Improved predictive control approach to networked control systems

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Abstract: A predictive control-based approach is proposed to networked control systems. In this approach, an improved predictive controller is designed using delayed sensing data and a compensation scheme is proposed to overcome the negative effects of the network-induced delays and data packet dropouts in both the forward and backward channels. The proposed approach is easy to be implemented in practice compared with previous results in that only delayed data of the control inputs are used to derive the forward control predictions. The stability of the closed-loop system is obtained by modelling the system as a time delay system with structural uncertainties. Simulations show that the proposed approach is superior to the previous results in the situation where only delayed data are used.

1 Introduction

Networked control systems (NCSs) are control systems where the links from sensor to controller ('backward channel') and from controller to actuator ('forward channel') are not connected directly but closed through some form of network. In NCSs, the network introduces communication constraints to the control system such as the network-induced delay, data packet dropout, data packet disorder, data rate constraint and so on, which requires the designer to develop novel approaches to overcome these negative effects to retain stability and meet other performance requirements [1–8].

One technique recently proposed for NCSs is the predictive control approach, as, for example, in [9–11]. It is noticed that most of the existing results using predictive control consider it as merely a control approach, whereas in [11], the dynamics of the plant is explicitly used to derive a sequence of forward control predictions, which are sent to the actuator simultaneously and the actuator chooses the appropriate one to compensate for the delays. In this way, the delays in both channels can be theoretically exactly compensated for and, thus, better performance can be expected. In [11], the design of the predictive controller is based on the previous control inputs up to the last step. However, this information is not easy to obtain for certain conditions because of the delays in both channels, which restricts the application of the approach proposed in practice.

In this paper, an improved predictive control-based approach is proposed to NCSs. In this approach, the deriving predictive controller is only based on delayed sensing data which is always available to the controller, thus enabling the approach to be feasible in practice. The compensation scheme proposed in [11] is redesigned to handle delays (data packet dropout as well) in both channels at the same time, which cannot be obtained using the previous approach if only the delayed data are used. The corresponding closed-loop system is modelled as a time delay system with structural uncertainties, for which fruitful results of delay-dependent stability analysis can be applied to analyse the stability [12, 13]. This can be compared with the previous results in [11], where the stability of the derived closed-loop system is obtained using switched system theory and the number of the LMIs required to guarantee stability is proportional to the size of the delays.

The remainder of the paper is organised as follows. Section 2 presents the design details of the proposed approach; Section 3 analyses the stability of the corresponding closed-loop system; Section 4 gives examples
to illustrate the effectiveness of the proposed approach and Section 5 concludes the paper.

## 2 Design of the proposed approach

The following multi-input multi-output (MIMO) linear time-invariant system $S$ is considered in this paper

$$ S: \begin{cases} x(k + 1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} $$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^r$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{r \times n}$.

For the system considered above, the design details of the improved predictive controller are presented first in this section, following which the compensation scheme in [11] is redesigned to deal with communication constraints in both channels. The proposed approach relaxes the requirements compared with previous approaches and thus is easier to be implemented in practice.

### 2.1 Design of the predictive controller

In a typical model predictive control implementation, the predictive controller determines a sequence of forward control signals at each control interval that optimise future open-loop plant behaviour and only the first control input is actually applied to the plant. The entire optimisation is repeated at every subsequent control interval, which enables the controller to deal with uncertainties. In such a typical implementation, the forward predictive outputs are based on previous outputs up to the last step. However, in NCSs, the previous outputs to a certain time are unavailable to the controller because of the network-induced delay and data packet dropout in the backward channel. The objective function for open-loop optimisation in this paper is therefore defined as follows

$$ J_{k,\tau_{ud}} = \tilde{Y}^T(k-k_{\tau_{ud}})Q\tilde{Y}(k-k_{\tau_{ud}}) + \Delta U^T(k \in k_{\tau_{ud}}) \times R\Delta U(k \in k_{\tau_{ud}}) $$

where $J_{k,\tau_{ud}}$ is the objective function at time $k$, $k_{\tau_{ud}}$ the corresponding delay in the backward channel, $\Delta U^T(k \in k_{\tau_{ud}}) = [\Delta u(k - \tau_{ud})k - \tau_{ud} \ldots \Delta u(k + N_p - 1)k - \tau_{ud}]^T$ the forward control increment sequence, $\tilde{Y}(k \in k_{\tau_{ud}}) = [\tilde{Y}(k + 1)k - \tau_{ud} \ldots \tilde{Y}(k + N_p k - \tau_{ud})]^T$ the backward output trajectory which will be shown to be based only on the state at time $k - \tau_{ud}$ and $\Delta U^T(k \in k_{\tau_{ud}}); Q$ and $R$ are constant weighting matrices and $N_p$ and $N_u$ are the prediction and control horizons respectively. The so-called 'forward' predictive control increment sequence here actually includes as part of it the 'backward' control increment signals from time $k - \tau_{ud}$ to $k - 1$. The objective function is chosen in this way because of the fact that those 'backward' control increment signals are unavailable for the controller at time $k$ under the proposed predictive-based approach in this paper and in [11], even though they have already been applied to the plant. See Remark 2 and Section 2.2 for more details.

The objective here is to minimise the objective function (2) to derive the optimal control predictions. For this purpose, we rewrite system $S$ as $S'$ by letting $\tilde{x}(k) = [x(k) u(k - 1)]^T$,

$$ S': \begin{cases} \tilde{x}(k + 1) = \tilde{A}\tilde{x}(k) + \tilde{B}\Delta u(k) \\ y(k) = \tilde{C}\tilde{x}(k) \end{cases} $$

where $\tilde{A} = (A \ B \ 0 \ I)$, $\tilde{B} = (B \ I)$, $\tilde{C} = (C \ 0)$ and $\Delta u(k) = u(k) - u(k - 1)$.

The predictive outputs at time $k$ based on the state at time $k - \tau_{ud}$ and the control increment sequences from $k - \tau_{ud}$ can then be obtained by iteration

$$ \tilde{y}(k + j\mid k - \tau_{ud}) = \tilde{C}\tilde{A}^{j-1} \Delta u(k + j\mid k - \tau_{ud}), \quad j = 1, 2, \ldots, N_p $$

Thus

$$ \tilde{Y}(k \mid k - \tau_{ud}) = E_{\tau_{ud}} \tilde{x}(k - \tau_{ud}) + F_{\tau_{ud}} \Delta U(k \mid k - \tau_{ud}) (4) $$

where $F_{\tau_{ud}}$ is a block lower triangular matrix with its non-null elements defined by $F_{i,j} = \tilde{C}\tilde{A}^{i-j} \Delta \tilde{B}^j$, $i - j \leq \tau_{ud}$ and $E_{\tau_{ud}} = [(\tilde{C}\tilde{A}^{\tau_{ud}+i-1})^T \ldots (\tilde{C}\tilde{A}^{\tau_{ud}+N_p})^T]^T$.

Let $\Delta U^*(k \mid k - \tau_{ud}) = [\Delta u^*(k \mid k - \tau_{ud}) \ldots \Delta u^*(k + N_u - 1)k - \tau_{ud})]^T$ denote the optimal control increment sequence from $k$ to $k + N_u - 1$. It can be calculated by substituting (4)–(2) and optimising $\tilde{J}_{k,\tau_{ud}}$ which turns out to be state feedback control

$$ \Delta U^*(k \mid k - \tau_{ud}) = -K_{\tau_{ud}} \tilde{x}(k - \tau_{ud}) $$

where $K_{\tau_{ud}} = M_{\tau_{ud}}(F_{\tau_{ud}}^T Q F_{\tau_{ud}} + R)^{-1} F_{\tau_{ud}}^T Q E_{\tau_{ud}}$ and $M_{\tau_{ud}} = (0_{mN_u \times mN_u} I_{mN_u \times mN_u})$.

**Remark 1 (state observer):** If the state vector $x$ is not available, an observer must be included

$$ \tilde{x}(k + 1) = \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k) + L(C\tilde{x}(k) - C\tilde{z}(k)) (6) $$

where $\tilde{z}(k)$ is the observed state at time $k$.

**Remark 2:** In [11], state feedback $u = K\tilde{x}_{\bar{u}_k\tau_{ud}}$ is also used, where $K$ is artificially chosen without consideration of the communication constraints and $\tilde{x}_{\bar{u}_k\tau_{ud}}$ depends on the state estimation $\tilde{x}_{\bar{u}_k\tau_{ud}}(k-\tau_{ud})$, the past control input up to $u_{k-1}$ and the past output up to $y_{k-1}$ of the system'. However, as we will specify later in the following
subsection, under the compensation scheme in the forward channel in [11], the whole sequence of the optimal forward control increments \( \Delta U^*(k|k - \tau_{\text{nc}}) \) is sent to the actuator and only one of them is chosen to be applied to the plant. Thus, unless information from the actuator is received, we have no idea which control prediction was really used if the data packets in the forward channel were randomly delayed. Hence, the use of the previous control inputs implies an additional communication channel which can send the applied control inputs to the controller efficiently. Without such a channel, the approaches proposed in these publications are only applicable to such a situation where there is no delay or data packet dropout in the forward channel. To relax this requirement, we redesigned the predictive controller in this paper by optimising an objective function which includes as part of it the previous control increment sequence from \( k - \tau_{\text{nc}} \) to \( k - 1 \) [i.e. \( \Delta U^*(k|k - \tau_{\text{nc}}) \)]. As a result, the forward control predictions at time \( k \) are only based on data up to time \( k - \tau_{\text{nc}} \) [see (5)], which is always feasible in practice. In the next section, it is shown that such an improved predictive controller with the modified compensation scheme gives state feedback control where the feedback gain varies with the delays in both channels. This varying feedback gain scheme is shown to be superior to the fixed feedback gain approach in [11] in such a situation where only the delayed data are used.

2.2 Design of the compensation scheme for communication constraints

To take advantage of the characteristics of the packet-based transmission in a networked environment and the proposed predictive controller to compensate for the network constraints considered in this paper, that is, the network-induced delay and data packet dropout, we make the following two assumptions similar to those in [11],

(A1) All the components in the system including the controller, the actuator and the sensor are time-synchronised and each data packet transmitted through the network uses a time stamp to notify the time when it was sent;

(A2) Each optimal control increment sequence \( \Delta U^*(k|k - \tau_{\text{nc}}) \) is packed into one data packet to be sent to the actuator;

and a further assumption that is not explicitly included in [11], and yet very important to implement the proposed approach, with which the compensation for the data packet dropout can also be realised.

(A3) The sum of the maximum network-induced delays in the forward channel (backward channel) and the maximum number of continuous data packet dropouts is upper bounded by \( \bar{\tau}_{\text{nc}} \) (\( \bar{\tau}_{\text{nc}} \) accordingly) and

\[
\bar{\tau}_{\text{nc}} \leq N_{\text{c}} - 1
\]  

Remark 3: The network-induced delay in the backward channel for each data packet is known to the controller under assumption (A1) and the network-induced delays in both channels for each control predictive sequence are known to the actuator under assumptions (A1) and (A2).

Remark 4: In this paper, the dropped data packets are simply ignored which is common in a real-time application but not regarded as an infinite delay. The so-called 'maximum network-induced delay' in assumption (A3) is only measured over those data packets that are received successfully. Hereafter, the term 'network-induced delay' instead of 'network-induced delay and data packet dropout' is used for simplicity since the effect of the data packet dropout does not need any special treatment using the compensation scheme described below under assumption (A3).

With the above assumptions, we propose a new compensation scheme for the network-induced delay in the backward channel, and redesign the compensation scheme at the actuator side compared with [11] as follows.

1. Compensation for the communication constraints in the backward channel: The network-induced delay in the backward channel is known to the controller (Remark 3), which enables the predictive control sequence to be calculated by using (5). However, as the matrices \( E_{\tau_{\text{c}}} \), \( F_{\tau_{\text{c}}} \) and \( M_{\tau_{\text{c}}} \) in (5) vary with \( \tau_{\text{nc}} \), it would be a great computation burden for the predictive controller if these matrices are calculated online. Fortunately, these matrices, actually, can be calculated offline since all the matrices are fixed for a given delay. This advantage enables us to calculate offline all the matrices with respect to the specific \( \tau_{\text{nc}} \), store them in the controller and just choose the appropriate ones when calculating online the predictive control increments, according to the current value of the delay \( \tau_{\text{nc}} \).

Let \( \mathcal{E}_c = \{ E_{c_0}, E_{c_1}, \ldots, E_{c_s} \} \), \( \mathcal{F}_c = \{ F_{c_0}, F_{c_1}, \ldots, F_{c_s} \} \) and \( \mathcal{M}_c = \{ M_{c_0}, M_{c_1}, \ldots, M_{c_s} \} \), then we have for any \( k \) (or \( \tau_{\text{nc}} \)), \( E_{\tau_{\text{nc}} k} \in \mathcal{E}_c \), \( F_{\tau_{\text{nc}} k} \in \mathcal{F}_c \) and \( M_{\tau_{\text{nc}} k} \in \mathcal{M}_c \), respectively. For a practical implementation, these matrices are calculated offline and stored in the matrix selector for online use.

2. Compensation for the communication constraints in the forward channel: As in [11], a cache is used at the actuator side which can only store one predictive control increment sequence (or one data packet, see assumption (A2)) at any one time. When a new sequence arrives at the actuator side, it is compared with the one already in the cache according to the time stamps and only the latest one sent from the controller is stored. This comparison process is introduced since different data packets may experience different delays in the forward channel, thereby producing a situation where, for example, a data packet sent earlier from
the controller may arrive at the actuator later or may never
arrive in the case of a data packet dropout. As a result of
the comparison process, the predictive control sequence
stored in the cache of the actuator is always the latest one
available at any specific time.

At every execution time instant, the actuator selects the
appropriate control prediction which can compensate for
the current network-induced delay in the forward channel
from the predictive control increment sequence in the cache
and applies it to the plant. It is necessary to point out that
the appropriate control increment is always available
provided assumption (A3) holds.

The algorithm of the predictive control-based approach
can now be summarized as follows.

(S1) At time \( k \), if the predictive controller receives the delayed
data of state \( x(k - \tau_{u,k}) \) or \( y(k - \tau_{u,k}) \) if the state is
unavailable] and the control input \( \Delta u(k - \tau_{u,k}) \), then the
following are done.

(S1a) The current network-induced delay in the backward
channel \( \tau_{u,k} \) is read

(S1b) The predictive control increment sequence \( \Delta U^*(k | k - \tau_{u,k}) \)
using (5) is calculated.

(S1c) \( \Delta U^*(k | k - \tau_{u,k}) \) is packed and sent to the actuator
simultaneously with the time stamps \( k \) and \( \tau_{u,k} \).

If no data packet is received at time \( k \), then let \( k = k + 1 \), and
wait for the next time instant.

(S2) The cache of the actuator updates its predictive control
increment sequence according to the time stamps once a data
packet arrives.

(S3) An appropriate control increment prediction is selected
from the predictive control increment sequence and applied
to the plant.

(S4) The current sensing data with the control input are sent
to the controller.

The structure of the proposed approach is illustrated in
Fig. 1.

3 Stability of the proposed
approach
In this section, we first give the explicit expression of the
closed-loop system under the proposed predictive control-
based approach in this paper, and then analyse the stability
of the closed-loop system using the results of the delay-
dependent stability analysis [12].

3.1 Closed-loop system
It has already been stated that there may be more than one
predictive control increment sequence available for the
actuator at time \( k \) because of different delays those
sequences experienced, but only the latest one is stored in
the cache after the comparison process. If we denote the
delay in the forward channel of this predictive control
increment sequence by \( \tau_{u,k} \) and the corresponding delay in
the backward channel by \( \tau_{u,k}^* \), then the predictive control
increment sequence applied at time \( k \) is calculated at time
\( k - \tau_{u,k} \) based on data up to time \( k - \tau_{u,k}^* \), that is,
\( \Delta U^*(k - \tau_{u,k} | k - \tau_{u,k}^*) \), where \( \tau_{u,k} = \tau_{u,k}^* + \tau_{u,k}^* \). Thus, the
predictive control increment really applied at time \( k \) is

\[
\Delta u(k) = \Delta U^*(k | k - \tau_{u,k}^*)
\]

\[
= d_{ca,k}^T \Delta U^*(k - \tau_{u,k}^* | k - \tau_{u,k}^*)
\]

\[
= -d_{ca,k}^T K u_{ca} \tilde{x}(k - \tau_{u,k}^*)
\]

where \( d_{ca,k}^T \) is a \( 1 \times N_u \) block matrix with all entries 0
except the \( u_{ca} \) block, the identity matrix with rank \( m \).

Notice that in practice there is at least a one-step delay in
both the forward and backward channels, and the possible
control increment input at time \( k \) should be one entry out

![Figure 1 Structure of the improved predictive control-based approach](image-url)
of the following matrix

\[
U_k = -\begin{pmatrix}
  k_{11} \bar{z}(k - 2) & k_{12} \bar{z}(k - 3) & \cdots & k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) \\
k_{12} \bar{z}(k - 3) & k_{22} \bar{z}(k - 4) & \cdots & k_{2 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) & \cdots & k_{\tau_u \tau_u} \bar{z}(k - \tau_u - \tau_u) & 0 \\
k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) & \cdots & k_{\tau_u \tau_u} \bar{z}(k - \tau_u - \tau_u) & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) & \cdots & k_{\tau_u \tau_u} \bar{z}(k - \tau_u - \tau_u) & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) & \cdots & k_{\tau_u \tau_u} \bar{z}(k - \tau_u - \tau_u) & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) & \cdots & k_{\tau_u \tau_u} \bar{z}(k - \tau_u - \tau_u) & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) & \cdots & k_{\tau_u \tau_u} \bar{z}(k - \tau_u - \tau_u) & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
k_{1 \tau_u} \bar{z}(k - \tau_u - 1) & k_{2 \tau_u} \bar{z}(k - \tau_u - 2) & \cdots & k_{\tau_u \tau_u} \bar{z}(k - \tau_u - \tau_u) & 0 \\
\vspace{0.5cm}
\end{pmatrix}(K_1 \cdots K_{\tau_u})
\]

where the gain matrix of \(U_k\) is

\[
\begin{pmatrix}
k_{11} & k_{12} & \cdots & k_{1 \tau_u} \\
k_{12} & k_{22} & \cdots & k_{2 \tau_u} \\
\vdots & \vdots & \ddots & \vdots \\
k_{1 \tau_u} & k_{2 \tau_u} & \cdots & k_{\tau_u \tau_u}
\end{pmatrix}
\]

where \(K_i, 1 \leq i \leq \tau_u\) are obtained using (5).

The control increment applied at time \(k\) can then be represented by

\[
\Delta u(k) = -k_{\tau_u \tau_u} \bar{z}(k - \tau_u), \quad 1 \leq \tau_u \leq \tau_u, \quad 1 \leq \tau_u \leq \tau_u
\]

(9)

Let \(\Gamma_{\tau_u} = \{K_{\tau_u} \in \mathbb{R}^{(N - N)}: 2 \leq \tau_u \leq \tau_u, \quad \tau_u \leq \tau_u, \quad \tau_u \leq \tau_u\};\) then the closed-loop system under the proposed approach can now be written as

\[
\bar{z}(k + 1) = \tilde{A}\bar{z}(k - \tau_u) + \tilde{B}k_{\tau_u \tau_u} \bar{z}(k - \tau_u), \quad k_{\tau_u \tau_u} \in \Gamma_{\tau_u}
\]

(10)

Remark 5: In [11], the feedback gain is designed without consideration of the network-induced delays and, thus, fixed for different delays. This can be compared with the approach in this paper where for different delays, different feedback gains apply (9). Simulations illustrate the superiority of this varying gain scheme compared with the fixed gain scheme in [11].

3.2 Stability analysis

Let \(\bar{k} = \max_{2 \leq \tau_u \leq \tau_u} k_{\tau_u \tau_u} \bar{z}(k - \tau_u)\) where \(|\cdot|\) denotes the Euclidean norm. Then \(k_{\tau_u \tau_u} \bar{z}(k - \tau_u)\) can be represented by

\[
\tilde{B}k_{\tau_u \tau_u} \bar{z}(k - \tau_u) = \tilde{B}_m \cdot D_{\tau_u \tau_u} \bar{z}(k - \tau_u)
\]

(11)

where \(\tilde{B}_m = \tilde{k}\tilde{B}\) is a constant matrix and \(D_{\tau_u \tau_u} \bar{z}(k - \tau_u) = \tilde{k}\tilde{B}\). It is easy to conclude that \(D_{\tau_u \tau_u} \bar{z}(k - \tau_u) \leq 1, \forall 1 \leq \tau_u \leq \tau_u, \quad 1 \leq \tau_u \leq \tau_u\).

With the modelling of the closed-loop system in (10) and (11), the delay-dependent analysis which has been explored a lot recently (see, e.g., in [14–16]) can now be applied to derive a stability criterion.

**Theorem 1:** If there exists \(P_i = P_i^T > 0, i = 1, 2, 3, \quad \lambda > 0\) with appropriate dimensions and \(\lambda > 0\) satisfying the following two LMI

\[
\begin{pmatrix}
X_{11} & X_{12} & N_1 \\
X_{12}^T & X_{22} & N_2 \\
N_1^T & N_2^T & P_3
\end{pmatrix} > 0
\]

(12)

\[
\begin{pmatrix}
\Phi_{11} & \Phi_{12} & (\tilde{A} - I)^T \tilde{H} & P_1 \tilde{B}_m \\
\Phi_{12}^T & \Phi_{22} + \lambda I & 0 & 0 \\
H^T (\tilde{A} - I) & 0 & -H & H \tilde{B}_m \\
\tilde{B}_m^T P_1 & 0 & \tilde{B}_m^T H & -\lambda I
\end{pmatrix} > 0
\]

(13)

then the closed-loop system (10) is stable. Here

\[
\Phi_{11} = (\tilde{A} - I)P_1 + P_1 (\tilde{A} - I) \\
\Phi_{12} = -P_1 + \tilde{A}X_{12} + \tilde{X}_{12} \\
\Phi_{22} = -P_2 - N_2 - N_2^T + \tilde{X}_{22} \\
H = P_1 + \tilde{X}_{12}
\]

**Proof:** Let \(d_1 = 2, d_2 = \tau, A = \tilde{A}\), and \(\Delta A(t) = -\tilde{k}\tilde{B}\) in Theorem 7 in [12], then the above theorem can be obtained using the same techniques as in [12].

Remark 6: The key idea to find the stability criteria in [11] is to model the closed-loop system as an augmented switched system \(X(k + 1) = \Lambda(\tau)X(k)\) where the system matrix \(\Lambda(\tau)\) varies with different delays. As a result, the number of the LMI’s that guarantee the stability of the closed-loop system is proportional to the size of the delays (Theorem 2 in [11]). In this paper, by designing a different predictive controller (5), the closed-loop system can then be modelled as a time delay system with structural uncertainties and the LMI’s required to ensure the stability are reduced to two.

Remark 7: If a state observer as in Remark 1 is also involved, let \(z(k) = [\tilde{x}^T(k) \quad \tilde{x}^T(k)]^T = [\tilde{x}^T(k) \tilde{u}^T(k - 1) \tilde{x}^T(k)]^T\), then
the closed-loop system can be written as
\[ z(k + 1) = \hat{A}z(k) - \hat{B}_{r_{sc},r_{ca}}z(k - \tau_s) \]  
(14)

where
\[
\hat{A} = \begin{pmatrix}
A & B & 0 \\
0 & I & 0 \\
LC & R & A- LC
\end{pmatrix},
\]
\[
\hat{B}_{r_{sc},r_{ca}} = \begin{pmatrix}
Bk^{\infty}_{r_{sc},r_{ca}} & Bk^{1}_{r_{sc},r_{ca}} & 0 \\
k^{\infty}_{r_{sc},r_{ca}} & k^{1}_{r_{sc},r_{ca}} & 0 \\
-Bk^{\infty}_{r_{sc},r_{ca}} & -Bk^{1}_{r_{sc},r_{ca}} & 0
\end{pmatrix},
\]
and \( k_{r_{sc},r_{ca}} = [k^{\infty}_{r_{sc},r_{ca}} k^{1}_{r_{sc},r_{ca}}] \). Thus, a similar stability criterion to Theorem 1 can be obtained analogously.

### 4 Simulation

Two examples are presented in this section to illustrate the effectiveness of the proposed approach in this paper.

**Example 1 (Example 1 in [11]):** The system matrices adopted are as follows
\[
A = \begin{pmatrix}
1.0100 & 0.2710 & -0.4880 \\
0.4820 & 0.1000 & 0.2400 \\
0.0020 & 0.3681 & 0.7070
\end{pmatrix},
B = \begin{pmatrix}
5 \\
3 \\
5
\end{pmatrix},
C = \begin{pmatrix}
1 & 2 & 3 \\
4 & 3 & 1
\end{pmatrix}
\]

In [11], the above system is illustrated to be stable with \( \bar{\tau}_s = 2, \bar{\tau}_d = 1 \)
\[
L = \begin{pmatrix}
-0.3614 & 0.3326 \\
0.0332 & 0.0576 \\
0.2481 & -0.0750
\end{pmatrix}
\]

and a fixed feedback gain
\[
K = \begin{pmatrix}
0.5858 & -0.1347 & -0.4543 \\
-0.5550 & 0.0461 & 0.4721
\end{pmatrix}
\]

However, using the approach proposed in this paper, this system is unstable with the same \( \bar{\tau}_s, \bar{\tau}_d \) and \( L \). (see Fig. 2. Other parameters: \( N_s = 8 \) and \( N_d = 10 \)) This fact seems to mean the approach in [11] is better than the approach in this paper, but we need to remember that the approach in [11] takes advantage of more information to design the predictive controller and some of the information used is not easy to obtain in practice (Remark 2). On the other hand, the simulation results do illustrate that the varying feedback gain scheme in this paper is superior to the previous fixed feedback gain scheme (Remark 5), where the same system is stable using the approach in this paper.

![Figure 2 Example 1: The system is unstable using the approach in this paper when \( \bar{\tau}_s = 2 \) and \( \bar{\tau}_d = 1 \) ](image)

![Figure 3 Example 1: The system is stable using the varying feedback gain scheme when \( \bar{\tau}_s = 1 \) and \( \bar{\tau}_d = 1 \) ](image)

![Figure 4 Example 1: The system is unstable using the fixed feedback gain scheme when \( \bar{\tau}_s = 1 \) and \( \bar{\tau}_d = 1 \) ](image)
when $\bar{\tau}_c = \bar{\tau}_a = 1$ (Fig. 3) and yet is unstable using the same state feedback (10) with the fixed $K$ above (Fig. 4).

**Example 2**: The system matrices are set as

$$A = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.5 \end{pmatrix}, \quad B = \begin{pmatrix} 0.05 & 0.2 \end{pmatrix}, \quad C = (1 \ 0)$$

This system can be shown using Theorem 1 to be stable under $\bar{\tau}_c = 3$, $\bar{\tau}_a = 2$, $N_p = 8$, and $N_p = 10$. The simulation result is illustrated in Fig. 5.

5 Conclusion

A predictive control-based approach to NCSs was recently reported in [11], the implementation of which needs an additional communication channel and, thus, is not easy to be applied in practice. To deal with this problem, an improved predictive controller is designed with a modified compensation scheme, which is feasible in practice and can handle communication constraints in both the backward and forward channels. The stability criteria of the corresponding closed-loop system consists of only two LMIs which is easy to check compared with the previous results where the number of the LMIs required was proportional to the size of the delays. Simulations show that the proposed approach is superior to the previous results in the situation where only the delayed data are used.

6 References


