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# Edge-fault tolerance of hypercube-like networks ☆

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ABSTRACT

This paper considers a kind of generalized measure $\lambda_s^{(h)}$ of fault tolerance in a hypercube-like graph $G_n$ which contains several well-known interconnection networks such as hypercubes, varietal hypercubes, twisted cubes, crossed cubes, Möbius cubes and the recursive circulant $G(2^n, 4)$, and proves $\lambda_s^{(h)}(G_n) = 2^h(n-h)$ for any $h$ with $0 \leqslant h \leqslant n-1$ by the induction on $n$ and a new technique. This result shows that at least $2^h(n-h)$ edges of $G_n$ have to be removed to get a disconnected graph that contains no vertices of degree less than $h$. Compared with previous results, this result enhances fault-tolerant ability of the above-mentioned networks theoretically.



## 1. Introduction

It is well known that interconnection networks play an important role in parallel computing/communication systems. An interconnection network can be modeled by a graph $G = (V, E)$, where $V$ is the set of processors and $E$ is the set of communication links in the network. For graph terminology and notation not defined here we follow [20].

The edge-connectivity of a graph $G$ is an important measurement for fault tolerance of the network, and the larger the edge-connectivity is, the more reliable the network is. However, computing this parameter, one implicitly assumes that all links incident with the same processor may fail simultaneously. Consequently, this measurement is inaccurate for large-scale processing systems in which some subsets of system components cannot fail at the same time in real applications. To overcome such a shortcoming, Esfahanian [7] proposed the concept of restricted connectivity, in which the links incident with the same processor cannot fail at the same time. Latifi et al. [11] generalized it to the restricted $h$-connectivity, in which at least $h$ links incident with the same processor cannot fail. This parameter can measure fault tolerance of an interconnection network more accurately than the classical connectivity. The concepts stated here are slightly different from theirs.

For a given integer $h$ ($\geqslant 0$), an edge subset $F$ of a connected graph $G$ is called an *h-super edge-cut*, or *h-edge-cut* for short, if $G - F$ is disconnected and has the minimum degree $\delta(G - F) \geqslant h$. The *h-super edge-connectivity* of $G$, denoted by $\lambda_s^{(h)}(G)$, is defined as the minimum cardinality over all $h$-edge-cuts of $G$. It is clear that $\lambda_s^{(0)}(G) = \lambda(G)$, where $\lambda(G)$ is classical edge-connectivity of $G$. For $h \geqslant 1$, if $\lambda_s^{(h)}(G)$ exists, then $\lambda_s^{(h-1)}(G) \leqslant \lambda_s^{(h)}(G)$.

For any graph $G$ and a given integer $h$, determining $\lambda_s^{(h)}(G)$ is quite difficult since Latifi et al. [11] conjectured it is NP-hard, not proved so far. In fact, the existence of $\lambda_s^{(h)}(G)$ is an open problem so far when $h \geqslant 1$. Only few results have been known on $\lambda_s^{(h)}(G)$ for particular classes of graphs and small $h$'s, such as, Xu [19] determined $\lambda_s^{(h)}(Q_n) = 2^h(n-h)$ for $h \leqslant n-1$.

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It is widely known that the hypercube has been one of the most popular interconnection networks for parallel computer/communication system. However, the hypercube has the large diameter correspondingly. To minimize diameter, various networks are proposed by twisting some pairs of links in hypercubes, such as the varietal hypercube $VQ_n$ [5], the twisted cube $TQ_n$ [1,2], the locally twisted cube $LTQ_n$ [21], the crossed cube $CQ_n$ [8,10], the Möbius cube $MQ_n$ [6], the recursive circulant $G(2^n, 4)$ [13] and so on. Because of the lack of the unified perspective on these variants, results of one topology are hard to be extended to others. To make a unified study of these variants, Vaidya et al. [16] introduced the class of hypercube-like graphs $HL_n$, which contains all the above-mentioned networks. Thus, the hypercube-like graphs have received much attention in recent years [3,4,12,14,15,17,18].

In this paper, we determine $\lambda_s^{(h)}(G_n) = 2^h(n-h)$ for any $G_n \in HL_n$ and $0 \leqslant h \leqslant n-1$. Our result contains many know conclusions and enhances the fault-tolerant ability of the hypercube-like networks theoretically.

The proof of this result is in Section 3 by the induction on $n$ and a new technique. Section 2 recalls the definition and Section 4 gives a conclusion on our work.

## 2. Hypercube-like graphs

Let $G_0 = (V_0, E_0)$ and $G_1 = (V_1, E_1)$ be two disjoint graphs with the same order, $\sigma$ a bijection from $V_0$ to $V_1$. A 1-1 connection between $G_0$ and $G_1$ is defined as an edge-set $M_\sigma = \{x\sigma(x) \mid x \in V_0,\ \sigma(x) \in V_1\}$. Let $G_0 \oplus_\sigma G_1$ denote a graph $G = (V_0 \cup V_1, E_0 \cup E_1 \cup M_\sigma)$. Clearly, $M_\sigma$ is a perfect matching of $G$. Moreover, if $\sigma$ is the identical permutation on $V(G_0)$, then $G_0 \oplus_\sigma G_0 = G_0 \times K_2$, where $\times$ denotes the Cartesian product, and $K_2$ is a complete graph of order two.

Note that the operation $\oplus_\sigma$ may generate different graphs according to different $\sigma$. Applying the operation $\oplus_\sigma$ repeatedly, a set of $n$-dimensional *hypercube-like graphs* also called *bijective connection graphs* (in brief, *BC graphs*) [15], denoted by $HL_n$, can be recursively defined as follows.

(1) $HL_0 = \{G_0\}$, where $G_0 = K_1$, which is a single vertex;
(2) $G_n \in HL_n$ if and only if $G_n = G_{n-1} \oplus_\sigma G'_{n-1}$ for some $G_{n-1}, G'_{n-1} \in HL_{n-1}$, where $\sigma$ is a bijection from $V(G_{n-1})$ to $V(G'_{n-1})$.

It is clear that for a graph $G_n \in HL_n$, $G_n$ is an $n$-regular connected graph of order $2^n$ and $\lambda(G_n) = n$ (see [16]). A hypercube-like graph in $HL_4$ is shown in Fig. 1, which is isomorphic to $G(2^4, 4)$.

By definitions, it is easy to see that the hypercube $Q_n = Q_{n-1} \oplus_{\sigma_1} Q_{n-1}$, the varietal hypercube $VQ_n = VQ_{n-1} \oplus_{\sigma_2} VQ_{n-1}$, the twisted cube $TQ_n = TQ_{n-1} \oplus_{\sigma_3} TQ_{n-1}$, the locally twisted cube $LTQ_n = LTQ_{n-1} \oplus_{\sigma_4} LTQ_{n-1}$, the crossed cube $CQ_n = CQ_{n-1} \oplus_{\sigma_5} CQ_{n-1}$, the Möbius cube $MQ_n = MQ_{n-1} \oplus_{\sigma_6} MQ_{n-1}$, where $\sigma_i$ is a given permutation on the vertex-set of the corresponding $G_{n-1}$ in $HL_{n-1}$ for each $i \in \{1, 2, \ldots, 6\}$. As regards the recursive circulant $G(2^n, 4)$, when $n \in \{2, 3, 4\}$, there is a permutation $\sigma$ on $V(G(2^{n-1}, 4))$ such that $G(2^n, 4) = G(2^{n-1}, 4) \oplus_\sigma G(2^{n-1}, 4)$ (see Fig. 1). In general, $G(2^n, 4)$ cannot be obtained from the operation $\oplus_\sigma$ on two recursive circulants. In other words, for an arbitrary $G(2^n, 4)$, there is no a permutation $\sigma$ on $V(G(2^{n-1}, 4))$ such that $G(2^n, 4) = G(2^{n-1}, 4) \oplus_\sigma G(2^{n-1}, 4)$. However, Kim et al. [9] pointed out that there is a permutation $\sigma$ on $V(G(2^{n-2} \times K_2, 4))$ such that $G(2^n, 4) = [G(2^{n-2}, 4) \times K_2] \oplus_\sigma [G(2^{n-2}, 4) \times K_2]$. Thus, $\{Q_n, VQ_n, TQ_n, LTQ_n, CQ_n, MQ_n, G(2^n, 4)\} \subseteq HL_n$.

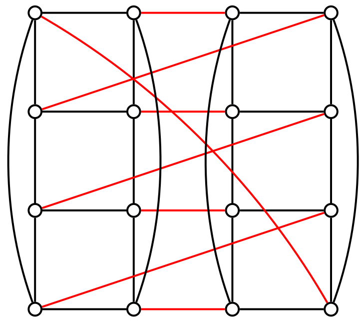

**Fig. 1.** A hypercube-like graph in $HL_4$.

For convenience, let $I_n = \{0, 1, \ldots, n\}$. For a graph $G$, we write $|G|$ for $|V(G)|$, for a subgraph $X \subseteq G$, write $X$ for $V(X)$. For each $i \in I_{n-1}$, if $G_i, G'_i \in HL_i$ and $G_{i+1} = (V(G_i) \cup V(G'_i), E(G_i) \cup E(G'_i) \cup M_{\sigma_i})$, we write $M_i$ for $M_{\sigma_i}$, and say that $G_i$ and $G'_i$ are the $i$-dimensional underlying graphs of $G_{i+1}$ with $\sigma_i$.

**Lemma 2.1.** *For given $h \in I_{n-1}$ and $G_n \in HL_n$, there is a sequence of graphs $\{G_h, G_{h+1}, \ldots, G_{n-1}, G_n\}$ such that $G_i$ is one of the $i$-dimensional underlying graphs of $G_{i+1}$ for each $i$ with $h \leqslant i \leqslant n-1$.*

**Proof.** From the recursive definition of $HL_n$, for the given graph $G_n \in HL_n$, there are two $(n-1)$-dimensional underlying graphs $G_{n-1}, G'_{n-1} \in HL_{n-1}$ with $\sigma_{n-1}$ such that $G_n = G_{n-1} \oplus_{\sigma_{n-1}} G'_{n-1}$; for the graph $G_{n-1} \in HL_{n-1}$ there are two $(n-2)$-dimensional underlying graphs $G_{n-2}, G'_{n-2} \in HL_{n-2}$ with $\sigma_{n-2}$ such that $G_{n-1} = G_{n-2} \oplus_{\sigma_{n-2}} G'_{n-2}$. In general, for each $i$ with $h \leqslant i \leqslant n-1$ and the graph $G_{i+1} \in HL_{i+1}$, there are two $i$-dimensional underlying graphs $G_i, G'_i \in HL_i$ with $\sigma_i$ such that $G_{i+1} = G_i \oplus_{\sigma_i} G'_i$. Thus the lemma follows. $\square$

## 3. Main results

In this section, our aim is to prove that $\lambda_s^{(h)}(G_n) = 2^h(n-h)$ for any $G_n \in HL_n$ and $h \in I_{n-1}$.

**Lemma 3.1.** $\lambda_s^{(h)}(G_n) \leqslant 2^h(n-h)$ *for any $G_n \in HL_n$ and $h \in I_{n-1}$.*

**Proof.** Let $G_n \in HL_n$. By Lemma 2.1 there is a sequence of graphs $\{G_h, G_{h+1}, \ldots, G_{n-1}, G_n\}$ such that $G_i$ is one of the $i$-dimensional underlying graphs of $G_{i+1}$ for each $i$ with $h \leqslant i \leqslant n-1$. Let $F$ be the set of edges between $G_h$ and $G_n - G_h$. Then $F$ is an edge-cut of $G_n$. Since $G_n$ is $n$-regular and $G_h$ is $h$-regular, $|F| = |G_h|(n-h) = 2^h(n-h)$.

We now show that $F$ is an $h$-edge-cut of $G_n$ by proving $\delta(G_n - F) \geqslant h$. Let $x$ be a vertex in $G_n - F$. If $x$ is in $G_h$, then $x$ in $G_n - F$ has degree $h$ clearly since $G_h \in HL_h$. If $x$ is in $G_n - G_h$, it can be matched at most one vertex in $G_h$ by the matching $M_i$ for some $i \in \{h, h+1, \ldots, n-1\}$, which implies that $x$ has degree at least $n-1$ ($\geqslant h$) in $G_n - F$. By the arbitrariness of $x$, $\delta(G_n - F) \geqslant h$, which shows that $F$ is an $h$-edge-cut of $G_n$, and so

$$\lambda_s^{(h)}(G_n) \leqslant |F| = 2^h(n-h).$$

The lemma follows. □

For $G_n \in HL_n$, let $X \subseteq G_n$ be a non-empty subgraph of $G_n$, $Y = G_n - X$, and $E_n(X)$ denote the set of edges between $X$ and $Y$ in $G_n$. For convenience, let $G_n = H_0 \oplus_\sigma H_1$, where $H_0 = G_{n-1}$ and $H_1 = G'_{n-1}$, $G_{n-1}, G'_{n-1} \in HL_{n-1}$, $\sigma$ is a bijection from $V(G_{n-1})$ to $V(G'_{n-1})$. For each $i \in I_1$, let

$$X_i = X \cap H_i, \qquad Y_i = Y \cap H_i,$$

$$F_i = E_n(X) \cap E(H_i) \quad \text{and} \quad F_2 = E_n(X) \cap M_{n-1}.$$

**Lemma 3.2.** $|X| \geqslant 2^h$ *if* $\delta(X) \geqslant h$ *for any* $n$ *and given* $h \in I_n$.

**Proof.** If $h = 0$, then the conclusion holds immediately, so we proceed by induction on $n$ ($\geqslant 1$) for fixed $h \in I_n \setminus \{0\}$ by the recursive structure of $G_n$. Clearly, if $n = 1$ then the conclusion is true for $h = 1$. Assume that the conclusion holds for $n-1$ with $n \geqslant 2$.

If $X \subseteq H_0$ or $X \subseteq H_1$, then we have done by the induction hypothesis. Assume $X_i = X \cap H_i \neq \emptyset$ for each $i \in I_1$ below. Then $\delta(X_i) \geqslant h-1$ in $H_i$ for each $i \in I_1$ since $\delta(X) \geqslant h$ and there is at most one edge linking a vertex in $X_0$ and a vertex in $X_1$ in $G_n$. By the induction hypothesis, $|X_i| \geqslant 2^{h-1}$ for each $i \in I_1$. It follows that

$$|X| = |X_0| + |X_1| \geqslant 2 \cdot 2^{h-1} = 2^h.$$

By the induction principle, the lemma follows. □

**Lemma 3.3.** $|X| + |E_n(X)| \geqslant 2^h(n+1-h)$ *if* $\delta(X) \geqslant h$ *for any* $n$ *and* $h \in I_n$.

**Proof.** Since $X$ is non-empty subgraph of $G_n$ and $G_n$ is $n$-regular, $|X| + |E_n(X)| \geqslant n+1$, and so the conclusion is true for $h = 0$. Assume $h \geqslant 1$ below, and prove the conclusion by induction on $n\,(\geqslant 1)$ for fixed $h \in I_n \setminus \{0\}$. Clearly, the conclusion hold for $n = 1$. Assume the induction hypothesis for $n-1$ ($n \geqslant 2$). There are two cases.

**Case 1.** $X \subseteq H_0$ or $X \subseteq H_1$.

Assume $X \subseteq H_0$ without loss of generality. Since every vertex in $X$ has exactly one neighbor in $H_1$ matched by a perfect matching $M_\sigma$, we have $|F_2| = |X|$, and so $|F_2| = |X| \geqslant 2^h$ by Lemma 3.2.

Since $\delta(X) \geqslant h$ and $h \in I_{n-1} \setminus \{0\}$, using the induction hypothesis in $H_0$, we have $|X| + |E_{n-1}(X)| \geqslant 2^h(n-h)$. Combining this with $|F_2| \geqslant 2^h$, we have

$$\begin{aligned}|X| + \left|E_n(X)\right| &= |X| + \left|E_{n-1}(X)\right| + |F_2| \\ &\geqslant 2^h(n-h) + 2^h \\ &= 2^h(n+1-h),\end{aligned}$$

and so the conclusion holds.

**Case 2.** $X_i = X \cap H_i \neq \emptyset$ for each $i \in I_1$.

For each $i \in I_1$, since $X_i \subseteq H_i$ and $\delta(X) \geqslant h$ in $G_n$, $\delta(X_i) \geqslant h-1$ in $H_i$. Using the induction hypothesis in $H_i$, we have

$$|X_i| + \left|E_{n-1}(X_i)\right| \geqslant 2^{h-1}(n+1-h) \quad \text{for each } i \in I_1.$$

It follows that

$$\begin{aligned}|X| + \left|E_n(X)\right| &\geqslant |X_0| + \left|E_{n-1}(X_0)\right| + |X_1| + \left|E_{n-1}(X_1)\right| \\ &\geqslant 2^h(n+1-h).\end{aligned}$$

By the induction principle, the lemma follows. □

**Lemma 3.4.** $|E_n(X)| \geqslant 2^h(n-h)$ *if* $\delta(X) \geqslant h$ *and* $\delta(Y) \geqslant h$ *for any* $n$ *and* $h \in I_{n-1}$.

**Proof.** Since $|E_n(X)| \geqslant \lambda(G_n) = n$, so the conclusion is true for $h = 0$, assume $h \geqslant 1$ below. By symmetry of $X$ and $Y$, we can assume $|X| \leqslant |Y|$. We prove the conclusion by induction on $n$ ($\geqslant 1$). The conclusion is true for $n = 1$ clearly. Assume the induction hypothesis for $n-1$ with $n \geqslant 2$. There are two cases.

**Case 1.** $X \subseteq H_0$ or $X \subseteq H_1$.

Assume $X \subseteq H_0$ without loss of generality. If $h = n-1$, then $|X| \geqslant 2^{n-1}$ by Lemma 3.2. Thus $X = H_0, Y = H_1$, and $|E_n(X)| = 2^{n-1}$, so the conclusion is true. Thus, assume $h \in I_{n-2}$ below.

Clearly, $|F_2| = |X|$ since every vertex in $X$ has exactly one neighbor in $H_1$ matched by a perfect matching $M_{n-1}$. Since $\delta(X) \geqslant h$, $X \subseteq H_0$ and $h \in I_{n-2}$, using Lemma 3.3 in $H_0$, we have

$$|X| + \left|E_{n-1}(X)\right| \geqslant 2^h(n-h).$$

It follows that

$$\begin{aligned}\left|E_n(X)\right| &= \left|E_{n-1}(X)\right| + |F_2| \\ &= \left|E_{n-1}(X)\right| + |X| \\ &\geqslant 2^h(n-h),\end{aligned}$$

and so the conclusion holds.

**Case 2.** $X_i = X \cap H_i \neq \emptyset$ for each $i \in I_1$.

Clearly, for each $i \in I_1$, $Y_i \neq \emptyset$ since $X_i \subseteq H_i$, and $\delta(X_i) \geqslant h-1$ and $\delta(Y_i) \geqslant h-1$ in $H_i$ since $\delta(X) \geqslant h$ and $\delta(Y) \geqslant h$ in $G_n$. Thus, $X_i \subseteq H_i$ and $Y_i \subseteq H_i$, satisfy our hypothesis for each $i \in I_1$. By the induction hypothesis,

$$|F_i| = \left|E_{n-1}(X_i)\right| \geqslant 2^{h-1}(n-h) \quad \text{for each } i \in I_1.$$

It follows that

$$\left|E_n(X)\right| \geqslant |F_0| + |F_1| \geqslant 2 \cdot 2^{h-1}(n-h) = 2^h(n-h),$$

and so the conclusion holds.

By the induction principle, the lemma follows. □

**Theorem 3.5.** $\lambda_s^{(h)}(G_n) = 2^h(n-h)$ *for any* $G_n \in HL_n$ *and any* $n$ *and* $h \in I_{n-1}$.

**Proof.** Let $G_n \in HL_n$. By Lemma 3.1, we need only to show that $\lambda_s^{(h)}(G_n) \geqslant 2^h(n-h)$ for any $h \in I_{n-1}$. Let $F$ be an $h$-edge-cut of $G_n$ with $|F| = \lambda_s^{(h)}(G_n)$, $X$ a connected component of $G_n - F$, and $Y = G_n - X$. Clearly, $\delta(X) \geqslant h$ and $\delta(Y) \geqslant h$ since $F$ is an $h$-edge-cut. By Lemma 3.4, we immediately have

$$\lambda_s^{(h)}(G_n) = |F| = \left|E_n(X)\right| \geqslant 2^h(n-h),$$

and so the theorem follows. □

**Corollary 3.6.** *If $G_n \in \{Q_n, VQ_n, CQ_n, MQ_n, TQ_n, LTQ_n, G(2^n, 4)\}$, then $\lambda_s^{(h)}(G_n) = 2^h(n-h)$ for any $h \in I_{n-1}$.*

## 4. Conclusions

In this paper, we consider the generalized measures of edge-fault tolerance for the hypercube-like networks, called the $h$-super edge-connectivity $\lambda_s^{(h)}$. For the hypercube-like graph $G_n \in HL_n$, we prove that $\lambda_s^{(h)}(G_n) = 2^h(n-h)$ for any $n$ and $h \in I_{n-1}$. This result shows that at least $2^h(n-h)$ edges of $G_n$ have to be removed to get a disconnected graph that contains no vertices of degree less than $h$. Thus, when the hypercube-like networks are used to model the topological structure of a large-scale parallel processing system, this result provides a more accurate measurement for fault tolerance of the system.

Similarly, we can define $h$-super connectivity $\kappa_s^{(h)}(G_n)$ of a graph $G_n \in HL_n$ by considering vertices rather than edges. One may ask if $\kappa_s^{(h)}(G_n) = 2^h(n-h)$ for any $n$ and $h \in I_{n-1}$ with $0 \leqslant h \leqslant n-1$, or how many vertices of $G_n$ have to be removed to get a disconnected graph that contains no vertices of degree less than $h$. In fact, there are (or is) some $n$ and/or $h$ such that $\kappa_s^{(h)}(G_n)$ does not exist, that is, no matter how we remove vertices, we cannot get a disconnected graph that contains no vertices of degree less than $h$. The graph shown in Fig. 1 is an example for $n = 4$ and $h = 2$. It is worthwhile to research the existence of $\kappa_s^{(h)}(G_n)$ for some $G_n \in HL_n$ or $h \in I_{n-1}$, and to determine $\kappa_s^{(h)}(G_n)$ if $\kappa_s^{(h)}(G_n)$ exists.

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