Equilibrium and catastrophe of coronal flux ropes in axisymmetrical magnetic field

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1. Introduction

[1] Using a 2.5-dimensional, time-dependent ideal magnetohydrodynamic model in spherical coordinates, we present a numerical study of the property of magnetostatic equilibria associated with a coronal magnetic flux rope embedded in an axisymmetrical background magnetic field. The background field is potential (either closed or partly opened), a magnetic flux rope emerges out of the solar surface, and the resultant system is allowed to relax to equilibrium through numerical simulation. It is shown that the flux rope either sticks to the solar surface so that the whole magnetic configuration stays in equilibrium or escapes from the top of the computational domain, leading to the opening of the background field. Whether the rope remains attached to the solar surface or escapes to infinity depends on the magnetic energy of the system. The rope sticks to the solar surface when the magnetic energy of the system is less than a certain threshold, and it escapes otherwise. The threshold is slightly larger than the open limit, i.e., the magnetic energy of the corresponding fully opened field. The gravity, say, associated with the prominence mass, will raise the threshold by an amount that is approximately equal to the magnitude of the excess gravitational energy associated with the prominence. It implies that a catastrophe occurs when the magnetic energy of the system exceeds the threshold. The implication of such a catastrophe in coronal mass ejections is briefly discussed.


1. Introduction

[2] Coronal mass ejections (CME) are among the most important coronal active phenomena. A major portion of CMEs are characterized by a three-part structure: a dense helmet dome, a dark cavity, and a quiescent prominence, as observed in white light [Hundhausen, 1999; Dere et al., 1999; Low, 2001]. Prominences and their eruption are commonly considered as a possible candidate for triggering CMEs [Forbes, 2000; Low, 2001]. Theoretically, a coronal magnetic flux rope is believed to accompany most prominences whose magnetic field is opposite to the photospheric magnetic polarity beneath them, often referred to as the inverse prominence configuration [Low and Hundhausen, 1995]. In this situation, an eruption of the flux rope must lead to the eruption of the prominence and the opening of the ambient magnetic field, and the latter was usually considered as a necessary condition for CMEs [Hundhausen, 1988]. Many authors suggested that a catastrophic loss of mechanical equilibrium might be responsible for the rope eruption. Both analytical and numerical models were proposed to reveal possible catastrophic behaviors of an ideal magnetohydrodynamic (MHD) system containing magnetic flux ropes [Forbes and Isenberg, 1991; Forbes and Priest, 1995; Isenberg et al., 1993; Lin et al., 1998, 2001; Hu and Liu, 2000; Hu, 2001; Hu et al., 2001; Hu and Jiang, 2001; Li and Hu, 2001]. A common conclusion of these studies lies in that catastrophe exists under certain conditions. However, most of them were limited to Cartesian geometry with translational symmetry, for which only the energy of a finite length along the ignorable coordinate axis is meaningful. It takes an infinite amount of energy to open up a closed magnetic field, since the magnetic field along the current sheet in a fully or partly opened field decreases as fast as the reciprocal of the height at large distance and thus has an infinite energy per unit length. Consequently, any flux rope in the corona that always has finite energy per unit length along the ignorable coordinate axis can never open the background field. Therefore it is hard to make any physically reasonable analysis in energetics in terms of the 2.5-dimensional (2.5-D) approximation in Cartesian geometry. However, the problem of energetics is certainly important for active phenomena like CMEs (see reviews by Forbes [2000] and Low [2001]). If one persists in two-
dimensional analysis, one had better use spherical geometry, in which axisymmetrical magnetic configurations are easily found to have finite energy for both themselves and their corresponding fully opened fields [Aly, 1984, 1991; Sturrock, 1991]. In this case, the energy analysis is both meaningful and tractable.

[1] Lin et al. [1998] derived a two-dimensional flux rope model in the ideal MHD framework, using spherical geometry. In this model, the background field is created by a Sun-centered dipole and the flux rope is thin in the sense that its radius is much smaller than its heliocentric distance. As a result, an island-shaped region appears in the meridional plane, being sandwiched between the thin flux rope and the background field. The magnetic field is force free within the flux rope and potential outside. On this basis, they reduced the strength of the photospheric dipole source and let the loss of the photospheric flux enter the island surrounding the flux rope. A mechanical equilibrium was then achieved by a balance between the relevant forces. An interesting finding of that paper lies in that if the photospheric source is reduced below a critical value, the balance cannot be maintained, and the flux rope finds a new equilibrium attained with an abrupt rise into the atmosphere. In addition, the stored energy at the critical point is less than that of the corresponding fully opened field, so they argued that the jumped flux rope would reach a new equilibrium at a higher altitude, creating an electric current sheet below the associated island. Nevertheless, they failed to find the solution for the new equilibrium and, needless to say, to investigate its stability. Instead, an asymptotic analysis was carried out on the behavior of the system. The conclusion turned out to be also interesting: no equilibrium exists for the system with a large current sheet below the flux rope, and the field becomes fully opened as the strength of the background field is smaller than the threshold.

[2] Our paper presents a numerical study of problems similar to that treated analytically by Lin et al. [1998]. Following the procedures in our previous papers [Hu, 2001; Hu et al., 2001; Hu and Jiang, 2001], time-dependent numerical simulations are carried out first to produce a properly selected initial state that is not in equilibrium but includes the necessary ingredients (a background field and an embedded flux rope) and then to let it relax to equilibrium. Our concern is the final equilibrium state obtained but not the detailed dynamical process by which it is reached. This provides us the flexibility to take some special measures to improve the accuracy and reliability of the numerical results. The background field is still a dipole one if it is closed, but it can be partly opened with an equatorial current sheet that extends from somewhere in the corona to infinity. The flux rope is no longer “thin” but is stretched transversely throughout the island so as to be in direct contact with the background field. Such a configuration makes the comparison between our results and Lin et al.’s somewhat difficult but seems to be closer to reality. In addition, the magnetic field within the flux rope does not have to be force free; so other forces such as the gravitational force can be taken into account. On this basis, we can examine the effect of the gravity on the catastrophic behavior of the system.

[3] The basic equations and the initial boundary conditions are given in section 2. In section 3, we describe the numerical method, in which several special measures are taken to improve the accuracy and reliability of the simulations. Numerical results are discussed in section 4 with emphasis on the catastrophic behavior of the system and its relationship to the magnetic energy. We conclude our work in section 5.

2. Basic Equations and Initial Boundary Conditions

2.1. Basic Equations

[5] Take the spherical coordinate system (r, θ, ϕ) and consider 2.5-D problems in the meridional plane. Introducing a magnetic flux function ψ (t, r, θ) related to the magnetic field by

\[ \mathbf{B} = \nabla \times \left( \frac{\psi}{r \sin \theta} \right) + \mathbf{B}_0, \quad \mathbf{B}_0 = B_\phi \hat{\phi}, \]  

the 2.5-D ideal MHD equations may be cast in the nondimensional form

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{\beta}{2} \nabla T + \frac{\beta T}{2\rho} \nabla p + \frac{1}{\rho} \left[ \mathcal{L} \nabla \psi + \mathbf{B}_0 \times (\nabla \times \mathbf{B}_0) \right] \]

\[ + \frac{1}{\rho r \sin \theta} \nabla \psi \cdot (\nabla \times \mathbf{B}_0) \frac{\phi}{r^2} + \frac{g_s}{r^2} = 0 \]  

\[ \frac{\partial \mathbf{B}_0}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B}_0 = 0 \]  

\[ \frac{\partial \psi}{\partial t} + r \sin \theta \nabla \cdot \left( \frac{\mathbf{B}_0 \mathbf{v}}{r \sin \theta} \right) + \left[ \nabla \psi \times \nabla \left( \frac{\mathbf{v}}{r \sin \theta} \right) \right] = 0 \]  

\[ \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T + (\gamma - 1) T \nabla \cdot \mathbf{v} = 0, \]  

where

\[ \mathcal{L} \psi \equiv \frac{1}{r^2 \sin \theta} \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} - \cot \theta \frac{\partial \psi}{\partial \theta} \right). \]

\[ \gamma \] is the polytropic index (taken to be 1.05 in this study), and g_s is the gravitational acceleration at the solar surface (GM_s/R_0^2) normalized by v_\phi^2/R_0. Here G is the gravitational constant, M_s is the mass of the Sun, R_0 is the radius of the Sun (taken as the unit of length), and v_\phi is the characteristic Alfvén wave speed (taken as the unit of velocity). Besides, \beta is the characteristic ratio of gas pressure to magnetic pressure,

\[ \beta = 2\mu \rho_0 R_G T_0 R_0^4/v_\phi^2. \]  

where \mu is the vacuum magnetic permeability, R_G is the gas constant, \rho_0 and T_0 are the density and temperature at the base, respectively, and \psi_0 is the magnetic flux function at the equator of the Sun, i.e., the total magnetic flux.
emanating from per unit radian of the north hemisphere of the Sun. All symbols with subscript 0 represent the units of the corresponding quantities. Other units include $B_0 = \psi_0 R_0^2$ for field strength, $v_0 \equiv v_{10} = B_0 \sqrt{\mu_0}$ for velocity, $t_0 \equiv \tau_A = R_0/v_0$ for time, $M_0 = \rho_0 R_0^2$ for mass, $W_0 = M_0 v_0^2$ for energy, and so on.

[7] In the following numerical examples, the basic units are set to be

$$
T_0 = 2 \times 10^6 \text{ K} \\
\rho_0 = 1.67 \times 10^{-13} \text{ kg m}^{-3}
$$

in addition to $R_0 = 6.95 \times 10^8 \text{ m}$. Consequently, the units of other quantities are $v_0 = 257 \text{ km s}^{-1}$, $v_0 = 2.70 \times 10^4 \text{ km s}^{-1}$, $B_0 = 1.18 \times 10^{-4} \text{ Wb}$, $W_0 = 3.71 \times 10^{13} \text{ J}$, etc. All these units except $M_0$ depend on $\beta$.

### 2.2. Initial and Boundary Conditions

[8] The initial corona is assumed to be isothermal and static with

$$
T(0, r, \theta) = 1 \\
\rho(0, r, \theta) = \exp \left[ \frac{GM(1 - r)}{R_0 R_0 T_0 r} \right].
$$

Where

$$
\psi_0 = \psi_0(0, r, \theta) = r (1 - r^2) \left[ 1 + \frac{a^2}{2} \right] \arctan \left( \frac{1}{a} \right) - \frac{\pi a^2 \sin^2 \theta}{2r} + 2an_0
$$

and

$$
\psi_0 = \psi_0(0, r, \theta) = r (1 - r^2) \left[ 1 + \frac{a^2}{2} \right] \arctan \left( \frac{1}{a} \right) - \frac{\pi a^2 \sin^2 \theta}{2r} + 2an_0
$$

where

$$
a^2 = \frac{1}{2} \left[ 1 - \frac{a^2}{r^2} + \frac{4a^2}{r^2} \cos^2 \theta \right]^{1/2} - \frac{1}{2} \left( 1 - \frac{a^2}{r^2} \right)
$$

$$
v^2 = \frac{1}{2} \left[ 1 - \frac{a^2}{r^2} + \frac{4a^2}{r^2} \cos^2 \theta \right]^{1/2} + \frac{1}{2} \left( 1 - \frac{a^2}{r^2} \right)
$$

$$
v^2 = \frac{1}{2} \left[ 1 - \frac{a^2}{r^2} + \frac{4a^2}{r^2} \cos^2 \theta \right]^{1/2} + \frac{1}{2} \left( 1 - \frac{a^2}{r^2} \right)
$$

Starting from $\psi_0$, the initial magnetic flux function that we need is simply expressed by

$$
\psi(0, r, \theta) = \frac{\psi_0(0, r, \theta) - \psi_0(1, \theta)}{\psi_0(1, \pi/2) - \psi_0(1, 0)}.
$$

Then we have $\psi(0, r, 0) = 0$, $\psi(0, 1, \pi/2) = 1$, and

$$
\psi_0(0, a, \pi/2) = 2a \left[ \frac{1 + a^2}{a^2} - \sqrt{a^2 - 1} \right]^{-1}
$$

which is the value of the magnetic flux function at the current sheet. Consequently, $(1 - \psi_0)$ is the total magnetic flux per unit radian within the closed arcade. The initial field is controlled by a single parameter $a$, the heliocentric distance of the inner edge of the current sheet. As $a \to \infty$, equation (11) approaches a dipole field:

$$
\lim_{a \to \infty} \psi(0, r, 0) = \frac{\sin^2 \theta}{r}.
$$

The magnetic configurations for the dipole and the partly opened field with $a = 2$ are shown in Figures 1a and 1b, and they will be used as the initial background fields in the following simulations.

[10] For a given value of $a > 1$, we construct an initial state. Starting from it, we let a magnetic flux rope of circular cross section emerge from the base of the arcade in a time period of $T_E$. The half-width of the emerging flux rope is $\lambda_w$ in latitude, and the emergence is uniform in velocity. The emerging part of the flux rope is bounded by $\lambda = \theta - \pi/2 = \pm \lambda_E$, where

$$
\lambda_E = 2\lambda_w \left[ \frac{1}{T_E} \left( 1 - \frac{1}{T_E} \right) \right]^{1/2}.
$$

At the base of the emerging part of the rope ($r = 1, |\lambda| \leq \lambda_E$), the relevant quantities are specified as a function of $\lambda$ and $\theta$ as follows:

$$
\psi(t, 1, \lambda) = \psi(0, 1, \lambda) + \psi_E(t, \lambda) + \psi_0(t, \lambda)
$$

$$
\psi_E(t, \lambda) = C_E (\lambda - \lambda^2)/\lambda^2
$$

$$
B_0(t, 1, \lambda) = B_0 (\lambda - \lambda^2)^{1/2}/\lambda^2
$$

$$
v(t, 1, \lambda) = (2\nu_0/\tau_E)
$$

$$
\nu(t, 1, \lambda) = \nu_0(1, \lambda) = 0
$$

$$
T(t, 1, \lambda) = 0.02
$$

where $C_E$ and $B_0$ are constants and we use them to control the magnetic properties of the emerging rope and thus the magnetic energy of the whole system. Note that the contour of $\psi = 1$ depicts the outer boundary of the flux rope. After $t = T_E$, the state at the solar surface returns to original.

[11] The computational domain is taken to be $1 < r < 30$ (in the unit of the solar radius $R_0$), $0 \leq \theta \leq \pi/2$, discretized into $130 \times 90$ grid points. The grid spacing increases according to a geometric series of common ratio 1.03 along the radial direction from 0.02 at the solar surface ($r = 1$) to 0.86 at the top ($r = 30$). A uniform mesh is adopted in the $\theta$ direction with the equator ($\theta = \pi/2$) as a grid point and the polar axis ($\theta = 0$) as a half-grid point. Symmetrical boundary
conditions are used at the equator and the polar axis. Incidentally, taking the equator as a grid point is aimed at reducing the numerical diffusion of the magnetic field across it [Chen et al., 2000]. The top of the numerical box is high enough to contain the flux rope as a whole during the numerical simulation, so that \( B_z \) is set to be zero everywhere at the top. In addition, the magnetic field is nearly potential outside the flux rope so that we may safely set \( j_z = -r\sin \theta \frac{\partial \psi}{\partial \theta} = 0 \) at the top, from which \( \psi \) can be updated there from those at the inner grid points. Other quantities are calculated in terms of equivalent extrapolation except for the density that is fixed, and the reason will be given in the following section.

2.3. Emergence Parameters and Properties of the Flux Rope

[12] As can be seen from (14)–(18), we have four emergence parameters \((\tau_E, \lambda_m, C_E, \text{and } B_{\phi0})\) which control the magnetic properties of the emerging flux rope. For all examples presented below, we fix \( \tau_E = 1 \) and \( \lambda_m = 0.087 \) radian (i.e., \( 5^\circ \)) and leave \( C_E \) and \( B_{\phi0} \), free to change. Such a choice in the rope emergence parameters is somewhat artificial and arbitrary, but let us reemphasize that the emergence is performed only to create an appropriate initial state with a flux rope of certain properties in the corona, which is then relaxed to equilibrium. Our concern is the eventual equilibrium obtained rather than the detailed process toward it.

[13] Two parameters are used to characterize the geometrical properties of the flux rope: the height of the rope axis above the solar surface \((h_a)\) and the length of the newly formed current sheet below the rope \((h_l)\). Each of them evolves with time during the time-dependent simulation until a stable value is reached if the system approaches equilibrium. There is the possibility that the flux rope keeps ascending during the simulation and tends to escape from the top of the numerical box, implying that the relevant geometrical parameters approach infinity. As far as the physical properties of the flux rope are concerned, we have the following three parameters: the annular flux per unit radial \( \Phi_{an} \) defined as the difference of \( \psi \) between the rope axis and the rope boundary; the axial flux \( \Phi_{\phi} \), defined as the surface integration of \( B_{\phi} \) over the cross section of the flux rope; and the total mass \( M \) within the flux rope.

3. Numerical Method

[14] The multistep implicit scheme [Hu, 1989] is used to solve (2)–(6), but some special measures are taken to improve the accuracy and reliability of numerical results, as described below.

3.1. Substitute for the Magnetic Flux Function

[15] The magnetic flux function \( \psi \) defined by (1) is proportional to \( \sin^2 \theta \) near the polar axis, which will cause large numerical errors in the difference operation of terms like \( \frac{\partial \psi}{\partial \theta}/\sin \theta \). If \( \psi \) is substituted by \( \phi = \psi/\sin \theta \), then we have

\[
\mathcal{L}\psi = \frac{1}{r^2 \sin \theta} \left( \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial \phi}{\partial \theta} - \frac{\phi}{r^2 \sin^2 \theta} \right),
\]

and so on. The numerical accuracy and stability were substantially improved with the use of \( \phi \) instead of \( \psi \), as confirmed by its implementation into the numerical simulations. The magnetic flux function \( \psi \) is updated immediately after \( \phi \) is updated, so that we can plot the projected magnetic field lines in the meridional plane that are simply contours of \( \psi \).

3.2. Elimination of Numerical Reconnection

[16] Taking the equator as a grid point may efficiently reduce the pseudoreconnection in the current sheet located in the equatorial plane but cannot completely prevent it from occurring. In the present case, any numerical reconnection in the equatorial current sheets, either inherent by the initial partly opened background field or newly formed during the eruption, will cause a false transfer of annular magnetic flux from the background to the flux rope, which results in a topological change of the system and even an erroneous eruption of the flux rope. Therefore it is crucial to eliminate any numerical reconnection in the current sheets in order to achieve correct equilibrium solutions in the frame of ideal MHD. A special measure is taken to meet this need.

[17] As we know, the magnetic flux function \( \psi \) is constant along the current sheet. Any numerical reconnection reduces the value of \( \psi \) at the reconnection site on the sheet in the present simulations, and fixing the value of \( \psi \) at a constant along the current sheet will prohibit the numerical reconnection. As a matter of fact, we know a priori the value of \( \psi \) at the current sheet, that is, either \( \psi_o \) given in (12) at the original current sheet or \( \psi \) at the newly formed current
sheet. Therefore we may reassign the value of \( \psi \) at each time step simply by setting a minimum for \( \psi \) along the entire current sheet, either \( \psi_N \) or 1, so that it remains invariant and uniform there. By doing so, the numerical reconnection is completely prevented in the equatorial current sheets.

### 3.3. Conservation of Magnetic Fluxes and Mass in the Flux Rope

[18] In the frame of ideal MHD, the annular and axial magnetic fluxes and the total mass within the rope should be conserved during the evolution of the system [Hu et al., 1997]. These quantities are inherent qualities of the flux rope and thus affect the equilibrium properties of the system. However, their numerical values may change owing to numerical errors as the computation proceeds. To maintain their invariance, we calculate them at two adjacent time steps, say, \( t_n \) and \( t_{n+1} = t_n + \Delta t \), where \( \Delta t \) is the time step length, and the ratios between their values at \( t_n \) and \( t_{n+1} \) are well, which may deviate from unity owing to numerical errors. We denote these ratios \( \alpha_{\psi} \) for the annular flux, \( \alpha_B \) for the axial flux, and \( \alpha_{\rho} \) for the mass. Then \( \psi, B, \) and \( \rho \) updated at \( t_{n+1} \) within the flux rope (\( \psi > 1 \)) are replaced by

\[
\begin{align*}
\psi' &= 1 + \alpha_{\psi}(\psi - 1) \\
B' &= \alpha_B B \\
\rho' &= \alpha_{\rho} \rho,
\end{align*}
\]

respectively. By doing so successively, the solution obtained must have the two fluxes and the total mass within the flux rope strictly conserved. The ratios, i.e., the modification factors in (20), are very close to unity at each time step; the final solution does not represent any real physical process but is immaterial for the final solution obtained.

### 3.4. Prevention of Solar Wind Solutions

[19] The set of basic equations (2)–(6) may result in solar wind solutions that are not what we want for the purpose of our paper. Although a “solid wall” condition specified at the solar surface generally hinders solar wind flows from occurring, a coarse mesh near the base, as taken in our paper and most previous two-dimensional simulations in spherical geometry, may still result in a solar wind solution. The occurrence of the solar wind flow may open part of the closed component of the background field, leading to an unwanted eruption of the flux rope. Therefore we must prevent any solar wind flow from occurring. An appropriate way to this end is to fix the density at the top of the numerical box. The reason is simple: the decrease of the density at the top is a prerequisite to the development of a solar wind flow, and to keep the density invariant there will suppress the solar wind accordingly. This measure will also lead to reflection of MHD waves on the top, too, which is undesirable but has little influence on the final equilibrium of the system, as demonstrated by test calculations.

### 3.5. Maintaining Same Isothermal Equilibrium Outside the Flux Rope

[20] During the emergence and the eruption of the flux rope, the background field expands outward, leading to a polytropic variation of the initial isothermal coronal plasma. However, our purpose is not to trace the evolution of a real flux rope emergence. Instead, we are just interested in a possible equilibrium consisting of a flux rope and its background. When the system reaches equilibrium, we certainly hope that the state of the coronal plasma outside the flux rope remains the same as original, i.e., in an isothermal static equilibrium with the same temperature, and the magnetic field there remains to be potential. Such a state cannot be reached by a spontaneous evolution of the system of interest; some artificial measures must be taken to achieve it. To this end, we recover the density and temperature outside the flux rope at each time step. This seems to make the background plasma “incompressible” and will certainly distort the true evolution of the system. Nevertheless, since we are interested in the final equilibrium, it does not matter how the final state is reached. In fact, iteration method was often used to obtain steady state or magnetostatic equilibrium solutions based on an initial guess, and the transition that it involves from the guess to the final solution does not represent any real physical process but is immaterial for the final solution obtained.

### 4. Numerical Results

#### 4.1. Magnetic Energy and Excess Gravitational Energy

[21] The present code failed to maintain the total energy conservation during the simulation; so a comprehensive energy analysis is impossible. Numerical diffusion is exerted on the flow velocity all the time to stabilize the calculation so that the kinetic energy is dissipated in vain. The variation of both thermal and gravitational energies of the coronal plasma outside the flux rope due to the work done by the moving and expanding rope is eliminated, since we have recovered the density and temperature distribution there during the simulation. However, an approximate and asymptotic energy analysis is still meaningful. For instance, if the magnetic field is close to force free, then the magnetic energy dominates, and other forms of energy can be neglected. The magnetic energy normalized by \( 4\pi W_0 = 4\pi B_0^2 r_0^3 / \mu \) is evaluated by

\[
W_m = \frac{1}{2} \int_0^{30} dr \int_0^{r/3} 21 B^2 r^2 \sin \theta \, d\theta + \frac{30^2}{2} \int_0^{r/3} (B^2 - B_0^2) \sin \theta \, d\theta,
\]

where the first term on the right-hand side is the magnetic energy in the numerical box (1 \( \leq r \leq 30 \)) and the second term is that above the box, which has been transformed into a surface integral over the top under the assumption that the
magnetic field above the box is potential and may have a neutral current sheet at the equator [see Low, 1999]. The magnetic energy thus obtained is 1/3 for the dipole field (shown in Figure 1a) and 0.382 for the partly opened field with $a = 2$ (shown in Figure 1b); the corresponding fully opened fields have an energy of 0.554 (1.662 times that of the dipole field [see Low and Smith, 1993]) and 0.553, respectively. The difference between the two values stems from the slightly different distributions of $\psi$ at the solar surface inherent by the two fields.

On the other hand, the gravity is believed to play an important role in the development of CMEs (see a review by Low [2001]). First, since the escape speed is of the same order of magnitude as the median speed of CMEs, the work done against the gravity to lift the CME mass should be comparable with the kinetic energy. Second, the weight of the prominence serves as a confinement agent in maintaining the system in stable equilibrium, whereas a drainage of plasma from the prominence may destabilize the system. Taking the two CME events reported by Illing and Hundhausen [1986] and Gopalswamy and Hanaoka [1998] as examples, the prominence had a mass in excess of $5 \times 10^{16}$ g prior to eruption. Most of the mass was drained during the eruption, and only $10^{16}$ g was detected in the ejected part of the prominence. On this basis, Low [2001] argued that the prominence as a confinement agent may be as important as the helmet dome. We do not include a helmet dome–like structure in our model, but we do introduce a high-density structure in the emerging flux rope, which is presumably representative of the prominence. Consequently, we may explore the effect of the excess gravitational energy associated with the prominence in our model. The so-called excess gravitational energy in the unit of $4\pi W_0$ is given by

$$W_g = -\int_{1}^{30} dr \int_{0}^{\pi/2} g_r [\rho - \rho(0, r, 0)] r \sin \theta d\theta,$$

where $\rho(0, r, 0)$ is the density distribution for the background, given in (9). The integration may be limited to the flux rope ($\psi > 1$), since we have recovered the background density outside at each time step in the numerical simulations.

### 4.2. Case of $\beta = 0.001$

In this case, $\beta$ is so low that the initial magnetic field is close to the force-free state at least in the region near the solar surface, say, within $5 R_\odot$, where the ratio of gas pressure to magnetic pressure increases along the equator from its characteristic value of 0.001 at the base to about 0.15 at $r = 5$. As a result, the magnetic energy dominates, and other forms of energy are approximately negligible.
Moreover, we take the emergence parameter $C_E = 1$ for the dipole field case and 0.5 for the partly opened field case. Relevant units are $B_0 = 3.73 \times 10^{-3}$ T, $\psi_0 = 1.80 \times 10^{15}$ Wb, $W_0 = 3.71 \times 10^{27}$ J, and so on. Figure 2 shows the magnetic configurations at several separate time steps for the dipole field case with $B_{\phi 0} = 3.0$ (Figures 2a–2c) and 3.1 (Figures 2d–2f). The flux rope initially ascends and becomes separated from the solar surface (Figure 2a) but eventually falls back to the solar surface for $B_{\phi 0} = 3.0$. On the other hand, it keeps ascending and tends to escape from the numerical box for a slightly larger value of $B_{\phi 0} = 3.1$. Figure 3a shows the geometrical parameters of the flux rope as a function of time during the simulations; the case of $B_{\phi 0} = 3.0$ is depicted by the thick curves, and the case of $B_{\phi 0} = 3.1$ is shown by the thin curves. The different evolutionary behaviors indicate that a catastrophe occurs when $B_{\phi 0}$ increases from 3.0 to 3.1. The erupting speed of the flux rope is about 0.2–0.3 times the Alfvén speed at the beginning of the eruption, but it decreases with the height of the rope owing to the increase of both the plasma beta and the magnetic tension force produced by the magnetic field lines that pass over the top of the flux rope. Note that the deceleration has been greatly exaggerated by the magnetostatic approximation of the background coronal plasma. In fact, the occurrence of the solar wind will slow down the increase rate of the plasma beta with heliocentric distance and, in addition, will dissolve a part of the magnetic field that produces the tension force and entraps the flux rope at large altitude. Similar results were obtained for the partly opened field case, as shown in Figure 3b. The catastrophe occurs when $B_{\phi 0}$ increases from 3.9 to 4.0. These four examples are referred to as cases A, B, C, and D hereinafter.

[24] Lin et al. [1998] argued that an equilibrium state should exist with a current sheet of finite length below the magnetic island, but they did not really obtain such a solution because of mathematical difficulties. Instead, they made an asymptotic analysis assuming a sufficiently large current sheet below the island that is suspended highly in the corona. Their solution shows that no mechanical equilibrium exists for the highly levitating flux rope: the resultant force is always downward. This conclusion happens to be agreeable with ours that the flux rope can either stick to the solar surface in equilibrium or go to infinity but cannot stably levitate high in the corona. We point out in passing that our model is different from Lin et al.’s: the flux rope spreads throughout the magnetic island in our case, but it is thin and located at the central axis of the island in their model.

[25] We also calculated by using equations (21) and (22) the magnetic energy of the whole system and the excess gravitational energy associated with the surplus prominence mass over the background within the flux rope. The results are updated at $t = 500\tau_A$ when the flux rope has either reached equilibrium or gone away from the Sun with $h_s > 10$ and are listed for cases A, B, C, and D in the first four rows of Table 1, along with the relevant parameters ($a$, $b$, $C_E$, and $B_{\phi 0}$), three conserved quantities of the flux rope (the annular flux per unit radian $\phi_{\rho}$, the axial flux $\phi_{\psi}$, and the total mass $M$ within the flux rope), and the open limit $W_{\text{open}}$.

[26] It can be seen that $W_g$ is negligible in comparison with $W_m$ as expected. The thermal energy associated with the plasma pressure should be small in comparison with $W_m$, too; so the state of the system is mainly determined by the
magnetic energy. As Table 1 shows, \( W_m \) is slightly larger than the open limit for the eruption cases (B and D) and slightly smaller for the equilibrium cases (cases A and C). Therefore we may simply use \( W_m \), which is more meaningful in physics than \( B_{\infty} \), to characterize the catastrophic point of the system, i.e., the threshold of \( W_m \), at which an infinitesimal increase of \( W_m \) leads to a catastrophe of the whole system. The threshold should be close to and actually larger than the open limit for the following two reasons. First, to determine the threshold exactly, one must make the system undergo a quasi-static evolution with slowly increasing magnetic energy. In the present simulations, however, the flux rope grows out of nothing, the magnetic energy of the system increases abruptly, and the whole process is far from quasi-static. To some extent, this is equivalent to giving an impulse to the system so that a catastrophe occurs even if the magnetic energy is slightly below the threshold. In other words, one may underestimate the threshold based on the numbers of \( W_m \) listed in Table 1. Second, these numbers for the eruption cases (B and D) were evaluated at \( t = 500 \tau_A \), when the flux rope axis is more than 10 solar radii away from the solar surface. In fact, \( W_m \) decreases with increasing \( h_a \), as shown in Figure 4 for case B. The loss of the magnetic energy is presumably attributed to the wave and mass motions caused by the eruption of the flux rope. The curve starts at \( t = 10 \tau_A \) with \( h_a = 1.75 \), when the annular magnetic flux of the flux rope begins to be fixed. The magnetic energy at that point is 0.564, which is larger than 0.558 at \( t = 500 \tau_A \). The threshold should be the magnetic energy at a time right before the flux rope begins to erupt. Although we are presently unable to obtain a definite value of the threshold, we argue that it is larger than the open limit for the present case. This means that a force-free magnetic configuration in equilibrium with detached field lines may have an energy larger than the open limit. This conclusion is not new but was previously predicted by Aly [1991].

### 4.3. Case of \( \beta = 0.1 \)

In this case, the gravitational energy cannot be ignored as compared with the magnetic energy and should have significant effect on the equilibrium properties of the system. Relevant units are \( B_0 = 3.73 \times 10^{-4} \) T, \( \psi_0 = 1.80 \times 10^{14} \) Wb, \( W_0 = 3.71 \times 10^{25} \) J, and so on. We take \( C_E = 1 \) for this case. For the dipole field case, the geometrical parameters as a function of time are shown in Figure 3c for \( B_{\infty 0} = 5.4 \) by the thick curves and for \( B_{\infty 0} = 5.5 \) by the thin curves. The flux rope reaches equilibrium for \( B_{\infty 0} = 5.4 \), whereas it escapes to infinity for \( B_{\infty 0} = 5.5 \). This indicates that when \( B_{\infty 0} \) increases from 5.4 to 5.5, a catastrophe occurs. Similar behavior exists for the partly opened field case, as shown in Figure 3d. The catastrophe takes place when \( B_{\infty 0} \) increases from 4.4 (thick curves) to 4.5 (thin curves). The relevant parameters and energies are listed in the last four rows of Table 1 for these cases, labeled E, F, G, and H. It can be seen that the sum of the magnetic energy and the excess gravitational energy is larger than the open limit for the eruption cases F and H, whereas it is smaller than the open limit when the rope stays in equilibrium attached to the solar surface (cases E and G).

So far we have not considered the effect of the plasma pressure. Unfortunately, we are presently unable to make a quantitative analysis of this effect. Nevertheless, on the basis of the conclusion reached in section 4.2 and the numbers in Table 1, at least we can say that the energy threshold is raised by the gravity by an amount that is approximately equal to the magnitude of the excess gravitational energy associated with the prominence. A catastrophe occurs when the magnetic energy of the system exceeds the threshold.

### 5. Concluding Remarks

Using a 2.5-D ideal MHD model in the meridional plane, we conducted a numerical study to find equilibrium solutions associated with a coronal magnetic flux rope embedded in a dipole or partly opened background field. Our focus is on the catastrophic behavior of the system and its relation to the magnetic energy of the system. The main conclusions are summarized as follows:

1. For an axisymmetrical magnetic configuration in spherical geometry with an embedded flux rope whose field...
lines are detached from the solar surface, there exists a threshold for the magnetic energy of the system. When the magnetic energy is less than the threshold, the whole system stays in equilibrium with the flux rope attached to the solar surface. Otherwise, the flux rope escapes to infinity and the background field becomes fully opened. This means that a catastrophe occurs as the magnetic energy exceeds the threshold. For the cases investigated in the present study, the flux rope cannot levitate stably in the corona with a vertical current sheet below; namely, only the configurations that do not have a current sheet below the flux rope can be in equilibrium.

[11] 2. For force-free magnetic configurations, the energy threshold is slightly larger than the open limit, i.e., the magnetic energy of the corresponding fully opened field. Therefore force-free magnetic configurations with detached field lines may have an energy larger than the open limit, which confirms the conclusion previously predicted by Aly [1991]. The gravity associated with the prominence will raise the threshold by an amount that is approximately equal to the magnitude of the excess gravitational energy carried by the prominence mass.

[12] 3. Since the magnetic energy plays a crucial role in the catastrophic behavior of the system, any disturbances such as photospheric motions or prominence mass losses, which increase the magnetic energy of the system or decrease the gravitational energy in magnitude within the flux rope so that the energy threshold is exceeded, will cause an eruption of the flux rope and thus an opening of the ambient field. This is what a coronal mass ejection requires, as suggested by many authors [e.g., Hundhausen, 1988; Low, 2001].

We did not investigate the effect of anchoring the ends of flux rope in the photosphere due to our 2.5-D approximation. If this effect is taken into account, say, by a 3-D simulation, the major part of the stressed flux rope might expand outward and leave the solar surface even if the magnetic energy is less than the threshold. In this situation, the closed arcade right above the flux rope will be pushed up so as to form a current sheet below.

Another problem of our study is related to the plasma pressure. We did not look into this issue in this study, but such an effect does change the magnetic configuration from a force-free to a non-force-free regime and thus affects the equilibrium property and catastrophic behavior of the system, as addressed by Low and Smith [1993] and Low [2001]. Furthermore, the construction of different magnetic configurations is implemented through a time-dependent process that is far from quasi-static. As a result, we failed to determine a definite threshold for each case. Finally, the solar wind fully develops outside several solar radii; so its effect should be incorporated for any reasonable coronal flux rope models. Further simulations are necessary to solve these problems, and we relegate this task to future efforts.

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References