The Non-attractor Beginning of Inflation and Features in the Power Spectrum and Non-Gaussianity

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Introduction

- **Non-attractor inflation** model indicates some interesting *features* in power spectrum and local f_{NL}, differing from the standard slow-roll results.
- Observations favor a class of inflation models with **plateau potentials** (R^2 & α-attractor), which is more likely to be in a *non-attractor stage* at the beginning.

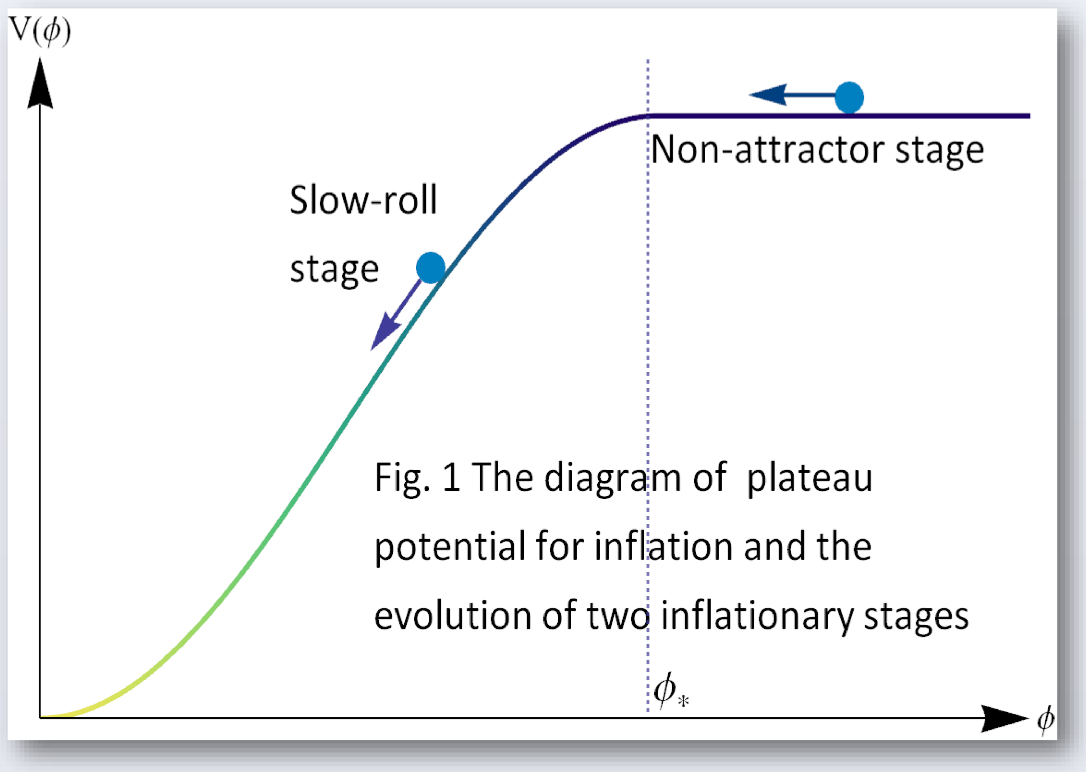

Fig. 1 The diagram of plateau potential for inflation and the evolution of two inflationary stages

Questions we try to answer in this work:

- Q1. Is non-attractor inflation a viable model?
- Q2. If not, how to make it complete?
- Q3. What is the implications of a non-attractor initial condition?

Non-attractor Inflation

- **Background Dynamics:**

The non-attractor inflation is driven by a constant potential, as shown in Fig.1 (for $\phi > \phi_*$). The background equations in this stage are

$$3M_p^2H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \simeq V_0 \qquad \ddot{\phi} + 3H\dot{\phi} = 0$$

Its solution results in the following slow-roll parameters

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \propto a^{-6}$$

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} = -6$$

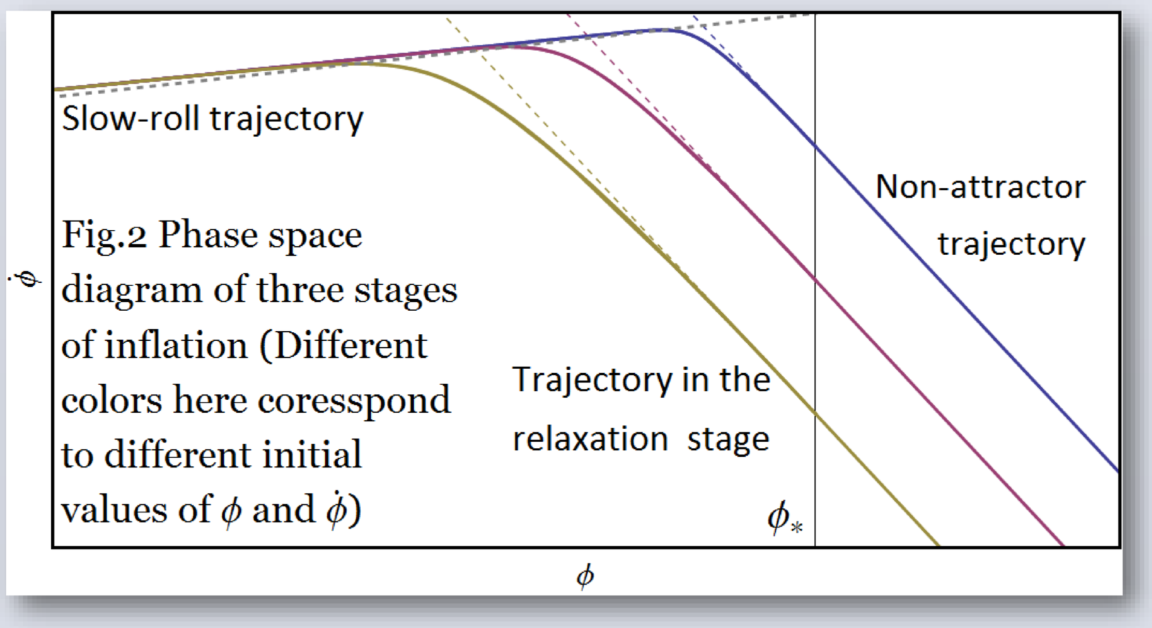

Fig.2 Phase space diagram of three stages of inflation (Different colors here coresspond to different initial values of ϕ and $\dot{\phi}$)

And the trajectory of inflaton in the phase space is given by

$$\dot{\phi} + 3H\phi = C(\dot{\phi}_*, \phi_*)$$

- **Curvature Perturbation ζ:**

Note here $\eta < -3$, thus ζ is growing after horizon exit and the spectrum is defined at the end of this stage:

$$P_\zeta = \frac{H^2}{\dot{\phi}_*}\frac{H^2}{4\pi^2}$$

which is exactly scale invariant.

- **E-folding number problem of non-attractor model**

During the non-attractor stage, the kinetic energy of inflaton drops quickly $\dot{\phi}^2 \propto e^{-6N}$, thus we have

$$\frac{H^2}{\dot{\phi}_*^{\,2}} = \frac{V_0/3M_p^2}{\dot{\phi}_i^2/e^{6N}} = \frac{e^{6N}}{3M_p^2}\frac{V_0}{\dot{\phi}_i^2}$$

Since for an inflationary solution we have $V_0 > \dot{\phi}_i^{\,2}$, thus the e-folding number N must be small, and we get $N{\sim}3$.

Conclusion1: The non-attractor inflation only lasts limited e-folding numbers. A slow-roll stage is needed to complete inflation!

Slow-Roll Stage of Inflation

- **Which slow-roll models can be attached to non-attractor?**

The slow-roll potential is shown as the $\phi < \phi_*$ part in Fig.1. In this stage we have $\epsilon, \eta \ll 1$, and the trajectory in the phase space is

$$\dot{\phi}_a = -\frac{M_p}{\sqrt{3}}\frac{V'}{V^{1/2}}$$

Two possible cases at the transition point and their implications on the large-scale power spectrum:

- $|\dot{\phi}_*| < |\dot{\phi}_a| \xrightarrow{\text{yields}}$ enhancement ✗
- $|\dot{\phi}_*| \geq |\dot{\phi}_a| \xrightarrow{\text{yields}}$ suppressions ✓

Since at the end of the non-attractor stage, $\dot{\phi}_* \to 0$, thus

$\dot{\phi}_a = 0$ at $\phi = \phi_*$ ⟹ $V'(\phi_*) = 0$ ⟹ Hilltop Inflation

Conclusion2: The non-attractor phase should be followed by a stage with Hilltop potentials, e.g. Natural Inflation and small field models.

The slow-roll potential can be expanded around the transition ϕ_*

$$V(\phi) = V_0 - \frac{\alpha}{2}(\phi - \phi_*)^2 + \cdots$$

From the phase space trajectory in Fig.2, we find a **relaxation process** before joining the slow-roll evolution. In summary, we have

Stages	ϵ	η	N	$n_s - 1$	f_{NL}
Non-attractor	$\propto a^{-6}$	-6	~ 3	0	5/2
Relaxation	decay	step	~ 3	step	step feature
Slow-roll	$\sim const$	$\ll 1$	$50 \sim 60$	$-2\epsilon - \eta$	$5(n_s - 1)/12$

Features in the Power Spectrum and f_{NL}

Finally, we use the δN formalism to calculate the power spectrum and bispectrum with a non-attractor initial condition.

The curvature perturbation outside of the horizon can

✓ remain constant ($\eta > -3$), spectrum is defined at horizon-exit

✓ or grow ($\eta < -3$) , spectrum is defined at $\eta = -3$

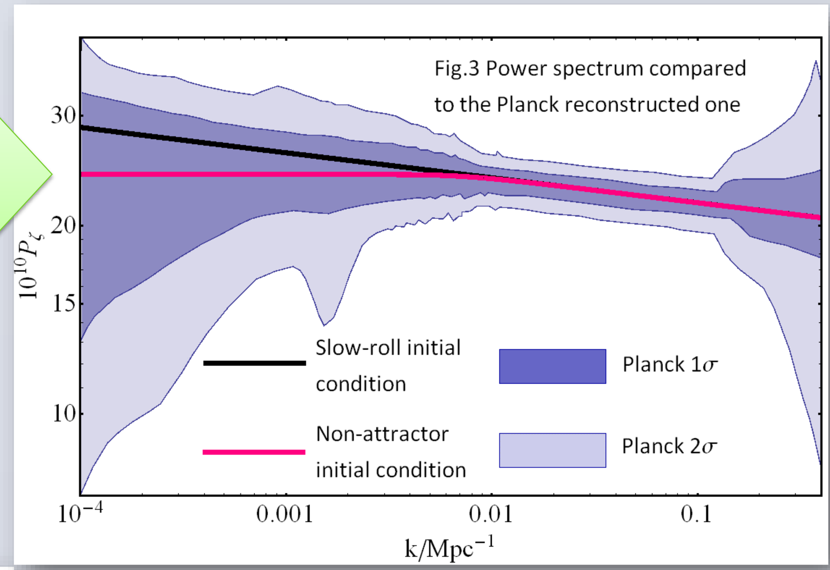

Fig.3 Power spectrum compared to the Planck reconstructed one

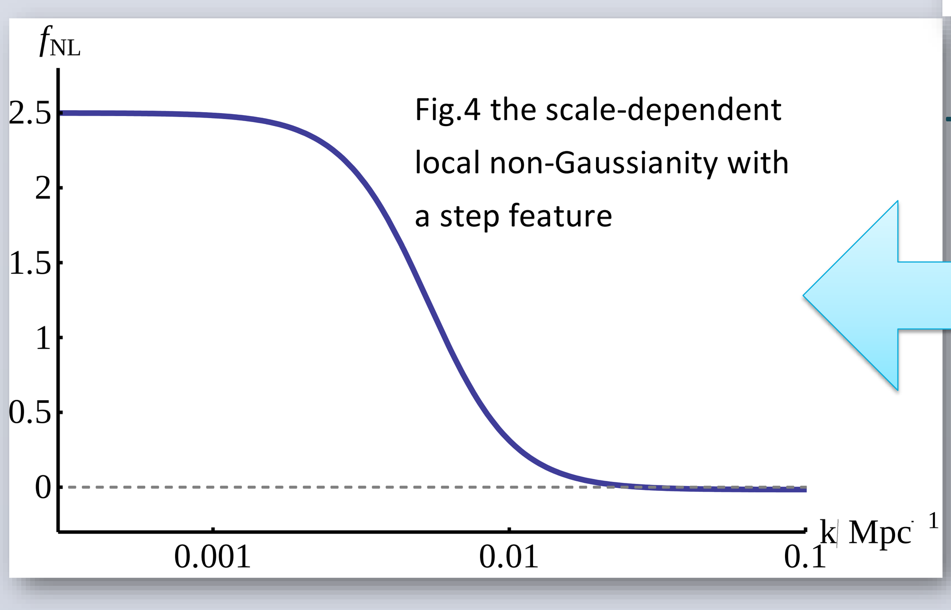

Fig.4 the scale-dependent local non-Gaussianity with a step feature

The local f_{NL} is calculated as

$$f_{NL} = -\frac{5}{6}\left(\frac{\partial N}{\partial \phi}\right)^2 \Big/ \frac{\partial^2 N}{\partial \phi^2}$$

$$= -\frac{5}{12}\eta$$

Conclusion3: The non-attractor beginning of inflation leads to large-scale suppression in the power spectrum, while results in step features in the squeezed limit of local Non-Gaussiantiy.

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