G)$ and $\lambda^{(h)}(G)$ is quite difficult. In fact, the

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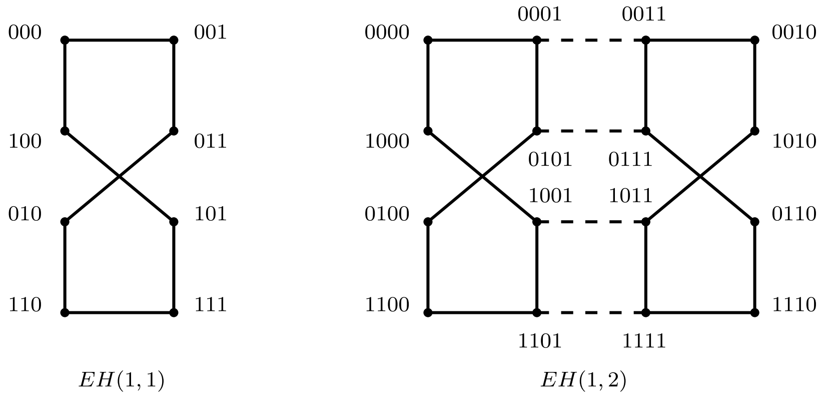

Fig. 1. Two exchanged hypercubes $EH(1,1)$ and $EH(1,2)$.

existence of $\kappa^{(h)}(G)$ and $\lambda^{(h)}(G)$ is an open problem so far when $h \geqslant 1$. Only a little knowledge of results have been known on $\kappa^{(h)}$ and $\lambda^{(h)}$ for particular classes of graphs and small h's, such as [2,4,5,8,10–14,16,17,19,20].

It is widely known that the hypercube Q_n has been one of the most popular interconnection networks for parallel computer/communication system. Xu [14] determined $\lambda^{(h)}(Q_n) = 2^h(n-h)$ for $h \leqslant n-1$, and Oh et al. [11] and Wu et al. [13] independently determined $\kappa^{(h)}(Q_n) = 2^h(n-h)$ for $h \leqslant n-2$.

This paper is concerned about the exchanged hypercubes $EH(s,t)$, proposed by Loh et al. [7]. As a variant of the hypercube, $EH(s,t)$ is a graph obtained by removing edges from a hypercube Q_{s+t+1}. It not only keeps numerous desirable properties of the hypercube, but also reduced the interconnection complexity. Very recently, Ma et al. [10] have determined $\kappa^{(1)}(EH(s,t)) = \lambda^{(1)}(EH(s,t)) = 2s$. We, in this paper, will generalize this result by proving that $\kappa^{(h)}(EH(s,t)) = \lambda^{(h)}(EH(s,t)) = 2^h(s+1-h)$ for any h with $0 \leqslant h \leqslant s$.

The proof of this result is in Section 3. In Section 2, we recall the structure of $EH(s,t)$ and some lemmas used in our proofs.

2. Definitions and lemmas

For a given position integer n, let $I_n = \{1,2,\ldots,n\}$. The sequence $x_n x_{n-1} \cdots x_1$ is called a binary string of length n if $x_r \in \{0,1\}$ for each $r \in I_n$. Let $x = x_n x_{n-1} \cdots x_1$ and $y = y_n y_{n-1} \cdots y_1$ be two distinct binary strings of length n. *Hamming distance* between x and y, denoted by $H(x,y)$, is the number of r's for which $|x_r - y_r| = 1$ for $r \in I_n$.

For a binary string $u = u_n u_{n-1} \cdots u_1 u_0$ of length $n+1$, we call u_r the r-th bit of u for $r \in I_n$, and u_0 the last bit of u, denote sub-sequence $u_j u_{j-1} \cdots u_{i+1} u_i$ of u by $u[j:i]$, i.e., $u[j,i] = u_j u_{j-1} \cdots u_{i+1} u_i$. Let

$$V(s,t) = \bigl\{u_{s+t} \cdots u_{t+1} u_t \cdots u_1 u_0 \bigm| u_0, u_i \in \{0,1\}, \\ i \in I_{s+t}\bigr\}.$$

Definition 2.1. The exchanged hypercube is an undirected graph $EH(s,t) = (V,E)$, where $s \geqslant 1$ and $t \geqslant 1$ are integers. The set of vertices V is $V(s,t)$, and the set of edges E is composed of three disjoint types E_1, E_2 and E_3:

$$E_1 = \bigl\{uv \in V \times V \bigm| u[s+t:1] = v[s+t:1],\ u_0 \neq v_0\bigr\},$$
$$E_2 = \bigl\{uv \in V \times V \bigm| u[s+t:t+1] = v[s+t:t+1], \\ H\bigl(u[t:1],\ v[t:1]\bigr) = 1,\ u_0 = v_0 = 1\bigr\},$$
$$E_3 = \bigl\{uv \in V \times V \bigm| u[t:1] = v[t:1],\ H\bigl(u[s+t:t+1], \\ v[s+t:t+1]\bigr) = 1,\ u_0 = v_0 = 0\bigr\}.$$

Now we give an alternative definition of $EH(s,t)$.

Definition 2.2. An exchanged hypercube $EH(s,t)$ consists of the vertex-set $V(s,t)$ and the edge-set E, two vertices $u = u_{s+t} \cdots u_{t+1} u_t \cdots u_1 u_0$ and $v = v_{s+t} \cdots v_{t+1} v_t \cdots v_1 v_0$ linked by an edge, called r-dimensional edge, if and only if the following conditions are satisfied:

(a) u and v differ exactly in one bit on the r-th bit or on the last bit,
(b) if $r \in I_t$, then $u_0 = v_0 = 1$,
(c) if $r \in I_{s+t} - I_t$, then $u_0 = v_0 = 0$.

The exchanged hypercubes $EH(1,1)$ and $EH(1,2)$ are shown in Fig. 1.

From Definition 2.2, it is easy to see that $EH(s,t)$ can be obtained from a hypercube Q_{s+t+1} with vertex-set $V(s,t)$ by removing all r-dimensional edges that link two vertices with the last bit 0 if $r \in I_t$ and two vertices with the last bit 1 if $r \in I_{s+t} - I_t$. Thus, $EH(s,t)$ is a bipartite graph with minimum degree $\min\{s,t\}+1$ and maximum degree $\max\{s,t\}+1$. The following three lemmas obtained by Loh et al. [7] and Ma [8] are very useful for our proofs.

Lemma 2.3. *(See Loh et al. [7].) $EH(s,t)$ is isomorphic to $EH(t,s)$.*

By Lemma 2.3, without loss of generality, we can assume $s \leqslant t$ in the following discussion, and so $EH(s,t)$ has the minimum degree $s+1$. For fixed $r \in I_{s+t}$ and $i \in \{0,1\}$, let H_i^r denote a subgraph of $EH(s,t)$ induced by all vertices whose r-th bits are i.

Lemma 2.4. *(See Loh et al. [7].) For a fixed $r \in I_{s+t}$, $EH(s,t)$ can be decomposed into 2 isomorphic subgraphs H_0^r and H_1^r,* *which are isomorphic to* $EH(s,t-1)$ *if* $r \in I_t$ *and* $t \geqslant 2$*, and isomorphic to* $EH(s-1,t)$ *if* $r \in I_{s+t} - I_t$ *and* $s \geqslant 2$*. Moreover, there are* 2^{s+t-1} *independent edges between* H_0^r *and* H_1^r*.*

Lemma 2.5. *(See Ma [8].)* $\kappa(EH(s,t)) = \lambda(EH(s,t)) = s+1$ *for any* s *and* t *with* $1 \leqslant s \leqslant t$*.*

3. Main results

In this section, we present our main results, that is, we determine the h-connectivity and h-edge-connectivity of the exchanged hypercube $EH(s,t)$.

Lemma 3.1. $\kappa^{(h)}(EH(s,t)) \leqslant 2^h(s+1-h)$ *and* $\lambda^{(h)}(EH(s,t)) \leqslant 2^h(s+1-h)$ *for* $h \leqslant s$*.*

Proof. Let X be a subset of vertices in $EH(s,t)$ whose the rightmost $s+t+1-h$ bits are zeros and the leftmost h bits do not care, denoted by

$$X = \{ *^h 0^{s+t+1-h} \mid * \in \{0,1\} \}.$$

Then the subgraph of $EH(s,t)$ induced by X is a hypercube Q_h. Let S be the neighbor-set of X in $EH(s,t) - X$ and F the edge-sets between X and S. By Definition 2.2, S has the form

$$S = \{ *^h \underbrace{0^p 1 0^{s-h-p-1}}_{s-h} 0^{t+1} \mid 0 \leqslant p \leqslant s-h-1,\ h \leqslant s-1 \} \cup \{ *^h 0^{s+t-h} 1 \},$$

where $* \in \{0,1\}$. On the one hand, since every vertex of X has degree $s+1$ in $EH(s,t)$ and h neighbors in X, it has exactly $s-h+1$ neighbors in S. On the other hand, every vertex of S has exactly one neighbor in X. It follows that

$$|S| = |F| = 2^h(s+1-h).$$

We show that S is an h-vertex-cut of $EH(s,t)$. Clearly, S is a vertex-cut of $EH(s,t)$ since $|X \cup S| = 2^h(s+2-h) < 2^{s+t+1}$. Let $Y = EH(s,t) - (X \cup S)$ and v be any vertex in Y. We only need to show that the vertex v has degree at least h in Y. In fact, it is easy to see from the formal definition of S that if v is adjacent to some vertex in S then it has only the form

$$v = *^h \underbrace{0^p 1 0^{s-h-p-1}}_{s-h} 0^t 1 \text{ or } *^h 0^{s-h} \underbrace{0^r 1 0^{t-r-1}}_{t} 1 \text{ or } *^h \underbrace{0^p 1 0^q 1 0^{s-h-p-q-2}}_{s-h} 0^{t+1}.$$

If v has the former two forms, then v has one neighbor in S, thus v has at least $(s+1-1 = s \geqslant) h$ neighbors in Y. If v has the last form, then $s-h \geqslant 2$ and v has two neighbors in S. Thus, v has at least $(s+1-2 = s-1 >) h$ neighbors in Y.

By the arbitrariness of $v \in Y$, S is an h-vertex-cut of $EH(s,t)$, and so

$$\kappa^{(h)}(EH(s,t)) \leqslant |S| = 2^h(s+1-h)$$

as required.

We now show that F is an h-edge-cut of $EH(s,t)$. Since every vertex v in $EH(s,t) - X$ has at most one neighbor in X, then v has at least $(s+1-1 = s \geqslant) h$ neighbors in $EH(s,t) - X$. By the arbitrariness of $v \in EH(s,t) - X$, F is an h-edge-cut of $EH(s,t)$, and so

$$\lambda^{(h)}(EH(s,t)) \leqslant |F| = 2^h(s+1-h).$$

The lemma follows. □

Corollary 3.2. $\kappa^{(1)}(EH(1,t)) = \lambda^{(1)}(EH(1,t)) = 2$ *for* $t \geqslant 1$*.*

Proof. On the one hand, $\kappa^{(h)}(EH(1,t)) \leqslant 2$ and $\lambda^{(h)}(EH(1,t)) \leqslant 2$ by Lemma 3.1 when $s = 1$. On the other hand, by Lemma 2.5, $\kappa(EH(1,t)) = \lambda(EH(1,t)) = 2$, thus $\kappa^{(h)}(EH(1,t)) \geqslant \kappa(EH(1,t)) = 2$ and $\lambda^{(h)}(EH(1,t)) \geqslant \lambda(EH(1,t)) = 2$. The results hold. □

Theorem 3.3. *For* $1 \leqslant s \leqslant t$ *and any* h *with* $0 \leqslant h \leqslant s$*,*

$$\kappa^{(h)}(EH(s,t)) = \lambda^{(h)}(EH(s,t)) = 2^h(s+1-h).$$

Proof. By Lemma 3.1, we only need to prove that,

$$\kappa^{(h)}(EH(s,t)) = \lambda^{(h)}(EH(s,t)) \geqslant 2^h(s+1-h).$$

We proceed by induction on $h \geqslant 0$. The theorem holds for $h = 0$ by Lemma 2.5. Assume the induction hypothesis for $h-1$ with $h \geqslant 1$, that is,

$$\kappa^{(h-1)}(EH(s,t)) = \lambda^{(h-1)}(EH(s,t)) \geqslant 2^{h-1}(s+2-h). \tag{3.1}$$

Note $h = 1$ if $s = 1$. By Corollary 3.2, $\kappa^{(1)}(EH(1,t)) = \lambda^{(1)}(EH(1,t)) = 2$ for any $t \geqslant 1$, the theorem is true for $s = 1$. Thus, we assume $s \geqslant 2$ below.

Let S be a minimum h-vertex-cut (or h-edge-cut) of $EH(s,t)$ and X be the vertex-set of a minimum connected component of $EH(s,t) - S$. Then

$$|S| = \begin{cases} \kappa^{(h)}(EH(s,t)) & \text{if } S \text{ is a vertex-cut;} \\ \lambda^{(h)}(EH(s,t)) & \text{if } S \text{ is an edge-cut.} \end{cases}$$

Thus, we only need to prove that

$$|S| \geqslant 2^h(s+1-h). \tag{3.2}$$

To the end, let Y be the set of vertices in $EH(s,t) - S$ not in X, and for a fixed $r \in I_{s+t}$ and each $i = 0, 1$, let

$$X_i = X \cap H_i^r,$$
$$Y_i = Y \cap H_i^r \quad \text{and}$$
$$S_i = S \cap H_i^r.$$

Let $J = \{i \in \{0,1\} \mid X_i \neq \emptyset\}$ and $J' = \{i \in J \mid Y_i \neq \emptyset\}$. Clearly, $0 \leqslant |J'| \leqslant |J| \leqslant 2$ and $|J'| = 0$ only when $|J| = 1$. We choose $r \in I_{s+t}$ such that $|J|$ is as large as possible. For each $i \in \{0,1\}$, we write H_i for H_i^r for short. We first prove the following inequality

$$|S_i| \geqslant 2^{h-1}(s+1-h) \quad \text{if } X_i \neq \emptyset \text{ and } Y_i \neq \emptyset \text{ for } i \in \{0,1\}. \tag{3.3}$$

In fact, for some $i \in \{0,1\}$, if $X_i \neq \emptyset$ and $Y_i \neq \emptyset$, then S_i is a vertex-cut (or an edge-cut) of H_i. Let u be any vertex in $X_i \cup Y_i$. Since S is an h-vertex-cut (or h-edge-cut) of $EH(s,t)$, u has degree at least h in $EH(s,t) - S$. By Lemma 2.4, u has at most one neighbor in H_j, where $j \neq i$. Thus, u has degree at least $h-1$ in H_i, which implies that S_i is an $(h-1)$-vertex-cut (or edge-cut) of H_i, that is,

$$|S_i| \geqslant \kappa^{(h-1)}(H_i) \quad \big(\text{or } |S_i| \geqslant \lambda^{(h-1)}(H_i)\big). \tag{3.4}$$

If $r \in I_{s+t} - I_t$, then $H_i \cong EH(s-1,t)$ by Lemma 2.4. By the induction hypothesis (3.1), $\kappa^{(h-1)}(H_i) = \lambda^{(h-1)}(H_i) \geqslant 2^{h-1}(s+1-h)$, from which and (3.4), we have that $|S_i| \geqslant 2^{h-1}(s+1-h)$.

If $r \in I_t$, then $H_i \cong EH(s,t-1)$ by Lemma 2.4.

If $t \geqslant s+1$, by the induction hypothesis (3.1),

$$\kappa^{(h-1)}(H_i) = \lambda^{(h-1)}(H_i) \geqslant 2^{h-1}(s+2-h) > 2^{h-1}(s+1-h),$$

from which and (3.4), we have that $|S_i| > 2^{h-1}(s+1-h)$.

If $t = s$, then $EH(s,t-1) \cong EH(s-1,t)$ by Lemma 2.3. By the induction hypothesis (3.1),

$$\kappa^{(h-1)}(H_i) = \lambda^{(h-1)}(H_i) \geqslant 2^{h-1}(s+1-h),$$

from which and (3.4), we have that $|S_i| \geqslant 2^{h-1}(s+1-h)$. The inequality (3.3) follows.

We now prove the inequality in (3.2).

If $|J| = 1$ then, by the choice of J, no matter what $r \in I_{s+t}$ is chosen, the r-th bits of all vertices in X are the same. In other words, the r-th bits of all vertices in X are the same for any $r \in I_{s+t}$, and possible different in the last bit. Thus $|X| \leqslant 2$ and $h \leqslant 1$. By the hypothesis of $h \geqslant 1$, we have $h = 1$ and $|X| = 2$. The subgraph of $EH(s,t)$ induced by X is an edge in E_1, thus

$$|S| = s + t \geqslant 2s = 2^h(s+1-h),$$

as required. Assume $|J| = 2$ below, that is, $X_i \neq \emptyset$ for each $i = 0,1$. In this case, $|J'| \geqslant 1$.

If $|J'| = 2$ then, for each $i = 0,1$, since $X_i \neq \emptyset$ and $Y_i \neq \emptyset$, we have that $|S_i| \geqslant 2^{h-1}(s+1-h)$ by (3.3). Note that $|S| = |S_0| + |S_1|$ if S is an h-vertex-cut and $|S| \geqslant |S_0| + |S_1|$ if S is an h-edge-cut. It follows that

$$\begin{aligned} |S| &\geqslant |S_0| + |S_1| \\ &\geqslant 2 \times 2^{h-1}(s+1-h) \\ &= 2^h(s+1-h), \end{aligned}$$

as required.

If $|J'| = 1$, then one of Y_0 and Y_1 must be empty. Without loss of generality, assume $Y_1 = \emptyset$ and $Y_0 \neq \emptyset$.

Clearly, S is not an h-edge-cut, otherwise, $|Y| < |H_0| < |X|$, a contradiction with the minimality of X. Thus, S is an h-vertex-cut. By (3.3), $|S_0| \geqslant 2^{h-1}(s+1-h)$. Since $Y_1 = \emptyset$, we have

$$|X_1| = |H_1| - |S_1| \text{ and } |Y| = |H_0| - |X_0| - |S_0|. \tag{3.5}$$

If $|S_1| < |S_0|$ then, by (3.5), we obtain that $|Y| < |X_1| < |X|$, which contradicts to the minimality of X. Thus, $|S_1| \geqslant |S_0|$, from which and (3.3) we have that

$$\begin{aligned} |S| &= |S_0| + |S_1| \geqslant 2|S_0| \\ &\geqslant 2 \times 2^{h-1}(s+1-h) \\ &= 2^h(s+1-h), \end{aligned}$$

as required. Thus, the inequality in (3.2) holds, and so the theorem follows. □

Corollary 3.4. *(See Ma and Zhu [10].) If $1 \leqslant s \leqslant t$, then $\kappa^{(1)}(EH(s,t)) = \lambda^{(1)}(EH(s,t)) = 2s$.*

A dual-cube $DC(n)$, proposed by Li and Peng [6] constructed from hypercubes, preserves the main desired properties of the hypercube. Very recently, Yang and Zhou [18] have determined that $\kappa^{(h)}(DC(n)) = 2^n(n+1-h)$ for each $h = 0,1,2$. Since $EH(n,n)$ is isomorphic to $DC(n)$, the following result is obtained immediately.

Corollary 3.5. *For dual-cube $DC(n)$, $\kappa^{(h)}(DC(n)) = \lambda^{(h)}(DC(n)) = 2^n(n+1-h)$ for any h with $0 \leqslant h \leqslant n$.*

4. Conclusions

In this paper, we consider the generalized measures of fault tolerance for a network, called the h-connectivity κ^h and the h-edge-connectivity λ^h. For the exchanged hypercube $EH(s,t)$, which has about half edges of the hypercube Q_{s+t+1}, we prove that $\kappa^{(h)} = \lambda^{(h)} = 2^h(s+1-h)$ for any h with $0 \leqslant h \leqslant s$ and $s \leqslant t$. The results show that at least $2^h(s+1-h)$ vertices (resp. $2^h(s+1-h)$ edges) of $EH(s,t)$ have to be removed to get a disconnected graph that contains no vertices of degree less than h. Thus, when the exchanged hypercube is used to model the topological structure of a large-scale parallel processing system, these results can provide more accurate measurements for fault tolerance of the system.

Otherwise, Ma and Liu [9] investigated bipancyclicity of $EH(s,t)$. However, there are many interesting combinatorial and topological problems, e.g., wide-diameter, fault-diameter, panconnectivity, spanning-connectivity, which are still open for the exchanged hypercube network.

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References

[1] A.H. Esfahanian, Generalized measures of fault tolerance with application to n-cube networks, IEEE Transactions on Computers 38 (11) (1989) 1586–1591.

[2] A.H. Esfahanian, S.L. Hakimi, On computing a conditional edge connectivity of a graph, Information Processing Letters 27 (1988) 195–199.

[3] S. Latifi, M. Hegde, M. Naraghi-Pour, Conditional connectivity measures for large multiprocessor systems, IEEE Transactions on Computers 43 (1994) 218–222.
[4] X.-J. Li, J.-M. Xu, Generalized measures of fault tolerance in (n,k)-star graphs, http://arxiv.org/abs/1204.1440, 2012.
[5] X.-J. Li, J.-M. Xu, Generalized measures of edge fault tolerance in (n,k)-star graphs, Mathematical Science Letters 1 (2) (2012) 133–138.
[6] Y. Li, S. Peng, Dual-cubes: a new interconnection network for high-performance computer clusters, in: Proceedings of the 2000 International Computer Symposium, Workshop on Computer Architecture, 2000, pp. 51–57.
[7] P.K.K. Loh, W.J. Hsu, Y. Pan, The exchanged hypercube, IEEE Transactions on Parallel and Distributed Systems 16 (9) (2005) 866–874.
[8] M. Ma, The connectivity of exchanged hypercubes, Discrete Mathematics Algorithms and Applications 2 (2) (2010) 213–220.
[9] M. Ma, B. Liu, Cycles embedding in exchanged hypercubes, Information Processing Letters 110 (2) (2009) 71–76.
[10] M. Ma, L. Zhu, The super connectivity of exchanged hypercubes, Information Processing Letters 111 (2011) 360–364.
[11] A.D. Oh, H. Choi, Generalized measures of fault tolerance in n-cube networks, IEEE Transactions on Parallel and Distributed Systems 4 (1993) 702–703.
[12] M. Wan, Z. Zhang, A kind of conditional vertex connectivity of star graphs, Applied Mathematics Letters 22 (2009) 264–267.
[13] J. Wu, G. Guo, Fault tolerance measures for m-ary n-dimensional hypercubes based on forbidden faulty sets, IEEE Transactions on Computers 47 (1998) 888–893.
[14] J.-M. Xu, On conditional edge-connectivity of graphs, Acta Mathematicae Applicatae Sinica 16 (4) (2000) 414–419.
[15] J.-M. Xu, Topological Structure and Analysis of Interconnection Networks, Kluwer Academic Publishers, Dordrecht/Boston/London, 2001.
[16] J.-M. Xu, M. Xu, Q. Zhu, The super connectivity of shuffle-cubes, Information Processing Letters 96 (2005) 123–127.
[17] W.-H. Yang, H.-Z. Li, X.-F. Guo, A kind of conditional fault tolerance of (n,k)-star graphs, Information Processing Letters 110 (2010) 1007–1011.
[18] X. Yang, S. Zhou, On conditional fault tolerant of dual-cubes, International Journal of Parallel, Emergent and Distributed Systems 28 (3) (2013) 199–213, http://dx.doi.org/10.1080/17445760.2012.704631.
[19] Q. Zhu, J.-M. Xu, X.-M. Hou, X. Xu, On reliability of the folded hypercubes, Information Sciences 177 (8) (2007) 1782–1788.
[20] Q. Zhu, J.-M. Xu, M. Lü, Edge fault tolerance analysis of a class of interconnection networks, Applied Mathematics and Computation 172 (1) (2006) 111–121.