

The Errata Given by the Authors

(<http://quoll.uwaterloo.ca/agt/errors/>)

Chapter 1: Graphs

page 2, line 10: replace “ordered” by “unordered”.

Chapter 3: Transitive Graphs

page 37, line 9/10: in the last line before the diagram, replace “arc-transitive” by “arc transitive” [this is a biggie!]

page 50, line 2: The permutation τ is defined as $x^\tau = \beta^{-1}x^\pi$. This means that τ is the composition of π FIRST followed by left-multiplication by β^{-1} ; The proof on page 50 refers to “left multiplication by β^{-1} followed by π ” which is the wrong way round; it does not affect the consequence, which is that $r + \ell$ is even. In addition, the statement of the theorem is unnecessarily unwieldy; in this situation k and ℓ are simply the index of α and β in

Chapter 4: Arc-Transitive Graphs

Lemma 4.2.1 does not hold as stated. We should add the hypothesis that f is locally onto, i.e., that it maps the out-neighbours of a vertex v onto the out-neighbours of $f(v)$. In Theorem 4.2.2, we need to make a corresponding adjustment to the appeal to this lemma.

The following comment should be added to the notes: Building on hard work by many people (including the classification of finite simple groups), Richard Weiss proved that if X is an s -transitive graph with valency at least three, then $s \leq 7$. See: Richard Weiss, The nonexistence of 8-transitive graphs. *Combinatorica* 1 (1981), no. 3, 309–311.

Chapter 5: Generalized Polygons and Moore Graphs

At line 7/8 on page 77, and at line -8 on page 91, we forgot that the odd circuits are Moore graphs. (We should probably add the constraint: valency > 2 .) [Thanks: Theo Grundhoefer]

Chapter 6: Homomorphisms

page 107: the last paragraph starts: (The proof is left as an exercise.) One of the students (Alastair Farrugia) has just asked, the proof of what? The simplest fix would be to delete this line. The intent was to get a proof that the subdirect product is the direct product in the category of graphs with maps to F . The last parenthetical sentence in this paragraph will need to be fixed too

In exercise 14, replace “Show” by “If X has at least one edge, show”. (The object is to verify Lemma 6.4.4, which has the extra qualification. The problem is that F_1^K is a complete graph with a loop on each vertex.)

Exercise 8 is false. Forget it!

Exercise 27: change “Show that” to “Prove or disprove” (The claim is false, C_7 is a counterexample.)

Exercise 29: We need X to be connected and G to be a 2-group.

Exercise 46: replace the first reference to r by “the girth r of X ”.

Exercise 48: there is a problem with the case $n = 2$. I think the simplest fix is to replace $n \geq 2$ by $n \geq 3$, and then delete the second sentence. Further, in (b), replace “Show that” by “Show that if”, and replace “if and only if” by “,then”.

Chapter 7: Kneser Graphs

page 145: In the proof of Lemma 7.7.1, we missed the most important case. The correct result is - if $v \geq 2r$, then an independent set of $C(v, r)$ has size at most r - if $v > 2r$, then it is also the case that any independent set of size r consists of all the r -subsets through a point. Proof: For $x \in \{0, \dots, v-1\}$ let s_x denote the r -subset $\{x, \dots, x+r-1\}$ (wrapping round as necessary). Let S be an independent set of size r , and assume wlog that $s_0 \in S$ and $s_{v-1} \notin S$. Then the subsets that intersect s_0 (that is, the non-neighbours of s_0 in $C(v, r)$) are the $2r-2$ subsets $s_1 s_2 \dots s_{r-1} s_{v-1} s_{v-2} \dots s_{v-(r-1)}$. Now if $v \geq 2r$, then s_1 does NOT overlap $s_{v-(r-1)}$, s_2 does NOT overlap $s_{v-(r-2)}$ and so on, and so we can only take at most one member from each such pair to belong to S . Hence S contains at most $r-1$ of these elements, and so at most r in total. Furthermore, if $v > 2r$, then $C(v, r)$ contains the following path of length $2r-2$ among the non-neighbours of s_0 : $(s_1, s_{v-(r-1)}, s_2, s_{v-(r-2)}, s_3, \dots, s_{r-1}, s_{v-1})$ and so the remaining elements of S must form an independent set of size $r-1$ in this path. By assumption we have that s_{v-1} is not in S , and the only way to select an independent set of size $r-1$ from the remaining $(2r-1)$ -path is to take every alternate vertex starting at s_1 - namely s_1, s_2, \dots, s_{r-1} . QED.

page 152, last displayed equation: should read: $g(t) := \prod_{x_i < 0} (t - a_i)$. We currently have $x - a_i$.

page 154: the definition of Cartesian product is wrong! It should read “where (x_1, y_1) is adjacent to (x_2, y_2) if and only if $x_1 = x_2$ and $y_1 \sim y_2$ or $x_1 \sim x_2$ and $y_1 + 1 = y_2$.”

page 156: The definition of lexicographic product is incorrect. If $x_1 \sim x_2$, then $(x_1, y_1) \sim (x_2, y_2)$. (There is no restriction on the second coordinate in this case.)

page 159, ex26: in the second last line of this exercise, Y should be $Y[X]$. (Well, I’m sure that the Y is wrong.)

page 159, ex 27: we should add the assumption $v \geq 5$.

Chapter 8: Matrix Theory

page 168, proof of 8.3.1: In the first line of the proof we should have “null space of D^T ”. In the second sentence of the proof, on the next line, we should have “ $z^T D = 0$ ”.

page 173, last para of Section 8.5: delete the parenthetic sentence “(Indeed...and semiregular.)” A-invariance of the bipartition is equivalent to semiregularity for bipartite graphs, but that is not what we say. It may be that the entire paragraph is a bit of a mess.

page 178, line -10: The two partitioned vectors (x, y) and $(x, -y)$ should both have transposes on them, i.e., they should read $(x, y)^T$ and $(x, -y)^T$.

page 179: the 12-entry of the 2×2 matrix in the final display of the proof of 8.9.1 should be R^T , not R .

page 190, exer 15: it will be clearer if we write “be a symmetric $n \times n$ matrix.”

Chapter 9: Interlacing

page 204: in the displayed expression for the matrix B , the $(2,1)$ -entry should be $\frac{|S|k}{n-|S|}$. And just before this display, we should replace “each vertex has exactly $|S|k/(n-|S|)$ neighbours” by “the average number of neighbours in S of a vertex not in S is $|S|k/(n-|S|)$ ”.

page 209: in the second display, the third permutation should be (15432) , i.e., the inverse

of (12345)

Chapter 10: Strongly Regular Graphs

page 224, line 6 (of text): instead of “occurs twice”, I think we need “occurs once”. Or else say “no ordered pair...occurs twice”. (I prefer the latter.)

page 227, line -3 of Section 10.5: if a conference graph on n vertices exists, then n must be the sum of two squares. (It is not known that this condition is sufficient!)

page 245, exercise 10: k should be 3.

page 245, exercise 13: in line -3, we should have “if their product contains exactly seven cells”.