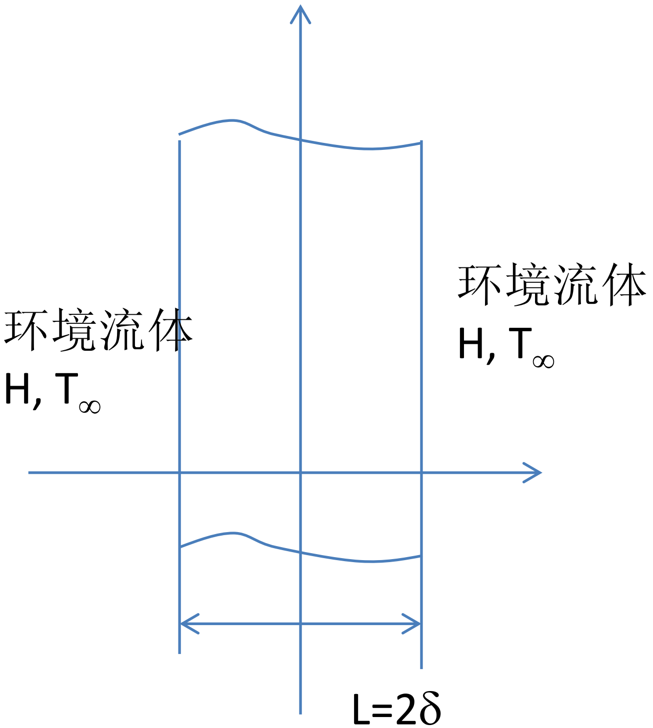
2014/3/14

内容提要

- 非稳态导热概要
- 非稳态导热的集中参数法
- 一维平板非稳态导热的分析解

3.3 一维平板非稳态导热的分析解



$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \qquad (0 < x < \delta, t > 0)$$

初始和边界条件

$$T(x,0) = T_0 \qquad (0 \le x \le \delta)$$

$$\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = 0 \qquad \textit{symmetric condition}$$

$$-\lambda \frac{\partial T(x,t)}{\partial x} = h[T(\delta,t) - T_\infty]$$

引入过余温度

$$\theta = T(x,t) - T\infty$$

一维平板非稳态导热的控制方程

$$\alpha \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial \theta}{\partial t} \qquad (0 < x < \delta, t > 0)$$

初始和边界条件

$$\theta(x,0) = \theta_0 \qquad (0 \le x \le \delta)$$

$$\left. \frac{\partial \theta(x,t)}{\partial x} \right|_{x=0} = 0 \qquad symmetric\ condition$$

$$-\lambda \frac{\partial \theta(x,t)}{\partial x} = h\theta(\delta, t) \qquad x = \delta$$

运用分离变量，令

$$\theta = \theta_1(t)\theta_2(x)$$

只是时间的函数，只是空间的函数

代入微分方程

$$\theta_2 \frac{\partial \theta_1}{\partial t} = \theta_1 \alpha \frac{\partial^2 \theta_2}{\partial x^2}$$

两边除以 $\alpha\theta_1\theta_2$

$$\frac{1}{\alpha\theta_1}\frac{\partial \theta_1}{\partial t} = \frac{1}{\theta_2}\frac{\partial^2 \theta_2}{\partial x^2}$$

引入分离变量 $\dfrac{P}{\sqrt{\alpha}}$

求解常微分方程

$$\frac{1}{\alpha\theta_1}\frac{\partial\theta_1}{\partial t}=-\frac{P^2}{\alpha}$$

$$\theta_1=e^{-P^2 t}$$

$$\frac{1}{\theta_2}\frac{\partial^2\theta_2}{\partial x^2}=-\frac{P^2}{\alpha}$$

$$\theta_2=C_1(\cos\frac{P}{\sqrt{\alpha}}x+C_2\sin\frac{P}{\sqrt{\alpha}}x)$$

通解

$$\theta=e^{-P^2 t}[C_1(\cos\frac{P}{\sqrt{\alpha}}x+C_2\sin\frac{P}{\sqrt{\alpha}}x)]$$

微分（边界条件中有这一项）

$$\frac{\partial\theta}{\partial x}=\frac{P}{\sqrt{\alpha}}e^{-P^2 t}[C_2\cos\frac{P}{\sqrt{\alpha}}x-C_1\sin\frac{P}{\sqrt{\alpha}}x)]$$

代入对称边界条件

$$\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0}=0 \qquad \textit{symmetric condition}$$

$$C_2=0$$

代入对流换热边界条件 $$-\lambda\frac{\partial\theta(x,t)}{\partial x}=h\theta(\delta,t) \qquad x=\delta$$

$$\frac{P}{\sqrt{\alpha}}\cancel{e^{-P^2t}}(-\cancel{C_1}\sin\frac{P}{\sqrt{\alpha}}\delta)=-\frac{h}{\lambda}\cancel{e^{-P^2t}}(\cancel{C_1}\cos\frac{P}{\sqrt{\alpha}}\delta)$$

令 $$\mu=\frac{P\delta}{\sqrt{\alpha}}$$

$$\tan\mu = \frac{h/\lambda}{P/\sqrt{\alpha}} = \frac{B_i}{\mu}$$

$$c\tan\mu = \frac{\mu}{B_i}$$

超越方程

通解的形式

$$\theta = e^{-P^2 t}\cos\frac{P}{\sqrt{\alpha}}x$$

$$= \sum_{n=1}^{\infty} C_n e^{-P^2 t}\cos\frac{P}{\sqrt{\alpha}}x$$

$$= \sum_{n=1}^{\infty} C_n e^{-\frac{P^2\delta^2}{\alpha}\frac{\alpha t}{\delta^2}}\cos\frac{P\delta}{\sqrt{\alpha}}\frac{x}{\delta}$$

$$= \sum_{n=1}^{\infty} C_n e^{-\mu^2 F_o}\cos(\mu\eta)$$

根据正交函数性质及傅里叶级数展开

$$C_n = \frac{2\sin\mu_n}{\mu_n + \cos\mu_n \sin\mu_n}$$

温度与无量纲参数的关系

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty} = func(Fo(t), \frac{x}{\delta}, Bi)$$

2014/03/18

内容提要

◆半无限平板的非稳态导热
◆对流传热理论基础
◆教科书5.1
1. 影响对流传热的因素
2. 表面传热系数的确定
◆ 教科书5.3 **对流边界层**

1. 速度边界层
2. 热边界层
3. 层流和湍流
4. 雷诺数
5. 努赛尔数

半无限平板的温度变化

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

$$\eta \equiv \frac{x}{\sqrt{4\alpha t}}$$

$$\frac{d^2 T}{d\eta^2} = -2\eta \frac{dT}{d\eta}$$

$$\frac{T - T_s}{T_0 - T_s} = \frac{2}{\sqrt{\pi}} \int_0^{\eta} \exp(-\eta^2) d\eta \equiv erf(\eta)$$

$$q_x = -\lambda \frac{\partial T}{\partial x} = -\lambda (T_0 - T_s) \frac{\partial erf\,(-\eta^2)}{\partial x}$$

$$= -\lambda (T_0 - T_s) \frac{2}{\sqrt{\pi}} \frac{\partial \int_0^{\eta} \exp(-\eta^2) d\eta}{\partial \eta} \frac{\partial \eta}{\partial x}$$

$$= -\lambda (T_0 - T_s) \frac{2}{\sqrt{\pi}} \exp(-\eta^2) \frac{1}{2\sqrt{\alpha t}}$$

$$= \lambda \frac{T_s - T_0}{\sqrt{\pi \alpha t}} \exp(-\eta^2)$$

$$= \lambda \frac{T_s - T_0}{\sqrt{\pi \alpha t}} \exp\left[-\frac{x^2}{4\alpha t} \right]$$

教科书 P135 Eq 3-42

通过表面的热流密度

$$q_{x=0} = \lambda \frac{T_s - T_0}{\sqrt{\pi a t}}$$

一段时间内表面上的传热量

$$Q = A\int_0^t \lambda \frac{T_s - T_0}{\sqrt{\pi a t}} dt$$

$$= A\frac{\lambda (T_s - T_0)}{\sqrt{\pi a}} 2\sqrt{t} = A\sqrt{\frac{t}{\pi}}\sqrt{\rho c \lambda}(T_s - T_0)$$

表面传热系数

$$h=-\frac{\lambda}{\Delta T_s}\frac{\partial T(x,t)}{\partial x}\bigg|_{x=0}$$

◆表面传热系数，由流体导热系数，贴壁处壁面法线方向上的流体温度变化率，壁面与流体的温度差决定

◆速度边界层的主要表现形式为**表面摩擦**，影响表面摩擦系数

◆热边界层的主要表现形式为**对流换热**，会影响对流换热系数

对流传热在工业中的应用

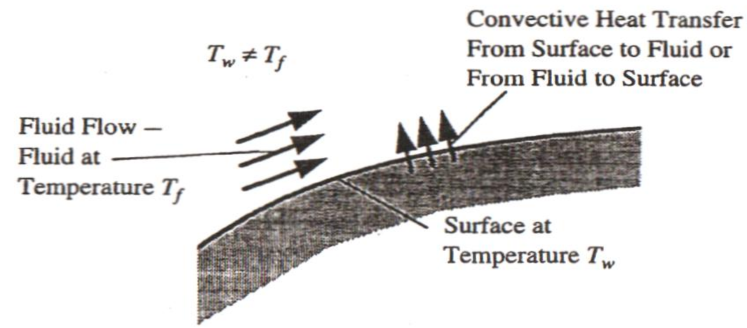


FIGURE 1.1
Convective heat transfer.

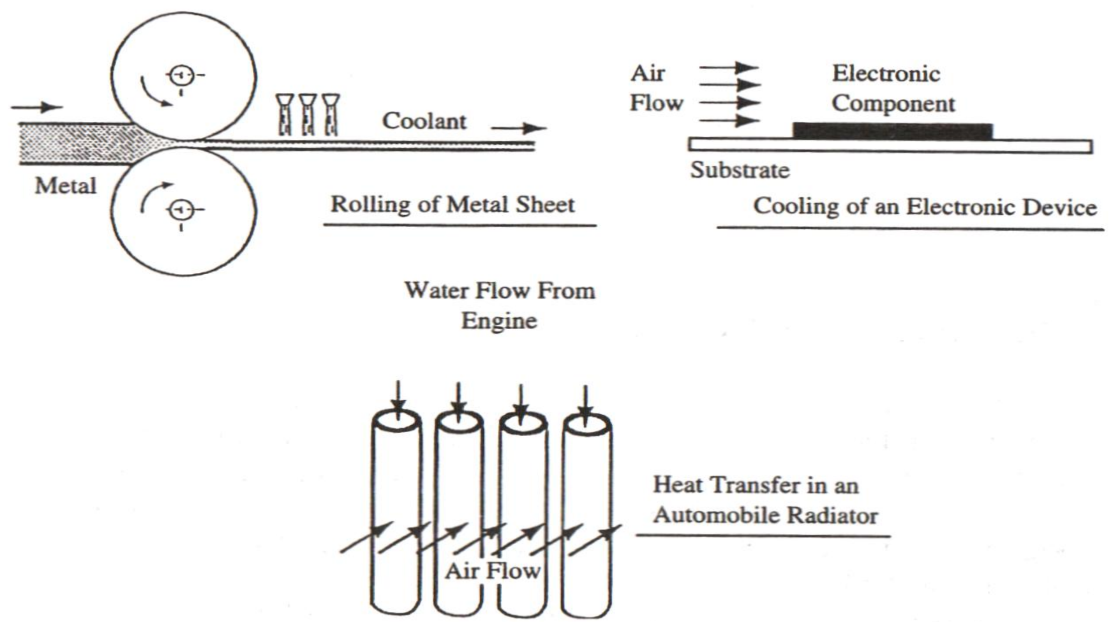
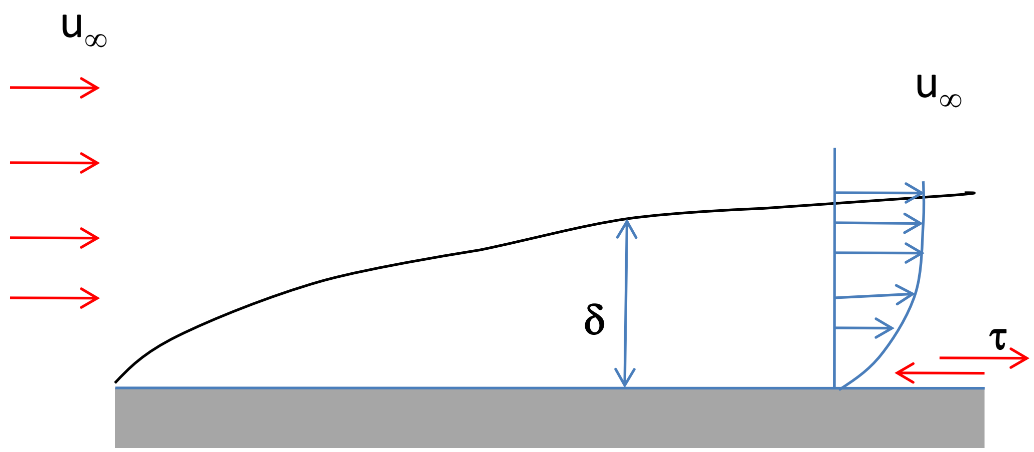
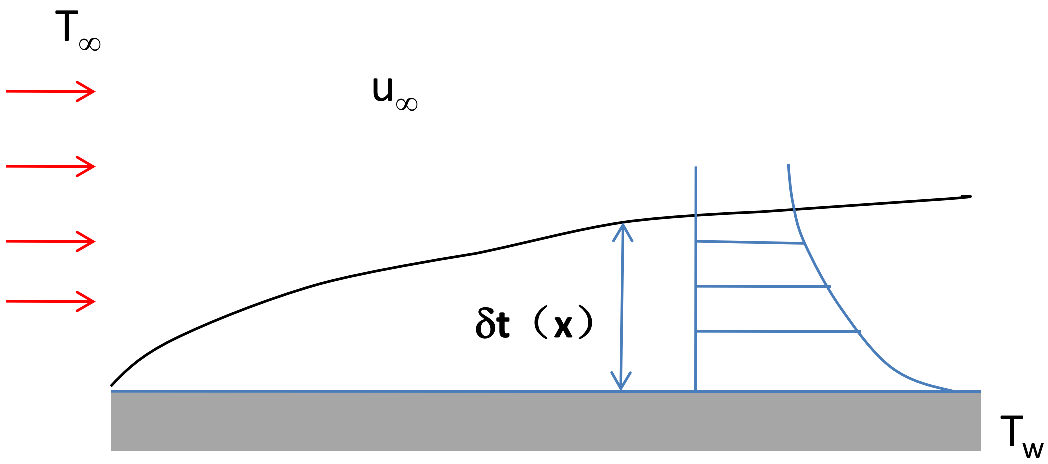


FIGURE 1.2
Some situations that involve convective heat transfer.

速度边界层



热边界层



层流到湍流的过渡

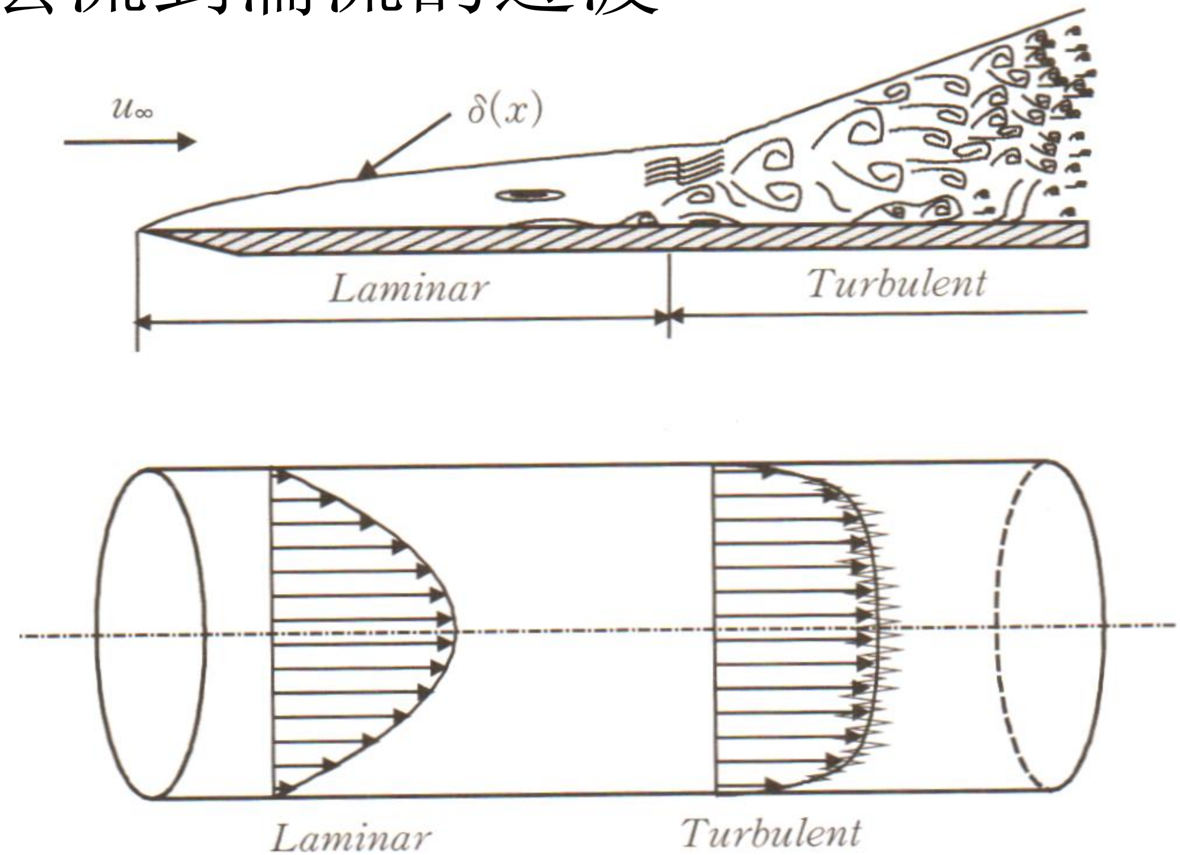


図 4.2　層流から乱流への遷移

努赛尔数Nu的物理意义

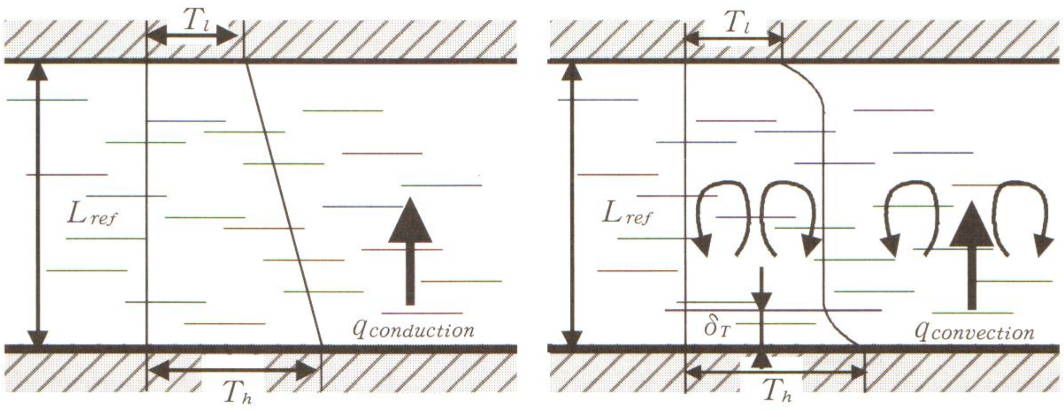


図 4.3　ヌッセルト数の意味

由导热引起的流体热流密度

$$q_{conduction} = k_f \frac{(T_h - T_L)}{L_{ref}}$$

$$\frac{q_{convection}}{q_{conduction}} = \frac{h(T_h - T_l)}{k_f \dfrac{(T_h - T_L)}{L_{ref}}} = \frac{hL_{ref}}{k_f} = Nu_{ref}$$

$$q_{convection} = h(T_h - T_l)$$